Abstract

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Contents

1 Introduction

2 Agda proof assistant

3 Propositional calculus

Propositional calculus is a formal system that consists of a set of propositional constants, symbols, inference rules, and axioms. The symbols in propositional calculus represent logical connectives and parenthesis.

The inference rules of propositional calculus define how these symbols can be used. These inference rules specify how to construct well-formed formulas that follow the syntax of the system.

The axioms of propositional calculus are the initial statements or assumptions from which we can derive additional statements using the inference rules.

The semantics of propositional calculus define how the expressions in the system correspond to truth values, typically "true" or "false".

3.1 Logical connectives

[Argue choice of logical connectives]

3.2 Formulas

Definition 3.1 (Well formed formula)**.** *The set of well formed formulas is inductively defined as*

- *propositional constants* $p_0, p_1, p_2, \ldots, p_n$ *are well formed formulas*
- $\top$ *and* $\bot$ *are well formed formulas*
- *if* p_i *and* p_j *are well formed formulas, then so are*

$$\neg p_i, \quad p_i \wedge p_j, \quad p_i \vee p_j.$$

We can represent the concept of a well formed formula in `Agda` as a Type

```
data Formula : Type where
  _∧_ : Formula → Formula → Formula
  _∨_ : Formula → Formula → Formula
  ¬_  : Formula → Formula
  const : ℕ → Formula
  ⊥   : Formula
  ⊤   : Formula
```

3.3 Context

Definition 3.2 (Context)**.**

```
data ctxt : Type where
  ∅   : ctxt
  _:_ : ctxt → Formula → ctxt
```

```
data _∈_ : Formula → ctxt → Type where
  Z : ∀ {Γ ϕ} → ϕ ∈ (Γ : ϕ)
  S_ : ∀ {Γ ϕ ψ} → ϕ ∈ Γ → ϕ ∈ (Γ : ψ)
```

3.4 Inference rules

```
data _⊢_ : ctxt → Formula → Type where
  ∧-intro : {Γ : ctxt} {ϕ ψ : Formula} → Γ ⊢ ϕ → Γ ⊢ ψ → Γ ⊢ ϕ ∧ ψ
  ∧-elimˡ : {Γ : ctxt} {ϕ ψ : Formula} → Γ ⊢ ϕ ∧ ψ → Γ ⊢ ϕ
  ∧-elimʳ : {Γ : ctxt} {ϕ ψ : Formula} → Γ ⊢ ϕ ∧ ψ → Γ ⊢ ψ
  ∨-introˡ : {Γ : ctxt} {ϕ ψ : Formula} → Γ ⊢ ψ → Γ ⊢ ϕ ∨ ψ
  ∨-introʳ : {Γ : ctxt} {ϕ ψ : Formula} → Γ ⊢ ϕ → Γ ⊢ ϕ ∨ ψ
  ∨-elim : {Γ : ctxt} (ϕ ψ γ : Formula) → Γ ⊢ ϕ ∨ ψ → (Γ : ϕ) ⊢ γ → (Γ : ψ) ⊢ γ → Γ ⊢ γ
  ¬-intro : {Γ : ctxt} {ϕ : Formula} → (Γ : ϕ) ⊢ ⊥ → Γ ⊢ ¬ ϕ
  RAA : {Γ : ctxt} {ϕ : Formula} → (Γ : ¬ ϕ) ⊢ ⊥ → Γ ⊢ ϕ
  ⊥-intro : {Γ : ctxt} {ϕ : Formula} → Γ ⊢ ϕ ∧ ¬ ϕ → Γ ⊢ ⊥
  ⊥-elim : {Γ : ctxt} {ϕ : Formula} → (Γ : ⊥) ⊢ ϕ
  ⊤-intro : ∅ ⊢ ⊤
  axiom : {Γ : ctxt} {ϕ : Formula} → ϕ ∈ Γ → Γ ⊢ ϕ
  LEM : {Γ : ctxt} {ϕ : Formula} → Γ ⊢ ϕ ∨ ¬ ϕ
  weakening : {Γ : ctxt} {ϕ ψ : Formula} → Γ ⊢ ψ → (Γ : ϕ) ⊢ ψ
  exchange : {Γ : ctxt} {ϕ ψ γ : Formula} → ((Γ : ϕ) : ψ) ⊢ γ → ((Γ : ψ) : ϕ) ⊢ γ
  contraction : {Γ : ctxt} {ϕ ψ : Formula} → ((Γ : ϕ) : ϕ) ⊢ ψ → (Γ : ϕ) ⊢ ψ
```

3.5 Properties of a propositional calculus

4 Boolean algebra

5 Lindenbaum-Tarski algebra

[What is LT?]

5.1 Definining Lindenbaum Tarski algebra

5.2 Proof that the Lindenbaum Tarski algebra is Boolean

5.3 Soundness

References