Online Appendix to "Overborrowing and Systemic Externalities in the Business Cycle Under Imperfect Information"*

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A.1 Calibration Details

As mentioned in the paper, we use the Kalman filter and its statistical properties to estimate the structural parameters governing the income processes included in our model.

In particular, since we assume the innovations, $\left\{\varepsilon_t^T,\, \varepsilon_t^N,\, \varepsilon_t^g\right\}$ are Gaussian, we can derive a likelihood function $\mathcal{L}(\Theta,\, s_t)$, where s_t is a two-column matrix that contains the observable signals Δ_t^T and Δ_t^N; and (θ) is a vector containing the structural parameters of the model (Hamilton, 1994). The log-likelihood function is given by

$$l(\Theta,\, s_t) = -\frac{Tn}{2}\ln(2\pi) - \frac{T}{2}\ln\Big(\det(\mathbf{ZPZ'})\Big) + \frac{1}{2}\sum_{t=1}^{T}\Big((\mathbf{s^{\boldsymbol{\cdot}}t} - \mathbf{Za}_{t|t-1})'(\mathbf{ZPZ'})^{-1}(\mathbf{s^{\boldsymbol{\cdot}}t} - \mathbf{Za}_{t|t-1})\Big) \quad \text{(A.1.1)}$$

which can be maximized with respect to Θ to find the maximum likelihood estimates of the parameters that form the state transition matrix $\mathbf{A}$ and the noise covariance matrix $\mathbf{Q}$. As shown by equations 12 and 13, the output of this process is a vector

*The views and conclusions presented in this paper are exclusively those of the authors and do not represent the views of the Central Bank of Chile or of its Board members. *Instructions and Matlab codes for replication are available at: https://shorturl.at/DJPXN.*

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$\Theta^* = \left(\rho_{z^T,z^T},\, \rho_{z^T,z^N},\, \rho_{z^N z^T},\, \rho_{z^N,z^N},\, \rho_g,\, \sigma_{z^T z^T},\, \sigma_{z^N z^T},\, \sigma_{z^N,z^N}, \sigma_g\right)$ plus the corresponding forecasts for the unobservable components of income Z_t^T, Z_t^N and g_t.

Following Garcia-Cicco et al. (2010), estimating trend shocks in the data requires long samples. We use annual data for Argentina from 1903 to 2018 from Ferreres (2020). We compute *tradable output* (Y_t^T) as the sum of GDP from the following categories: farming, livestock, hunting, and forestry; fisheries; mining and quarrying; and manufacturing. *Non-tradable output* is computed as the sum of GDP from construction; electricity, gas, and water; transport, storage, and communications; financial intermediation; real estate activities; and other services. Equivalently, non-tradable output equals total GDP minus tradable output.

Following equations (10) and (11), we define the observable signals Δ_t^T and Δ_t^N as $\ln \frac{Y_t^T}{Y_{t-1}^T}$ and $\ln \frac{Y_t^N}{Y_{t-1}^T}$, respectively. We detrend Δ_t^N using a quadratic trend. We find the maximum likelihood estimates using the following computational algorithm:

1. Set an initial value Θ_0.[1]

2. Set matrices $\mathbf{A}$ and $\mathbf{Q}$ to form the state-space described in (13).

3. Using the Kalman Filter, compute $\mathbf{a}_{t|t-1}$ and P following (15),(16), and (17).

4. Compute the log-likelihood function value using (36).

5. Iterate over values for Θ until a local maximum, denoted as $\hat{\Theta}$, is found.[2]

6. Define the information matrix as the negative hessian of $l(\Theta^*,\, s_t)$ divided by the length of Δ_t^T and Δ_t^N.

7. Compute the standard errors of Θ^* as the squared root of the diagonal elements of the inverted information matrix.

The Matlab code required to implement this routine is available at https://shorturl.at/DJPXN.

[1]For this step, we use Matlab's *fmincon* minimization routine and simulated annealing. The bounds are set to prevent negative numbers from appearing in the diagonal elements of matrix $\mathbf{Q}$.

[2]For this step, we use Matlab's *patternsearch* command.

A.1.1 Robustness Checks

A substantial body of literature in economics has been dedicated to studying the role of permanent and transitory income components and how to decompose macroeconomic aggregates.[3] Moreover, it has been well-established that identification is a delicate issue. For instance, Quah (1992) showed that infinite decompositions can separate the same time series into permanent and transitory components.

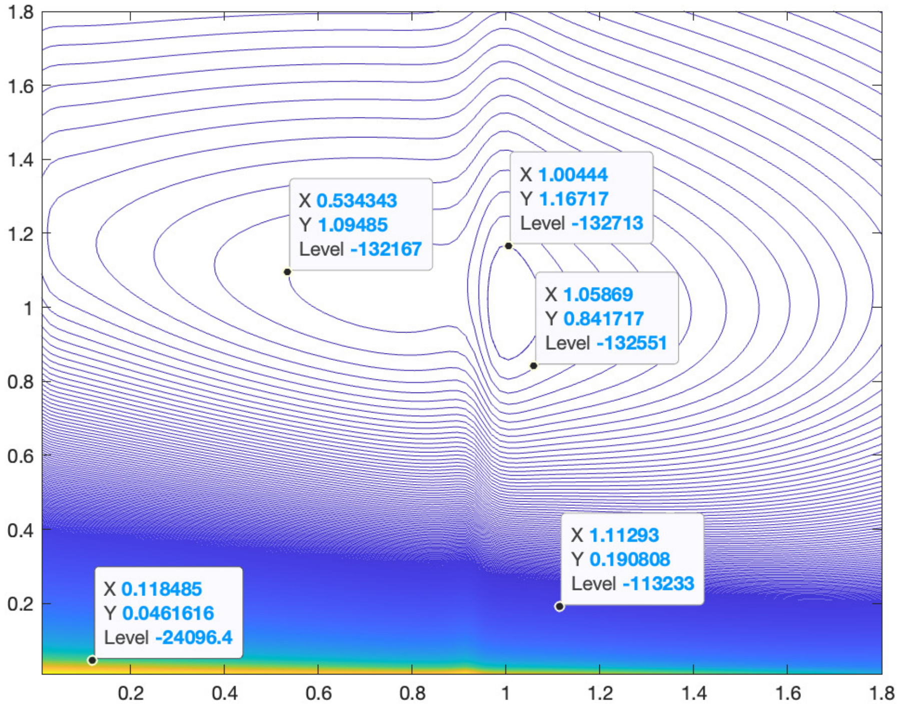
To address this concern, we conduct a robustness check to ensure that the estimated parameters effectively maximize the log-likelihood function. Specifically, we simulate the function, varying the values of the two crucial parameters that determine the relevance of shocks to the permanent income component—the persistence of those shocks (ρ_g) and the volatility of trend shocks (σ_g). Meanwhile, we keep the values of the remaining parameters at their estimated targets. This exercise aims to determine whether the log-likelihood function exhibits additional local or global minima across a wide range of parameter values. We varied the two parameters of interest from 1% to 180% of their calibrated values. The upper restriction on the values is imposed to maintain the assumption that shocks to the trend mean revert.

Figure A.1 illustrates the main takeaways from this exercise. First, the negative of the log-likelihood function is minimized precisely at the point (1, 1)—our calibrated values. This outcome is not surprising, as the log-likelihood function was the criterion used to select these parameters. Second, the log-likelihood function is well-behaved and exhibits no other local minima. Third, the value of the log-likelihood function rapidly decreases as we move towards combinations of parameter values where trend shocks are relatively less important. In other words, the maximization algorithm favors our estimated parameters over a limit scenario where ρ_g and σ_g are arbitrarily small.

Similar exercises conducted for the other parameters in our model yielded comparable results. The log-likelihood function attained a maximum value smaller than in our benchmark calibration. In those perturbed simulations, the algorithm still preferred to

[3] See, for example, Cuddington and Winters (1987); Quah (1992); Harvey and Koopman (2000); Garratt et al. (2006); Aguiar and Gopinath (2007); Oh et al. (2008) among others.

Figure A.1: Maximum Likelihood Function: Contour Plot



Note: Monte Carlo simulation of the Log-likelihood function implied by our calibration. The values of (ρ_g) and (σ_g) vary within 1% and 180% of their point estimates. The other parameters in the model remain at their estimated targets.

choose points away from the aforementioned limit scenario.

A.2 Solution Method

In this appendix, we explain in detail the methods used to solve for the equilibria under perfect and imperfect information. Regarding perfect information, we follow the algorithm proposed by Bianchi (2011) to find the solutions for both the decentralized equilibrium and the Social Planner's problem. However, we extend the solution method to include a different state space for shocks to account for growth shocks.

Under imperfect information, the state space changes as the agent only observes the signals and not the fundamental components of income. In this sense, the state space under imperfect information is larger as it includes not only the exogenous processes for Z_t^T, Z_t^N, and g_t but also the processes for the signals Δ_t^T, and Δ_t^N. Moreover, the resulting transition matrix incorporates the information friction.

We use the following algorithm to discretize the state-space under imperfect information:

1. Simulate a time series of 1,000,000 periods for the unobservable states Z_t^T, Z_t^N, and g_t.
2. Compute the value of the signals Δ_t^T and Δ_t^N using the system of equations (12).
3. Apply the Kalman filter to Δ_t^T and Δ_t^N to compute forecasts for the underlying values of $\tilde{z}_t^T$, $\tilde{z}_t^N$, and $\tilde{z}_t$.
4. Using distance minimization, approximate each forecast and the realization of the observable signals to the values of five equally spaced grids of 19 points.
5. With the resulting discrete-valued time series, estimate the probability of transitioning from a given quintet $\{z_t^T, z_t^N, g_t, \Delta_t^T, \Delta_t^N\}$ to another.
6. Reduce the resulting transition matrix by eliminating those rows and columns representing cases with zero probability of occurrence.

With the calculated transition matrix and corresponding grids, we can proceed to solve for the equilibrium in each of the proposed models. Under perfect and imperfect information, we use standard value function iteration to solve the Social Planner's problem. For the competitive equilibrium, we use time iteration. The process includes an equally spaced grid for the endogenous state B_{t+1} with 501 points. The algorithm is as follows:

1. For a conjecture of B_{t+1}, and given the endowment, solve for the price of relative price p, and tradable consumption c_t^T.
2. Compute the marginal utility of consumption: this will give you a mapping $z^T \times z^N \times z \times g^T \times g^N \times B$ into $\mathbb{R}$.
3. Compute the Euler equation for each point of the mapping.

4. Get the optimal value of the Lagrange multiplier associated to the occasionally binding borrowing constraint $\mu^*(b_{t+1})$ as the $\underset{b_{t+1}\in B}{arg\,min}|\mu\left(b_{t+1}\right)|$

5. Update your initial conjecture of the marginal utility of consumption.

6. Iterate until you reach a fixed point.

All the Matlab code is available at https://shorturl.at/DJPXN.

A.3 Sensitivity Analysis

We conducted a series of exercises to evaluate alternative parameter values, examining their impact on the model's outcomes. This comprehensive sensitivity analysis enabled us to assess the robustness of our results and gain deeper insights into the model's behavior. The analysis was divided into three sets, with the key findings summarized in Table A.1.

First, we tested the parameters governing the stochastic processes of the underlying income components. Specifically, we examined alternative values for the persistence and volatility of both the permanent and transitory components, considering deviations of ± 15 percent from the estimated parameters in each case.

Our findings indicate that while the results vary quantitatively, the qualitative conclusions remain robust. Notably, the welfare gains under imperfect information are approximately twice as large, and the mean tax is about six times higher than the respective values in the perfectly informed economy. Additionally, the level of overborrowing is consistently around one percentage point, and the difference in the frequency of financial crises remains similar across calibrations.

Second, we solved the model using a higher discount factor, β, to examine how impatience affects our results. Specifically, we set $\beta = 0.90$, a value commonly used in related literature. While our baseline model assumes a relatively impatient household to match the data on debt-to-GDP and the frequency of crises, our findings remain qualitatively robust under this more traditional calibration of the agent's impatience.

Third, we present a sensitivity analysis on the elasticity of substitution between tradable and nontradable goods, ϵ. This parameter is critical to the model, as the magnitude of price adjustments is directly linked to the substitutability between tradable and non-tradable goods. A higher ϵ increases the sensitivity of the relative price to changes in the ratio C_t^T/C_t^N. In our baseline calibration, we follow Bianchi (2011) and set $\epsilon = 0.83$. However, related literature estimates the plausible range of ϵ to be between 0.40 and 0.83 (see, e.g., Akinci (2011); Mendoza et al. (2005); Stockman and Tesar (1995)). Our sensitivity analysis examines $\epsilon = 0.5$, reflecting a lower substitution rate. Under this new calibration, we find that most of our results remain qualitatively robust. Both the informed and uninformed economies exhibit similar levels of overborrowing, and the welfare gain from implementing the optimal macroprudential policy under imperfect information is approximately 1.6 times greater than the welfare improvement achieved under perfect information.

Finally, we recalibrate the model to a perfectly informed decentralized economy, ensuring it matches the same moments as the baseline model. The primary finding of this exercise is that, in the recalibrated economy, the informed Planner implements a mean tax comparable to the average macroprudential tax chosen by the uninformed Planner. However, as shown in Figure 13, the increase in the mean tax arises because the recalibrated Planner imposes taxes ranging from 20 to 60 percent of external debt for a larger portion of the time.

These differences highlight a key aspect of the recalibration process. Given the estimated stochastic processes, achieving the observed average debt-to-GDP ratio and frequency of financial crises requires a very high degree of impatience. Specifically, the recalibrated economy requieres an annual discount factor (β) of 0.53, which is notably lower than the standard values typically used in the literature. As a result of this higher degree of impatience, both the Planner and private households in the recalibrated economy accumulate more debt than in the baseline calibration.

Table A.1: Sensitivity Analysis

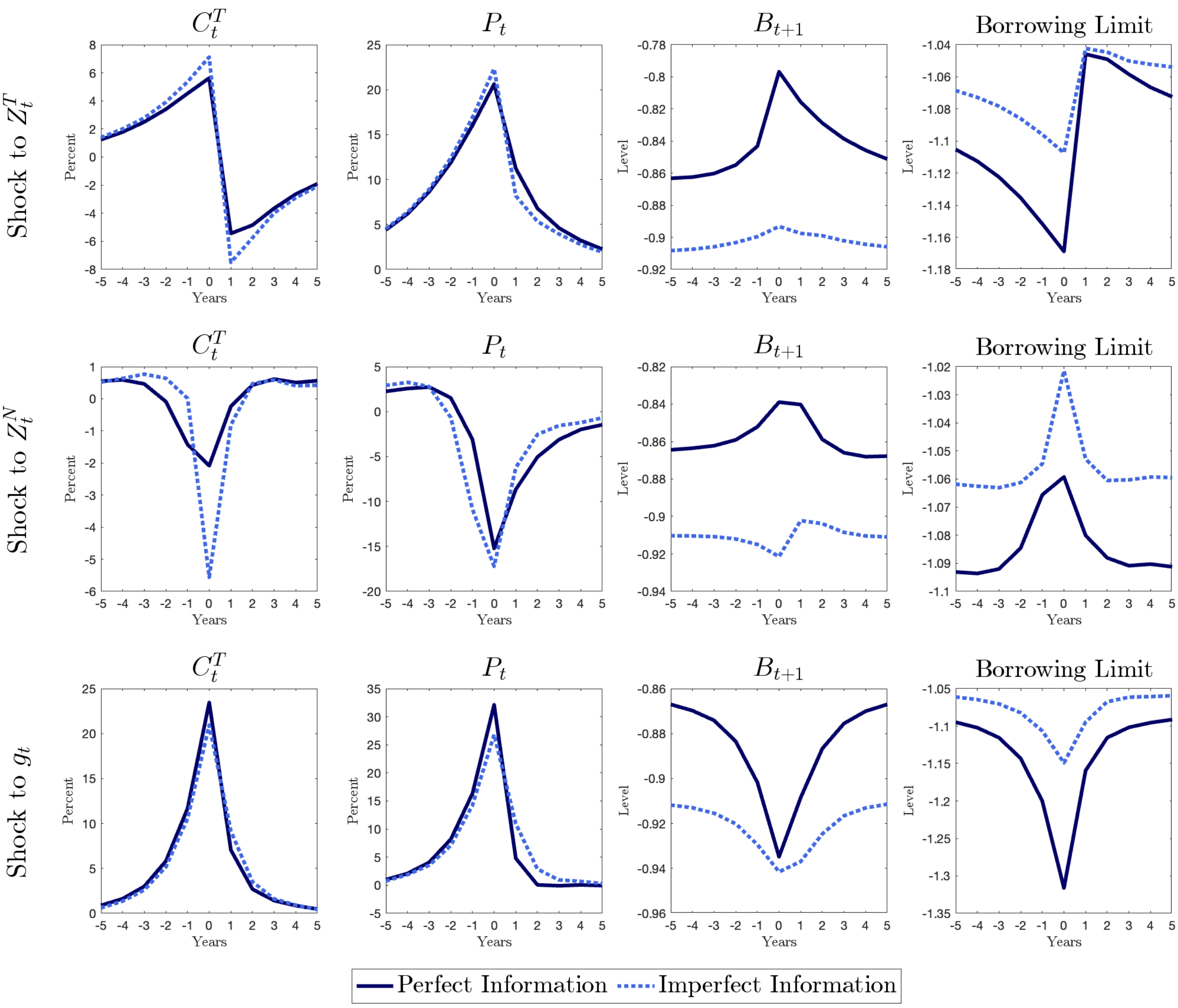
	Welfare Gains		Tax on Debt		Debt-to-Output Ratio				Probability of Crises				Severity of Financial Crises: Consumption				Severity of Financial Crises: RER				Severity of Financial Crises: Current Account-to-Output			
					Perf. Info		Imp. Info		Perf. Info		Imp. Info		Perf. Info		Imp. Info		Perf. Info		Imp. Info		Perf. Info		Imp. Info	
	Perf. Info	Imp. Info	Perf. Info	Imp. Info	D.E	S.P	D.E	S.P	D.E	S.P	D.E	S.P	D.E	S.P	D.E	S.P	D.E	S.P	D.E	S.P	D.E	S.P	D.E	S.P
Baseline ($\beta = 0.83,\ \kappa = 0.335$)	0.11	0.24	9.75	11.40	-27.15	-26.16	-29.02	-28.06	4.15	1.98	5.53	1.73	-25.06	-24.55	-24.71	-21.08	66.57	62.20	70.90	59.15	17.41	14.20	15.48	8.28
$\beta = 0.90$	0.06	0.11	4.33	4.83	-25.92	-24.85	-28.12	-27.17	2.42	1.02	2.95	0.78	-22.94	-22.64	-22.15	-18.86	65.17	62.90	68.75	61.52	15.14	12.52	12.59	7.26
$\epsilon = 0.50$	0.17	0.27	9.66	11.33	-23.74	-22.55	-25.38	-24.37	2.96	1.19	3.09	0.73	-28.13	-26.17	-29.00	-22.46	140.13	124.84	159.29	134.40	23.94	18.84	24.91	17.12
Recalibrated F.I Economy ($\beta = 0.53,\ \kappa = 0.3525$)	0.15	0.24	37.27	11.40	-28.95	-28.60	-29.02	-28.06	5.50	4.16	5.53	1.73	-30.53	-29.15	-24.71	-21.08	86.25	77.00	70.90	59.15	25.89	21.91	15.48	8.28
Autocorrelation ρ_g *(15 % less)*	0.12	0.25	9.97	11.59	-27.17	-26.15	-29.05	-28.06	4.25	2.01	5.54	1.75	-24.98	-24.34	-24.70	-21.10	65.93	61.03	71.35	58.77	17.32	13.92	15.59	8.08
Autocorrelation ρ_g *(15 % more)*	0.11	0.22	9.55	11.17	-27.11	-26.15	-28.99	-28.04	4.03	1.95	5.40	1.76	-25.08	-24.68	-24.51	-21.18	67.44	63.68	70.94	59.29	17.55	14.55	15.52	8.46
Volatility σ_g *(15 % less)*	0.12	0.25	10.19	11.69	-27.49	-26.49	-29.15	-28.19	4.40	2.03	5.80	1.81	-24.12	-23.41	-23.85	-20.15	65.80	61.23	69.85	58.18	17.45	14.08	15.37	8.11
Volatility σ_g *(15 % more)*	0.10	0.23	9.32	11.07	-26.82	-25.85	-28.90	-27.93	3.93	1.92	5.23	1.68	-25.98	-25.66	-25.51	-21.98	67.42	63.58	72.42	-24.53	17.38	14.38	15.68	8.40

Note: This table summarizes the key descriptive statistics for each alternative calibration tested. We measure the welfare costs of the pecuniary externality as the percentage of lifetime consumption lost relative to a Social Planner sharing the same information set. The probability of a financial crisis is defined as the frequency of crises occurring every 100 years. The consumption and RER changes during a financial crisis are the percentage changes relative to their respective ergodic means when the collateral constraint binds and the economy experiences a one-standard deviation current account reversal.

A.4 Responses to Positive Shocks to the Underlying Components of Income

Each row of Figure A.2 compares the dynamics of the main aggregate variables under perfect and imperfect information when the economy is hit by a one-standard-deviation positive shock to the unobserved exogenous state (as indicated in the left-axis of each row). The horizontal axis covers five years before and after the shock occurrence.

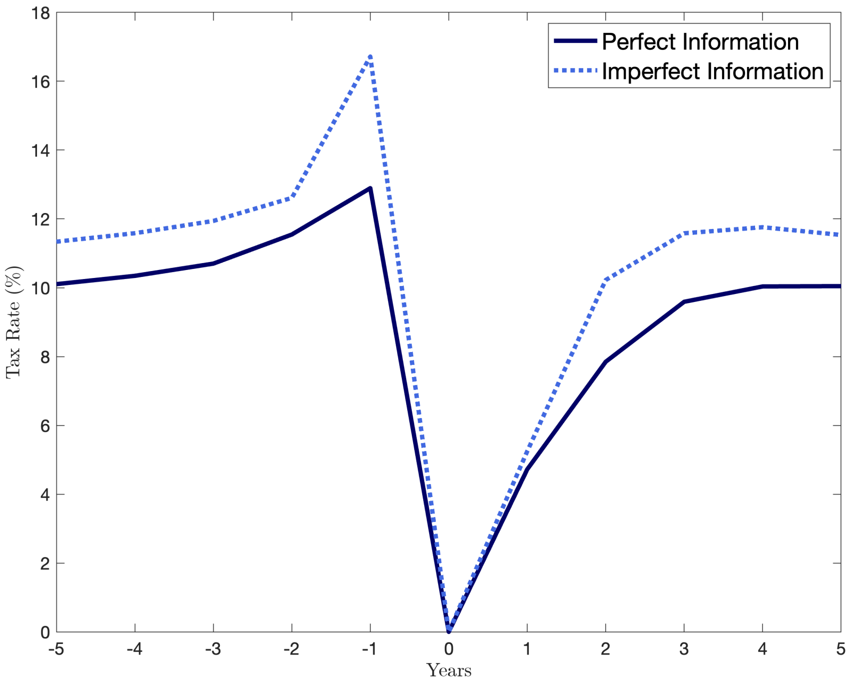
Figure A.2: Endogenous Responses to Shocks to the Underlying Components of Income



A.5 Optimal Tax Policy During a Sudden Stop

Conditional moments reveal the more noticeable differences between the tax policies of informed and uninformed economies. Figure A.3 illustrates the dynamics of the optimal tax in the years before and after a Sudden Stop. Specifically, it depicts a relatively consistent gap between the tax rates during tranquil times. However, a wider gap emerges in the year preceding the crisis. Negative shocks to both permanent and transitory components prompt households to increase their debt for consumption smoothing. Uninformed agents inaccurately predict the shocks to be more transitory than they are, leading to a greater surge in debt holdings. Consequently, the uninformed Planner increases its tax rate by 4.1 percentage points (from 12.6 percent to 16.7 percent), while the informed Planner does so by 1.3 percentage points (from 11.6 to 12.9 percent).

Figure A.3: Optimal Tax Policy in a Sudden Stop.

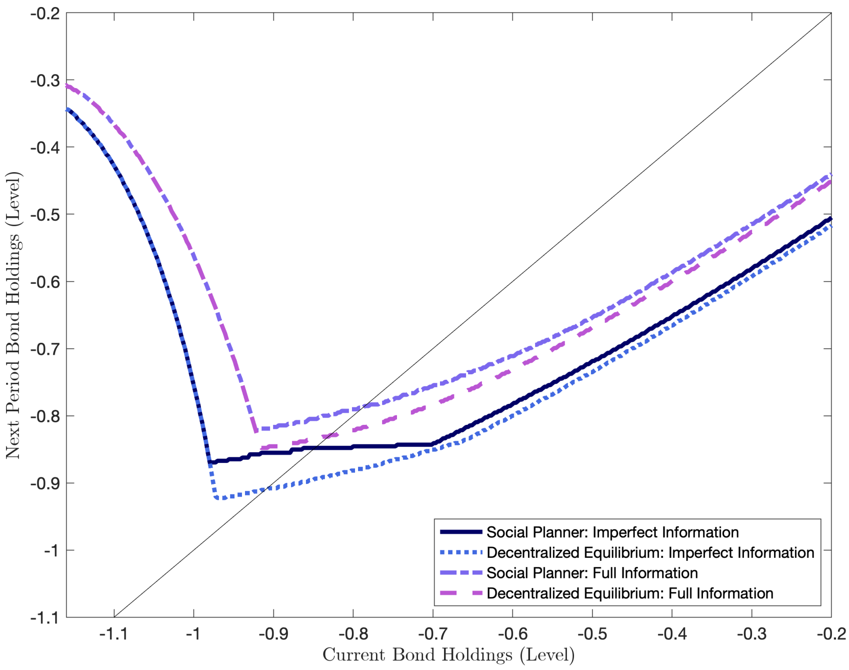


Note: In this figure, we compare the dynamics of the optimal tax policy under perfect and imperfect information during a Sudden Stop. The horizontal axis covers the five years before and after the crisis. The Sudden Stop occurs in period 0, where the optimal tax rate is equal to zero.

A.6 Policy Functions

This section describes the bond decision rules for the decentralized and constrained-efficient equilibrium based on current bond holdings. Figure A.4 compares the policy functions for the informed and uninformed cases when the economy experiences a one-standard-deviation negative shock to the permanent income component. Like the perfectly informed case in Bianchi (2011), the policy functions exhibit a non-monotonic behavior characterized by a kink due to the borrowing constraint under imperfect information. The kink is located precisely where the financial constraint is satisfied with equality but is not binding. It becomes binding for any debt level to the left of this point, while for any point to the right, the constraint is slack.

Figure A.4: Policy Functions for a Negative One Standard-Deviation Shock to the Permanent Component of Income



Note: This figure highlights the distinct decision rules followed by the Social Planner and the decentralized equilibrium in the informed and uninformed economies, especially as current debt levels approach the critical region where the collateral constraint becomes binding. Debt increases to the left.

Given our calibration for the exogenous processes of the transitory and permanent income components, both the Social Planner and the decentralized equilibrium in the uninformed economy maintain higher debt levels than their counterparts in the informed economy. Furthermore, under both information sets, the Social Planner consistently opts for lower levels of the next period's debt than private households. However, as current debt levels approach the region where the collateral constraint becomes binding, the uninformed Planner deviates further from the decentralized equilibrium calibration than the informed Planner. As detailed in Section 3.1, this precautionary behavior from the uninformed Planner relates to how it adjusts its marginal utility of wealth to reflect that the increased uncertainty due to the information friction affects its valuation of how the collateral value changes with consumption.

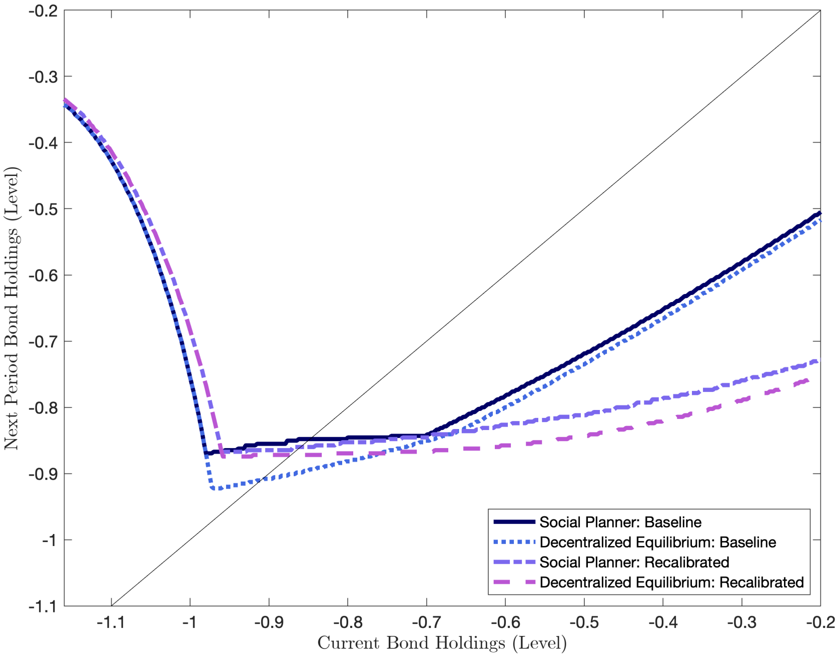
A.6.1 Policy Rules in the Recalibrated Economy

In section 4.3.3, we studied the optimal tax policy of a perfectly informed economy calibrated to match the moments in the data. We showed that the informed Planner in the recalibrated economy chooses a tax policy that implies an average tax similar to the mean tax implemented by the uninformed Planner.

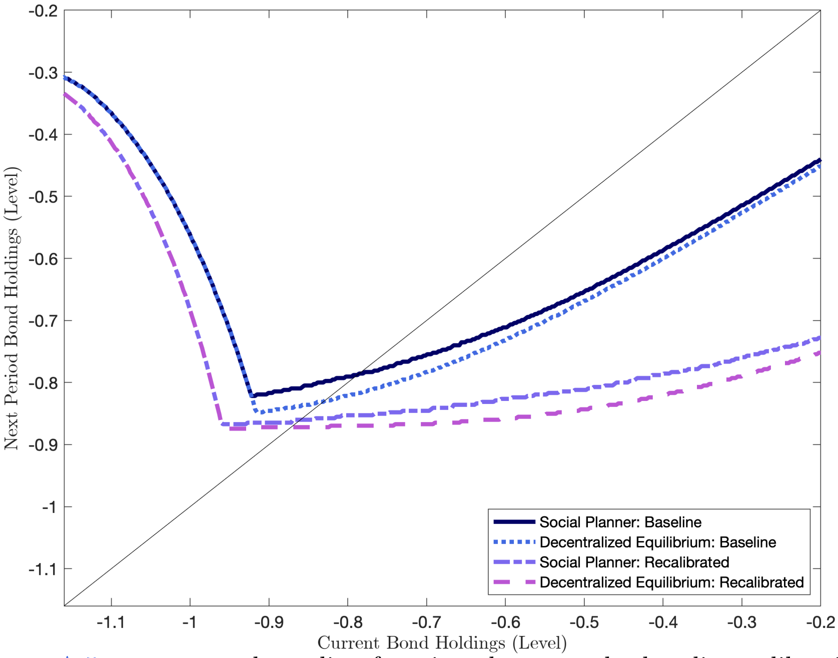
Figure A.5a compares policy functions between the baseline calibration under imperfect information and the recalibrated perfectly informed economy. Due to the lower degree of impatience implied by the recalibration, the Planner and private households in this scenario hold more debt than in the baseline calibration. However, in the region where the collateral constraint is more likely to bind, the policy rules for both Planners become very similar. This similarity explains why, in both calibrations, the tax tends to activate at similar debt levels. Nevertheless, the degree of overborrowing implied by both models differs significantly, accounting for the distinct distributions shown in Figure **??**. Additionally, Figure A.5b illustrates differences in policy functions between the baseline and the recalibration under perfect information.

Figure A.5: Policy Functions: Recalibrated Economy

(a) Imperfect Information



(b) Perfect Information



Note: Figure A.5a compares the policy functions between the baseline calibration under imperfect information and the recalibrated perfectly informed economy. Figure A.5b illustrates the differences between the policy functions for the baseline and the recalibration under perfect information. Debt increases to the left.

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