# Probability of Recession

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## Summary

Forecast the probability of a recession in the next 126 trading days using the following predictors:

- 1. Spread between 10Y CMT and Effective Federal Funds Rate
- 2. Lags of the spread
- 3. Adstock transformations of the spread

There are between 250 and 253 trading days in a year.

#### Extract Historical Data

Refer to this vignette for FRED data access.

```
library(tidyverse)
library(lubridate)
library(fredr)
library(car)
library(MLmetrics)
library(caret)
library(pdp)
library(gridExtra)
library(mboost)
library(gbm)
library(randomForest)
library(glmnet)
library(gtsummary)
randSeed <- 1983
startTestDate <- "1978-01-01"
startTrainDate <- "1988-01-01"
```

```
# series_id <- c("FEDFUNDS", "GS10", "USREC", "UNRATE", "CPIAUCSL")
series_id <- c("DFF", "DGS10") # daily
response_id <- "USREC" # monthly
full_data <- map_dfr(series_id, function(x) {</pre>
```

```
fredr(
    series_id = x,
    observation_start = as.Date("1950-01-01"),
    observation_end = as.Date("2023-12-01")
)

recession_dates <- map_dfr(response_id, function(x) {
    fredr(
        series_id = x,
        observation_start = as.Date("1950-01-01"),
        observation_end = as.Date("2023-12-01")
)
})</pre>
```

#### Pivot Wider

## Calculate Features/Predictors

```
full_data_wide_features <- full_data_wide_raw %>%
  arrange(date) %>%
  mutate(SPRD_10YCMT_FEDFUNDS = DGS10 - DFF
         ) %>%
  mutate(across(
    .cols=c(SPRD_10YCMT_FEDFUNDS),
    .fns=list(lag1m = ~lag(.x, 1*21),
         lag3m = ~lag(.x, 3*21),
         lag6m = ~lag(.x, 6*21),
         lag9m = ~lag(.x, 9*21),
         lag12m = ~lag(.x, 12*21),
         lag5d = \sim lag(.x, 5),
         lag10d = \sim lag(.x, 10),
         lag15d = \sim lag(.x, 15)
  )) %>%
  drop_na()
```

#### Calculate Adstock

The adstock transformation is an auto-regressive transformation of a time series. The transformation takes into account past values of the time series. The intuition is that past values of the time series has a contemporaneous effect on the outcome.

$$AdStock(x_t) = x_t + \theta AdStock(x_{t-1})$$

where

 $0 < \theta < 1$ 

.

The parameters cannot be estimated easily with least squares or logistic regression. Instead, we assume a range of potential values.

```
full_data_wide_features_adstock <- full_data_wide_features %>%
  arrange(date) %>%
    mutate(across(
    .cols=c(SPRD_10YCMT_FEDFUNDS),
    .fns=list(adstk001 = ~stats::filter(.x,
                                      filter=0.001,
                                      method="recursive") ,
         adstk10 = ~stats::filter(.x,
                                      filter=0.10,
                                      method="recursive") ,
         adstk20 = ~stats::filter(.x,
                                      filter=0.20,
                                      method="recursive"),
         adstk40 = ~stats::filter(.x,
                                      filter=0.40,
                                      method="recursive"),
         adstk75 = ~stats::filter(.x,
                                      filter=0.75,
                                      method="recursive"),
         adstk95 = ~stats::filter(.x,
                                      filter=0.95,
                                      method="recursive")
  ))) %>%
  mutate(constant=1)
```

### Calculate Moving Average

```
sides=1) ,
        ma15d = ~stats::filter(.x,
                                     filter=ma_fun(15),
                                    method="convolution",
                                    sides=1),
        ma20d = ~stats::filter(.x,
                                      filter=ma_fun(20),
                                    method="convolution",
                                    sides=1),
        ma25d = ~stats::filter(.x,
                                     filter=ma_fun(25),
                                    method="convolution",
                                    sides=1),
        ma2m = ~stats::filter(.x,
                                      filter=ma_fun(2*21),
                                    method="convolution",
                                    sides=1),
        ma3m = ~stats::filter(.x,
                                      filter=ma_fun(3*21),
                                    method="convolution",
                                    sides=1),
        ma6m = ~stats::filter(.x,
                                      filter=ma_fun(6*21),
                                    method="convolution",
                                    sides=1),
        ma9m = ~stats::filter(.x,
                                      filter=ma_fun(9*21),
                                    method="convolution",
                                    sides=1),
        ma12m = ~stats::filter(.x,
                                      filter=ma_fun(12*21),
                                    method="convolution",
                                    sides=1)
)))
```

#### Recession in next 6 months

```
by = c("date_month" = "date_month",
                   "date_year" = "date_year")) %>%
  mutate(USREC = value)
df_FUTREC = as.data.frame(
  data.table::shift(
    full_data_wide$USREC,
    n = 1:(6 * 21),
    type = "lead",
    give.names = TRUE,
    fill = NA
  )
) %>%
 rowwise() %>%
 mutate(FUTREC = max(c_across(V1_lead_1:V1_lead_126)))
full_data_wide$FUTREC <- df_FUTREC$FUTREC</pre>
full_data_wide <- full_data_wide %>%
  select(date=date.x, everything(), -date_month,
         -date_year, -date.y,
         -value) %>%
  drop_na()
full_data_wide$constant <- 1</pre>
full_data_wide_noUSREC <- full_data_wide %>%
 select(-USREC)
```

#### Remove the last 12 months of historical data

Since the NBER often dates recessions after they have already occurred (and sometimes ended), remove the last 12 months of historical data from both the training and test data sets.

```
recent_data <- tail(full_data_wide_noUSREC, 12*21)
train_test <- head(full_data_wide_noUSREC, -12*21)</pre>
```

# Split Train/Test

```
train_data <- train_test %>%
  filter(date >= startTrainDate)

test_data <- train_test %>%
  filter(date >= startTestDate) %>%
  filter(date < startTrainDate)

train_yes_no <- train_data %>%
  mutate(FUTREC = case_when(FUTREC == 1 ~ "yes",
```

Characteristic	N = 8,357
date	1988-01-04 to 2021-05-26
DFF	2.72 (0.36, 5.29)
DGS10	4.48 (2.64, 6.21)
SPRD_10YCMT_FEDFUNDS	$1.49 \ (0.48, \ 2.53)$
SPRD_10YCMT_FEDFUNDS_lag1m	$1.49 \ (0.48, \ 2.53)$
SPRD_10YCMT_FEDFUNDS_lag3m	$1.49 \ (0.48, \ 2.53)$
SPRD_10YCMT_FEDFUNDS_lag6m	$1.52\ (0.48,\ 2.53)$
SPRD_10YCMT_FEDFUNDS_lag9m	$1.54 \ (0.48, \ 2.53)$
SPRD_10YCMT_FEDFUNDS_lag12m	$1.54 \ (0.48, \ 2.53)$
SPRD_10YCMT_FEDFUNDS_lag5d	$1.49 \ (0.48, \ 2.53)$
SPRD_10YCMT_FEDFUNDS_lag10d	$1.49 \ (0.48, \ 2.53)$
SPRD_10YCMT_FEDFUNDS_lag15d	$1.49 \ (0.48, \ 2.53)$
SPRD_10YCMT_FEDFUNDS_adstk001	$1.49 \ (0.48, \ 2.53)$
SPRD_10YCMT_FEDFUNDS_adstk10	$1.65 \ (0.54, \ 2.81)$
SPRD_10YCMT_FEDFUNDS_adstk20	$1.86 \ (0.60, \ 3.16)$
SPRD_10YCMT_FEDFUNDS_adstk40	$2.48 \ (0.80,  4.22)$
SPRD_10YCMT_FEDFUNDS_adstk75	5.9(2.0, 10.1)
SPRD_10YCMT_FEDFUNDS_adstk95	$30\ (10,\ 51)$
constant	8,357 (100%)
SPRD_10YCMT_FEDFUNDS_ma5d	$1.48 \ (0.49, \ 2.53)$
SPRD_10YCMT_FEDFUNDS_ma10d	$1.47 \ (0.50, \ 2.53)$
SPRD_10YCMT_FEDFUNDS_ma15d	$1.47 \ (0.50, \ 2.53)$
SPRD_10YCMT_FEDFUNDS_ma20d	$1.47 \ (0.50, \ 2.54)$
SPRD_10YCMT_FEDFUNDS_ma25d	$1.47 \ (0.51, \ 2.54)$
SPRD_10YCMT_FEDFUNDS_ma2m	$1.47 \ (0.50, \ 2.56)$
SPRD_10YCMT_FEDFUNDS_ma3m	$1.48 \ (0.47, \ 2.55)$
SPRD_10YCMT_FEDFUNDS_ma6m	$1.51 \ (0.48, \ 2.57)$
SPRD_10YCMT_FEDFUNDS_ma9m	$1.51 \ (0.48, \ 2.60)$
SPRD_10YCMT_FEDFUNDS_ma12m	$1.49 \ (0.50, \ 2.60)$
FUTREC	1,248 (15%)

### Remove stale data from test set

Exclude historical data prior to 1978-01-01 because the economy changed dramatically (due to computational innovation).

```
summary(test_data$date)
```

```
## Min. 1st Qu. Median Mean 3rd Qu. Max. ## "1978-01-03" "1980-07-02" "1983-01-04" "1983-01-01" "1985-07-02" "1987-12-31"
```

```
test_data <- test_data %>%
    filter(date >= startTestDate)

summary(test_data$date)

## Min. 1st Qu. Median Mean 3rd Qu. Max.
## "1978-01-03" "1980-07-02" "1983-01-04" "1983-01-01" "1985-07-02" "1987-12-31"
```

### Setup Parallel Processing

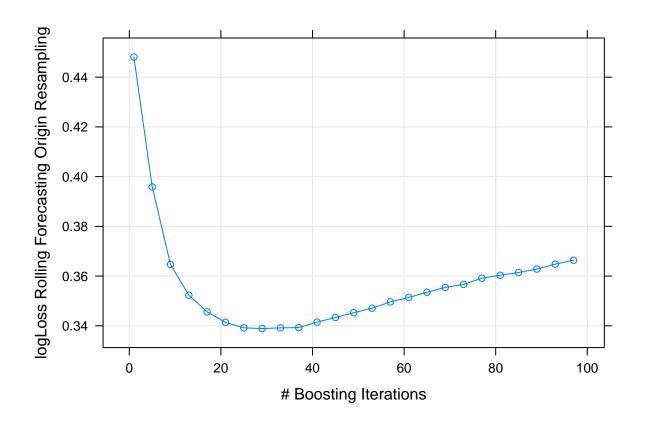
```
library(doParallel)

cl <- makePSOCKcluster(3)
registerDoParallel(cl)</pre>
```

#### **Cross-Validation Framework**

```
fcstHorizon <- 6*21
initWindow <- 120*21</pre>
param_skip <- fcstHorizon - 1</pre>
if(initWindow < 100){</pre>
  stop("Too few observations.")
}
fitControl_oneSE <- trainControl(method = "timeslice",</pre>
                            initialWindow=initWindow,
                            horizon=fcstHorizon,
                            fixedWindow=FALSE,
                            skip=param_skip,
                            ## Estimate class probabilities
                            classProbs = TRUE,
                            ## Evaluate performance using
                            ## the following function
                            summaryFunction = mnLogLoss,
                            selectionFunction="oneSE")
fitControl_best <- trainControl(method = "timeslice",</pre>
                            initialWindow=initWindow,
                            horizon=fcstHorizon,
                            fixedWindow=FALSE,
                            skip=param_skip,
                            ## Estimate class probabilities
                            classProbs = TRUE,
                            ## Evaluate performance using
                            ## the following function
                            summaryFunction = mnLogLoss,
                            selectionFunction="best")
```

# Gradient Boosting for Additive Models



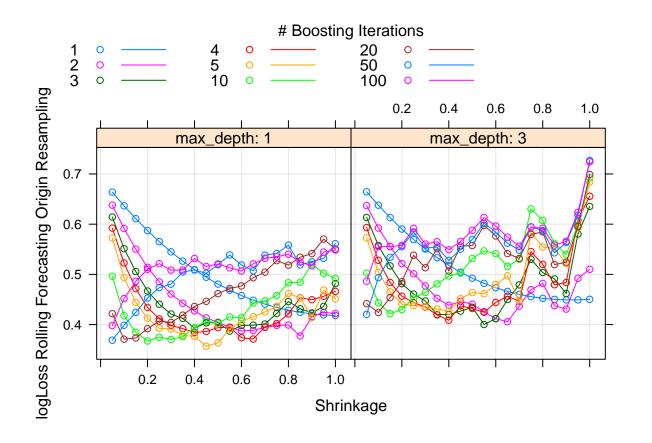
```
gam_mod$bestTune
## mstop prune
```

## 2

5

# eXtreme Gradient Boosting Trees

```
grid_xgb <- expand.grid(nrounds=c(1,2,3,4,5,10,20,</pre>
                                    50,100),
                         max_depth=c(1,3),
                         eta = seq(0.05, 1, 0.05),
                         gamma=0,
                         colsample_bytree=1,
                         min_child_weight=10,
                         subsample=1
set.seed(randSeed)
xgb_mod <- train(</pre>
  FUTREC ~ . - date - constant,
  data = train_yes_no,
  method = "xgbTree",
  trControl = fitControl_oneSE,
  metric = "logLoss",
  tuneGrid = grid_xgb,
  objective = "binary:logistic"
plot(xgb_mod)
```



```
xgb_mod$bestTune
```

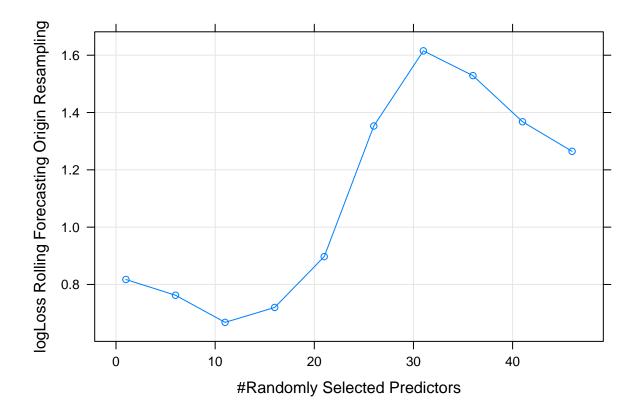
```
## nrounds max_depth eta gamma colsample_bytree min_child_weight subsample ## 271 1 1 0.8 0 1 1 10 1
```

### **Random Forest**

```
grid_rf <- data.frame(mtry=seq.int(1,50,5))

set.seed(randSeed)

rf_mod <- train(
  FUTREC ~ . - date - constant,
  data = train_yes_no,
  method = "rf",
  trControl = fitControl_oneSE,
  metric = "logLoss",
  tuneGrid = grid_rf,
  importance = TRUE
)</pre>
```



#### rf\_mod\$bestTune

```
## mtry
## 1 1
```

#### Stepwise Regression

The glmStepAIC method uses the glm() function from the stats package. The documentation for glm() says:

For binomial and quasibinomial families the response can also be specified as a factor (when the first level denotes failure and all others success) or as a two-column matrix with the columns giving the numbers of successes and failures.

However, for most methods (that do not invoke glm()) in train, the first level denotes the success (the opposite of glm()). This behavior causes the coefficient signs to flip. Be highly suspicious when interpreting coefficients from models that are fit using train.

```
set.seed(randSeed)

stepwise_mod <- train(
   FUTREC ~ . - date - constant,
   data = train_yes_no,
   method = "glmStepAIC",
   trControl = fitControl_oneSE,
   metric = "logLoss",
   tuneLength = 10,
   family = binomial,
   trace = 0,
   k = 10*log(nrow(train_yes_no)),
   direction = "forward"
)</pre>
```

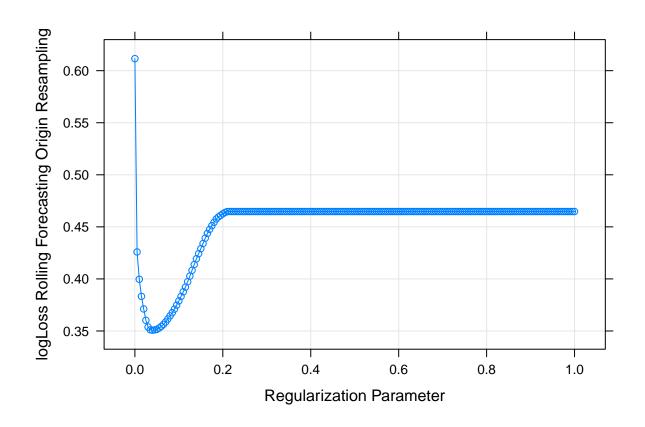
# Elastic Net (Lasso)

```
grid_glmnet <- expand.grid(
   alpha = 1,
   lambda = seq(0, 1, 0.005)
)

set.seed(randSeed)

glmnet_mod <- train(
   FUTREC ~ . - date - constant,
   data = train_yes_no,
   method = "glmnet",
   trControl = fitControl_best,
   metric = "logLoss",
   tuneGrid = grid_glmnet,</pre>
```

```
family = "binomial"
)
plot(glmnet_mod)
```



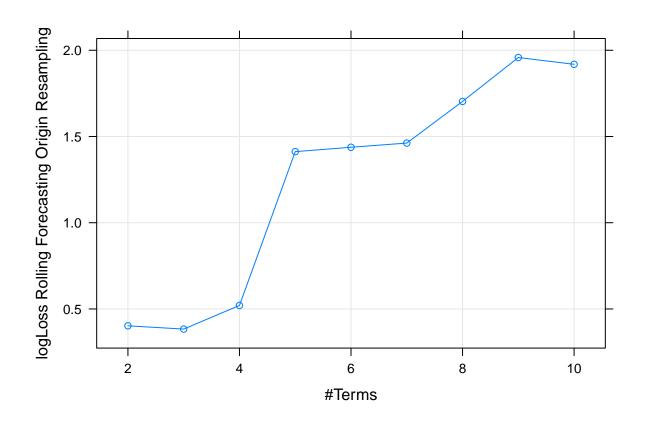
#### glmnet\_mod\$bestTune

```
## alpha lambda
## 9 1 0.04
```

# Multivariate Adaptive Regression Splines

```
metric = "logLoss",
  tuneGrid = grid_mars,
  glm = list(family = binomial)
)

plot(earth_mod)
```



```
earth_mod$bestTune
```

```
## nprune degree
## 1 2 1
```

# Null Model: Intercept-only Model

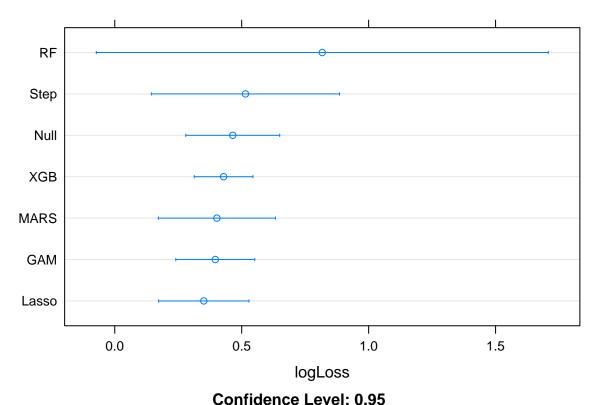
```
set.seed(randSeed)
null_mod <- train(
  FUTREC ~ constant,
  data = train_yes_no,
  method = "glm",
  trControl = fitControl_oneSE,
  metric = "logLoss",</pre>
```

```
family = binomial
)
```

### Compare Models

```
## Call:
## summary.resamples(object = resamps)
## Models: XGB, GAM, RF, Step, Lasso, MARS, Null
## Number of resamples: 46
##
## logLoss
##
                          1st Qu.
                                                                      Max. NA's
                Min.
                                     Median
                                                  Mean
                                                         3rd Qu.
## XGB
        2.010778e-01 0.217319219 0.22510756 0.4286741 0.4253161 1.649584
        9.649594e-02 0.125957153 0.14820984 0.3958571 0.2270635 1.762089
## GAM
        9.536533e-05 0.037201063 0.11551094 0.8173299 0.4148020 19.946658
                                                                              0
## RF
## Step 3.228520e-06 0.004524957 0.03375977 0.5148946 0.3415982 6.880424
                                                                              0
## Lasso 5.687156e-03 0.033942492 0.07558526 0.3507532 0.3453562 2.541241
                                                                              0
## MARS 7.184630e-04 0.056736427 0.06736552 0.4022505 0.3873593 3.418330
                                                                              0
## Null 1.004490e-01 0.154440199 0.17462598 0.4647126 0.2209028 2.167452
```

```
dotplot(resamps, metric = "logLoss", conf.level=0.95)
```



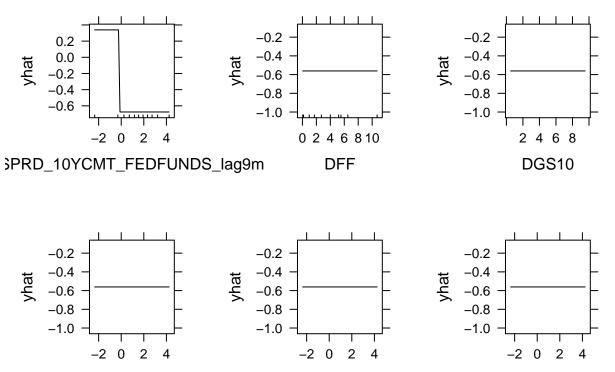
Communice Level. 0.3

# Explore XGB Model

variable	e			Overall
SPRD	10YCMT	FEDFUNDS	lag9m	100

variable	Overall
DFF	0
DGS10	0
SPRD_10YCMT_FEDFUNDS	0
SPRD_10YCMT_FEDFUNDS_lag1m	0
SPRD_10YCMT_FEDFUNDS_lag3m	0
SPRD_10YCMT_FEDFUNDS_lag6m	0
SPRD_10YCMT_FEDFUNDS_lag12m	0
SPRD_10YCMT_FEDFUNDS_lag5d	0
SPRD_10YCMT_FEDFUNDS_lag10d	0
SPRD_10YCMT_FEDFUNDS_lag15d	0
SPRD_10YCMT_FEDFUNDS_adstk001	0
SPRD_10YCMT_FEDFUNDS_adstk10	0
SPRD_10YCMT_FEDFUNDS_adstk20	0
SPRD_10YCMT_FEDFUNDS_adstk40	0
SPRD_10YCMT_FEDFUNDS_adstk75	0
SPRD_10YCMT_FEDFUNDS_adstk95	0
SPRD_10YCMT_FEDFUNDS_ma5d	0
SPRD_10YCMT_FEDFUNDS_ma10d	0
SPRD_10YCMT_FEDFUNDS_ma15d	0
SPRD_10YCMT_FEDFUNDS_ma20d	0
SPRD_10YCMT_FEDFUNDS_ma25d	0
SPRD_10YCMT_FEDFUNDS_ma2m	0
SPRD_10YCMT_FEDFUNDS_ma3m	0
SPRD_10YCMT_FEDFUNDS_ma6m	0
SPRD_10YCMT_FEDFUNDS_ma9m	0
SPRD_10YCMT_FEDFUNDS_ma12m	0

```
pdp.top1 <- partial(xgb_mod,</pre>
          pred.var = df_imp$variable[1],
          plot = TRUE,
          rug = TRUE)
pdp.top2 <- partial(xgb_mod,</pre>
          pred.var = df_imp$variable[2],
          plot = TRUE,
          rug = TRUE)
pdp.top3 <- partial(xgb_mod,</pre>
    pred.var = df_imp$variable[3],
    plot = TRUE,
    chull = TRUE
  )
pdp.top4 <- partial(xgb_mod,</pre>
    pred.var = df_imp$variable[4],
    plot = TRUE,
    chull = TRUE
  )
pdp.top5 <- partial(xgb_mod,</pre>
pred.var = df_imp$variable[5],
```



SPRD\_10YCMT\_FEDFUNDS.PR.D\_10YCMT\_FEDFUNDS.PR.D.M.10YCMT\_FEDFUNDS.

### Peeking

Peeking means we use the insights from the automated models to choose variables in subsequent models. This is technically cheating and causes the cross-validation errors to be artificially low. This is addressed in the test set which does not have peeking bias.

```
top_predictors <- head(df_imp$variable)
best_predictor <- head(top_predictors, 1)
top_fmla <- as.formula(paste0("FUTREC ~",</pre>
```

# Logistic Regression (with peeking)

As mentioned early, train and glm treat the reference level differently for binary outcomes. Hence, the coefficients are flipped when training a logistic regression inside train.

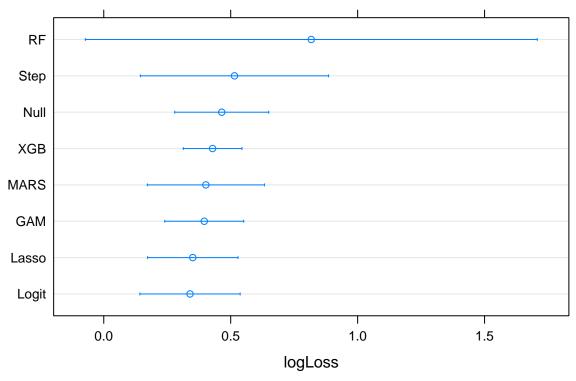
```
logit_mod <- train(
  top1_fmla,
  data = train_yes_no,
  method = "glm",
  trControl = fitControl_oneSE,
  metric = "logLoss",
  family=binomial
)</pre>
```

```
##
## Call:
## NULL
##
## Deviance Residuals:
##
                   1Q
       \mathtt{Min}
                         Median
                                       3Q
                                                 Max
## -3.15979
             0.06882
                        0.17978
                                  0.42060
                                             2.70315
##
## Coefficients:
##
                              Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                                0.4533
                                            0.0402
                                                     11.28
                                                             <2e-16 ***
## SPRD_10YCMT_FEDFUNDS_lag9m
                                1.7364
                                            0.0481
                                                     36.10
                                                             <2e-16 ***
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 7045.9 on 8356 degrees of freedom
## Residual deviance: 4452.8 on 8355 degrees of freedom
## AIC: 4456.8
##
## Number of Fisher Scoring iterations: 6
```

### **Compare Models**

CV errors for models with peeking are misleadingly low. This will be addressed with a test set.

```
mymods <- list(XGB = xgb_mod,</pre>
                          GAM = gam_mod,
                          RF = rf_mod,
                          Step = stepwise_mod,
                          Lasso = glmnet_mod,
                          MARS = earth_mod,
                          Null = null_mod,
                          Logit = logit_mod) ## peeking
resamps <- resamples(mymods)</pre>
summary(resamps)
##
## Call:
## summary.resamples(object = resamps)
## Models: XGB, GAM, RF, Step, Lasso, MARS, Null, Logit
## Number of resamples: 46
##
## logLoss
##
                          1st Qu.
                                      Median
                                                                       Max. NA's
                 Min.
                                                  Mean
                                                         3rd Qu.
         2.010778e-01 0.217319219 0.22510756 0.4286741 0.4253161 1.649584
## XGB
         9.649594e-02 0.125957153 0.14820984 0.3958571 0.2270635 1.762089
## GAM
                                                                               0
         9.536533e-05 0.037201063 0.11551094 0.8173299 0.4148020 19.946658
## Step 3.228520e-06 0.004524957 0.03375977 0.5148946 0.3415982 6.880424
                                                                               0
## Lasso 5.687156e-03 0.033942492 0.07558526 0.3507532 0.3453562 2.541241
                                                                               0
## MARS 7.184630e-04 0.056736427 0.06736552 0.4022505 0.3873593 3.418330
                                                                               0
## Null 1.004490e-01 0.154440199 0.17462598 0.4647126 0.2209028 2.167452
## Logit 4.929317e-04 0.011633527 0.04909710 0.3397412 0.3670520 2.966334
```



Confidence Level: 0.95

### Test Set Performance

```
perf <-
  function(lst_mods,
           f_metric = caTools::colAUC,
           metricname = "ROC-AUC",
           dat=test_data,
           response="FUTREC") {
    lst_preds <- map(</pre>
      .x = lst_mods,
      .f = function(x) {
        if (class(x)[1] != "train") {
          predict(x, newdata = dat, type = "response")
        } else
            predict(x, newdata = dat, type = "prob")[, "yes"]
      }
    map_dfr(lst_preds, function(x) {
      f_metric(x, dat[,response, drop=TRUE])
    }) %>%
```

```
pivot_longer(everything(), names_to = "model", values_to = metricname)
}

perf(mymods, caTools::colAUC, "ROC-AUC") %>%
    arrange(desc(`ROC-AUC`)) %>%
    knitr::kable()
```

model	ROC-AUC
Step	0.9230727
Lasso	0.9229897
GAM	0.9033686
Logit	0.8677480
MARS	0.8381275
RF	0.8333087
XGB	0.8227510
Null	0.5000000

```
perf(mymods, MLmetrics::LogLoss, "LogLoss") %>%
    arrange(LogLoss) %>%
    knitr::kable()
```

model	LogLoss
Lasso	0.3750170
XGB	0.4313091
GAM	0.4366629
Step	0.4695585
RF	0.4789201
Logit	0.5909993
Null	0.6541396
MARS	0.8957176

# Probability of Recession (Most Recent Trading Day)

```
curr_data <- tail(full_data_wide_features_adstock, 1)

curr_data$date

## [1] "2022-12-01"

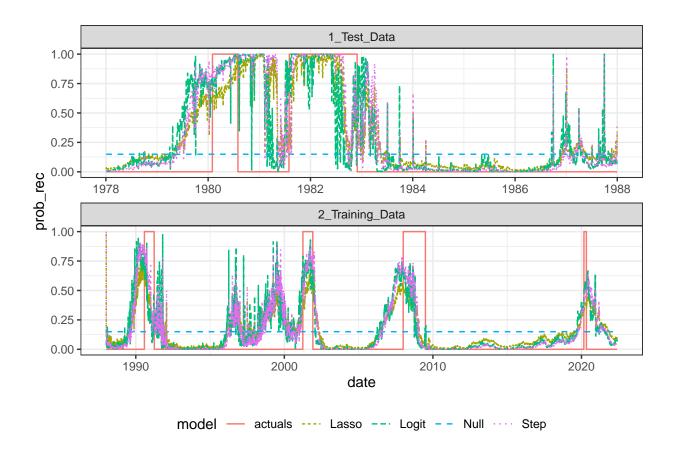
score_fun <- function(mods, dat) {
  output <- map_dfc(.x = mods, .f = function(x) {
    if(class(x)[1] != "train"){
      predict(x, newdata = dat, type = "response")
    } else(
      predict(x, newdata = dat, type = "prob")[,"yes"]
</pre>
```

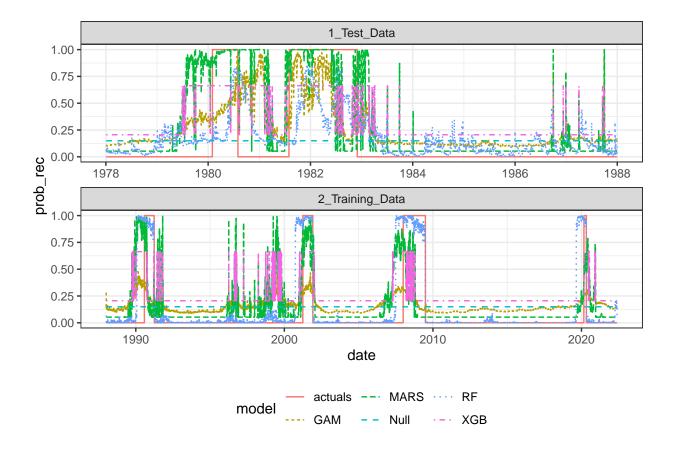
model	prob_rec
XGB	20.56%
GAM	12.87%
RF	3.00%
Step	3.34%
Lasso	8.16%
MARS	5.26%
Null	14.93%
Logit	3.55%

# Backtesting

```
full_data_bktst <- full_data_wide %>%
  filter(date >= startTestDate)
bkst_fun <- function(mods, dat) {</pre>
  output <- map_dfc(.x = mods, .f = function(x) {</pre>
    if(class(x)[1] != "train"){
      predict(x, newdata = dat, type = "response")
    } else(
       predict(x, newdata = dat, type = "prob")[,"yes"]
    )
  })
  output$date <- dat$date</pre>
  output <- output%>%
    pivot_longer(-date, names_to = "model",
                 values_to = "prob_rec")
 return(output)
df_plot <- bkst_fun(mymods, full_data_bktst)</pre>
```

```
actuals <- full_data_bktst %>%
  mutate(model="actuals") %>%
  select(date, model, prob_rec=USREC)
df_plot_final <- bind_rows(df_plot, actuals)</pre>
end_test_date <- max(test_data$date)</pre>
df_plot_final <- df_plot_final %>%
  mutate(epoc = case_when(date <= end_test_date ~ "1_Test_Data",</pre>
                           TRUE ~ "2_Training_Data")
  )
df_plot_logit_scam <- df_plot_final %>%
  filter(model %in% c('actuals', 'Null',
                       'Logit', 'Step', 'Lasso',
                       'LogitKnot'))
df_plot_knots_gbm <- df_plot_final %>%
  filter(model %in% c('actuals', 'Null',
                       'XGB', 'RF',
                       'GAM',
                       'MARS'))
```





stopCluster(cl)