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CS 2130
Pro. Sonderegger
1.1-1.5

1.1 Exercises:

Exercise 1.1.1: Identifying propositions.
Determine whether each of the following sentences is a proposition. If the sentence is a proposition, then write its negation.
(a)
Have a nice day.

Not a proposition.

(b)
The soup is cold.

Proposition. "The soup is hot."

(c)
The patient has diabetes.

Proposition. "The patient does not have diabetes."

(d)
The light is on.

Proposition. "The light is off."
(e)
It's a beautiful day.

Proposition. "It is not a beautiful day.”

(f)
Do you like my new shoes?

Not a proposition.

(g)
The sky is purple.

Proposition. "The sky is not purple."

(h)
2 + 3 = 6

Proposition. "2 + 3 does not equal 6."

(i)
Every prime number is even.

Proposition. "Every prime number is odd."

(j)
There is a number that is larger than 17.

Proposition. "There is not a number larger than 17."

Exercise 1.1.2:

Express each English statement using logical operations ∨, ∧, ¬ and the propositional variables t, n, and m defined below. The use of the word "or" means inclusive or.

t: The patient took the medication.
n: The patient had nausea.
m: The patient had migraines.
(a)
The patient had nausea and migraines.

n v m

(b)
The patient took the medication, but still had migraines.

t ^ m

(c)
The patient had nausea or migraines.

n ^ m

(d)
The patient did not have migraines.

$\neg m$

(e)
Despite the fact that the patient took the medication, the patient had nausea.

t^n

(f)
There is no way that the patient took the medication.
$\neg t$

Exercise 1.1.3: Applying logical operations.

Assume the propositions p, q, r, and s have the following truth values:
p is false
q is true
r is false
s is true
What are the truth values for the following compound propositions?
(a)
$\neg p$

True

(b)
$p \lor r$

false

(c)
$q \land s$

false

(d)
$q \lor s$

true

(e)
$q \oplus s$

false

(f)
$q \oplus r$

true

Exercise 1.1.4: Truth values for statements with inclusive and exclusive or.

Indicate whether each statement is true or false, assuming that the "or" in the sentence means the inclusive or. Then indicate whether the statement is true or false if the "or" means the exclusive or.

(a)
February has 31 days or the number 5 is an integer.

True, true.
(b)
The number π is an integer or the sun revolves around the earth.

True, false
(c)
20 nickels are worth one dollar or whales are mammals.

True, true
(d)
There are eight days in a week or there are seven days in a week.

True, true
(e)
January has exactly 31 days or April has exactly 30 days.
True, false

1.2 Excercises

Exercise 1.2.1: Truth values for compound English sentences.

Determine whether the following propositions are true or false:

(a)
5 is an odd number and 3 is a negative number.

False

(b)
5 is an odd number or 3 is a negative number.

True
(c)
8 is an odd number or 4 is not an odd number.

True
(d)
6 is an even number and 7 is odd or negative.

True
(e)
It is not true that either 7 is an odd number or 8 is an even number (or both).

False

Exercise 1.2.2: Translating English statements into logic.

Express each statement in logic using the variables:

p: It is windy.
q: It is cold.
r: It is raining.

(a)It is windy and cold.

$p \land q$

(b)It is windy but not cold.

$p \land \neg q$

(c)It is not true that it is windy or cold.

$\neg (p \lor q)$

(d)It is raining and it is windy or cold.

$r \land (p \lor q)$

(e)It is raining and windy or it is cold.

$(r \land p) \lor q$

(f)It is raining and windy but it is not cold.

$r \land p \land \neg q$

Exercise 1.2.3: Truth values for compound propositions.

The propositional variables, p, q, and s have the following truth assignments: p = T, q = T, s = F. Give the truth value for each proposition.

(a)

$p \lor \neg q$: T

(b)

$(p \land q) \lor s$: T

(c)

$p \land (q \lor s)$: T

(d)

$p \land \neg(q \lor s)$: F

(e)

$\neg(q \land p \land \neg s)$: F

(f)

$\neg(p \land \neg(q \land s))$: F

Exercise 1.2.4: Writing truth tables.

Write a truth table for each expression.(a)

¬p ⊕ q

p	q	¬p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

(b)

¬(p ∨ q)

p	q	¬(p ∨ q)
T	T	F
T	F	F
F	T	F
F	F	T

(c)

r ∨ (p ∧ ¬q)

p	q	r	(p ∧ ¬q)	r ∨ (p ∧ ¬q)
T	T	T	F	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

(d)

(r ∨ p) ∧ (¬r ∨ ¬q)

p	q	r	(r ∨ p)	(¬r ∨ ¬q)	(r ∨ p) ∧ (¬r ∨ ¬q)
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	F	T	F
F	F	T	T	T	T
F	F	F	F	T	F

Exercise 1.2.6: Multiple disjunction or conjunction operations.

Suppose that p, q, r, s, and t are all propositional variables.

(a)

Describe in words when the expression p ∨ q ∨ r ∨ s ∨ t is true and when it is false.

It is true when at least one variable is T but false only when all variables are F

(b)

Describe in words when the expression p ∧ q ∧ r ∧ s ∧ t is true and when it is false.

It is true only when all variables are T, and false if at least one variable is F.

Exercise 1.2.7: Expressing a set of conditions using logical operations.

Consider the following pieces of identification a person might have in order to apply for a credit card:

B: Applicant presents a birth certificate.

D: Applicant presents a driver's license.

M: Applicant presents a marriage license.

Write a logical expression for the requirements under the following conditions:

(a)

The applicant must present either a birth certificate, a driver's license or a marriage license.

$B \land D \land M$

(b)

The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

$(B \land D) \lor (B \land M) \lor (D \land M)$

(c)

Applicant must present either a birth certificate or both a driver's license and a marriage license.

$B \lor (D \land M)$

Exercise 1.2.8: Finding truth values to make two logical expressions evaluate to different values.

Give truth values for the propositional variables that cause the two expressions to have different truth values.

For example, given $p \vee q$ and $p \oplus q$, the correct answer would be $p = q = T$, because when p and q are both true, $p \vee q$ is true but $p \oplus q$ is false. Note that there may be more than one correct answer.

(a)

$r \wedge (p \vee q)$

$(r \wedge p) \vee q$

$r = F, p = q = T$

(b)

$\neg p \wedge q$

$\neg(p \wedge q)$

$p = q = F$

(c)

$p \vee q$

$(\neg p \wedge q) \vee (p \wedge \neg q)$

$p = F, q = T$

Exercise 1.2.9: Boolean expression to express a condition on the input variables.

(a)

Give a logical expression with variables p, q, and r that is true if p and q are false and r is true and is otherwise false.

$\neg (p \wedge q) \wedge r$

Exercise 1.3.1

Which of the following conditional statements are true and why?

(a)
If February has 30 days, then 7 is an odd number.

True, the first statement is false so it doesn't matter what the outcome is.

(b)
If January has 31 days, then 7 is an even number.

False, the first statement is true but the second statement is false.

(c)
If 7 is an odd number, then February does not have 30 days.

True, both statements are true.

(d)
If 7 is an even number, then January has exactly 28 days.

True, the first statement is false.

Exercise 1.3.2: The inverse, converse, and contrapositive of conditional sentences in English.

About
Give the inverse, contrapositive, and converse for each of the following statements:

(a)
If she finished her homework, then she went to the party.

I: If she didn't finish her homework, then she didn't go to the party.
CP: If she didn't go to the party, she didn't finish her homework.
C: If she went to the party, she finished her homework.

(b)
If he trained for the race, then he finished the race.

I: If he didn't train for the race, then he didn't finish the race.
CP: If he didn't finish the race, he didn't train for the race.
C: If he finished the race, he trained for the race.

(c)
If the patient took the medicine, then she had side effects.

I: If the patient didn't take the medicine, she didn't have side effects.
CP: If she didn't have side effects, the patient didn't take the medicine.
C: If she had side effects, the patient took the medicine.

(d)

If it was sunny, then the game was held.

I: If it wasn't sunny, then the game wasn't held.
CP: If the game wasn't held, it wasn't sunny.
C: If the game was held, it was sunny.

(e)
If it snowed last night, then school will be cancelled

I: If it didn't snow last night, then school won't be cancelled.
CP: If school isn't canceled, it didn't snow last night.
C: If school is canceled, it snowed last night.

Exercise 1.3.3: Truth values for the inverse, contrapositive, and converse of a conditional statement.

State the inverse, contrapositive, and converse of each conditional statement. Then indicate whether the inverse, contrapositive, and converse are true.

(a)
If 3 is a prime number then 5 is an even number.

I: If 3 isn't a prime number, then 5 isn't an even number: TRUE
CP: If 5 isn't an even number, then 3 isn't a prime number: FALSE
C: If 5 is an even number, 3 is a prime number: TRUE

(b)
If $7 < 5$, then $5 < 3$.

I: If $7 > 5$, then $5 > 3$: TRUE
CP: If $5 > 3$, then $7 > 5$: TRUE
C: If $5 < 3$, then $7 < 5$: TRUE

(c)
If 5 is a negative number, then 3 is a positive number.

I: If 5 is a positive number, then 3 is a negative number : FALSE
CP: If 3 is a negative number, then 5 is a positive number : TRUE
C: If 3 is a positive number, then 5 is a negative number : FALSE

Exercise 1.3.4: Truth tables for logical expressions with conditional operations.

Give a truth table for each expression.

(a)

$(\neg p \land q) \to p$

p	q	$(\neg p \land q)$	$(\neg p \land q) \to p$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	F	T

(b)

$(p \to q) \to (q \to p)$

p	q	$(p \to q)$	$(q \to p)$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(c)

$(p \lor q) \leftrightarrow (q \to \neg p)$

p	q	$(p \lor q)$	$(q \to \neg p)$	$(p \lor q) \leftrightarrow (q \to \neg p)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	T	F

(d)

$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$(p \leftrightarrow q)$	$(p \leftrightarrow \neg q)$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

(e)
$(p \vee q) \leftrightarrow (q \wedge p)$

p	q	$(p \vee q)$	$(q \wedge p)$	$(p \vee q) \leftrightarrow (q \wedge p)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

Exercise 1.3.5: Expressing conditional statements in English using logic.

Define the following propositions:

c: I will return to college.
j: I will get a job.
Translate the following English sentences into logical expressions using the definitions above:

(a)
Not getting a job is a sufficient condition for me to return to college.
$\neg j \rightarrow c$

(b)
If I return to college, then I won't get a job.
$c \rightarrow \neg j$

(c)
I am not getting a job, but I am still not returning to college.
$\neg j \rightarrow \neg c$

(d)
I will return to college only if I won't get a job.
$c \leftrightarrow \neg j$

(e)
There's no way I am returning to college.
$\neg c$

(f)
I will get a job and return to college.
$j \land c$

Exercise 1.3.6: Expressing English sentences in if-then form.

Give an English sentence in the form "If...then...." that is equivalent to each sentence.

(a)

Maintaining a B average is sufficient for Joe to be eligible for the honors program.

If Joe maintains a B average, he will be eligible for the honors program.

(b)

Maintaining a B average is necessary for Joe to be eligible for the honors program.

Joe will be eligible for the honors program if and only if he maintains a B average.

(c)

Rajiv can go on the roller coaster only if he is at least four feet tall.

Rajiv can go on the roller coaster if and only if he is at least four feet tall.

(d)

Rajiv can go on the roller coaster if he is at least four feet tall.

If Rajiv is at least four feet tall, then he can go on the roller coaster.

Exercise 1.3.7: Expressing conditional statements in English using logic.

Define the following propositions:

s: a person is a senior
y: a person is at least 17 years of age

p: a person is allowed to park in the school parking lot

Express each of the following English sentences with a logical expression:

(a)

A person is allowed to park in the school parking lot only if they are a senior and at least seventeen years of age.

$(s \wedge y) \rightarrow p$

(b)

A person can park in the school parking lot if they are a senior or at least seventeen years of age.

$(s \vee y) \rightarrow p$

(c)

Being 17 years of age is a necessary condition for being able to park in the school parking lot.

$y \leftrightarrow p$

(d)

A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

$(s \wedge y) \leftrightarrow p$

(e)

Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

$(s \vee y) \leftrightarrow p$

Exercise 1.3.8: Translating logical expressions into English.

Define the following propositions:

w: the roads were wet
a: there was an accident
h: traffic was heavy

Express each of the logical expressions as an English sentence:

(a)

$w \rightarrow h$

If the roads were wet, traffic was heavy.

(b)

$w \land a$

The roads were wet and there was an accident.

(c)

$\neg(a \land h)$

There was not an accident and the traffic wasn't heavy.

(d)

$h \rightarrow (a \lor w)$

If traffic was heavy, there was an accident or the roads were wet.

(e)

$w \land \neg h$

The roads were wet and the traffic wasn't heavy.

Exercise 1.3.9: Translating English propositions into logical expressions.

Use the definitions of the variables below to translate each English statement into an equivalent logical expression.

- y: the applicant is at least eighteen years old
- p: the applicant has parental permission
- c: the applicant can enroll in the course

(a)

The applicant is not eighteen years old but does have parental permission.

$\neg y \land p$

(b)

If the applicant is at least eighteen years old or has parental permission, then the applicant can enroll in the course.

$(y \lor p) \rightarrow c$

(c)

The applicant can enroll in the course only if the applicant has parental permission.

$p \leftrightarrow c$

(d)

Having parental permission is a necessary condition for enrolling in the course.

$p \rightarrow c$

Exercise 1.3.10: Determining if a truth value of a compound expression is known given a partial truth assignment.

The variable p is true, q is false, and the truth value for variable r is unknown. Indicate whether the truth value of each logical expression is true, false, or unknown.

(a)

$p \rightarrow (q \land r)$: F

(b)

$(p \lor r) \rightarrow r$: Unknown

(c)

$(p \lor r) \leftrightarrow (q \land r)$: F

(d)

$(p \land r) \leftrightarrow (q \land r)$: F

(e)

p → (r ∨ q) : Unknown

(f)

(p ∧ q) → r : T

Exercise 1.3.11: Finding logical expressions to match a truth table.

For each table, give a logical expression whose truth table is the same as the one given.

p	q	?
T	T	F
T	F	T
F	T	F
F	F	F

p ∧ ¬ q

(b)

p	q	?
T	T	F
T	F	T
F	T	T
F	F	F

p ⊕ q

Exercise 1.4.1: Proving tautologies and contradictions.

Show whether each logical expression is a tautology, contradiction or neither.
(a)

$(p \lor q) \lor (q \rightarrow p)$: Tautology

(b)

$(p \rightarrow q) \leftrightarrow (p \land \neg q)$: Contradiction

(c)

$(p \rightarrow q) \leftrightarrow p$: Neither

(d)

$(p \rightarrow q) \lor p$: Tautology

(e)

$(\neg p \lor q) \leftrightarrow (p \land \neg q)$: Contradiction

(f)

$(\neg p \lor q) \leftrightarrow (\neg p \land q)$: Neither

Exercise 1.4.2: Truth tables to prove logical equivalence.

Use truth tables to show that the following pairs of expressions are logically equivalent.

(a)

$p \leftrightarrow q$ and $(p \rightarrow q) \land (q \rightarrow p)$

p	q	$p \leftrightarrow q$	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \land (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

(b)

$\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$

p	q	$\neg(p \leftrightarrow q)$	$\neg p \leftrightarrow q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	F	F

(c)

$\neg p \to q$ and $p \lor q$

p	q	$\neg p \to q$	$p \lor q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

Exercise 1.4.3: Proving two logical expressions are not logically equivalent.

Prove that the following pairs of expressions are not logically equivalent.

(a)

$p \to q$ and $q \to p$

If p is false, the first statement is T and the second statement is F.

(b)

$\neg p \to q$ and $\neg p \lor q$

If p = q = F, the first statement is F and the second statement is T.

(c)

$(p \to q) \land (r \to q)$ and $(p \land r) \to q$

If p = q = F and r = T, the first statement is F and the second statement is T.

(d)

$p \wedge (p \rightarrow q)$ and $p \vee q$

If p is F and q is T, the first statement is F and the second statement is T.

Exercise 1.4.4: Proving whether two logical expressions are equivalent.

Determine whether the following pairs of expressions are logically equivalent. Prove your answer. If the pair is logically equivalent, then use a truth table to prove your answer.

(a)

$\neg(p \vee \neg q)$ and $\neg p \wedge q$

If p = F and q = T, the first statement is F and the second statement is T.

(b)

$\neg(p \vee \neg q)$ and $\neg p \wedge \neg q$

p	q	$\neg(p \vee \neg q)$	$\neg p \wedge \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

(c)

$p \wedge (p \rightarrow q)$ and $p \rightarrow q$

If p = q = F, the first statement is F and the second statement is true.

(d)

$p \wedge (p \rightarrow q)$ and $p \wedge q$

p	q	$p \wedge q$	$p \wedge (p \rightarrow q)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

Exercise 1.4.5: Logical equivalence of two English statements.

Define the following propositions:

- j: Sally got the job.
- l: Sally was late for her interview
- r: Sally updated her resume.

Express each pair of sentences using logical expressions. Then prove whether the two expressions are logically equivalent.

(a)

If Sally did not get the job, then she was late for her interview or did not update her resume.

If Sally updated her resume and did not get the job, then she was late for her interview.

$\neg j \rightarrow (l \vee \neg r) : (r \wedge \neg j) \rightarrow l$

j	l	r	$(l \vee \neg r)$	$\neg j \rightarrow (l \vee \neg r)$	$(r \wedge \neg j) \rightarrow l$	$(r \wedge \neg j)$
T	T	T	T	T	T	F
T	T	F	T	T	T	F
T	F	T	F	T	T	F
T	F	F	T	T	T	F
F	T	T	T	T	T	T
F	T	F	T	T	T	F
F	F	T	F	F	F	T
F	F	F	T	T	T	F

(b)

If Sally did not get the job, then she was late for her interview or did not update her resume.

If Sally updated her resume and was not late for her interview, then she got the job.

$\neg j \rightarrow (l \lor \neg r) : (r \land \neg l) \rightarrow j$

j	l	r	$(l \lor \neg r)$	$\neg j \rightarrow (l \lor \neg r)$	$(r \land \neg l) \rightarrow j$	$(r \land \neg l)$
T	T	T	T	T	T	F
T	T	F	T	T	T	F
T	F	T	F	T	T	T
T	F	F	T	T	T	F
F	T	T	T	T	T	F
F	T	F	T	T	T	F
F	F	T	F	F	F	T
F	F	F	T	T	T	F

(c)

If Sally got the job then she was not late for her interview.

If Sally did not get the job, then she was late for her interview.

$j \rightarrow \neg l : \neg j \rightarrow l$

Not equivalent. If j = l = T, the first statement is F and the second statement is T.

(d)

If Sally updated her resume or she was not late for her interview, then she got the job.

If Sally got the job, then she updated her resume and was not late for her interview.

$(r \vee \neg l) \rightarrow j : j \rightarrow (r \wedge \neg l)$

Not equivalent. If j = l = r = T, the first statement is T and the second statement is F.

Exercise 1.4.6: Applying De Morgan's laws.

Translate each English sentence into a logical expression using the propositional variables defined below. Then negate the entire logical expression using parentheses and the negation operation. Apply De Morgan's law to the resulting expression and translate the final logical expression back into English.

- p: the applicant has written permission from his parents
- e: the applicant is at least 18 years old
- s: the applicant is at least 16 years old

(a)

The applicant has written permission from his parents and is at least 16 years old.

$p \wedge s$

$\neg (p \wedge s)$

The applicant does not have written permission from his parents, or the applicant is not at least 16 years old.

(b)

The applicant has written permission from his parents or is at least 18 years old.

$p \vee e$
$\neg (p \vee e)$

The applicant does not have written permission from his parents and the applicant is not at least 18 years old.

Exercise 1.5.1: Label the steps in a proof of logical equivalence.

Below are several proofs showing that two logical expressions are logically equivalent. Label the steps in each proof with the law used to obtain each proposition from the previous proposition. The first line in the proof does not have a label.

(a)

$(p \to q) \land (q \lor p)$
$(\neg p \lor q) \land (q \lor p)$
$(q \lor \neg p) \land (q \lor p)$
$q \lor (\neg p \land p)$
$q \lor (p \land \neg p)$
$q \lor F$
q

1. Conditional
2. Commutative
3. Distributive
4. Commutative
5. Complement

(b)

$(\neg p \lor q) \rightarrow (p \land q)$
$\neg(\neg p \lor q) \lor (p \land q)$
$(\neg\neg p \land \neg q) \lor (p \land q)$
$(p \land \neg q) \lor (p \land q)$
$p \land (\neg q \lor q)$
$p \land T$
p

1. Conditional
2. De Morgan’s
3. Double negation
4. Distributive
5. Complement

(c)

$r \lor (\neg r \rightarrow p)$
$r \lor (\neg\neg r \lor p)$
$r \lor (r \lor p)$

(r v r) v p
r v p

1. Conditional
2. Double Negation
3. Distributive
4. Identity

Exercises 1:5:2-1:5:5 in second attached document.