# Probability of Recession

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## Summary

Forecast the probability of a recession in the next 6 months using the spread between 10Y CMT and Effective Federal Funds Rate and its transformations:

- 1. Lags
- 2. Adstock
- 3. Moving average

## **Extract Historical Data**

Refer to this vignette for FRED data access.

```
library(tidyverse)
library(lubridate)
library(fredr)
library(car)
library(MLmetrics)
library(caret)
library(pdp)
library(gridExtra)
library(mboost)
library(gbm)
library(randomForest)
library(glmnet)
library(gtsummary)
randSeed <- 1983
startTestDate <- "1978-01-01"
startTrainDate <- "1988-01-01"
```

```
series_id <- c("FEDFUNDS", "GS10", "USREC")

full_data <- map_dfr(series_id, function(x) {
   fredr(
       series_id = x,
       observation_start = as.Date("1950-01-01"),
       observation_end = as.Date("2023-12-01")
   )
})</pre>
```

#### Pivot Wider

## Calculate Features/Predictors

#### Calculate Adstock

The adstock transformation is an auto-regressive transformation of a time series. The transformation takes into account past values of the time series. The intuition is that past values of the time series has a contemporaneous effect on the outcome.

$$AdStock(x_t) = x_t + \theta AdStock(x_{t-1})$$
$$0 < \theta < 1$$

where

The parameters cannot be estimated easily with least squares or logistic regression. Instead, we assume a range of potential values.

```
filter=0.91,
                                    method="recursive") ,
       adstk92 = ~stats::filter(.x,
                                    filter=0.92,
                                    method="recursive"),
       adstk93 = ~stats::filter(.x,
                                    filter=0.93,
                                    method="recursive"),
       adstk94 = ~stats::filter(.x,
                                    filter=0.94,
                                    method="recursive"),
       adstk95 = ~stats::filter(.x,
                                    filter=0.95,
                                    method="recursive"),
       adstk99 = ~stats::filter(.x,
                                    filter=0.99,
                                    method="recursive")
))) %>%
mutate(constant=1)
```

## Calculate Moving Average

```
ma_fun <- function(k_param){</pre>
  rep(1/k_param, k_param)
full_data_wide_features_adstock <- full_data_wide_features_adstock %%
  arrange(date) %>%
    mutate(across(
    .cols=c(SPRD_10YCMT_FEDFUNDS, D_SPRD),
    .fns=list(
         ma2m = ~stats::filter(.x,
                                       filter=ma_fun(2),
                                     method="convolution",
                                     sides=1),
         ma3m = ~stats::filter(.x,
                                       filter=ma_fun(3),
                                     method="convolution",
                                     sides=1),
         ma6m = ~stats::filter(.x,
                                       filter=ma_fun(6),
                                     method="convolution",
                                     sides=1),
         ma9m = ~stats::filter(.x,
                                       filter=ma_fun(9),
                                     method="convolution",
                                     sides=1),
         ma12m = ~stats::filter(.x,
                                       filter=ma_fun(12),
                                     method="convolution",
                                     sides=1)
 )))
```

#### Remove the last 12 months of historical data

Since the NBER often dates recessions after they have already occurred (and sometimes ended), remove the last 12 months of historical data from both the training and test data sets.

```
recent_data <- tail(full_data_wide_features_adstock, 12)
train_test <- head(full_data_wide_features_adstock, -12) %>%
    drop_na()
```

#### Recession in next 6 months

## Split Train/Test

Characteristic	N = 401
date	1988-01-01 to 2021-05-01
USREC	36 (9.0%)
GS10	$4.50 \ (2.65, 6.22)$
FEDFUNDS	$2.63 \ (0.36, 5.29)$
SPRD_10YCMT_FEDFUNDS	$1.49 \ (0.53, \ 2.56)$
D SPRD	-0.02 (-0.16, 0.14)
SPRD_10YCMT_FEDFUNDS_lag1	$1.49\ (0.53, 2.56)$
SPRD_10YCMT_FEDFUNDS_lag3	$1.49 \ (0.53, \ 2.56)$
SPRD_10YCMT_FEDFUNDS_lag6	$1.53\ (0.53,\ 2.56)$
SPRD_10YCMT_FEDFUNDS_lag9	$1.55 \ (0.53, \ 2.56)$
SPRD_10YCMT_FEDFUNDS_lag12	$1.55 \ (0.53, \ 2.56)$
D_SPRD_lag1	-0.02 (-0.16, 0.14)
D SPRD lag3	-0.02 (-0.16, 0.14)
D SPRD lag6	-0.02 (-0.16, 0.14)
D_SPRD_lag9	-0.02 (-0.16, 0.15)
D_SPRD_lag12	-0.02 (-0.16, 0.15)
SPRD 10YCMT FEDFUNDS adstk85	10 (4, 17)
SPRD 10YCMT FEDFUNDS adstk91	17 (7, 26)
SPRD 10YCMT FEDFUNDS adstk92	19 (8, 29)
SPRD 10YCMT FEDFUNDS adstk93	22 (9, 32)
SPRD 10YCMT FEDFUNDS adstk94	26 (11, 37)
SPRD 10YCMT FEDFUNDS adstk95	31 (15, 44)
SPRD_10YCMT_FEDFUNDS_adstk99	137 (111, 161)
D SPRD adstk85	-0.06 (-0.47, 0.42)
D SPRD adstk91	-0.09 (-0.62, 0.48)
D SPRD adstk92	-0.10 (-0.65, 0.48)
D SPRD adstk93	-0.10 (-0.68, 0.51)
D SPRD adstk94	-0.10 (-0.69, 0.58)
D SPRD adstk95	-0.14 (-0.72, 0.59)
D SPRD adstk99	0.00 (-0.80, 1.20)
constant	401 (100%)
SPRD_10YCMT_FEDFUNDS_ma2m	$1.49 \ (0.48, \ 2.54)$
SPRD 10YCMT FEDFUNDS ma3m	$1.49\ (0.49,\ 2.56)$
SPRD 10YCMT FEDFUNDS ma6m	$1.51\ (0.50,\ 2.59)$
SPRD 10YCMT FEDFUNDS ma9m	$1.50\ (0.49,\ 2.62)$
SPRD_10YCMT_FEDFUNDS_ma12m	$1.49\ (0.50,\ 2.60)$
D SPRD ma2m	-0.03 (-0.12, 0.11)
D SPRD ma3m	-0.02 (-0.12, 0.10)
D SPRD ma6m	-0.01 (-0.10, 0.08)
D_SPRD_ma9m	-0.01 (-0.09, 0.07)
D_SPRD_ma12m	-0.01 (-0.08, 0.08)
FUTREC	56 (14%)

## Remove stale data from test set

Exclude historical data prior to 1978-01-01 because the economy changed dramatically (due to computational innovation).

<pre>summary(test_data\$date)</pre>		

## Min. 1st Qu. Median Mean 3rd Qu. Max.

```
## "1978-01-01" "1980-06-23" "1982-12-16" "1982-12-16" "1985-06-08" "1987-12-01"

test_data <- test_data %>%
    filter(date >= startTestDate)

summary(test_data$date)

## Min. 1st Qu. Median Mean 3rd Qu. Max.
## "1978-01-01" "1980-06-23" "1982-12-16" "1982-12-16" "1985-06-08" "1987-12-01"
```

## Setup Parallel Processing

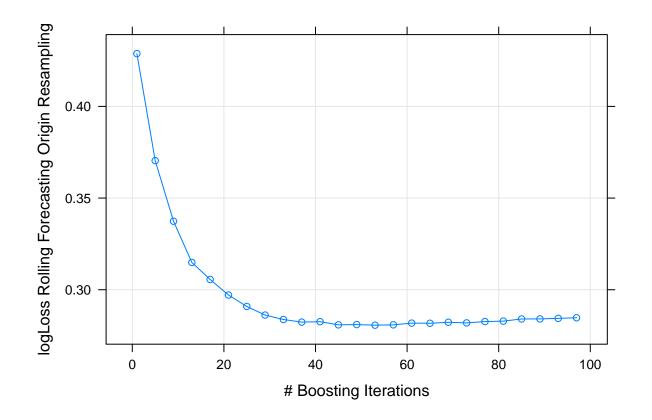
```
library(doParallel)

cl <- makePSOCKcluster(3)
registerDoParallel(cl)</pre>
```

#### **Cross-Validation Framework**

```
fcstHorizon <- 6
initWindow <- 120
param_skip <- fcstHorizon - 1</pre>
if(initWindow < 100){</pre>
  stop("Too few observations.")
}
fitControl_oneSE <- trainControl(method = "timeslice",</pre>
                            initialWindow=initWindow,
                            horizon=fcstHorizon,
                            fixedWindow=FALSE,
                            skip=param_skip,
                            ## Estimate class probabilities
                            classProbs = TRUE,
                            ## Evaluate performance using
                            ## the following function
                            summaryFunction = mnLogLoss,
                            selectionFunction="oneSE")
fitControl_best <- trainControl(method = "timeslice",</pre>
                            initialWindow=initWindow,
                            horizon=fcstHorizon,
                            fixedWindow=FALSE,
                            skip=param_skip,
                            ## Estimate class probabilities
                            classProbs = TRUE,
                            ## Evaluate performance using
                            ## the following function
                            summaryFunction = mnLogLoss,
                            selectionFunction="best")
```

## Gradient Boosting for Additive Models

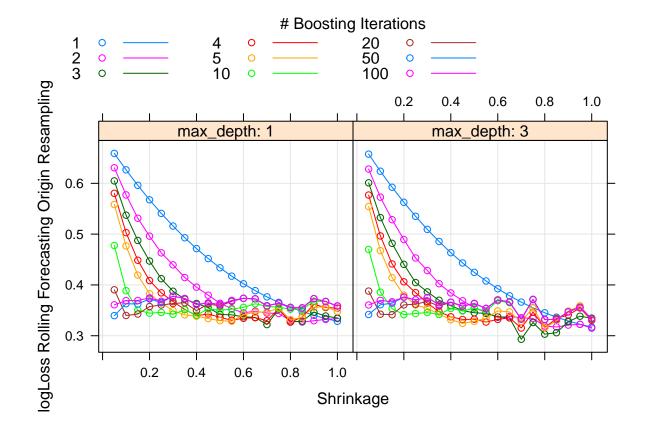


```
gam_mod$bestTune
```

```
## mstop prune
## 3 9 no
```

## eXtreme Gradient Boosting Trees

```
grid_xgb <- expand.grid(nrounds=c(1,2,3,4,5,10,20,</pre>
                                    50,100),
                         max_depth=c(1,3),
                         eta = seq(0.05, 1, 0.05),
                         gamma=0,
                         colsample_bytree=1,
                         min_child_weight=10,
                         subsample=1
set.seed(randSeed)
xgb_mod <- train(</pre>
  FUTREC ~ . - date - USREC - constant,
  data = train_yes_no,
  method = "xgbTree",
  trControl = fitControl_oneSE,
  metric = "logLoss",
  tuneGrid = grid_xgb,
  objective = "binary:logistic"
plot(xgb_mod)
```



```
xgb_mod$bestTune
```

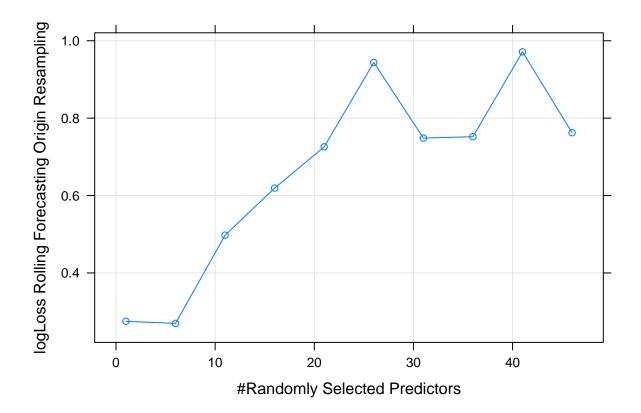
```
## nrounds max_depth eta gamma colsample_bytree min_child_weight subsample ## 253 1 1 0.75 0 1 1 10 1
```

## Random Forest

```
grid_rf <- data.frame(mtry=seq.int(1,50,5))

set.seed(randSeed)

rf_mod <- train(
  FUTREC ~ . - date - USREC - constant,
  data = train_yes_no,
  method = "rf",
  trControl = fitControl_oneSE,
  metric = "logLoss",
  tuneGrid = grid_rf,
  importance = TRUE
)</pre>
```



#### rf\_mod\$bestTune

```
## mtry
## 1 1
```

### Stepwise Regression

The glmStepAIC method uses the glm() function from the stats package. The documentation for glm() says:

For binomial and quasibinomial families the response can also be specified as a factor (when the first level denotes failure and all others success) or as a two-column matrix with the columns giving the numbers of successes and failures.

However, for most methods (that do not invoke glm()) in train, the first level denotes the success (the opposite of glm()). This behavior causes the coefficient signs to flip. Be highly suspicious when interpreting coefficients from models that are fit using train.

```
set.seed(randSeed)

stepwise_mod <- train(
  FUTREC ~ . - date - USREC - constant,
  data = train_yes_no,
  method = "glmStepAIC",
  trControl = fitControl_oneSE,
  metric = "logLoss",
  tuneLength = 10,
  family = binomial,
  trace = 0,
  k = 10*log(nrow(train_yes_no)),
  direction = "forward"
)</pre>
```

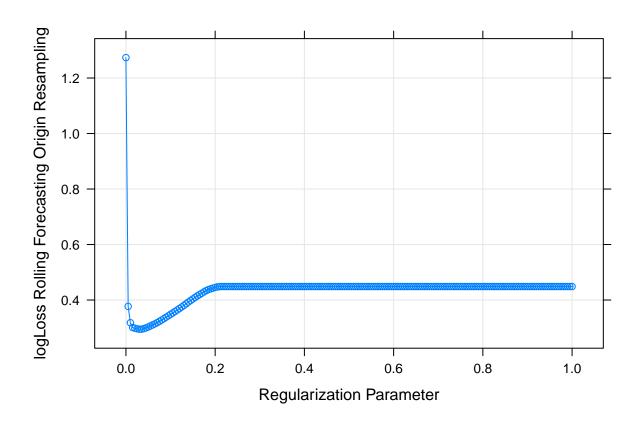
### Elastic Net (Lasso)

```
grid_glmnet <- expand.grid(
   alpha = 1,
   lambda = seq(0, 1, 0.005)
)

set.seed(randSeed)

glmnet_mod <- train(
   FUTREC ~ . - date - USREC - constant,
   data = train_yes_no,
   method = "glmnet",
   trControl = fitControl_best,
   metric = "logLoss",
   tuneGrid = grid_glmnet,</pre>
```

```
family = "binomial"
)
plot(glmnet_mod)
```



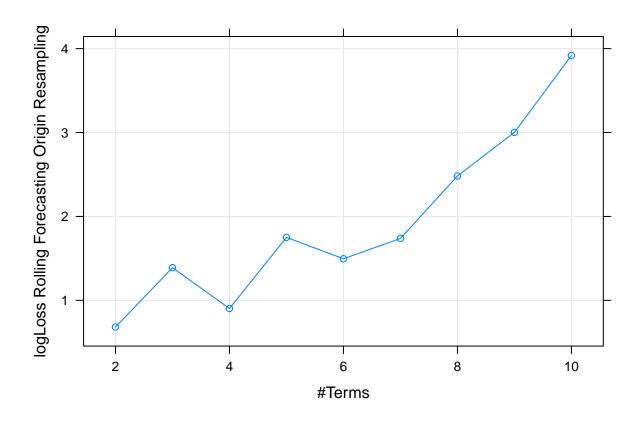
## glmnet\_mod\$bestTune

```
## alpha lambda
## 7 1 0.03
```

## Multivariate Adaptive Regression Splines

```
metric = "logLoss",
  tuneGrid = grid_mars,
  glm = list(family = binomial)
)

plot(earth_mod)
```



```
\verb| earth_mod\$bestTune| \\
```

```
## nprune degree
## 1 2 1
```

## Null Model: Intercept-only Model

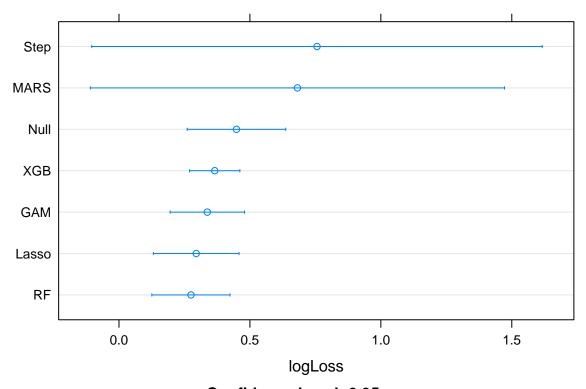
```
set.seed(randSeed)
null_mod <- train(
  FUTREC ~ constant,
  data = train_yes_no,
  method = "glm",
  trControl = fitControl_best,
  metric = "logLoss",</pre>
```

```
family = binomial
)
```

## Compare Models

```
##
## Call:
## summary.resamples(object = resamps)
## Models: XGB, GAM, RF, Step, Lasso, MARS, Null
## Number of resamples: 46
##
## logLoss
##
                 Min.
                           1st Qu.
                                         Median
                                                     Mean
                                                              3rd Qu.
## XGB
         2.073761e-01 2.133103e-01 0.2322580636 0.3656524 0.24149644 1.654720
         7.048603e-02 8.713700e-02 0.0989558601 0.3373635 0.22842154
## GAM
                                                                      1.662805
         2.003341e-03 9.816079e-03 0.0330654175 0.2749450 0.26223426 2.075257
## RF
## Step 9.992007e-16 3.654836e-12 0.0005376345 0.7561703 0.01570571 17.269388
## Lasso 2.021251e-04 8.571883e-03 0.0277020576 0.2949257 0.16335390 2.238882
## MARS 9.992007e-16 4.631315e-03 0.0288428833 0.6814256 0.05149754 17.269388
## Null 9.461598e-02 1.418640e-01 0.1612905053 0.4486199 0.19905716 2.277267
##
        NA's
## XGB
            0
## GAM
            0
## RF
            0
## Step
            0
## Lasso
## MARS
            0
## Null
            0
```

```
dotplot(resamps, metric = "logLoss", conf.level=0.95)
```

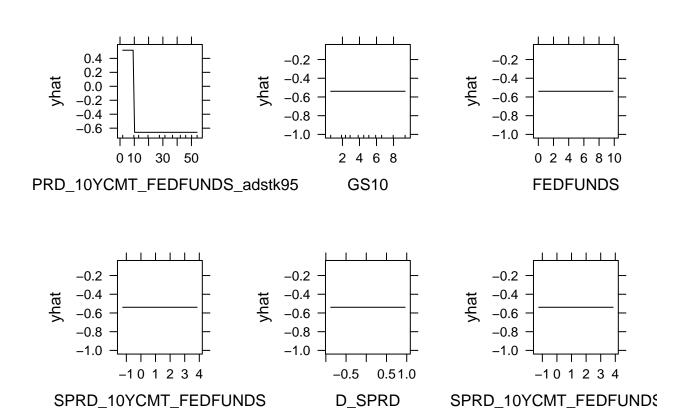


**Confidence Level: 0.95** 

## Explore XGB Model

```
variable
                                 Overall
GS10
                                     0
                                     0
FEDFUNDS
SPRD_10YCMT_FEDFUNDS
                                     0
                                     0
D SPRD
SPRD 10YCMT FEDFUNDS lag1
                                     0
SPRD\_10YCMT\_FEDFUNDS\_lag3
                                     0
SPRD_10YCMT_FEDFUNDS_lag6
                                     0
                                     0
SPRD_10YCMT_FEDFUNDS_lag9
SPRD\_10YCMT\_FEDFUNDS\_lag12
                                     0
D SPRD lag1
                                     0
D_SPRD_lag3
                                     0
D SPRD lag6
                                     0
D_SPRD_{lag9}
                                     0
D_SPRD_lag12
                                     0
SPRD\_10YCMT\_FEDFUNDS\_adstk85
                                     0
SPRD 10YCMT FEDFUNDS adstk91
                                     0
SPRD\_10YCMT\_FEDFUNDS\_adstk92
                                     0
SPRD\_10YCMT\_FEDFUNDS\_adstk93
                                     0
                                     0
SPRD\_10YCMT\_FEDFUNDS\_adstk94
SPRD\_10YCMT\_FEDFUNDS\_adstk99
                                     0
                                     0
D SPRD adstk85
D\_SPRD\_adstk91
                                     0
                                     0
D_SPRD_adstk92
D SPRD adstk93
                                     0
D_SPRD_adstk94
                                     0
D_SPRD_adstk95
                                     0
D SPRD adstk99
                                     0
SPRD_10YCMT_FEDFUNDS_ma2m
                                     0
SPRD\_10YCMT\_FEDFUNDS\_ma3m
                                     0
SPRD\_10YCMT\_FEDFUNDS\_ma6m
                                     0
SPRD 10YCMT FEDFUNDS ma9m
                                     0
SPRD 10YCMT FEDFUNDS ma12m
                                     0
D SPRD ma2m
                                     0
D_SPRD_ma3m
                                     0
D SPRD ma6m
                                     0
D\_SPRD\_ma9m
                                     0
D_SPRD_ma12m
                                     0
```

```
chull = TRUE
  )
pdp.top4 <- partial(xgb_mod,</pre>
    pred.var = df_imp$variable[4],
    plot = TRUE,
    chull = TRUE
  )
pdp.top5 <- partial(xgb_mod,</pre>
    pred.var = df_imp$variable[5],
    plot = TRUE,
    chull = TRUE
  )
pdp.top6 <- partial(xgb_mod,</pre>
    pred.var = df_imp$variable[6],
    plot = TRUE,
    chull = TRUE
  )
grid.arrange(pdp.top1, pdp.top2, pdp.top3,
              pdp.top4, pdp.top5, pdp.top6, ncol = 3)
```



## Peeking

Peeking means we use the insights from the automated models to choose variables in subsequent models. This is technically cheating and causes the cross-validation errors to be artificially low. This is addressed in the test set which does not have peeking bias.

## Logistic Regression (with peeking)

As mentioned early, train and glm treat the reference level differently for binary outcomes. Hence, the coefficients are flipped when training a logistic regression inside train.

```
logit_mod <- train(
  top1_fmla,
  data = train_yes_no,
  method = "glm",
  trControl = fitControl_oneSE,
  metric = "logLoss",
  family=binomial
)</pre>
```

```
##
## Call:
## NULL
##
## Deviance Residuals:
       Min
                  1Q
                        Median
                                      3Q
                                                Max
                       0.05037
## -2.86620
            0.01589
                                 0.21584
                                            1.44313
##
## Coefficients:
##
                               Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                                -2.35835
                                           0.47437 -4.972 6.64e-07 ***
## SPRD_10YCMT_FEDFUNDS_adstk95 0.22847
                                           0.03453
                                                     6.617 3.67e-11 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## (Dispersion parameter for binomial family taken to be 1)
##
##
      Null deviance: 324.27 on 400 degrees of freedom
```

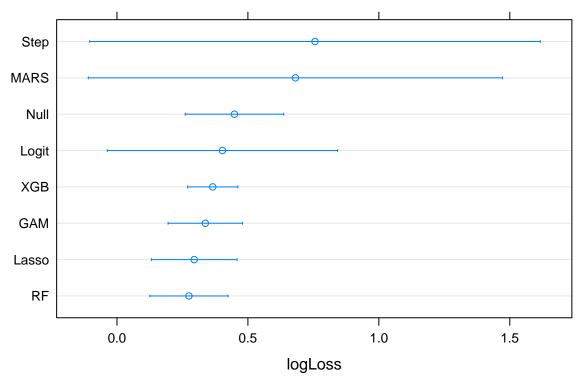
```
## Residual deviance: 169.90 on 399 degrees of freedom
## AIC: 173.9
##
## Number of Fisher Scoring iterations: 8
```

## **Compare Models**

CV errors for models with peeking are misleadingly low. This will be addressed with a test set.

```
##
## Call:
## summary.resamples(object = resamps)
## Models: XGB, GAM, RF, Step, Lasso, MARS, Null, Logit
## Number of resamples: 46
##
## logLoss
                                         Median
                                                              3rd Qu.
##
                 Min.
                           1st Qu.
                                                      Mean
                                                                           Max.
## XGB
         2.073761e-01 2.133103e-01 0.2322580636 0.3656524 0.24149644
                                                                       1.654720
         7.048603e-02 8.713700e-02 0.0989558601 0.3373635 0.22842154
## GAM
                                                                       1.662805
         2.003341e-03 9.816079e-03 0.0330654175 0.2749450 0.26223426
                                                                       2.075257
## Step 9.992007e-16 3.654836e-12 0.0005376345 0.7561703 0.01570571 17.269388
## Lasso 2.021251e-04 8.571883e-03 0.0277020576 0.2949257 0.16335390
## MARS 9.992007e-16 4.631315e-03 0.0288428833 0.6814256 0.05149754 17.269388
## Null 9.461598e-02 1.418640e-01 0.1612905053 0.4486199 0.19905716 2.277267
## Logit 9.992007e-16 1.102789e-04 0.0008839239 0.4028153 0.04970038 8.604311
         NA's
##
## XGB
## GAM
            0
## RF
## Step
            0
## Lasso
            0
## MARS
            0
## Null
            0
## Logit
            0
```

```
dotplot(resamps, metric = "logLoss", conf.level=0.95)
```



**Confidence Level: 0.95** 

## Test Set Performance

```
perf <-
  function(lst_mods,
           f_metric = caTools::colAUC,
           metricname = "ROC-AUC",
           dat=test_data,
           response="FUTREC") {
    lst_preds <- map(</pre>
      .x = lst_mods,
      .f = function(x) {
        if (class(x)[1] != "train") {
          predict(x, newdata = dat, type = "response")
        } else
            predict(x, newdata = dat, type = "prob")[, "yes"]
      }
    map_dfr(lst_preds, function(x) {
      f_metric(x, dat[,response, drop=TRUE])
    }) %>%
```

```
pivot_longer(everything(), names_to = "model", values_to = metricname)
}

perf(mymods, caTools::colAUC, "ROC-AUC") %>%
    arrange(desc(`ROC-AUC`)) %>%
    knitr::kable()
```

model	ROC-AUC
Step	0.9634233
Logit	0.9634233
GAM	0.9623580
Lasso	0.9477983
MARS	0.9318182
RF	0.9131747
XGB	0.8863636
Null	0.5000000

```
perf(mymods, MLmetrics::LogLoss, "LogLoss") %>%
  arrange(LogLoss) %>%
  knitr::kable()
```

model	LogLoss
MARS	0.2857389
GAM	0.3438593
RF	0.3544149
Lasso	0.3896416
XGB	0.4384522
Null	0.6352684
Step	0.6782045
Logit	0.6782045

## Probability of Recession (the 12 most recent months)

```
curr_data <- recent_data

curr_data$date

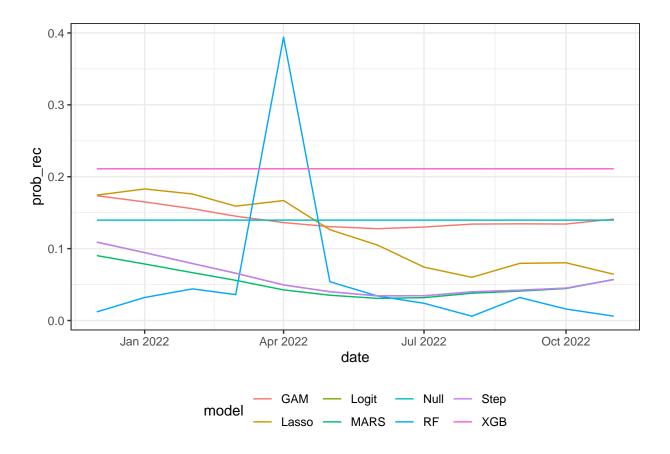
## [1] "2021-12-01" "2022-01-01" "2022-02-01" "2022-03-01" "2022-04-01"

## [6] "2022-05-01" "2022-06-01" "2022-07-01" "2022-08-01" "2022-09-01"

## [11] "2022-10-01" "2022-11-01"

score_fun <- function(mods, dat) {
    output <- map_dfc(.x = mods, .f = function(x) {
        if(class(x)[1] != "train") {
            predict(x, newdata = dat, type = "response")
        } else(
            predict(x, newdata = dat, type = "prob")[,"yes"]</pre>
```

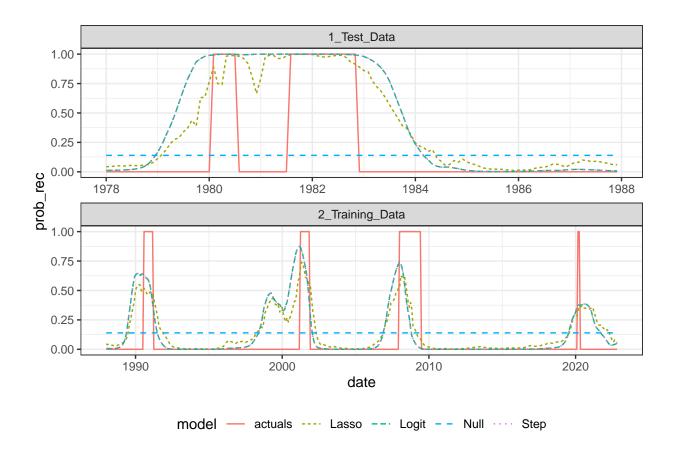
date	model	prob_rec
2022-10-01	XGB	0.2110378
2022-10-01	GAM	0.1341967
2022-10-01	RF	0.0160000
2022-10-01	Step	0.0450450
2022-10-01	Lasso	0.0803249
2022-10-01	MARS	0.0444954
2022-10-01	Null	0.1396509
2022-10-01	Logit	0.0450450
2022-11-01	XGB	0.2110378
2022-11-01	GAM	0.1410906
2022-11-01	RF	0.0060000
2022-11-01	Step	0.0568657
2022-11-01	Lasso	0.0643143
2022-11-01	MARS	0.0571497
2022-11-01	Null	0.1396509
2022-11-01	Logit	0.0568657

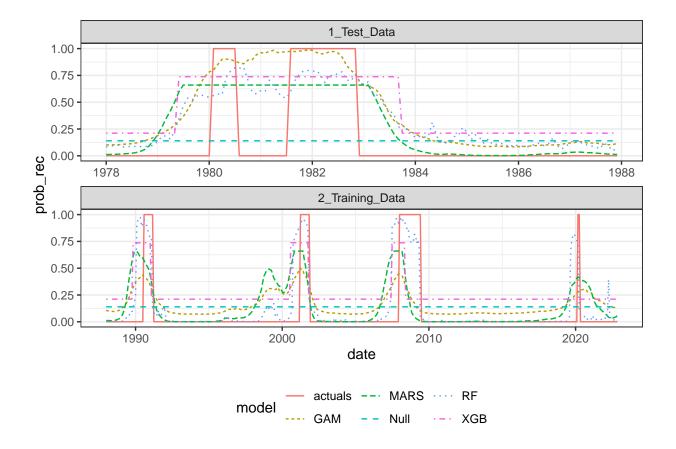


## Backtesting

```
full_data_bktst <- full_data_wide_features_adstock %>%
  filter(date >= startTestDate)
bkst_fun <- function(mods, dat) {</pre>
  output <- map_dfc(.x = mods, .f = function(x) {</pre>
    if(class(x)[1] != "train"){
      predict(x, newdata = dat, type = "response")
    } else(
       predict(x, newdata = dat, type = "prob")[,"yes"]
    )
  })
  output$date <- dat$date</pre>
  output <- output%>%
    pivot_longer(-date, names_to = "model",
                 values_to = "prob_rec")
  return(output)
}
```

```
df_plot <- bkst_fun(mymods, full_data_bktst)</pre>
actuals <- full_data_bktst %>%
  mutate(model="actuals") %>%
  select(date, model, prob_rec=USREC)
df_plot_final <- bind_rows(df_plot, actuals)</pre>
end_test_date <- max(test_data$date)</pre>
df_plot_final <- df_plot_final %>%
  mutate(epoc = case_when(date <= end_test_date ~ "1_Test_Data",</pre>
                           TRUE ~ "2_Training_Data")
  )
df_plot_logit_scam <- df_plot_final %>%
  filter(model %in% c('actuals', 'Null',
                       'Logit', 'Step', 'Lasso',
                       'LogitKnot'))
df_plot_knots_gbm <- df_plot_final %>%
  filter(model %in% c('actuals', 'Null',
                       'XGB', 'RF',
                       'GAM',
                       'MARS'))
```





stopCluster(cl)