# MPC for Group Reconstruction Circuits

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**Abstract**

In this paper, we present a thing.

# 1 Introduction

Write the introduction

# 2 Background

Throughout this paper, we let $\mathbb{G}$ denote a group of prime order $q$, with generators $G$ and $H$. Let $\mathbb{F}_q$ denote the scalar field associated with this group, and let $\mathbb{Z}/(q)$ denote the additive group of elements in this field.

We make heavy use of group homomorphisms throughout this paper. We let

$$\varphi(P_1, \ldots, P_m) : \mathbb{A} \to \mathbb{B}$$

denote a homomorphism from $\mathbb{A}$ to $\mathbb{B}$, parameterized by some public values $P_1, \ldots, P_m$. Commonly $\mathbb{A}$ will be a product of several groups $\mathbb{G}_1, \ldots, \mathbb{G}_n$, in which case we'd write:

$$\varphi(P_1, \ldots, P_m)(x_1, \ldots, x_n)$$

to denote the application of $\varphi$ to an element $(x_1, \ldots, x_n)$ of the product group. We also often leave the public values $P_i$ implicit.

## 2.1 Pedersen Commitments

## 2.2 Sigma Protocols

## 2.3 Maurer's $\varphi$-Proof

In [Mau09], Maurer generalized Schnorr's sigma protocol for knowledge of the discrete logarithm [Sch90] to a much larger class of relations. In particular, Maurer provided a sigma protocol for proving knowledge of the pre-image of a group homomorphism $\varphi$. We denote this protocol as a "$\varphi$-proof", and recapitulate the scheme here.

Given a homomorphism $\varphi : \mathbb{A} \to \mathbb{B}$, and a public value $X \in \mathbb{B}$, the prover wants to demonstrate knowledge of a private value $x \in \mathbb{A}$ such that $\varphi(x) = X$. The prover does this by means of Protocol 2.1:

**Protocol 2.1: $\varphi$-Proof**

| **Prover** | | **Verifier** |
|---|---|---|
| knows $x \in \mathbb{A}$ | | public $X \in \mathbb{B}$ |
| $k \xleftarrow{R} \mathbb{A}$ | | |
| $K \leftarrow \varphi(k)$ | | |
| | $\xrightarrow{K}$ | |
| | | $c \xleftarrow{R} \mathbb{Z}/(p)$ |
| | $\xleftarrow{c}$ | |
| $s \leftarrow k + c \cdot x$ | | |
| | $\xrightarrow{s}$ | |
| | | $\varphi(s) \stackrel{?}{=} K + c \cdot X$ |

Here, $p$ is chosen such that $\forall B \in \mathbb{B}.\ p \cdot B = 0$. In practice, we'll set $p = q$, which will work perfectly for the groups we use, which are all products of $\mathbb{G}$ or $\mathbb{Z}/(q)$.

**Claim 2.1.** Protocol 2.1 is a valid sigma protocol.

Completeness follows directly from the fact that $\varphi$ is a homomorphism.

For the HVZK property, the simulator $\mathcal{S}(X, c)$ works by generating a random $s \xleftarrow{R} \mathbb{A}$, and then setting $K := \varphi(S) - c \cdot X$.

Finally, we prove 2-extractability. Given two verifying transcripts $(K, c, s)$ and $(K, c', s')$ sharing the first message, we extract a value $\hat{x}$ satisfying $\varphi(\hat{x}) = X$ as follows:

$$
\begin{aligned}
\varphi(s) - c \cdot X = K &= \varphi(s') - c' \cdot X \\
\varphi(s) - \varphi(s') &= c \cdot X - c' \cdot X \\
\frac{1}{c - c'} \cdot \varphi(s - s') &= X \\
\varphi\left(\frac{s - s'}{c - c'}\right) &= X
\end{aligned}
$$

Thus, defining $\hat{x} := (s - s')/(c - c')$, we successfully extract a valid pre-image.

We conclude that the protocol is a valid sigma protocol.

■

Maurer's protocol can also work even in the case where the order of the groups are not known, but this makes the challenge generation a bit more complicated, and we don't need this functionality in this work.

## 2.4 UC Security and the Hybrid Model

## 2.5 Ideal Functionalities for Sigma Protocols

**Functionality 2.1: Zero-Knowledge Functionality** $\mathcal{F}(\texttt{ZK}, \varphi)$

A functionality $\mathcal{F}$ for parties $P_1, \ldots, P_n$.

On receiving $(\texttt{prove}, \texttt{sid}, X, x)$ from $P_i$:
$\mathcal{F}$ checks that $\texttt{sid}$ has not been used by $P_i$ before.
$\mathcal{F}$ checks that $\varphi(x) \stackrel{?}{=} X$.
$\mathcal{F}$ sends $(\texttt{proof}, \texttt{pid}_i, \texttt{sid}, X)$ to every party $P_j$.

### 2.6 Broadcast Functionalities

## 3 Group Reconstruction Circuits

### 3.1 Formal Definition

### 3.2 Normalized Form

## 4 MPC Protocol for GRCs

## 5 Security Analysis

## 6 Applications

## 7 Limitations and Further Work

## 8 Conclusion

## References

[Mau09] Ueli Maurer. Unifying Zero-Knowledge Proofs of Knowledge. In *AFRICACRYPT 2009*, volume 5580 of *LNCS*, pages 272–286. Springer, Berlin, Heidelberg, 2009.

[Sch90] C. P. Schnorr. Efficient Identification and Signatures for Smart Cards. In *CRYPTO 1989*, volume 435 of *LNCS*, pages 239–252, New York, NY, 1990. Springer.