Overborrowing and Systemic Externalities in the Business Cycle - Javier Bianchi (AER 2011)

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Motivation

- Periods of sustained increases in borrowing are often followed by a devastating disruption in financial markets
  - Great Depression, 1929
  - Tequila Crisis, 1994
  - Asian Tigers Crisis, 1999
  - Argentinean Crisis, 2002
  - Great Recession, 2008
- Research Question:
  - Why does the private sector becomes exposed to the dire consequences of financial crises?
  - What is the appropriate policy response to reduce these vulnerabilities?

Contribution

- DSGE model in SOE with financial constraints.
  - Credit constraint limits the amount of borrowing to a multiple of total income.
  - Constraint is occasionally binding.

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- Quantitative analysis to evaluate the macroeconomic and welfare effects of macroprudential policy

  Policy intervention motivated by an externality

Theoretical Model

Household Problem

- Two-sector small open endowment economy (tradable and non-tradable sectors)
- Collateral constraint: Borrowing $\leq$ than share κ of Y_t
- Incomplete credit markets
- Exogenous and constant interest rate

Household Problem

$$\max_{\{C_t^T, C_t^N, B_{t+1}\}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \right\}$$

where

$$C_t = \left(\omega \left(C_t^T \right)^{\frac{\eta-1}{\eta}} + (1-\omega) \left(C_t^N \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

Subject to

$$B_{t+1} + C_t^T + p_t C_t^N \geq Y_t^T + p_t Y_t^N + (1+r) B_t$$

$$B_{t+1} \geq -\kappa \left(Y_t^T + p_t Y_t^N \right)$$

Household Problem: FOC

Let Λ_t and μ_t be the multipliers associated with budget and collateral constraints. The first-order conditions are:

- Euler equation for bonds:

$$\Lambda_t \left[1 - \mu_t\right] = \beta(1+r)\mathbb{E}_t^j \Lambda_{t+1}$$

- Marginal utility of tradable and nontradable consumption:

$$p_t = \frac{1-\omega}{\omega}\left(\frac{C_t^N}{C_t^T}\right)^{-\left(\frac{1}{\eta}\right)}$$

- Budget and collateral constraint:

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Notice something weird here?

Endowments

- Households receive stochastic endowments of tradable, Y_t^T, and non-tradable, Y_t^N, income
- Endowment shocks are the only source of uncertainty in this model

Equilibrium

A decentralized competitive equilibrium is given by a set of allocations C_t^T, C_t^N, B_{t+1}, a vector of beliefs $\mathcal{B}_t$, and a pair of prices $r,\ p_t$, such that households maximize their utility, all the constraints are satisfied, and the market for bonds and goods clear

- Market clearing condition:

$$C_t^N = Y_t^N$$

- Equilibrium price:

$$p_t = \frac{1-\omega}{\omega}\left(\frac{C_t^T}{Y_t^N}\right)^{\frac{1}{\eta}}$$

-Borrowing Constraint:

$$B_{t+1} \geq -\kappa\left(Y_t^T + \frac{1-\omega}{\omega}\left(\frac{C_t^T}{Y_t^N}\right)^{\frac{1}{\eta}} Y_t^N\right)$$

The Social Planner

Constrained Social Planner

$$V(\mathcal{B}_t, B_t, Y_t) = \max_{C_t, B_{t+1}} U(C_t(C_t^T, Y_t^N)) + \beta \mathbb{E}_t \left[V(\mathcal{B}_{t+1}, B_{t+1}, Y_{t+1}) \right]$$

Subject to

$$B_{t+1} = B_t(1+r) + Y_t^T - C_t^T$$

$$B_{t+1} \geq -\kappa \left(1 + \frac{1-\omega}{\omega} \left(\frac{Y_t^N}{C_t^T} \right)^{-\frac{1}{\eta}} \left(\frac{Y_t^N}{Y_t^T} \right) \right)$$

Constrained Social Planner: FOC

$$\lambda_t^{sp}\left[1-\mu_t^{sp}\Psi_t\right] = u_T(t)$$
$$\lambda_t^{sp}\left[1-\mu_t^{sp}\Psi_t\right] = \beta(1+r)E_t\lambda_{t+1}^{sp}$$
$$\mu_t^{sp} \geq 0$$
$$\mu_t^{sp}(b_{t+1}+\kappa((\frac{1-\omega}{\omega})(\frac{c_t^T}{y_t^N})^{\eta+1}y^N+y^T)) = 0$$

where $\Psi_t \equiv \boxed{\frac{\partial BC_t}{\partial C_t^T}} = \kappa(\frac{\rho_t^N c_t^N}{c_t^T})(1+\eta) > 0$

How does the borrowing constraint change when I change C^T?

Decentralized Equilibrium vs. Constrained Efficient S.P

The Social Planner valuates wealth according to:

$$\lambda_t^{sp}\left[1-\mu_t^{sp}\Psi_t\right]=u_T(t)$$

While households:

$$\Lambda_t\left[1-\mu_t\right]=u_T(t)$$

Since HH's cannot internalize the equilibrium effects of their consumption behavior over the B.C, they undervalue wealth.

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Innefficient Allocations

Suppose the constraint is not binding, i.e. $\mu_t = 0$. The Euler equation in the decentralized equilibrium is:

$$\Lambda_t = \beta(1+r)\mathbb{E}_t^j \Lambda_{t+1}$$

While for the social planner:

$$\lambda_t^{sp} = \beta(1+r)\mathrm{E}\left[u_T(t+1)/\left[1 - \mu_{t+1}^{sp}\Psi_{t+1}\right]\right]$$

If the constraint is expected to bind in the future, the decentrilized equilibrium is not constrained efficient.

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Quantitative Analysis