Constant-Time Big Numbers (for Go)

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1 Introduction

As more activity takes place digitally, the importance of secure communication has never been greater. Thankfully, after 50 years of Public Key cryptography [13], we have good theoretical systems underpinning our security.

Most of these systems rely on modular arithmetic with large numbers. For example, RSA [30], or Elliptic Curve cryptography [23]. Working with these numbers is not natively supported by hardware. Instead, we use a "Big Number" software library to provide this functionality.

Unfortunately, although Public Key cryptosystems have been heavily scrutinized in theory, implementations often suffer from vulnerabilities in practice

One important class of vulnerability are timing side-channels [17]. This is where an implementation leaks information about secret values through its execution time. Big Numbers libraries designed without cryptography in mind suffer from these vulnerabilities.

In particular, Go [3] provides a general purpose Big Number type, `big.Int`, which does not provide constant-time operations. Unfortunately, this library gets used for cryptography [11], including inside of Go's own standard library, in the `go/crypto` package.

We've addressed this issue by creating a library [21] providing Big Numbers with constant-time operations. Our library provides the necessary operations for Public Key cryptography, all while avoiding leakage through timing side-channels. To demonstrate its utility, we've modified Go's `go/crypto` package, replacing the use of `big.Int` in the DSA and RSA systems, achieving a slowdown of only 2x in this scenario.

2 Background

In this section, we explain how Big Numbers are used in Public Key cryptography, what timing side-channels are, how we model their threat, as well as what kind of vulnerabilities are present in Go's `big.Int` type.

2.1 Big Numbers in Cryptography

Most Public Key cryptosystems rely on modular arithmetic.

In RSA [30], for example, a public key (e, N) consists of a modulus $N \in \mathbb{N}$, usually 2048 bits long, and an exponent $e \in [0, \varphi(N) - 1]$. To encrypt a

message $m \in [0, N-1]$, we calculate

$$c := m^e \mod N$$

Since N is much larger than a register, we need a Big Number library to implement this system. Modular exponentiation is also not supported in hardware, requiring extra support in software.

DSA [31] also relies on modular arithmetic, this time using a large prime p of around 2048 bits, and working in the multiplicative group $(\mathbb{Z}/p\mathbb{Z})^*$.

Elliptic Curve Cryptography [23] relies on complex formulas for adding points on an elliptic curve, built over a finite field K. This field is usually either a prime field $\mathbb{Z}/p\mathbb{Z}$, in which case arithmetic modulo p is used, or a binary extension field GF(2^n), in which case binary and polynomial arithmetic are used. For prime fields, a Big Number library necessary, because the size of the prime is greater than 200 bits.

2.1.1 Implementing Big Numbers

When a modulus is known in advance, a special purpose library implementing arithmetic with this fixed modulus can be used. This is the case for Elliptic Curve cryptography, where the prime field is a fixed parameter of the system. Using a fixed type makes it easier to provide constant-time operation.

A disadvantage is that since different systems require different moduli, it takes more work to implement and support each system. One way to address this is to automatically generate implementations of modular arithmetic, as done by FiatCrypto [14].

In some systems, you need support for dynamic moduli. Take RSA, for example. In this case, you need a library supporting dynamically sized numbers, as well as various operations.

2.1.2 Big Numbers in `go/crypto`

Go provides implementations of the Public Key cryptosystems we've mentioned so far in its `go/crypto` package.

Unfortunately [11], the general purpose `big.Int` is used, in part, to implement these systems, despite its potential vulnerability to timing attacks.

For DSA [31], Go uses `big.Int` for all operations, including key generation, signing, and verification.

For RSA [30], Go embeds `big.Int` as part of the API for this package. Key generation, encryption, decryption, signing, and verification all use `big.Int`.

For ECC [23], Go defines a general interface for Elliptic Curves, requiring operations like point addition, scalar multiplication, etc. All of these are defined in terms of `big.Int`. Some of the curves have specialized implementations of their prime field, only converting from `big.Int` to satisfy this interface. The remaining curves directly use `big.Int` for their field arithmetic.

2.2 Timing Attacks

A side-channel [16] leaks secret information indirectly, through the observable effects of a program's execution. For example, timing side-channels use the execution time of a program to gain information about the secret data it handles. A timing attack is the use of a timing side-channel to break the security of some program or cryptographic algorithm.

When a program takes a different number of steps based on the value of some secret, this is an obvious timing side-channel. For example, if a naive program for comparing inputs with a secret password stops as soon as a mismatch is found, then the algorithm itself has a timing side-channel. This side-channel can be exploited, to allow the secret password to be guessed byte-by-byte.

Not all timing side-channels are this simple. Algorithms that always take the same number of steps can still having timing side-channels because of the underlying hardware. For example, a processor may execute an operation faster for some inputs, or the presence of a cache could be used to infer what addresses are being accessed. These microarchitectural timing side-channels are also of concern. See [12] for a survey of these vulnerabilities.

2.2.1 Actual Attacks

Although the presence of a side-channel does not directly lead to attacks as early as 1995, Paul Kocher demonstrated the potential for timing side-channels to break crypto-systems [17, 18]. These attacks relied on algorithms that perform a varying number of operations based on secret data.

One objection to timing attacks is that while a timing side-channel is catastrophic in theory, in practice this channel is too noisy to exploit. Unfortunately, it's possible to exploit these attacks, even across a network [8, 7].

Noise only makes the channel more difficult to exploit, requiring more samples to detect the underlying signal.

The use of caches as a potential side-channel was identified early on as well [25]. Accessing data takes longer when that data is outside of the cache. If data accesses depend on a secret value, the observed execution time will also depend on this value. Additionally, an attacker located on the same machine can place data into the cache, and probe the cache themselves, to learn even finer information about the program's access patterns. This kind of colocation increasingly common, as more applications are run on cloud servers.

A wide variety of attacks involving caches have been mounted against various cryptosystems [4, 33, 9] . Accessing data is thus based on secret values fraught with peril.

2.2.2 Our Threat-Model

We can distill this variety of side-channels into a simple, albeit pessimistic, set of rules:

1. Any loop leaks the number of iterations taken.
2. Any memory access leaks the address accessed.
    (a) As a consequence, accessing an array leaks the index accessed.
3. Any conditional statement leaks which branch was taken.

Rule 1 is justified by theoretical concerns: a longer loop uses more operations. In practice, it's difficult to observe the iterations of each loop in a program from an overall timing signal, making this a pessimistic rule.

Rule 2 is justified by various cache based side-channels and attacks [4, 33, 9]. Since caches only load information an entire line at a time, it might seem that our rule is too pessimistic, and that only which cache line was accessed should be kept secret [6]. Unfortunately, it's possible to perform attacks on a much finer level [5, 24, 33]. This is why we take a pessimistic position, and assume that accesses leak their exact address.

Rule 3 is justified in two ways. First, if different branches of a conditional statement execute a different number of operations, we can easily observe which branch was taken. Second, even if both branches execute identitical operations, the CPU's branch predictor can be exploited to leak information about which branch was taken [2, 1, 10].

In addition to these rules, we assume that addition, multiplication, logical operations, and shifts, as implemented in hardware, are constant-time in their inputs. This is the case on most processors, one notable exception

being microprocessors [26]. This assumption is reasonable for the platforms targeted by Go and our library.

2.3 Vulnerabilities in `big.Int`

Go provides a general purpose type for Big Numbers: `big.Int`. This type focuses on being optimized, and useful in various situations. It does not focus on security, or on protecting against timing side-channels. Unfortunately, out of convenience, it gets used in cryptography, even inside of Go's standard library.

In this section, we look at some of the important implementation aspects of `big.Int`, and how they might be potentially vulnerable according to our threat model.

2.3.1 Padding

The `big.Int` type normalizes numbers internally, removing any leading zero limbs. Even if you initialize a number using bytes zero-padded to a certain length, the resulting value will immediately chop off these zeros. By discarding them, operations on this number will have fewer limbs to process, and will be faster.

Unfortunately, this means that `big.Int` pervasively leaks information about the padding of numbers. Since every operation on a number takes more time the more limbs the number uses, every operation leaks the padding of numbers. This has been exploited in OpenSSL [22], and might potentially be a vulnerability in Go's cryptography library.

2.3.2 Leaky Algorithms

Because `big.Int` is not written with cryptography in mind, its operations violate the rules in 2.2.2. Many methods take a different number of iterations , branch conditionally, or access memory differently depending on their values. Because `big.Int` is designed for general purpose use, this problem should only get worse as the library is further developed and optimized.

Ultimately, the problem is not the existence of `big.Int`, but its use in Go's cryptography library, and in the broader ecosystem.

2.3.3 Mitigations

Although `big.Int` gets used in Go's cryptography library, the authors are aware of its shortcomings, and have implemented several mitigations to try and make its timing side-channels harder to exploit.

One of the most important ones is a mitigation for RSA: blinding [18].

To decrypt a ciphertext $c = m^e \mod N$, we would normally calculate:

$$c^d \mod N$$

with d our private key, and (e, N) our public key. When exponentiation is not implemented in a constant-time way, like with `big.Int`, this process can leak information about m. If an adversary can choose c, then this can leak information about d as well.

To mitigate this, instead of decrypting c directly, we first generate a random integer $r \in (\mathbb{Z}/N\mathbb{Z})^*$. Then, we decrypt $r^e \cdot c$. This gives us the value $r \cdot m$, and we can recover m by multiplying by r^{-1}.

While this effectively mitigates the simplest attacks against exponentiation, a very leaky operation, other methods are left unprotected, and may have subtle exploits. We also have the unaddressed issue of padding, which has lead to attacks in OpenSSL [22].

3 Implementation

We've implemented a library, called `safenum` [21], intended to provide a replacement for `big.Int`, suitable for use in cryptography. In order to demonstrate its utility, we've replaced some of `go/crypto`'s usage of `big.Int` with our own library, in a separate repository [20].

In this section, we go over the design and implementation of our library.

3.1 The `safenum` library

Safenum defines a `Nat` type, intended to replace `big.Int`. This type represents arbitrary numbers in $\mathbb{N}$. Unlike `big.Int`, we do not handle negative numbers. Handling a sign bit in constant-time is exceedingly tricky. Thankfully, we haven't found this limitation to be restrictive when replacing `big.Int` in Go's cryptography library.

We represent numbers in base $W := 2^{64}$. Concretely, we store a number as a slice of type `[]uint`, in little endian order. We call these the "limbs" of a number. For example, the slice:

```
[]uint{13, 47, 52}
```

represents the number:

$$52 \cdot 2^{128} + 47 \cdot 2^{64} + 13$$

These limbs might be padded, to conceal the true value of a number, as we'll see later.

We provide operations for addition and multiplication of `Nats`. We also provide numerous operations for modular arithmetic, including modular addition, subtraction, multiplication, exponentiation, inversion, reduction, and taking square roots modulo prime numbers. We also provide the usual operations for serializing to and from bytes.

We try and structure the API in a similar way to `big.Int`, where an operation is performed on a separate `Nat` to receive the result. For example, this is the signature for modular addition:

```
func (z *Nat) ModAdd(x *Nat, y *Nat, m *Modulus) *Nat
```

This calculates $z \leftarrow x+y \mod m$, returning z. The advantage of structuring the API this way, instead of simply returning a new value, is that we can reuse the memory of z for the result.

We go one step further, in fact, and use the memory of the buffer `Nat` for all scratch space needed inside of an operation. Structuring our operations this way allows us to limit memory waste.

3.1.1 Handling Size

Unlike `big.Int`, a `Nat` doesn't truncate its limbs to remove any zero padding. Because of this, we distinguish between the *true size* of a number, which is how many significant bits or limbs it actually has, and the *announced size* of a number, which is how many limbs are actually used to store that number. The announced size is allowed to be leaked, while the true size should be kept secret. The true size is always at most the announced size

Because of this, we need to ensure that there's always a clear announced size to use for the results that we produce. For modular operations, we have an obvious choice: the size of the modulus. When doing a modular operation, the result will always receive the same announced size as the modulus does.

For example, when doing modular addition:

```
func (z *Nat) ModAdd(x *Nat, y *Nat, m *Modulus) *Nat
```

our result `z` will have the same announced size as `m`. After modular addition, we have that $z \in [0, m-1]$ by definition. Because of this, leaking the fact that the true size of z is at most that of m leaks no information about what value z actually has, beyond what's knowable just by inspecting the call graph in the source code of a program.

When serializing a `Nat`, we respect its announced size, and produce zeros for padding as necessary. This is done without any special handling, because we already store padded limbs anyways.

Similarly, when deserializing a `Nat` from bytes, we respect any padding, unlike `big.Int`. For example, if 32 big endian bytes are deserialized, we will end up with a `Nat` with an announced size of 256 bits, regardless of the value of those bytes.

This leaves us with non-modular addition and multiplication of numbers. One approach is to use the maximum possible resulting size for our result's announced length. For example, if we multiply numbers x_1 and x_2, of announced size b_1 and b_2, then our result will need a size of at most b_1+b_2. In situations where we know that our result will be smaller, this size explosion can be undesirable.

Because of this, we opt towards letting users specify exactly how many resulting bits they need in the output. For example, multiplication has the following signature:

```
func (z *Nat) Mul(x *Nat, y *Nat, cap uint) *Nat
```

Here `cap` is the number of bits that the result should have. We use this to determine the result's announced length. Any output beyond that capacity will simply be discarded.

In summary, the announced size of a `Nat` is always clear based on how it's produced, and results from deserializing a value, from using the same size as a modulus, or from manually deciding on an output size.

3.1.2 Moduli

In our library, we've decided to make a separate type for representing the moduli used in modular arithmetic: `Modulus`. There are several reasons for doing this.

First, various operations in modular arithmetic require different properties of the modulus which can be pre-computed. For example, montgomery multiplication requires us to know $m^{-1} \mod W$, with W our base, and m our modulus. By using a separate type for moduli, we can precompute these values.

Second, the true size of a modulus is considered to be leakable. As a consequence, moduli are stored *without* padding. This is desirable because modular reduction needs access to the most significant bits of a modulus, and fetching this information without leaking padding is exceedingly difficult. Furthermore, by storing moduli without padding, the announced size

of numbers produced through modular operations is as tight as possible, which speeds up operation.

This assumption is safe in cryptography. Moduli are often public, like with the public modulus N in RSA. In this case, leaking the true size is fine, since even the exact value is known. There are some cases where a secret modulus is necessary. For example, when generating an RSA key, we use the factorization $N = pq$ of the modulus, and calculate our private key modulo $\varphi(N) = (p-1)(q-1)$:

$$d := e^{-1} \mod \varphi(N)$$

Leaking the value of $\varphi(N)$ would be catastrophic. On the other hand, it's clear that the true size of $\varphi(N)$ is approximately that of N, which is known. In this case, leaking the true size of $\varphi(N)$ is fine.

Finally, moduli are also allowed to leak whether or not they are even. The motivation behind this is that certain modular operations have faster variants for odd moduli, or require different algorithms to support even moduli. Rather than providing different methods for these different cases, we choose to select the correct variant internally, making all modular operations work without any caveats. This requires dynamically checking whether or not a modulus is odd or even, in order to dispatch the right operation.

Fortunately, we're not aware of any realistic situation where the evennness of a modulus needs to be kept secret. In fact, for the cryptographic algorithms we know of, the evenness of the modulus used is known statically, making this check perfectly fine.

Because of these weaker constraints on what information can be leaked, we need a separate modulus type, in order to violate the stricter rules for normal numbers.

3.2 Constant-Time Operations

The rules we established in our threat model 2.2.2 are quite stringent. For many operations, we want to have conditional behavior depending on the values and results we see. Without access to branching, this would seem impossible. Thankfully, there are workarounds to enable us to have conditional behavior, all while not leaking information about which conditions are selected. The core idea here is that whenever we're faced with a choice, we perform both branches, and then combine the results together without revealing which result we end up using.

For example, a standard algorithm for modular subtraction would look like this (in pseudo-Go):

```
func (z *Nat) ModSub(x *Nat, y *Nat, m *Modulus) *Nat {
  borrow := z.Sub(x, y)
  if borrow == 1 {
    z.Add(z, m)
  }
}
```

The problem here is that by conditionally adding in m, we reveal whether or not $y > x$, and a borrow occurred. Our solution requires a new primitive:

```
func (z *Nat) ctCondCopy(v choice, y *Nat) *Nat
```

This function assigns y to z if $v = 1$, and does nothing otherwise. Furthermore, this primitive shouldn't leak any information about whether or not the condition was true.

With this in place, we can implement modular subtraction without leakage:

```
func (z *Nat) ModSub(x *Nat, y *Nat, m *Modulus) *Nat {
  borrow := z.Sub(x, y)
  scratch := new(Nat).Add(z, m)
  z.ctCondCopy(choice(borrow), scratch)
}
```

We always perform the addition, and copy over the result if necessary, without leaking the value of `borrow`.

This kind of rearrangement is the foundation that allows us to replace branching in algorithms with constant-time operations.

3.2.1 Building Primitives

The question remains: how do you build up the primitives like `ctCondCopy`, which let you choose results without leaking which result was chosen?

The methods for constant-time choice are analagous to the standard programming methods for conditional branching. Instead of using `bool` to represent the result of a condition, we use

```
type choice Word
```

The value of a choice is either 1 or 0, but we use the same type as full limbs, to try and avoid having the compiler turning or manipulations back into branches.

From this choice value, we can build a primitive that selects between two limbs without leaking which choice was selected:

```
func ctIfElse(v choice, x, y Word) Word {
  mask := -Word(v)
  return y ^ (mask & (y ^ x))
}
```

This routine returns x if $v = 1$, and y otherwise. Unlike a conditional statement, this is implemented through bitwise operations, and doesn't leak information about what was selected.

If $v = 0$, then `mask` contains only zeros, and we're left with `y`. When `v == 1`, then `mask` only contains ones. The `y`'s cancel eachother out, leaving us with `x`.

We can use this primitive to build up a larger selection primitive, allowing us to assign one slice to another, conditionally:

```
func ctCondCopy(v choice, x, y []Word) {
  for i := 0; i < len(x); i++ {
    x[i] = ctIfElse(v, y[i], x[i])
  }
}
```

These primitives allow us to use `choice` to introduce conditional behavior without leaking our choices, but we also need primitives to create `choice` values in the first place. These are built up in a similar way from small primitives to large primitives.

We can decide whether or not two limbs are equal using some bitwise trickery:

```
func ctEq(x, y Word) choice {
  q := uint64(x ^ y)
  return 1 ^ choice((q|-q)>>63)
}
```

To understand why this trick works, first realize that in two's complement, either the most significant bit of a number is set, or the most significant bit of a number's negation is set, unless that number is zero. Thus, the expression

```
choice((q|-q)>>63)
```

simply checks, using only bitwise operations, that `q` is not zero. Since `q` is zero precisely when `x` and `y` are equal, we can simply negate the non-zero check.

We can use this primitive to compare entire slices of limbs in constant time:

```
func cmpEq(x []Word, y []Word) choice {
  res := choice(1)
  for i := 0; i < len(x) && i < len(y); i++ {
    res &= ctEq(x[i], y[i])
  }
  return res
}
```

We can't exit early, since that would leak information about the value of x and y. Instead, we combine all the results together, using the fact that two slices are equal when every one of their matching limbs are equal.

3.3 Algorithm Choices

While going over how each operation works in detail is outside the scope of this report, describing some of the high-level techniques for certain trickier operations is nonetheless interesting.

Many of these operations were inspired by the excellent work of Thomas Pornin in BearSSL [27].

3.3.1 Modular Reduction

To reduce a number a modulo m, we first implement an operation that allows us to shift in a single limb. This lets us reduce a number of the form:

$$z := a \cdot W + b$$

with $a \in [0, m-1]$ and $b \in [0, W-1]$. With this in place, we can reduce an arbitrary number, by shifting in each of its limbs, from most significant, to least significant, reducing modulo m each time.

Implementing the shifting operation involves estimating the quotient $q := \lfloor z/m \rfloor$, by dividing the most significant 64 bits of m with the corresponding 128 bits of z, and then calculating $z - qm$, potentially fixing the result by adding or subtracting m again. This technique is further described in [27].

3.3.2 Inversion

For modular inversion, the method we use depends on whether or modulus is odd or even. As mentioned previously, we can check whether or not this is the case, and select the right method.

For odd moduli, we use the binary GCD algorithm, as described in [28]. We have yet to implement the most optimized version, which accumulates

intermediate results into single limb registers, and instead perform full width operations at each iteration.

For an even modulus m there's a standard trick to calculate $x^{-1} \mod m$. First, we calculate $u := m^{-1} \mod x$, using our method for odd moduli, and then we calculate our desired inverse as:

$$\frac{um - 1}{x}$$

3.3.3 Exponentiation

For exponentiation, we use left-to-right exponentiation, with a window size of 4 bits. In order to do window lookups in constant-time, we do a constant-time conditional assignment over each of the possible operands. Even though this requires traversing the entire window table every time, we've found that this method is still faster than using a window size of 2 bits.

3.3.4 Multiplication

For modular multiplication, we calculate the full product ab, and then reduce this result modulo m. This works for both odd and even moduli.

In the case of odd moduli, using Montgomery multiplication [15, 27] is faster, provided that the operands are already in their so-called Montgomery representation. For exponentiation, the cost of converting to and from this representation is amortized, since we do many multiplications between conversions, making it considerably faster to use this technique. Unfortunately, Montgomery multiplication doesn't work for even moduli, so exponentiation is slower in that case.

3.3.5 Modular Square Roots

To calculate $\sqrt{z} \mod p$, we assume that p is prime. Which algorithm we use depends on whether $p = 3 \mod 4$, or $p = 1 \mod 4$.

In the first case, we can calculate a root as:

$$\sqrt{z} = z^{(p+1)/4} \mod p$$

In the second case, we use a constant-time variant of the Tonnelli-Shanks algorithm, as described in [32].

3.4 Implementation Techniques

In this section, we describe a few remaining implementation choices and techniques of interest.

3.4.1 Saturated or Unsatured Limbs

As mentioned previously, we store numbers in base $W := 2^{64}$. This means that we use the full width of a register to store each limb. Because of this, we say that are limbs are *saturated*. It's also possible to store limbs *unsaturated*, by using fewer than 64 bits. BearSSL [27] takes this approach, using only 31 bits of the available 32 bits in the integers it uses. For 64 bit registers, using 63 bit unsaturated limbs would be the option of choice.

There are two compelling reasons for using unsaturated limbs.

The first is that this leaves an extra bit of space to hold a carry or borrow after an addition or subtraction, respectively. This allows us to chain together carries to implement operations over multiple limbs, without having to use assembly instructions. In Go, this isn't really an issue, because `bits.Add` and `bits.Sub` are provided to implement these intrinsics in a cross-platform way.

The second comes that if we use w bits for each limb, then montgomery multiplication needs to work with a value of size $2w + 1$ bits. With a fully saturated limb of 64 bits, we need 129 bits. This uses an extra register compared to unsaturated limbs of 63 bits. Because montgomery multiplication is called very often during exponentiation, this can yield considerable savings.

The disadvantage of using unsaturated limbs comes when converting numbers to and from bytes. With fully saturated limbs, our 64 bit limbs are nicely composed of 8 bytes. With 63 bit limbs, this isn't the case, making conversion to and from bytes more complicated, and expensive. Using unsaturated limbs would also require storing additional information about the exact announced size of a number, instead of being able to use the number of limbs directly.

Ultimately, we opted to use saturated limbs, in order to reuse the assembly routines already implemented for low level operations in `big.Int`. These were designed with saturated limbs in mind, and thankfully, are constant-time.

3.4.2 Redundant Reductions

Our library is defined to prevent misuse. Because of this, modular operations work even if their inputs are not already reduced. For example,

addition modulo m should return the right result, even if the inputs are greater than m. Unfortunately, the cost of reducing inputs modulo m when they were already in range is not desirable, since this operation is relatively expensive. Ideally, we'd like to avoid reducing inputs when we already know that they're correctly reduced.

To implement this, each number stores a pointer to a modulus, indicating that it is already reduced by this modulus. When we reduce a number modulo m, we check this pointer, and skip this reduction if it matches m. If we modify the value of a number, we update the modulus it points to accordingly, based on what method was called. For example:

```
z.ModAdd(x, y, m)
```

will set `z`'s modulus to `m`.

We only set this modulus pointer based on what methods are called, never on the actual value of a result. Because of this, the dynamic checks of this pointer only depend on the callgraph of our program. Since this graph is statically determined, these redundant reduction checks don't impact the constant-time properties of our library.

4 Results

We've compared the performance of our library with `big.Int` operation by operation, as well as in the context of the `go/crypto` package. Overall, our library is about 2.6x slower than using `big.Int` for most operations, but only 2x slower in realistic situations. In this section, we present these results in detail.

4.1 Comparison with `big.Int`

We've set up a series of benchmarks to compare the performance of `Nat` compared to `big.Int` on various operations.

The following operations are all implemented on values, exponents, and moduli of 2048 bits. For raw addition and multiplication, we use the full size necessary to represent the result in our benchmarks.

Operation	op / s (`big.Int`)	op / s (`Nat`)	ratio
Addition	10,980,842	12,164,599	0.90
Modular Addition	6,986,739	3,075,188	2.27
Multiplication	1,316,322	542,385	2.43
Modular Reduction	454,917	63,253	7.19
Modular Multiplication	1,000,000	44,596	22.42
Modular Inversion	1,000,000	621	1610
Modular Exponentiation	223	86	2.59

The most expensive operation, by far, is exponentiation. Because of this, it's fair to compare the performance on these two types mainly on this operation. We can see that `Nat` is 2.6x slower compared to `big.Int` for exponentiation, although some operations are much slower.

For comparing modular square roots, we used the primes $p_3 = 2^{244} + 79$, which is $3 \mod 4$, and $p_1 = 2^{244} + 153$ which is $1 \mod 4$. We use different primes to test the various codepaths for modular square roots:

Operation	op / s (`big.Int`)	op / s (`Nat`)	ratio
$\sqrt{z} \mod p_3$	40,464	26,886	1.50
$\sqrt{z} \mod p_1$	-	7,867	-

We didn't manage to find a large value where Go's Tonneli Shanks routine managed to find a square root without hanging, although we expect the ratio to be similar to the other case.

4.2 Comparison with `go/crypto`

We've created a forked package [20] of `go/crypto`, where we've replaced the usage of `big.Int` with our own `Nat` type for both RSA and DSA. All of the code using `big.Int` has been replaced, with the exception of primality checking. This endeavour demonstrates the utility of our package for writing cryptographic code.

We've also run benchmarks to assess the performance impact, as we show in the following table:

Operation	op / s (`big.Int`)	op / s (`Nat`)	ratio
RSA Decrypt	670	312	2.15
RSA Sign	675	372	1.81
RSA Decrypt (3 Prime)	1173	596	1.97
DSA Sign	6202	2625	2.36
DSA Parameters	0.89	1.64	0.54

We use a 2048 bit modulus for both RSA and DSA. For RSA, we use the CRT optimization, instead of using exponentiation directly. Our benchmarks

for `big.Int` use blinding, but our benchmarks for `Nat` do not. Because `Nat` is constant-time, we don't need to use blinding to mitigate timing attacks. We don't include DSA verification, since this can be safely done with `big.Int`.

Overall, we can see that in a real world scenario, the use of `Nat` is only 2x slower. This can surely be improved, but is already an encouraging result.

5 Further Work

While we're happy with the utility of our library, and the performance results we've managed to achieve, it's of course still possible to improve on this front.

5.1 Verifying Constant-Time Properties

Ultimately, we would like to have more assurance about the constant-time properties of our library. Our code hasn't undergone an audit, nor have we verified the assembly output produced by the Go compiler to ensure that it meets our demands.

Ideally, it would be nice to incorporate some kind of automated analysis of our code to detect timing side-channels. An approach similar to dudect [29] might be an interesting way to provide a form of fuzz testing to detect unwanted time-variation.

5.2 Optimizing Assembly Routines

Currently, we rely on some assembly routines pulled from `big.Int`, slightly modified to avoid jumping to non-constant-time routines. Unfortunately, not all of the primitive operations we would like to have are present. Furthermore, we could reduce memory usage in some places, by having these operations present a "conditional" variant. For example, we could have an add operation taking a `choice` flag, allowing us to choose whether or not to perform an addition, without leaking information. This would avoid having to use a scratch buffer and a conditional copy.

To gain similar speed to the other primitives, these new primitives would also need to be implemented in assembly. This would be time-consuming, but likely worth the effort. There are also new solutions to help with writing assembly routines in Go, such as the Avo library.

5.3 Upstreaming to `go/crypto`

While we believe our library is immediately useful for the broader ecosystem, it's not realistically going to be replacing the use of `big.Int` in Go's cryptography library any time soon.

The most likely path towards removing `big.Int` from `go/crypto` is likely to move towards specialized arithmetic implementations for each prime field involved in ECC. DSA is a legacy algorithm, where the security flaws introduced by `big.Int` are not of major concern.

This leaves RSA. Unfortunately, the nature of RSA, requiring dynamic moduli, makes it so that a big number library of some kind is necessary. Ideally, this library would be internal to RSA, allowing constant-time operation, and severing the bridge between Go's cryptography package, and `big.Int`.

As a proof of concept, we've implemented a fork of Go's RSA implementation [19], replacing `big.Int` for encryption and decryption, with an internal number type, using the minimal amount of code necessary to accomplish this.

Using unsaturated limbs, we've found that our fork of RSA suffers only a 1.67x slowdown, while implementing encryption and decryption in constant-time.

We hope to prepare a patch for Go's RSA implementation to merge in this work soon.

6 Conclusion

In summary, we have shown why Go's general purpose big number type, `big.Int`, is not suitable for Cryptography, for various reasons. Unfortunately, this type gets used out of convenience, even in Go's own cryptography library.

To address this, we've created a replacement library for `big.Int`, achieving a slowdown of only 2.6x for most operations, while attempting to provide constant-time operation.

To test the utility of this library, we've replaced the usage of `big.Int` in Go's implementation of RSA, and DSA, and found only a slowdown of 2x.

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