Euclid would approve with glee
How territoriality
Of bunting perched upon a stone
Is asymptote to hypercone

– Bruce G. Marcot, "ODE TO THE NICHE (AN ECOLOGIST'S NIGHTMARE)"

Hierarchical Modeling in Ecology

Andy Crosby

Cesar Estavo

Since that time, I have sometimes been accused of being a "modeler." I wish to state that I am not now nor have I ever been a "modeler." I was (and am) an ecologist who needed a model.

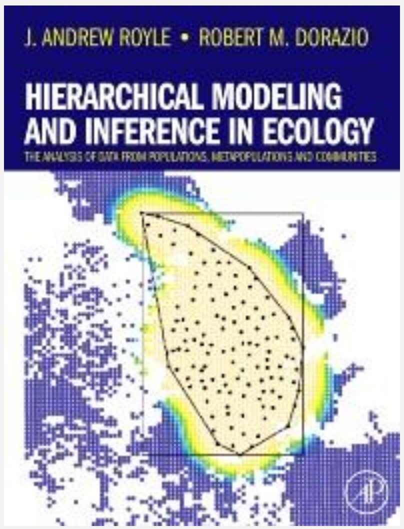
– David J. Mladenoff (Creator of the LANDIS forest simulation model)

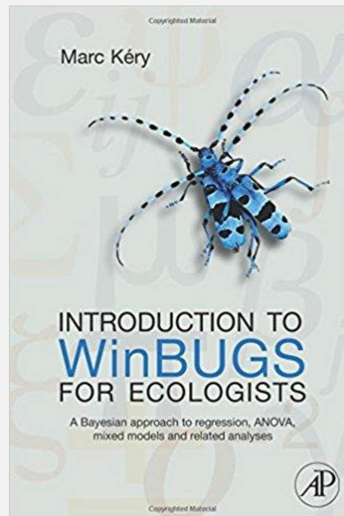
Likely describes the majority of us!

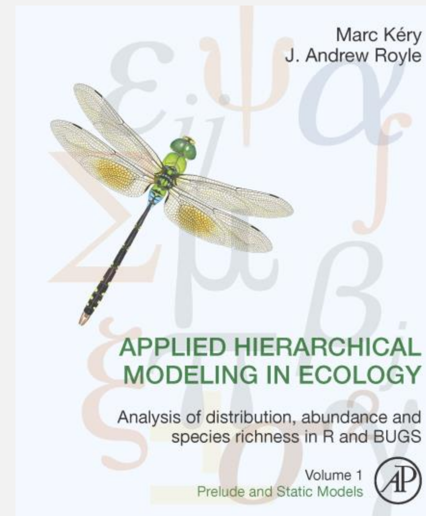
Objectives:

- Introduce hierarchical models as a way to think about ecological data
- De-mistify the modeling process
- Give some examples of HM applications

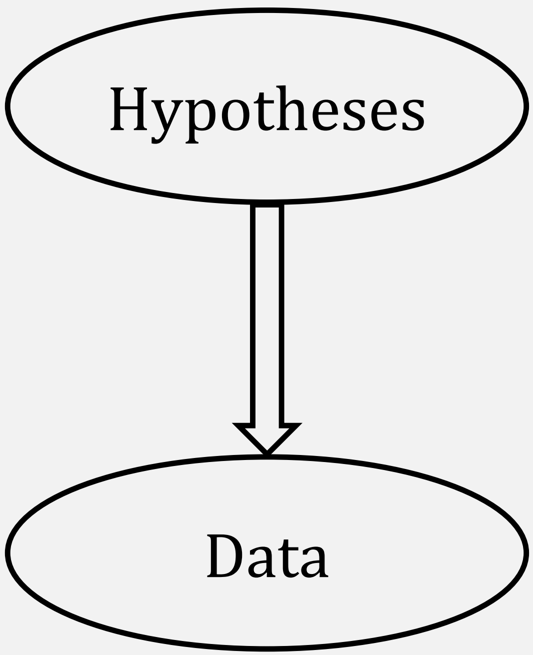
Resources:


J. ANDREW ROYLE • ROBERT M. DORAZIO
HIERARCHICAL MODELING AND INFERENCE IN ECOLOGY
THE ANALYSIS OF DATA FROM POPULATIONS, METAPOPULATIONS AND COMMUNITIES

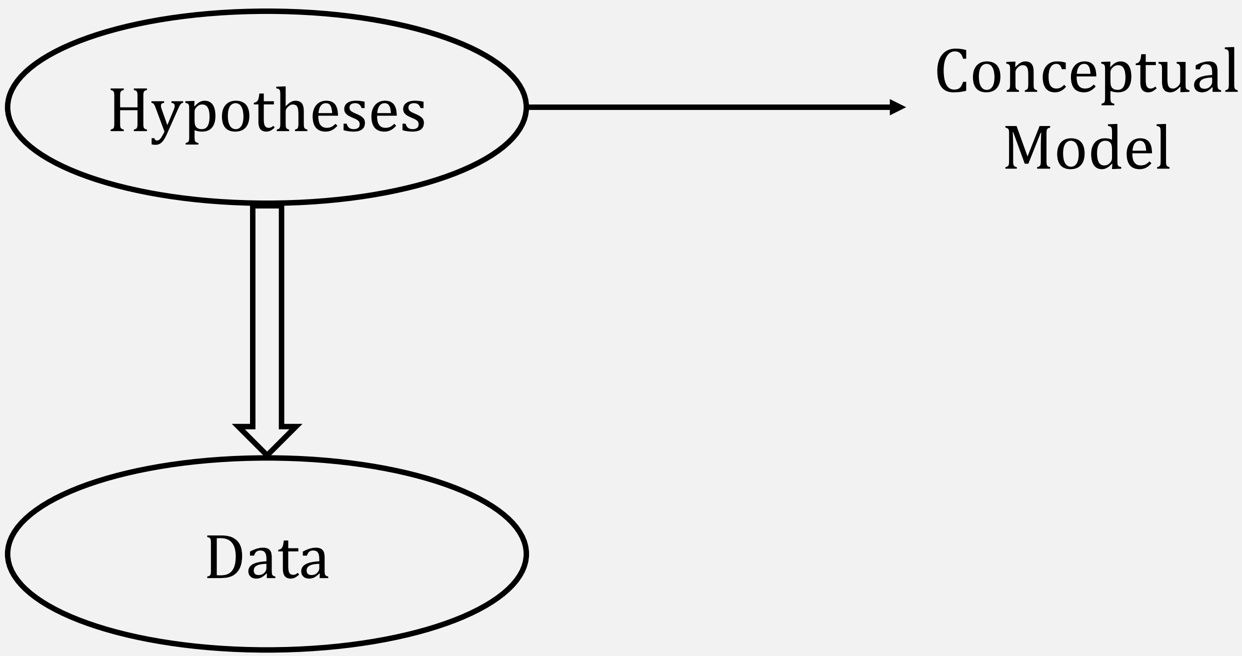

Marc Kéry
INTRODUCTION TO
WinBUGS
FOR ECOLOGISTS
A Bayesian approach to regression, ANOVA, mixed models and related analyses


Marc Kéry
J. Andrew Royle
APPLIED HIERARCHICAL MODELING IN ECOLOGY
Analysis of distribution, abundance and species richness in R and BUGS
Volume 1
Prelude and Static Models

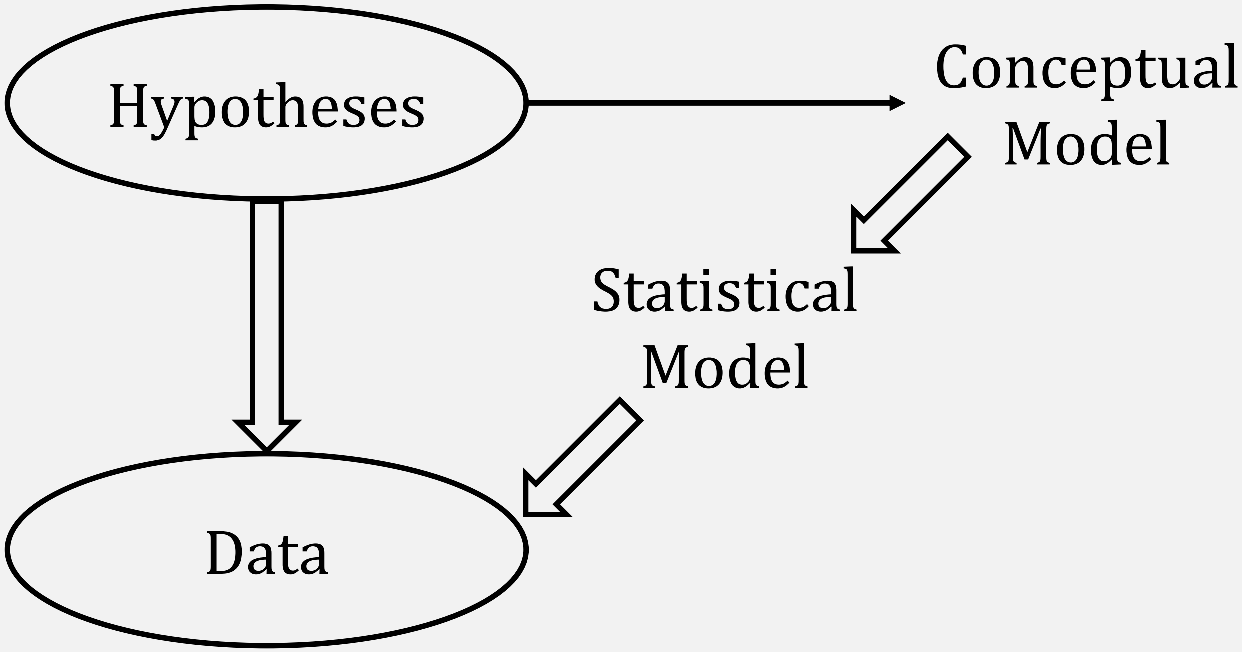
Why models?



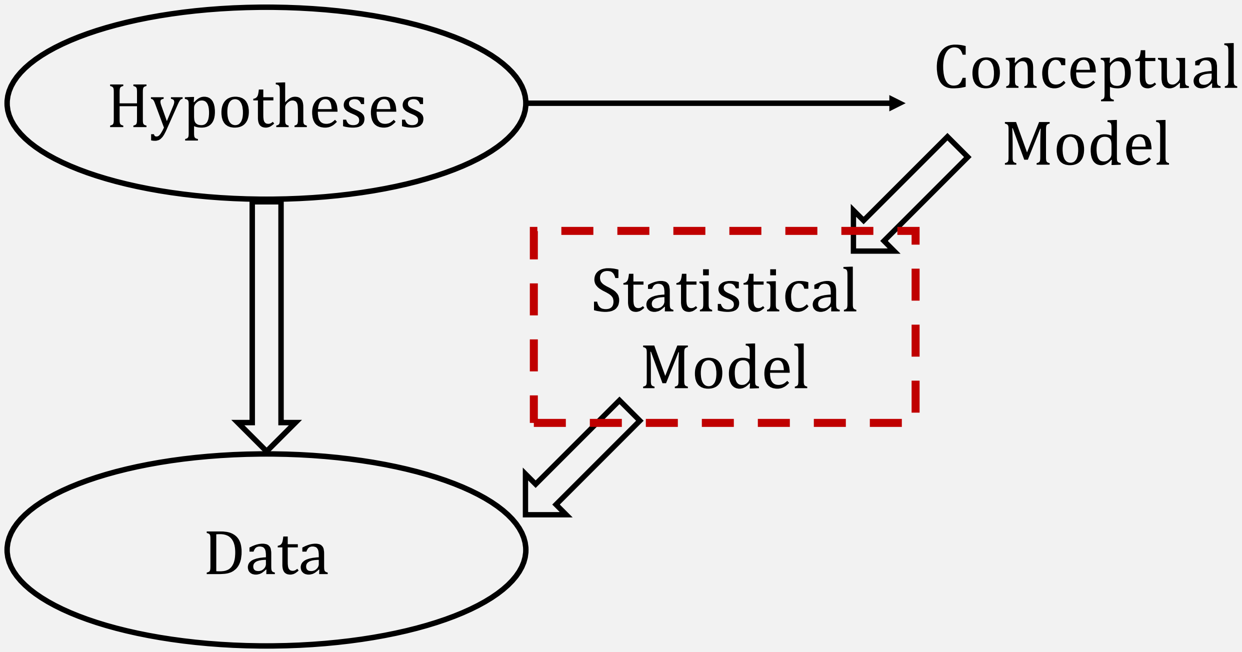
Why models?



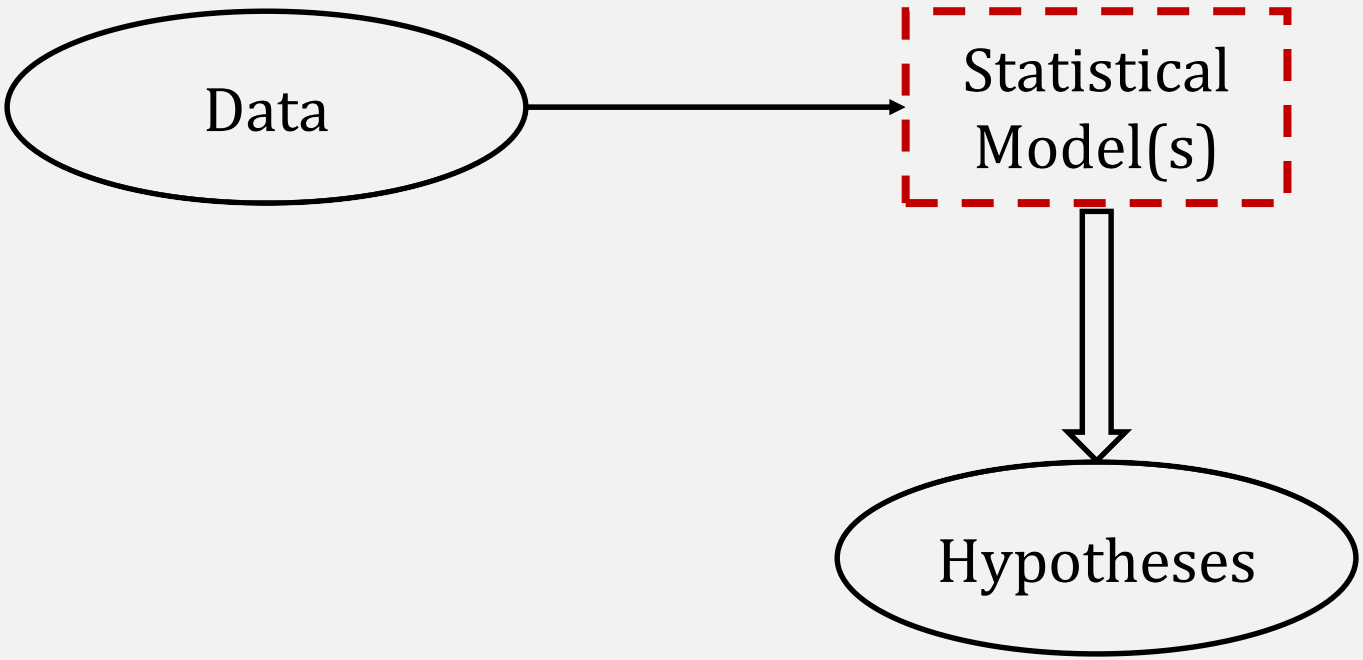
Why models?



H-D Method (Romesburg 1981)



Retroduction



Why hierarchical models?

1. Data reflects multiple distinct processes

    A. Ecological process of interest

    B. Observation process that produced the count

Why hierarchical models?

2. Hierarchically-structured systems

Why hierarchical models?

2. Hierarchically-structured systems

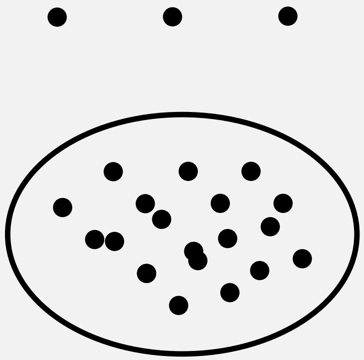
Individuals • • •

Why hierarchical models?

2. Hierarchically-structured systems

Individuals

Populations



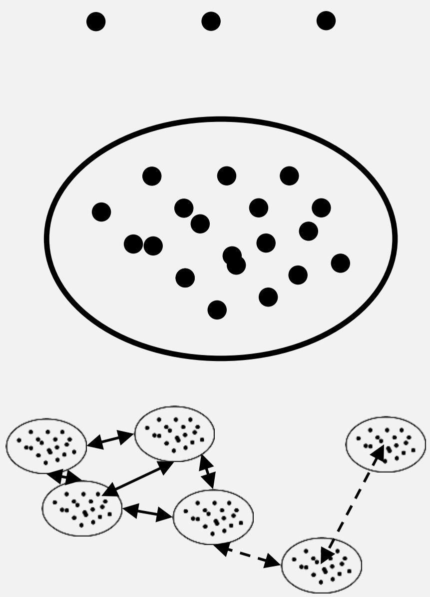
Why hierarchical models?

2. Hierarchically-structured systems

Individuals

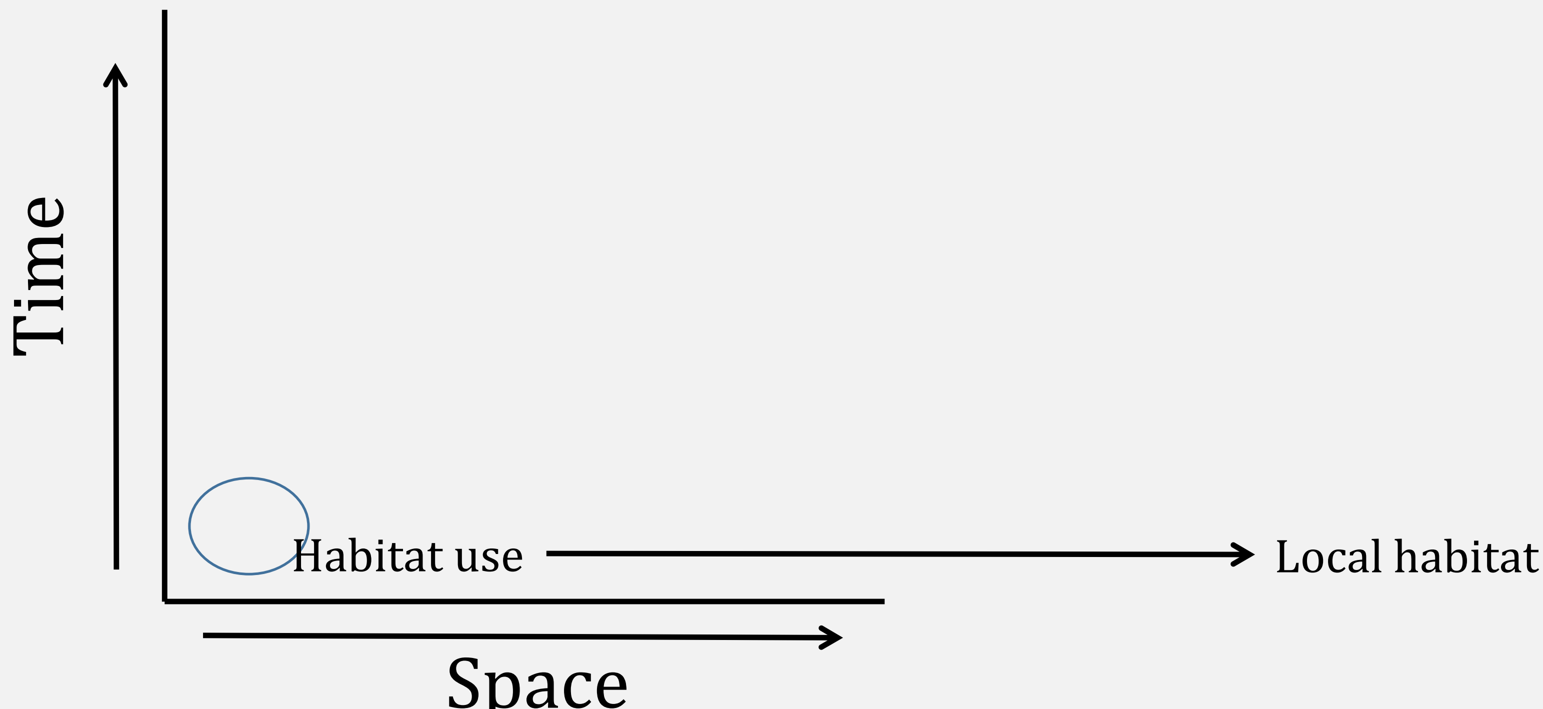
Populations

Metapopulations



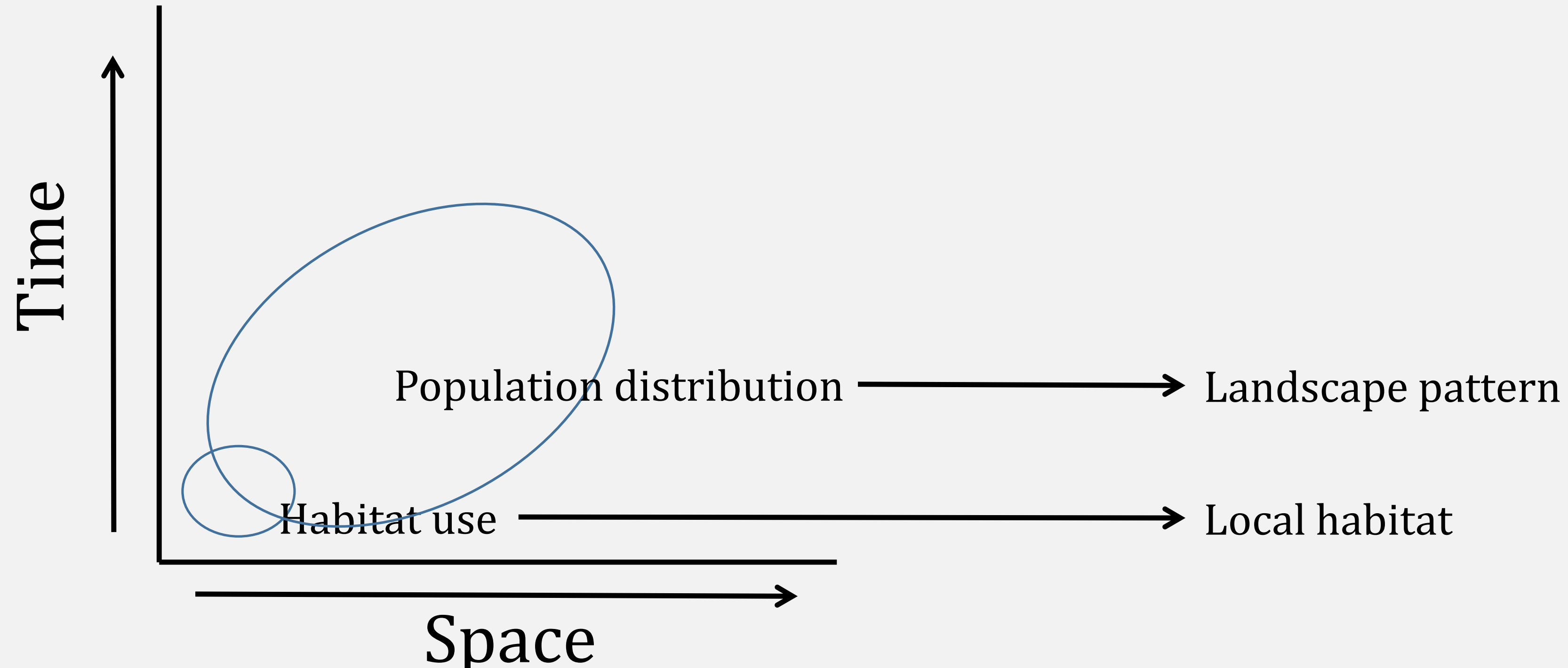
Why hierarchical models?

3. Scale-dependent processes



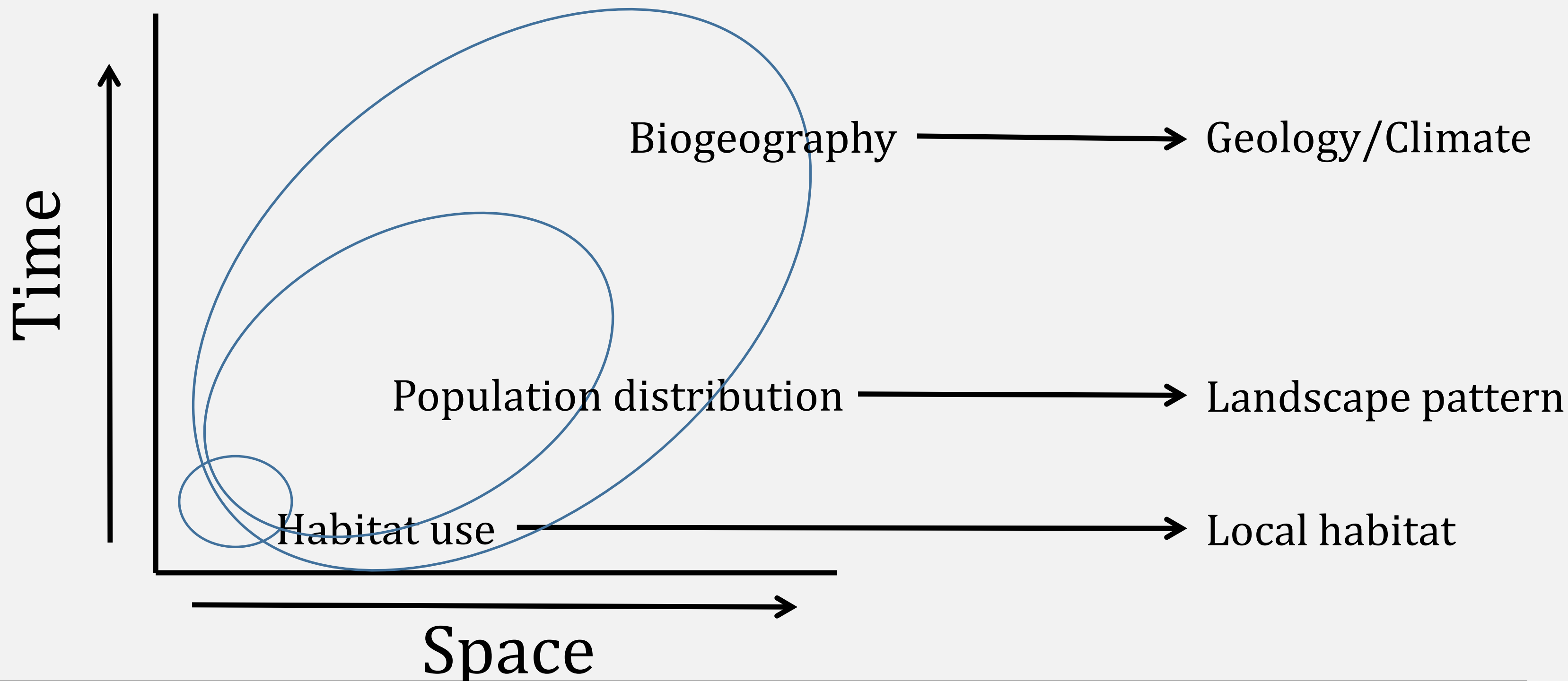
Why hierarchical models?

3. Scale-dependent processes



Why hierarchical models?

3. Scale-dependent processes



Conceptual correspondence

1. Levels of organization

2. Scales of different processes

3. **Hierarchical models**

What do we mean by hierarchical?

- Coupled sets of models that are conditionally related (e.g. the simplest occupancy model)

  - Model 1: the ecological process:

    $\boldsymbol{z_i \sim Bernoulli(\psi)}$ - where z is 0 or 1

  - Model 2: the observation process

    $\boldsymbol{y_{i,j} | z_i \sim Bernoulli(z_i p)}$ - where y is 0 or 1

  - **y is the data**

Translation: Ecological model

- ψ is the probability that a site is occupied
- z_i (occupied or not) is a random variable where success (z_i=1) has probability ψ

$$z_i \sim Bernoulli(\psi)$$

- $\boldsymbol{\psi}$ can be modeled as a function of covariates

$$logit(\psi_i) = \alpha + \boldsymbol{\beta X_i}$$

Translation: Observation model

- p is the probability of detection
- $y_{i,j}$ (detected or not) is a random variable where success ($y_{i,j}$=1), **conditional on z**, has probability $z_i \times p$

$$y_{i,j}|z_i \sim Bernoulli(z_i p)$$

- $\boldsymbol{p}$ can be modeled as a function of covariates

$$logit(p_{i,j}) = \alpha + \boldsymbol{\beta X_{i,j}}$$

English translation:

1. Site i has probability ψ of being occupied

$$z_i \sim Bernoulli(\psi)$$

2. Conditional on site i being occupied, we can detect the species with probability p

$$y_{i,j} | z_i \sim Bernoulli(z_i p)$$

What **don’t** we mean by hierarchical

- Doesn’t imply Bayesian or Frequentist
- Doesn’t have to be about detection
- Doesn’t have to be complicated

Example: zero-inflated abundance model

- Model 1: the occupancy process:

$$\boldsymbol{z_i \sim Bernoulli(\psi)}$$ - where z is 0 or 1

- Model 2: the abundance process

$$\boldsymbol{N_i | z_i \sim Poisson(z_i \lambda)}$$ - where N is an integer

- Model 3: the detection process

$$\boldsymbol{y_{i,j} | N_i \sim Binomial(N_i, p)}$$ - where y is the count

English translation:

1. Site i has probability ψ of being occupied

$$z_i \sim Bernoulli(\psi)$$

2. Conditional on site i being occupied, the site has an expected abundance of lambda

$$N_i | z_i \sim Poisson(z_i \lambda)$$

3. Conditional on abundance at site i, the expected count is binomial with N_i trials an probability of success p

$$y_{i,j} | N_i \sim Binomial(N_i, p)$$

English translation:

1. Site i has probability ψ of being occupied

$$z_i \sim Bernoulli(\psi)$$

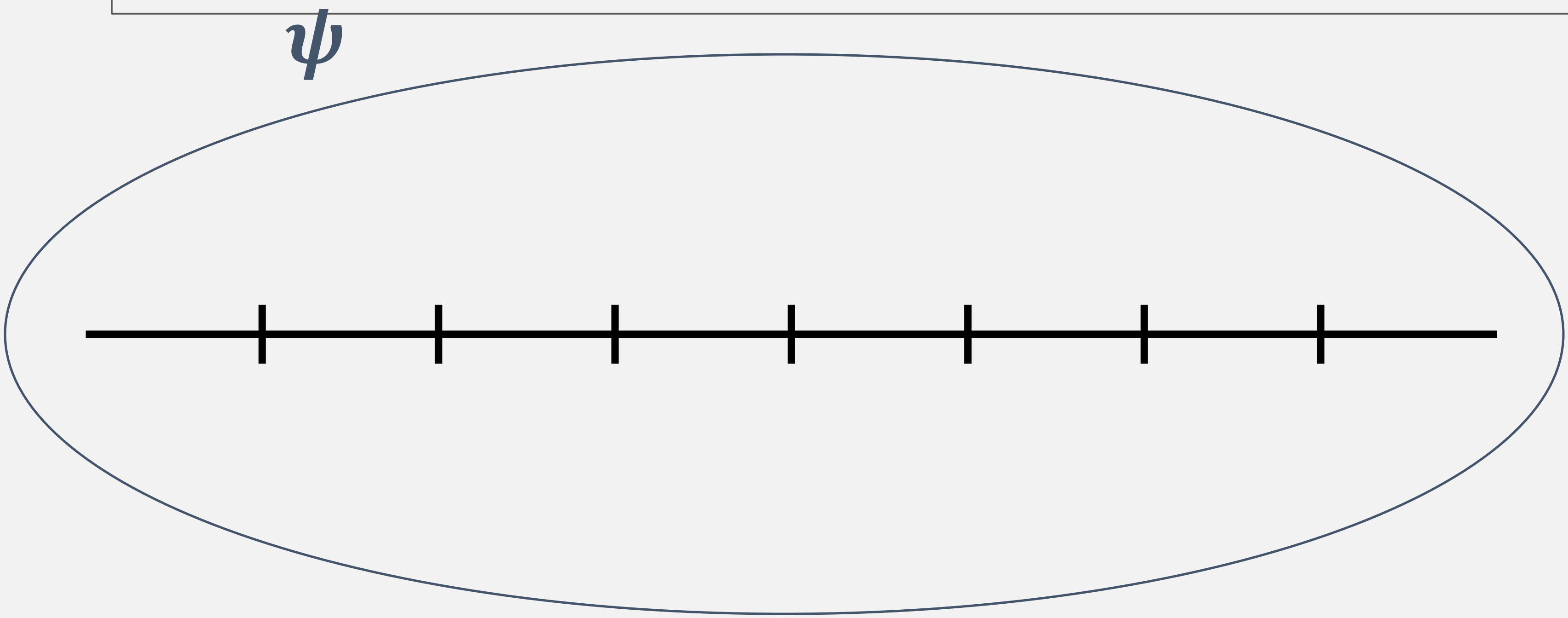
$$logit(\psi_i) = \alpha + \boldsymbol{\beta X_i}$$

2. Conditional on site i being occupied, the site has an expected abundance of lambda

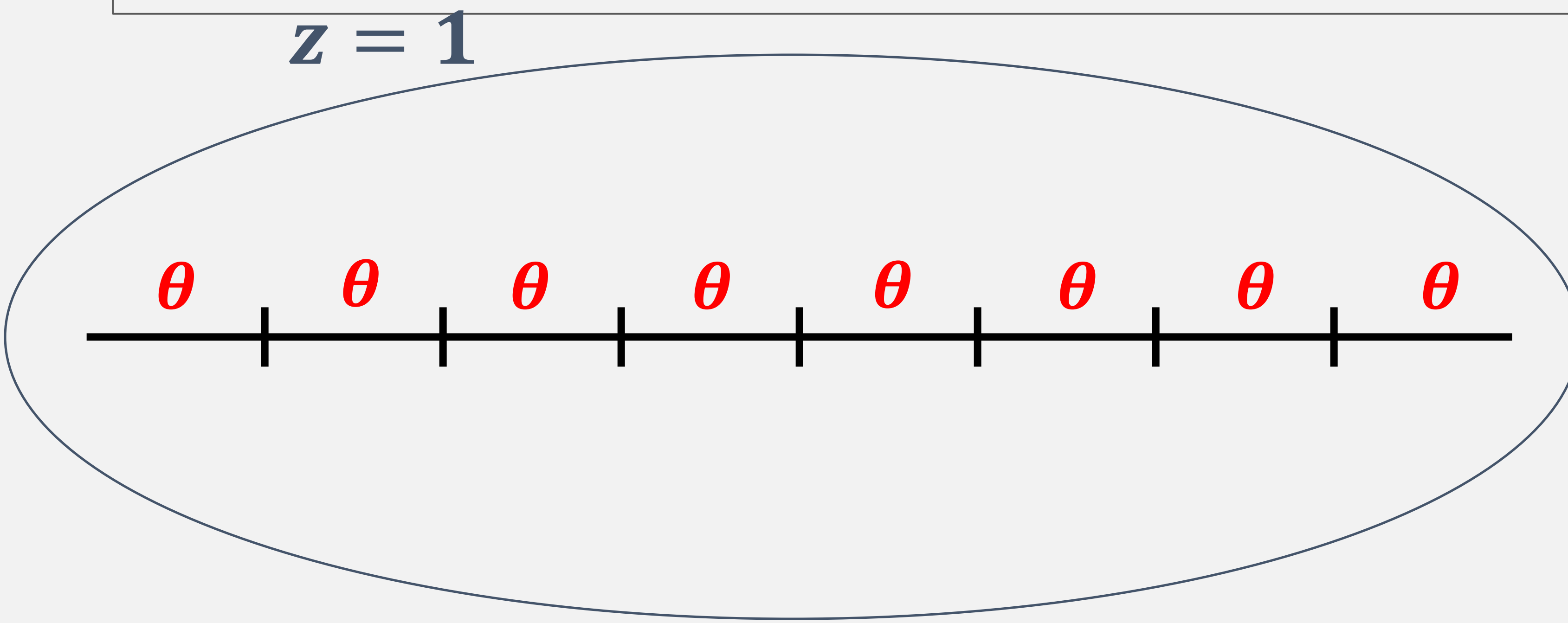
$$N_i | z_i \sim Poisson(z_i \lambda)$$

$$log\ (\lambda_i) = \alpha + \boldsymbol{\beta X_i}$$

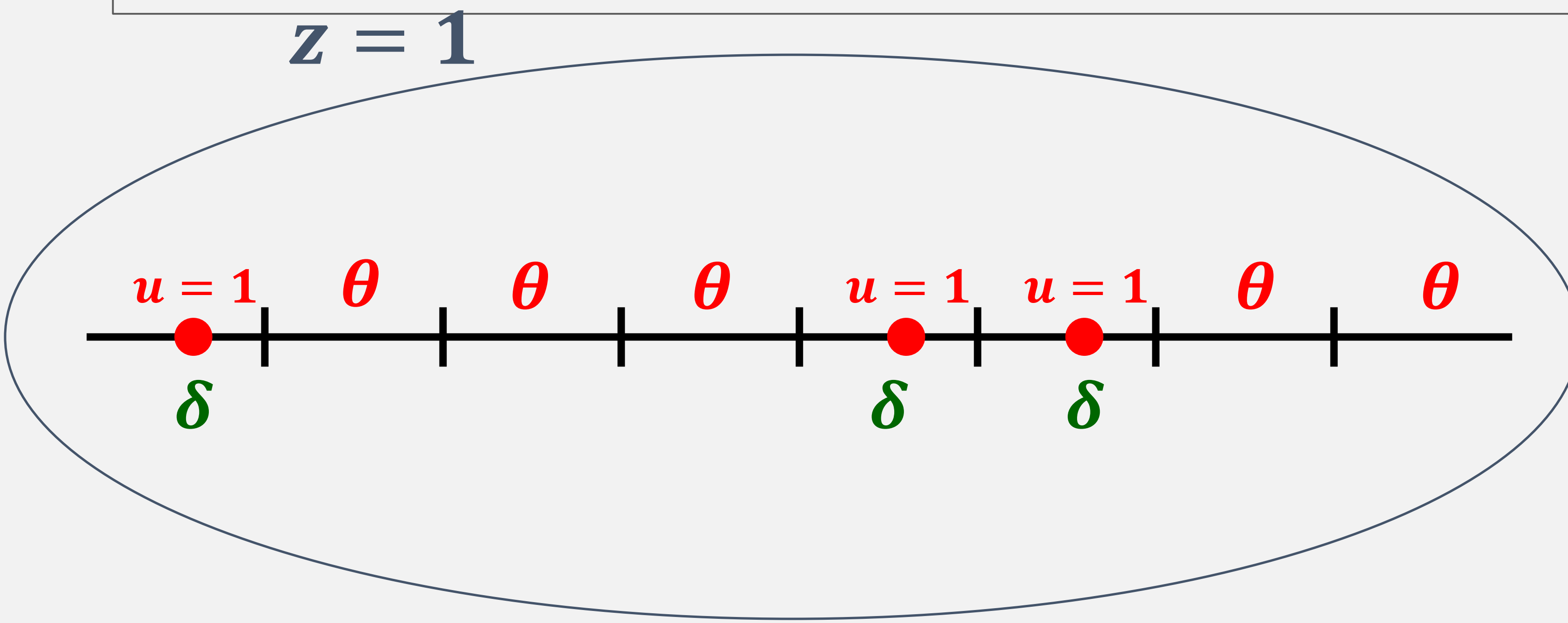
Model Formulation



Model Formulation

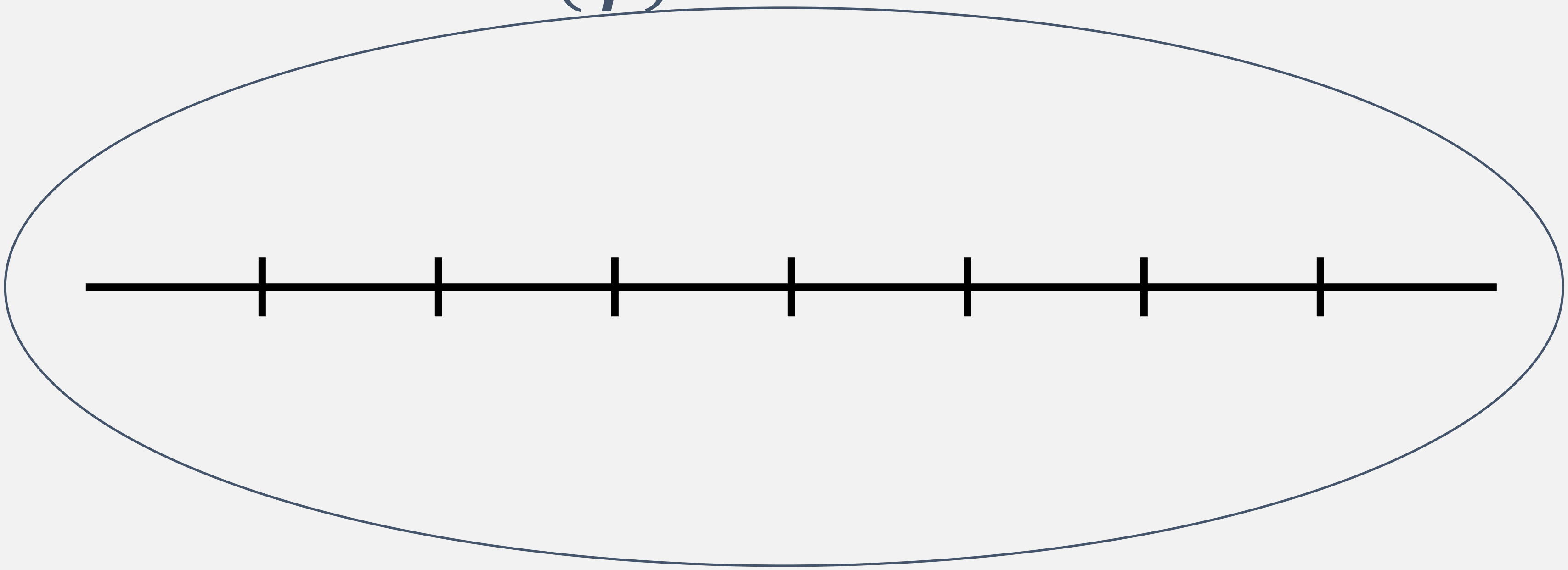


Model Formulation

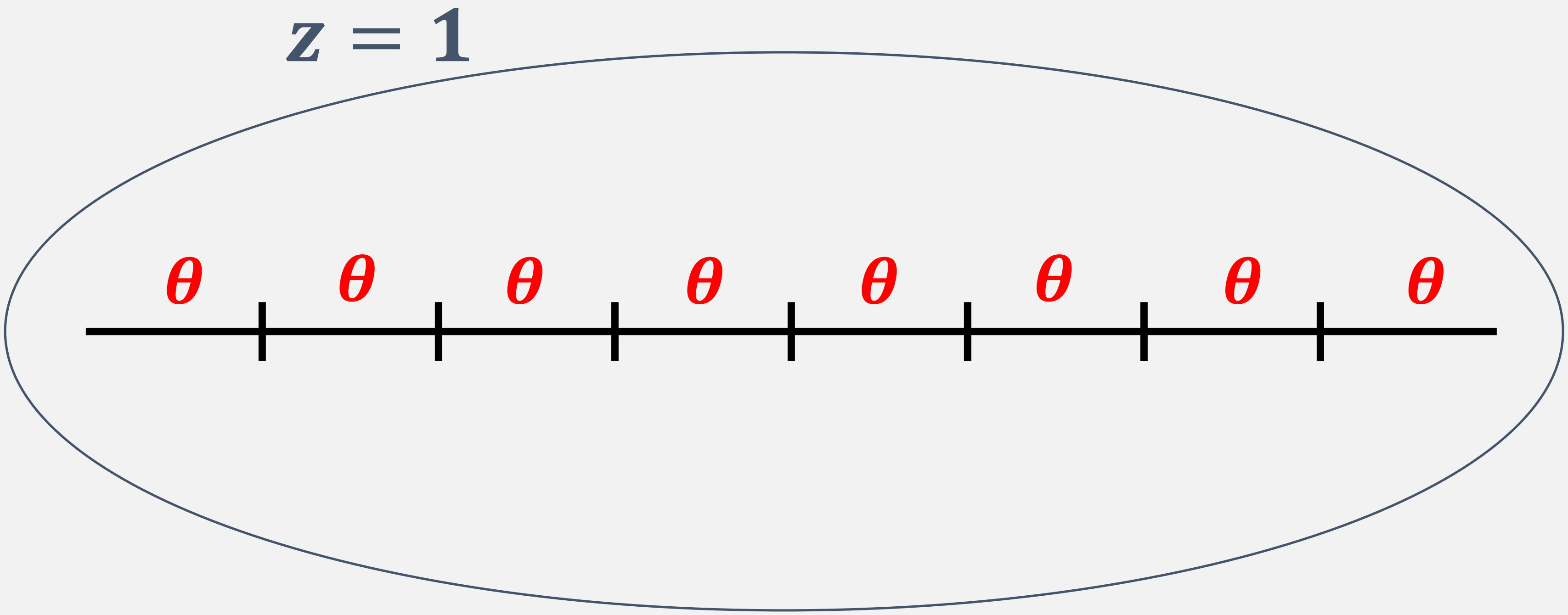


Tracking transect model

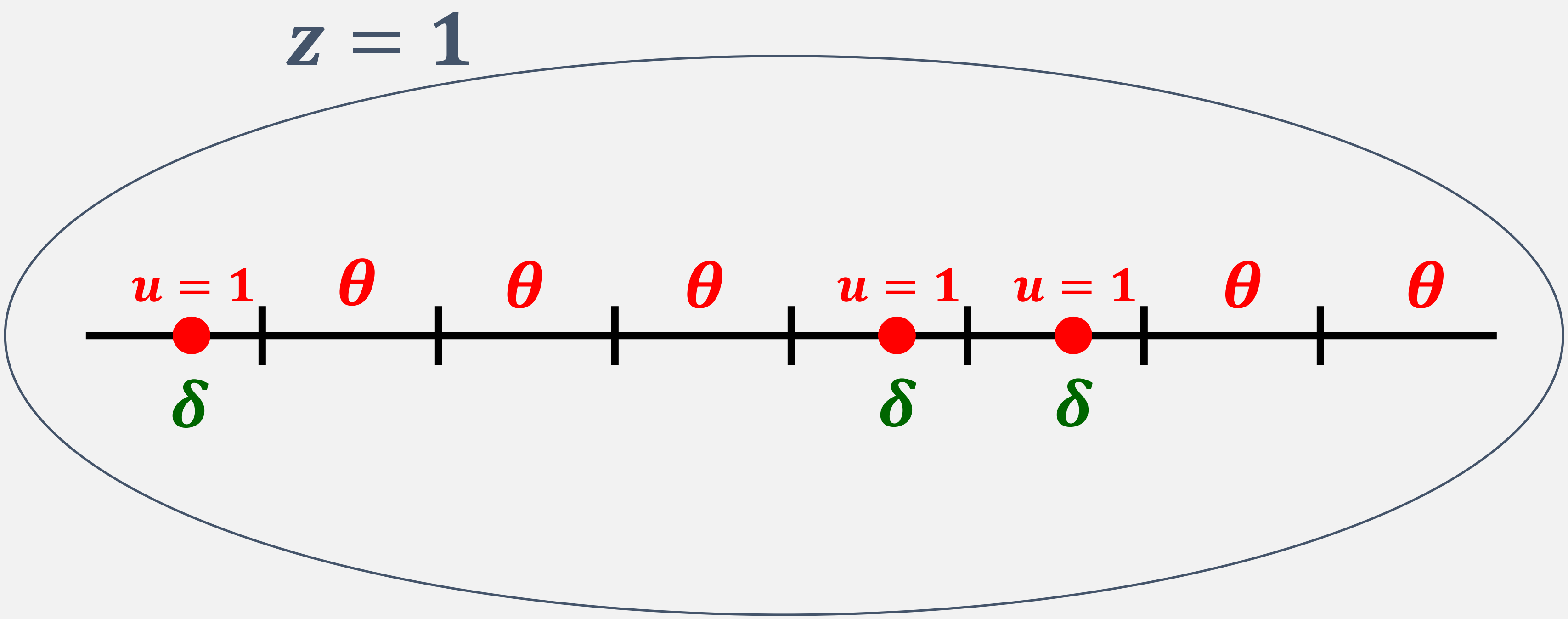
$z \sim Bernoullie(\psi)$



Tracking transect model



Tracking transect model



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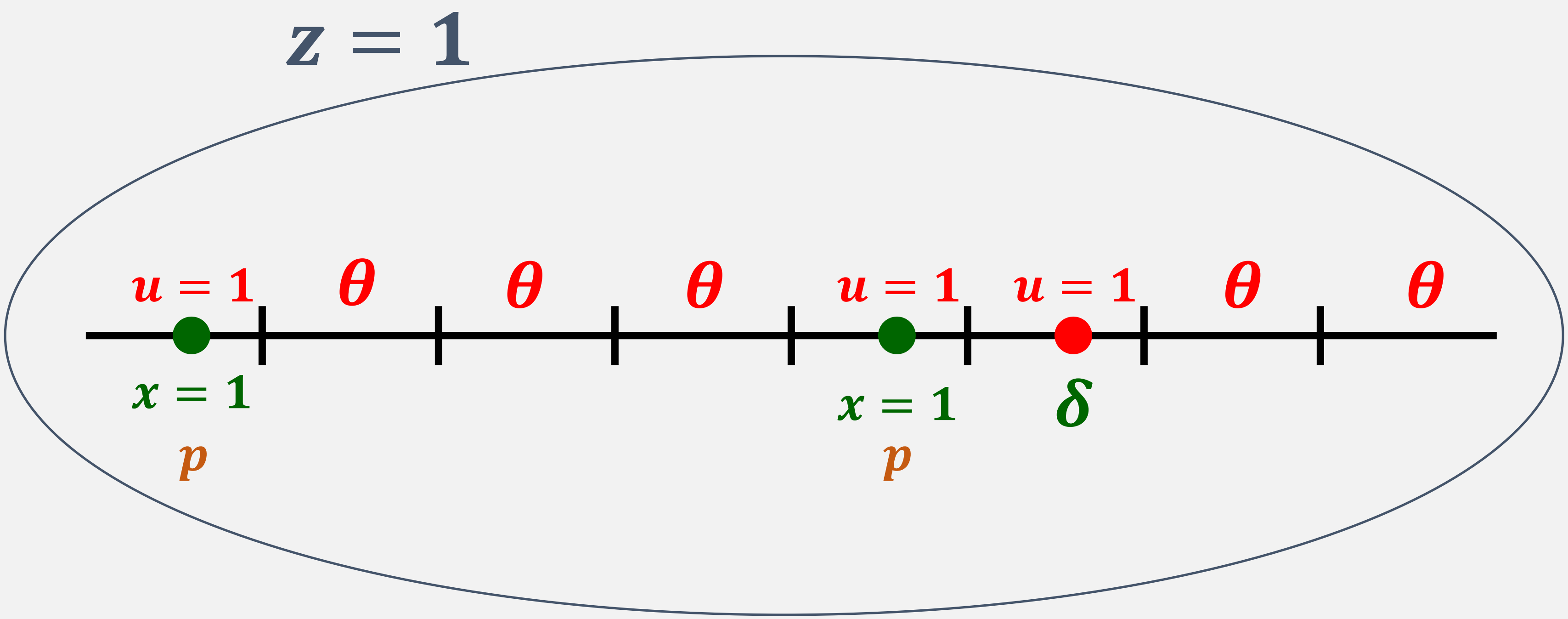
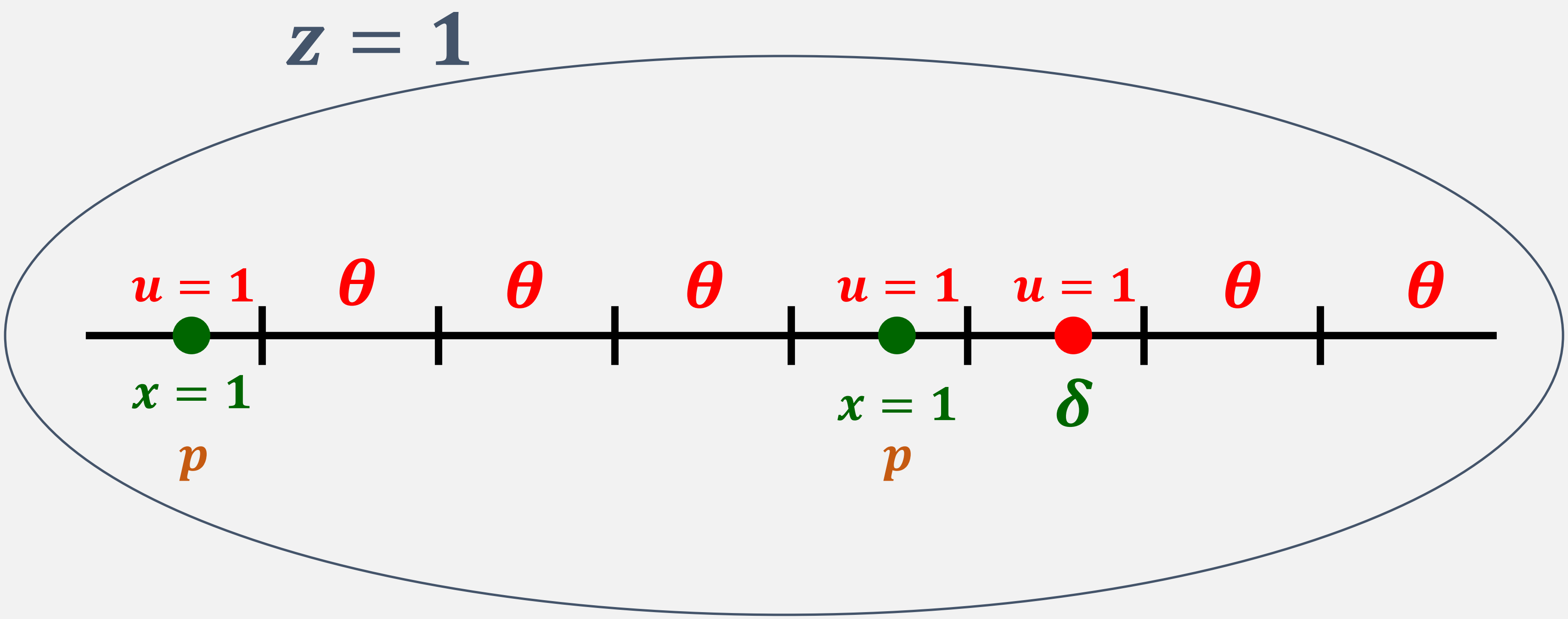
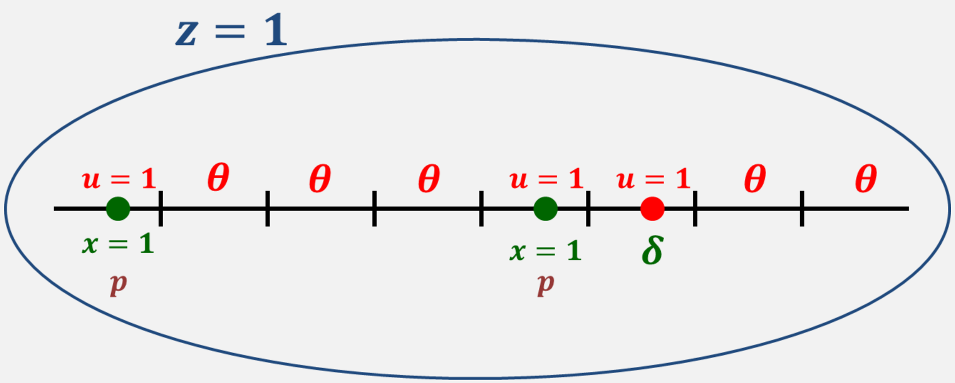


Tracking transect model

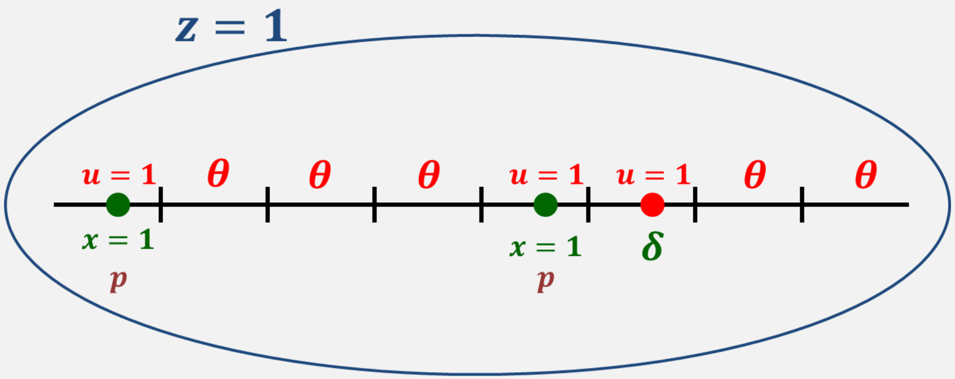


Model Formulation



$$z_i \sim Bernoulli(\psi_i)$$

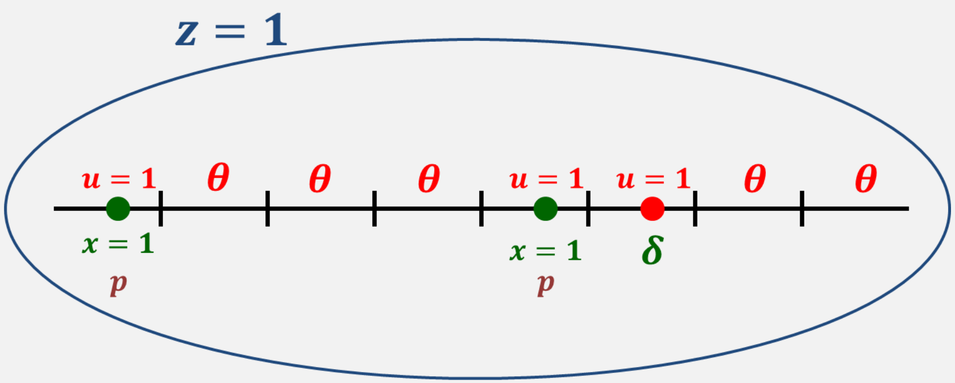
Model Formulation



$$z_i \sim Bernoulli(\psi_i)$$

$$u_{i,j} | z_i \sim Bernoulli(\theta_{i,j})$$

Model Formulation

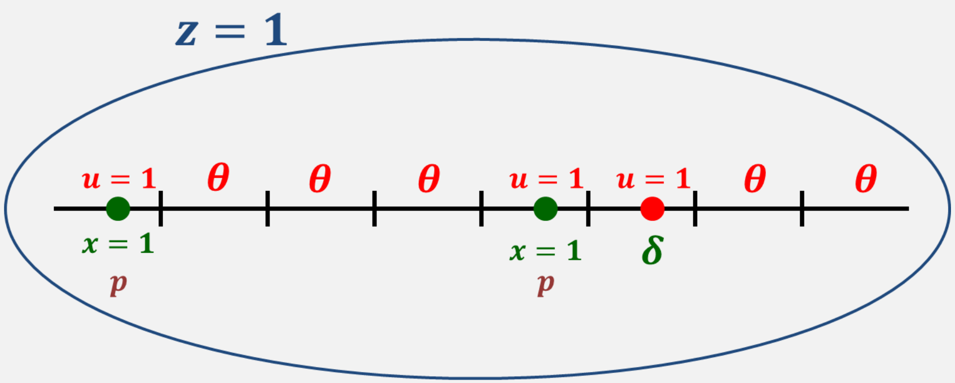


$$z_i \sim Bernoulli(\psi_i)$$

$$u_{i,j} | z_i \sim Bernoulli(\theta_{i,j})$$

$$x_{i,j,k} | u_{i,j} \sim Bernoulli(\delta_{i,j,k})$$

Model Formulation



$$z_i \sim Bernoulli(\psi_i)$$

$$u_{i,j} | z_i \sim Bernoulli(\theta_{i,j})$$

$$x_{i,j,k} | u_{i,j} \sim Bernoulli(\delta_{i,j,k})$$

$$y_{i,j,k,l} | x_{i,j,k} \sim Bernoulli(p_{i,j,k,l})$$

The Conceptual Middle Ground

- The Observation-driven View
  - e.g. Mark-recapture analysis

- The Process-driven View
  - e.g. Leslie matrix models

- The Hierarchical View
  - Accounts for both; includes all uncertainty

Bayesian vs. Frequentist

- I use Bayesian methods because:

1. Limits on MLE complexity
2. Intuitively easier to understand models
3. Freedom to develop more specific models

BUGS model formulation

```
for(i in 1:sites){
 z[i] ~ dbern(psi)
 for(j in 1:surveys){
  y[i,j] ~ dbern(z[i]*p)
  }
}
```

BUGS model with covariates

```
for(i in 1:sites){
  psi[i] <- alpha + beta*x[i]
  z[i] ~ dbern(psi[i])
  for(j in 1:surveys){
    p [i] <- alpha.p + beta.p*x[i]
    y[i,j] ~ dbern(z[i]*p[i])
    }
}
```