Natural Hedging between Longevity Risk and Mortality Risk

Crosby Hung Sing Yin

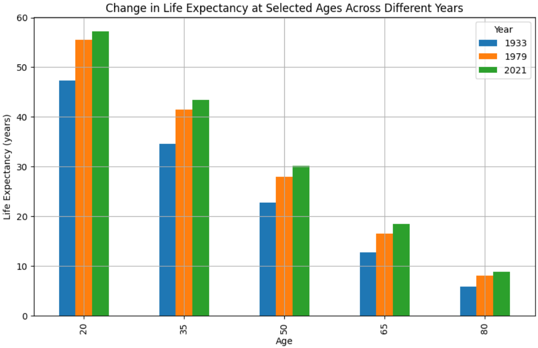
August 22, 2024

Table of Contents

Motivation

Global Life Expectancy at Birth

46.5 yr in 1950 $\Longrightarrow$ 71.7 yr in 2022 Source: United Nation



Pension & Life Insurance

Pension

- pays x at the beginning of each year
- participants accrued rights based on their age

Life Insurance

- receive F at the end of death year

Natural Hedging

Similar to financial risk, we can find derivatives such as mortality swaps to hedge it. However, due to its **over-counter, complex, modelling difficulty** which makes it **expensive**.

Hence, Cox and Lin propose the idea of natural hedging in 2007. Their empirical findings suggest that companies engaged in selling both life insurance and annuity policies typically offer annuities at **lower prices** compared to firms that operate solely in one line of business by markedly reduce the sensitivity of an insurance portfolio to mortality risk.

Why do we need stochastic mortality models?

Stochastic (LC) vs Static

- mortality at $t = T$ is only known by time $T + 1$
- Robust risk management
- affect contracts with embedded options

- Data Source: US Life table from mortality.org
- Base model: Lee Carter model (LC model)

Lee Carter Model

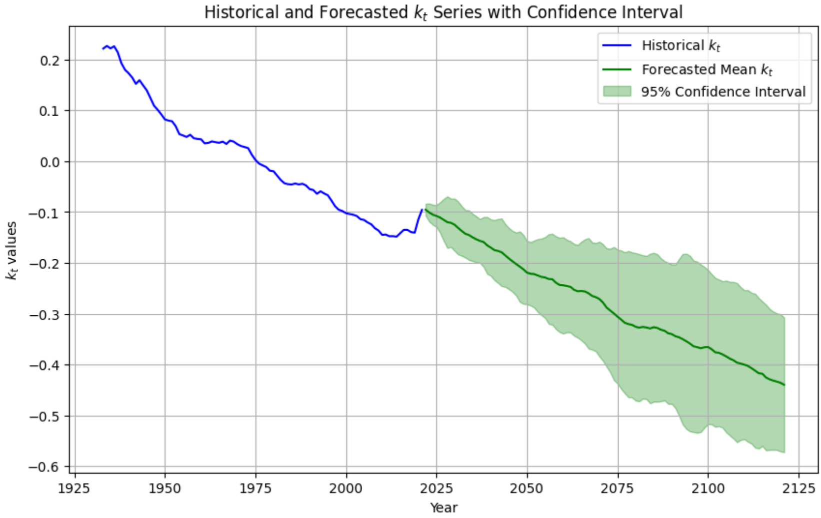
Model Setting

$$\ln m_{x,t} = \alpha_x + \beta_x K_t + \varepsilon_{x,t} \tag{1}$$

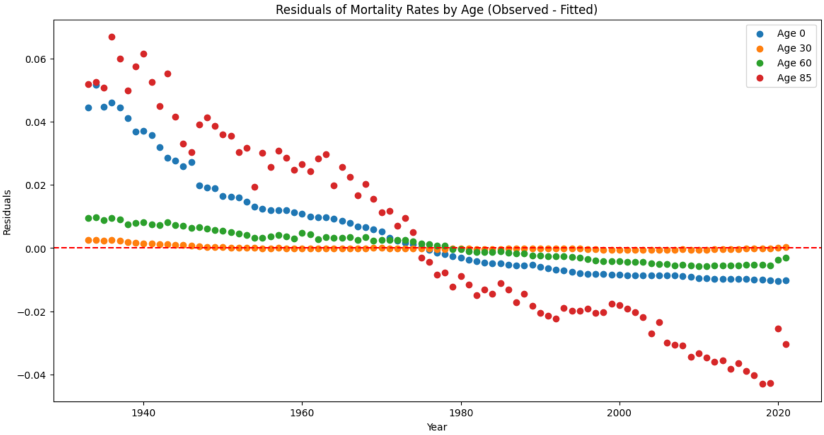
$$K_{t+1} = K_t + c + \sigma_k Z_t \tag{2}$$

- $m_{t,x}$: central death rate in year t at age x
- α_x: mean of the central mortality rates after logarithm transformation across age x.
- β_x: age-specific effect on mortality improvement
- K_t: stochastic mortality improvement
- $\varepsilon_{x,t} \overset{\text{iid}}{\sim} N(0, \sigma_x)^2$

Modeling of K_t



Evaluate the Fit



Kannisto Method

Motivation: The data in high ages is sparse and highly incredible.

Kannisto Method

$$logit(q_x) = \ln \frac{q_x}{1 - q_x} = \ln a + bx \tag{3}$$

We use $\{m_x : x \in [75, 85]\}$ to fit and extend the lifetable up to age 100.

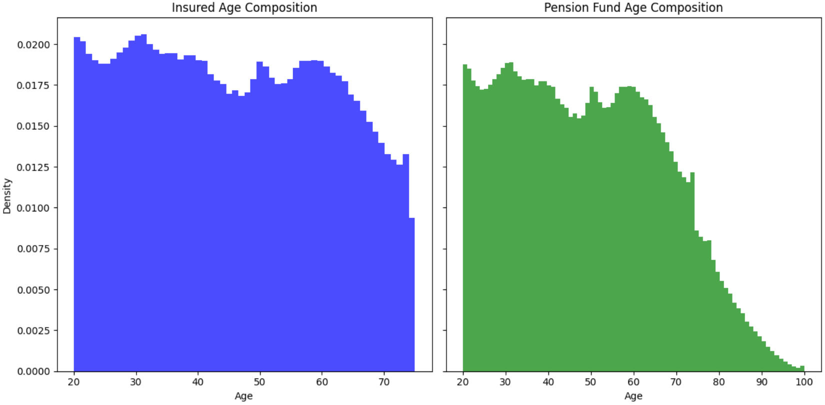
$$m_x = -ln(-c + \frac{1}{1 + ae^{bx}}) \tag{4}$$

Assumptions

1. Mortality data is based on the US Census Bureau HMD mortality data from 1933 to 2021.
2. The face value of the whole life insurance policy issued at age 45 is \$10 payable at the end of the year of death.
3. The pension fund provides an annual benefit of 1 starting from age 65. The present value of this liability is similar to the life one. The accrual of pension right increases linearly to a normalized value of 1 at age 65. For instance, for age 20, $\delta_x = \min(\frac{x-19}{46}, 1)$
4. Constant interest rate at $r = 4\%$. Denote $v = (1+r)^{-1}$ as the discounting factor.
5. No one survive above age 100.

Age Distribution

Due to limited access in data, we assume the age composition portfolio of insurer and pension fund is same as the demographic portfolio.



Present Value of Life Insurance and Pension

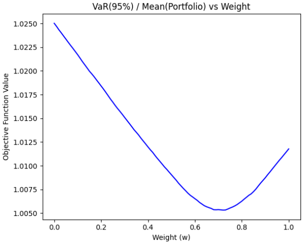
Let L be the present value of the loss.

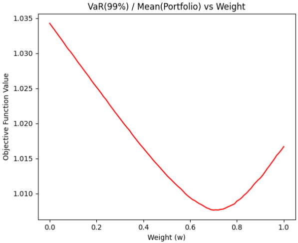
	PV Life	PV Pension
$\mathbb{E}[L]$	3.7544	3.8167
σ_L	0.0263	0.0582
Minimum	3.6507	3.5972
$Q_{25\%}$	3.7364	3.7775
Median	3.7539	3.8170
$Q_{75\%}$	3.7719	3.8566
Maximum	3.8594	4.0155
ρ	-0.78	

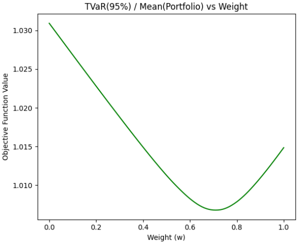
Table: Key Summary Statistics of PV Life and PV Pension

Value at Risk

Motivated by the risk capital regime, we consider $VaR_{95\%}(L)/\,\mathbb{E}[L]$, $VaR_{99\%}(L)/\,\mathbb{E}[L]$, $TVaR_{95\%}(L)/\,\mathbb{E}[L]$.



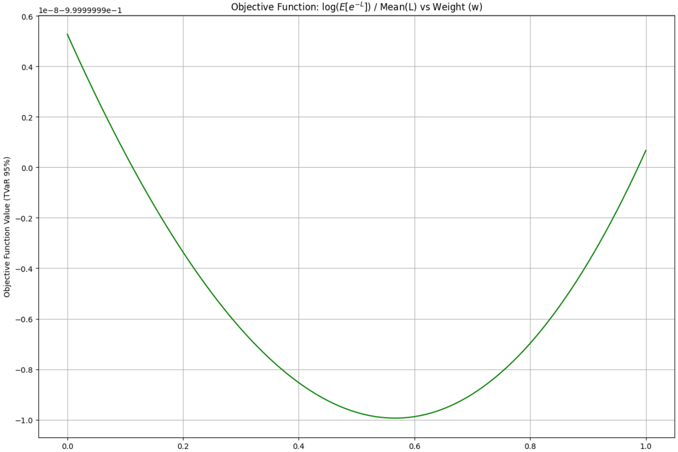




The optimal weights are (0.72, 0.70, 0.71).

Entropic Utility Function

As insurers are risk averse, we consider the entropic utility function $log(\mathbb{E}[e^{-L}])$ to reflect insurers' risk appetite. The optimal weight is 0.71.

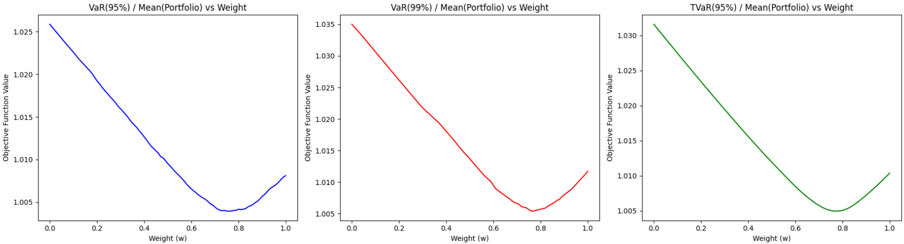


Summary

Objective Function (J)	w^*
$VaR_{95\%}(L)$	0.72
$VaR_{99\%}(L)$	0.70
$TVaR_{95\%}(L)$	0.71
$log(\mathbb{E}[e^{-L}])$	0.71

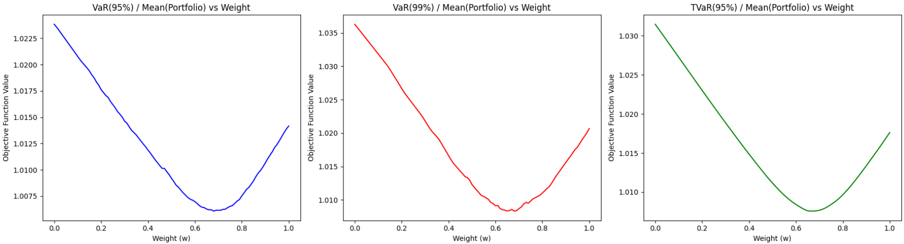
Table: Optimal Ratio under Different Objective Function J

Interest Rate Down (4% → 3%)



The optimal weights are (0.76, 0.77, 0.76).

Interest Rate Up (4% $\rightarrow$ 5%)



The optimal weights are (0.68, 0.68, 0.67).

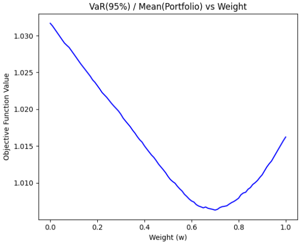
Optimal Weight under Interest Rate Shock

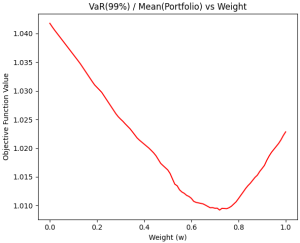
Objective Function (J)	$w^*(3\%)$	$w^*(4\%)$	$w^*(5\%)$
$VaR_{95\%}(L)$	0.76	0.72	0.68
$VaR_{99\%}(L)$	0.77	0.70	0.68
$TVaR_{95\%}(L)$	0.76	0.71	0.67
$log(e^{-L})$	0.77	0.71	0.68

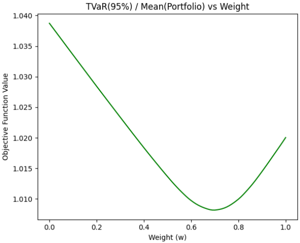
Table: Optimal Ratio under Different Objective Function J

Change of Population Basis

To show that it's effective in different population group. Same analysis is done to population in the Netherlands.







The optimal weights are (0.70, 0.72, 0.70).

Conclusion

Recommendation

- Recommend insurer to start a new line of business in annuity such as Qualifying Deferred Annuity Policy (QDAP)
- Recommend pension fund to absorb life insurance exposure
- Recommend to keep the ratio of life insurance to annutiy at 3:1 or keep the weight of life insurance within 0.7 to 0.8

Conclusion

Further Research

- Lee Carter model is often criticized by under estimating the variation due to its normality assumption on error. More sophisticated mortality model such as CBD model can be considered.
- Parameter uncertainty can be embedded in the model projection as well.