Probability of Recession

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Summary

Forecast the probability of a recession in the next 126 trading days using the following predictors:

- 1. Spread between 10Y CMT and Effective Federal Funds Rate
- 2. Lags of the spread
- 3. Adstock transformations of the spread

There are between 250 and 253 trading days in a year.

Extract Historical Data

Refer to this vignette for FRED data access.

```
library(tidyverse)
library(lubridate)
library(fredr)
library(car)
library(MLmetrics)
library(pdp)
library(gridExtra)
library(mboost)
library(gbm)
library(randomForest)
library(glmnet)
library(gtsummary)
```

```
# series_id <- c("FEDFUNDS", "GS10", "USREC", "UNRATE", "CPIAUCSL")
series_id <- c("DFF", "DGS10") # daily
response_id <- "USREC" # monthly
full_data <- map_dfr(series_id, function(x) {
   fredr(</pre>
```

```
series_id = x,
  observation_start = as.Date("1950-01-01"),
  observation_end = as.Date("2022-12-01")
)

recession_dates <- map_dfr(response_id, function(x) {
  fredr(
    series_id = x,
    observation_start = as.Date("1950-01-01"),
    observation_end = as.Date("2022-12-01")
)
})</pre>
```

Pivot Wider

Calculate Features/Predictors

```
full_data_wide_features <- full_data_wide_raw %>%
  arrange(date) %>%
  mutate(SPRD_10YCMT_FEDFUNDS = DGS10 - DFF
         ) %>%
  mutate(across(
    .cols=c(SPRD_10YCMT_FEDFUNDS),
    .fns=list(lag1m = ~lag(.x, 1*21),
         lag3m = ~lag(.x, 3*21),
         lag6m = ~lag(.x, 6*21),
         lag9m = ~lag(.x, 9*21),
         lag12m = ~lag(.x, 12*21),
         lag5d = \sim lag(.x, 5),
         lag10d = \sim lag(.x, 10),
         lag15d = \sim lag(.x, 15)
  )) %>%
  drop_na()
```

Calculate Adstock

The adstock transformation is an auto-regressive transformation of a time series. The transformation takes into account past values of the time series. The intuition is that past values of the time series has a contemporaneous effect on the outcome.

$$AdStock(x_t) = x_t + \theta AdStock(x_{t-1})$$

where

 $0 < \theta < 1$

.

The parameters cannot be estimated easily with least squares or logistic regression. Instead, we assume a range of potential values.

```
full_data_wide_features_adstock <- full_data_wide_features %>%
  arrange(date) %>%
    mutate(across(
    .cols=c(SPRD_10YCMT_FEDFUNDS),
    .fns=list(adstkL = ~stats::filter(.x,
                                      filter=0.01,
                                      method="recursive") ,
         adstkM = ~stats::filter(.x,
                                      filter=0.05,
                                      method="recursive") ,
         adstkHL = ~stats::filter(.x,
                                      filter=0.10,
                                      method="recursive"),
         adstkHM = ~stats::filter(.x,
                                      filter=0.15,
                                      method="recursive"),
         adstkHH = ~stats::filter(.x,
                                      filter=0.25,
                                      method="recursive"),
         adstkHHH = ~stats::filter(.x,
                                      filter=0.35,
                                      method="recursive")
  ))) %>%
  mutate(constant=1)
```

Calculate Moving Average

```
sides=1) ,
        ma15d = ~stats::filter(.x,
                                     filter=ma_fun(15),
                                    method="convolution",
                                    sides=1),
        ma20d = ~stats::filter(.x,
                                      filter=ma_fun(20),
                                    method="convolution",
                                    sides=1),
        ma25d = ~stats::filter(.x,
                                     filter=ma_fun(25),
                                    method="convolution",
                                    sides=1),
        ma2m = ~stats::filter(.x,
                                      filter=ma_fun(2*21),
                                    method="convolution",
                                    sides=1),
        ma3m = ~stats::filter(.x,
                                      filter=ma_fun(3*21),
                                    method="convolution",
                                    sides=1),
        ma6m = ~stats::filter(.x,
                                      filter=ma_fun(6*21),
                                    method="convolution",
                                    sides=1),
        ma9m = ~stats::filter(.x,
                                      filter=ma_fun(9*21),
                                    method="convolution",
                                    sides=1),
        ma12m = ~stats::filter(.x,
                                      filter=ma_fun(12*21),
                                    method="convolution",
                                    sides=1)
)))
```

Recession in next 6 months

```
by = c("date_month" = "date_month",
                   "date_year" = "date_year")) %>%
  mutate(USREC = value)
df_FUTREC = as.data.frame(
  data.table::shift(
    full_data_wide$USREC,
   n = 1:(6 * 21),
    type = "lead",
    give.names = TRUE,
    fill = NA
  )
) %>%
 rowwise() %>%
 mutate(FUTREC = max(c_across(V1_lead_1:V1_lead_126)))
full_data_wide$FUTREC <- df_FUTREC$FUTREC</pre>
full_data_wide <- full_data_wide %>%
  select(date=date.x, everything(), -date_month,
         -date_year, -date.y,
         -value) %>%
  drop_na()
full_data_wide_noUSREC <- full_data_wide %>%
  select(-USREC)
```

Remove the last 12 months of historical data

Since the NBER often dates recessions after they have already occurred (and sometimes ended), remove the last 12 months of historical data from both the training and test data sets.

```
recent_data <- tail(full_data_wide_noUSREC, 12*21)
train_test <- head(full_data_wide_noUSREC, -12*21)</pre>
```

Split Train/Test

```
full_size <- nrow(train_test)

test_size <- floor(full_size*0.5)

test_id <- seq.int(1,test_size,1)

train_test$constant <- 1

train_data <- train_test[-test_id,]

test_data <- train_test[test_id,]</pre>
```

Characteristic	N = 7,157
date	1992-09-18 to 2021-04-28
DFF	1.91 (0.18, 4.92)
DGS10	4.06(2.47, 5.49)
SPRD_10YCMT_FEDFUNDS	$1.51 \ (0.58, \ 2.58)$
SPRD_10YCMT_FEDFUNDS_lag1m	$1.52\ (0.58,\ 2.59)$
SPRD_10YCMT_FEDFUNDS_lag3m	$1.53\ (0.58,\ 2.61)$
SPRD_10YCMT_FEDFUNDS_lag6m	$1.57 \ (0.58, 2.65)$
SPRD_10YCMT_FEDFUNDS_lag9m	$1.61 \ (0.59, \ 2.67)$
SPRD_10YCMT_FEDFUNDS_lag12m	$1.66 \ (0.61, \ 2.68)$
SPRD_10YCMT_FEDFUNDS_lag5d	$1.52\ (0.58,\ 2.58)$
SPRD_10YCMT_FEDFUNDS_lag10d	$1.52\ (0.58,\ 2.58)$
SPRD_10YCMT_FEDFUNDS_lag15d	$1.52\ (0.58,\ 2.58)$
$SPRD_10YCMT_FEDFUNDS_adstkL$	$1.53 \ (0.59, \ 2.61)$
SPRD_10YCMT_FEDFUNDS_adstkM	$1.59 \ (0.61, \ 2.71)$
SPRD_10YCMT_FEDFUNDS_adstkHL	$1.68 \ (0.64, \ 2.86)$
SPRD_10YCMT_FEDFUNDS_adstkHM	1.78 (0.68, 3.03)
SPRD_10YCMT_FEDFUNDS_adstkHH	2.02(0.77, 3.44)
SPRD_10YCMT_FEDFUNDS_adstkHHH	$2.32\ (0.90,\ 3.97)$
constant	7,157 (100%)
SPRD_10YCMT_FEDFUNDS_ma5d	$1.51 \ (0.58, \ 2.58)$
SPRD_10YCMT_FEDFUNDS_ma10d	$1.51 \ (0.59, \ 2.59)$
SPRD_10YCMT_FEDFUNDS_ma15d	$1.50 \ (0.59, \ 2.59)$
SPRD_10YCMT_FEDFUNDS_ma20d	$1.50 \ (0.59, \ 2.60)$
SPRD_10YCMT_FEDFUNDS_ma25d	$1.50 \ (0.59, \ 2.61)$
SPRD_10YCMT_FEDFUNDS_ma2m	$1.51 \ (0.58, \ 2.61)$
SPRD_10YCMT_FEDFUNDS_ma3m	$1.51 \ (0.56, \ 2.62)$
SPRD_10YCMT_FEDFUNDS_ma6m	1.55 (0.57, 2.62)
SPRD_10YCMT_FEDFUNDS_ma9m	$1.56 \ (0.55, \ 2.66)$
SPRD_10YCMT_FEDFUNDS_ma12m	$1.57 \ (0.55, \ 2.69)$
FUTREC	959 (13%)

Remove stale data from test set

Exclude historical data prior to 1975-01-01 because the economy changed dramatically (due to computational innovation).

```
summary(test_data$date)

## Min. 1st Qu. Median Mean 3rd Qu. Max.
```

```
## "1964-01-09" "1971-03-14" "1978-05-16" "1978-05-16" "1985-07-23" "1992-09-17"

test_data <- test_data %>%
    filter(date >= startTestDate)

summary(test_data$date)

## Min. 1st Qu. Median Mean 3rd Qu. Max.
## "1975-01-02" "1979-06-09" "1983-11-14" "1983-11-13" "1988-04-20" "1992-09-17"
```

Setup Parallel Processing

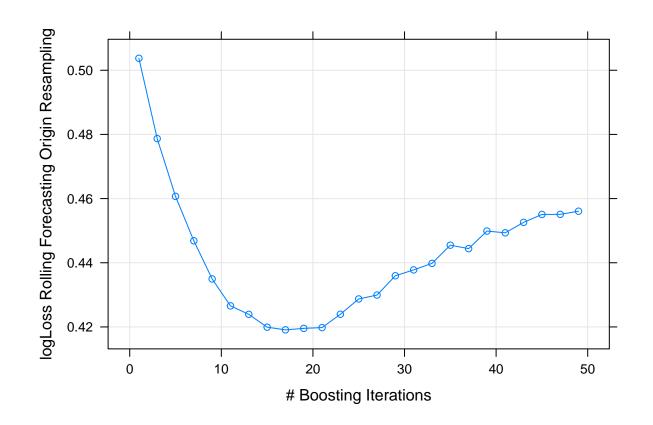
```
library(doParallel)

cl <- makePSOCKcluster(4)
registerDoParallel(cl)</pre>
```

Cross-Validation Framework

```
folds <- 20
fcstHorizon <- 12*21
initWindow <- nrow(train_yes_no) - fcstHorizon * folds</pre>
if(initWindow < 100){</pre>
  stop("Too few observations.")
}
fitControl_oneSE <- trainControl(method = "timeslice",</pre>
                            initialWindow=initWindow,
                            horizon=fcstHorizon,
                            fixedWindow=FALSE,
                            skip=fcstHorizon-1,
                            ## Estimate class probabilities
                            classProbs = TRUE,
                            ## Evaluate performance using
                            ## the following function
                            summaryFunction = mnLogLoss,
                            selectionFunction="oneSE")
fitControl_best <- trainControl(method = "timeslice",</pre>
                            initialWindow=initWindow,
                            horizon=fcstHorizon,
                            fixedWindow=FALSE,
                            skip=fcstHorizon-1,
                            ## Estimate class probabilities
                            classProbs = TRUE,
                            ## Evaluate performance using
                            ## the following function
                            summaryFunction = mnLogLoss,
                            selectionFunction="best")
```

Gradient Boosting for Additive Models

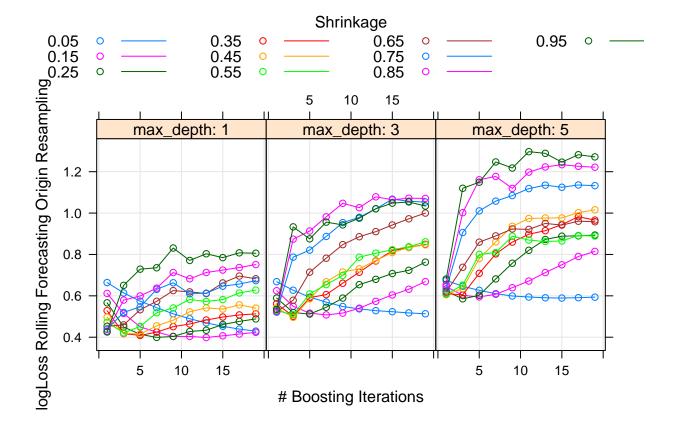


```
gam_mod$bestTune
```

```
## mstop prune
## 9 17 no
```

eXtreme Gradient Boosting Trees

```
grid_xgb <- expand.grid(nrounds=seq(1, 20, 2),</pre>
                         \max_{depth=c(1,3,5)},
                          eta = seq(0.05, 1, 0.1),
                          gamma=0,
                         colsample_bytree=1,
                         min_child_weight=10,
                          subsample=1
set.seed(randSeed)
xgb_mod <- train(</pre>
  FUTREC ~ . - date - constant,
  data = train_yes_no,
  method = "xgbTree",
  trControl = fitControl_best,
  metric = "logLoss",
  tuneGrid = grid_xgb,
  objective = "binary:logistic"
plot(xgb_mod)
```



```
xgb_mod$bestTune
```

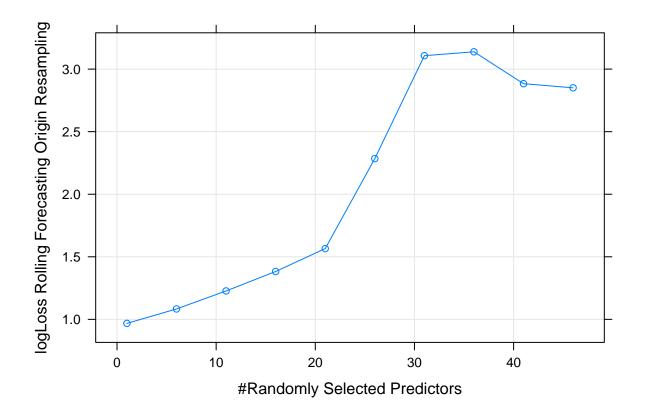
```
## nrounds max_depth eta gamma colsample_bytree min_child_weight subsample ## 37 13 1 0.15 0 1 10 1
```

Random Forest

```
grid_rf <- data.frame(mtry=seq.int(1,50,5))

set.seed(randSeed)

rf_mod <- train(
   FUTREC ~ . - date - constant,
   data = train_yes_no,
   method = "rf",
   trControl = fitControl_best,
   metric = "logLoss",
   tuneGrid = grid_rf,
   importance = TRUE
)</pre>
```



rf_mod\$bestTune

```
## mtry
## 1 1
```

Stepwise Regression

The glmStepAIC method uses the glm() function from the stats package. The documentation for glm() says:

For binomial and quasibinomial families the response can also be specified as a factor (when the first level denotes failure and all others success) or as a two-column matrix with the columns giving the numbers of successes and failures.

However, for most methods (that do not invoke glm()) in train, the first level denotes the success (the opposite of glm()). This behavior causes the coefficient signs to flip. Be highly suspicious when interpreting coefficients from models that are fit using train.

```
set.seed(randSeed)

stepwise_mod <- train(
  FUTREC ~ . - date - constant,
  data = train_yes_no,
  method = "glmStepAIC",
  trControl = fitControl_best,
  metric = "logLoss",
  tuneLength = 10,
  family = binomial,
  trace = 0,
  k = 10*log(nrow(train_yes_no)),
  direction = "forward"
)</pre>
```

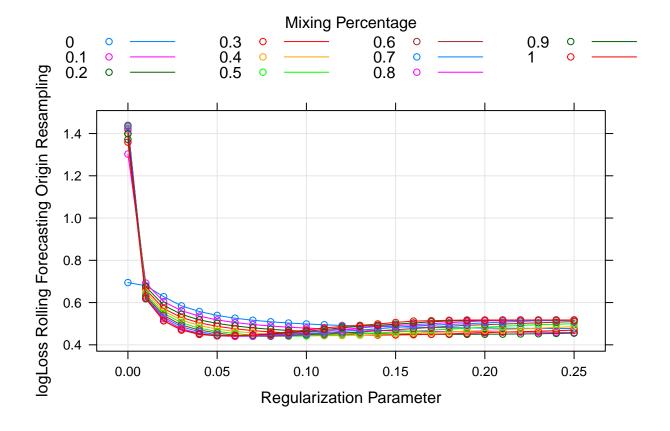
Elastic Net (Lasso)

```
grid_glmnet <- expand.grid(
   alpha = seq(0,1,0.1),
   lambda = seq(0, 0.25, 0.01)
)

set.seed(randSeed)

glmnet_mod <- train(
  FUTREC ~ . - date - constant,
   data = train_yes_no,
   method = "glmnet",
   trControl = fitControl_best,
   metric = "logLoss",
   tuneGrid = grid_glmnet,</pre>
```

```
family = "binomial"
)
plot(glmnet_mod)
```



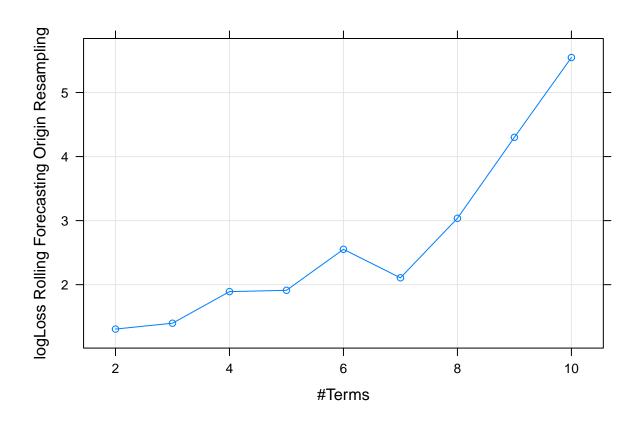
```
glmnet_mod$bestTune
```

```
## alpha lambda
## 215 0.8 0.06
```

Multivariate Adaptive Regression Splines

```
metric = "logLoss",
  tuneGrid = grid_mars,
  glm = list(family = binomial)
)

plot(earth_mod)
```



```
\verb| earth_mod\$bestTune| \\
```

```
## nprune degree
## 1 2 1
```

Null Model: Intercept-only Model

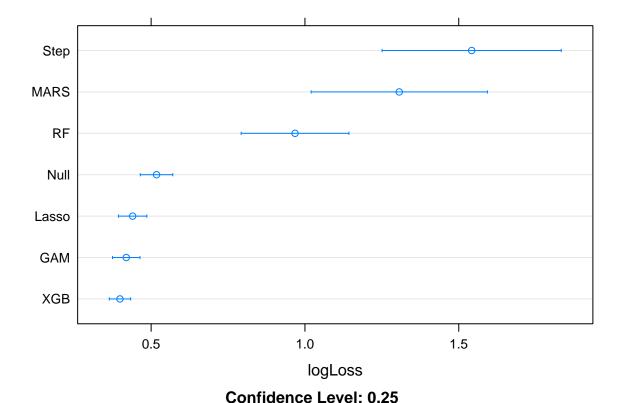
```
set.seed(randSeed)
null_mod <- train(
  FUTREC ~ constant,
  data = train_yes_no,
  method = "glm",
  trControl = fitControl_best,
  metric = "logLoss",</pre>
```

```
family = binomial
)
```

Compare Models

```
## Call:
## summary.resamples(object = resamps)
## Models: XGB, GAM, RF, Step, Lasso, MARS, Null
## Number of resamples: 20
##
## logLoss
##
                         1st Qu.
                                                       3rd Qu.
                                                                    Max. NA's
                 Min.
                                    Median
                                                Mean
## XGB
        9.192479e-02 0.11526830 0.1304653 0.3986430 0.4869654 1.703535
        4.987642e-02 0.08323740 0.1231974 0.4190890 0.4005984 2.122464
## GAM
        7.040866e-03 0.06073099 0.2466586 0.9677976 0.6102503 10.913992
                                                                            0
## RF
## Step 2.022540e-07 0.05006111 0.2384969 1.5420435 0.5442494 17.817623
                                                                            0
## Lasso 1.422861e-02 0.07611252 0.1499059 0.4398014 0.4424810 2.242503
                                                                            0
## MARS 7.981488e-05 0.05497337 0.0712627 1.3066759 0.6221208 17.817623
                                                                            0
## Null 9.011199e-02 0.13169314 0.1611508 0.5175254 0.2956142 2.193465
```

```
dotplot(resamps, metric = "logLoss", conf.level=0.25)
```

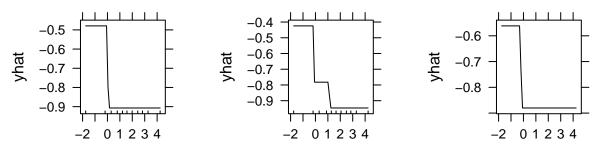


Explore XGB Model

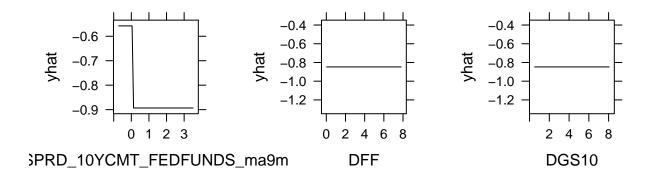
variable	e			Overall
SPRD	10YCMT	FEDFUNDS	lag6m	100.00000

variable	Overall
SPRD 10YCMT FEDFUNDS lag12m	79.69336
SPRD 10YCMT FEDFUNDS lag9m	63.85953
SPRD_10YCMT_FEDFUNDS_ma9m	53.70159
DFF	0.00000
DGS10	0.00000
SPRD_10YCMT_FEDFUNDS	0.00000
SPRD_10YCMT_FEDFUNDS_lag1m	0.00000
SPRD_10YCMT_FEDFUNDS_lag3m	0.00000
SPRD_10YCMT_FEDFUNDS_lag5d	0.00000
SPRD_10YCMT_FEDFUNDS_lag10d	0.00000
SPRD_10YCMT_FEDFUNDS_lag15d	0.00000
SPRD_10YCMT_FEDFUNDS_adstkL	0.00000
$SPRD_10YCMT_FEDFUNDS_adstkM$	0.00000
SPRD_10YCMT_FEDFUNDS_adstkHL	0.00000
SPRD_10YCMT_FEDFUNDS_adstkHM	0.00000
SPRD_10YCMT_FEDFUNDS_adstkHH	0.00000
SPRD_10YCMT_FEDFUNDS_adstkHHH	0.00000
SPRD_10YCMT_FEDFUNDS_ma5d	0.00000
SPRD_10YCMT_FEDFUNDS_ma10d	0.00000
SPRD_10YCMT_FEDFUNDS_ma15d	0.00000
SPRD_10YCMT_FEDFUNDS_ma20d	0.00000
SPRD_10YCMT_FEDFUNDS_ma25d	0.00000
SPRD_10YCMT_FEDFUNDS_ma2m	0.00000
SPRD_10YCMT_FEDFUNDS_ma3m	0.00000
SPRD_10YCMT_FEDFUNDS_ma6m	0.00000
SPRD_10YCMT_FEDFUNDS_ma12m	0.00000

```
pdp.top1 <- partial(xgb_mod,</pre>
          pred.var = df_imp$variable[1],
          plot = TRUE,
          rug = TRUE)
pdp.top2 <- partial(xgb_mod,</pre>
          pred.var = df_imp$variable[2],
          plot = TRUE,
          rug = TRUE)
pdp.top3 <- partial(xgb_mod,</pre>
    pred.var = df_imp$variable[3],
    plot = TRUE,
    chull = TRUE
  )
pdp.top4 <- partial(xgb_mod,</pre>
    pred.var = df_imp$variable[4],
    plot = TRUE,
    chull = TRUE
  )
pdp.top5 <- partial(xgb_mod,</pre>
pred.var = df_imp$variable[5],
```



3PRD_10YCMT_FEDFUNDSPROMM0YCMT_FEDFUNDSSPROMM0YCMT_FEDFUNDS.



Peeking

Peeking means we use the insights from the automated models to choose variables in subsequent models. This is technically cheating and causes the cross-validation errors to be artificially low. This is addressed in the test set which does not have peeking bias.

```
top_predictors <- head(df_imp$variable)
best_predictor <- head(top_predictors, 1)
top_fmla <- as.formula(paste0("FUTREC ~",</pre>
```

Logistic Regression (with peeking)

As mentioned early, train and glm treat the reference level differently for binary outcomes. Hence, the coefficients are flipped when training a logistic regression inside train.

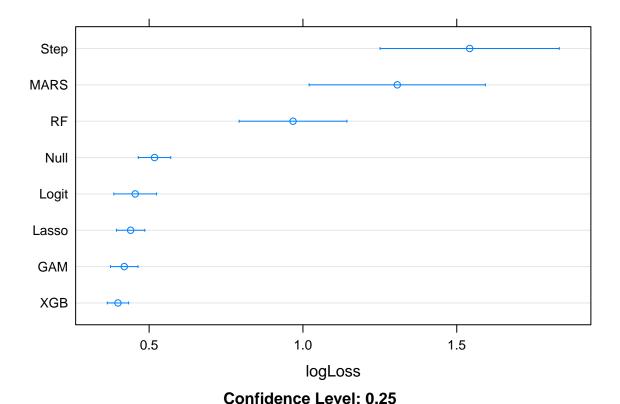
```
logit_mod <- train(
  top1_fmla,
  data = train_yes_no,
  method = "glm",
  trControl = fitControl_best,
  metric = "logLoss",
  family=binomial
)</pre>
```

```
##
## Call:
## NULL
##
## Deviance Residuals:
##
                1Q
                                   3Q
      Min
                     Median
                                           Max
## -3.3929
           0.1184
                     0.2504
                               0.4903
                                        1.9110
##
## Coefficients:
##
                              Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                               0.64204
                                          0.04400
                                                    14.59
                                                            <2e-16 ***
## SPRD_10YCMT_FEDFUNDS_lag6m 1.34846
                                          0.04443
                                                    30.35
                                                            <2e-16 ***
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##
      Null deviance: 5638.4 on 7156 degrees of freedom
## Residual deviance: 4144.0 on 7155 degrees of freedom
## AIC: 4148
##
## Number of Fisher Scoring iterations: 6
```

Compare Models

CV errors for models with peeking are misleadingly low. This will be addressed with a test set.

```
mymods <- list(XGB = xgb_mod,</pre>
                          GAM = gam_mod,
                          RF = rf_mod,
                          Step = stepwise_mod,
                          Lasso = glmnet_mod,
                          MARS = earth_mod,
                          Null = null_mod,
                          Logit = logit_mod) ## peeking
resamps <- resamples(mymods)</pre>
summary(resamps)
##
## Call:
## summary.resamples(object = resamps)
## Models: XGB, GAM, RF, Step, Lasso, MARS, Null, Logit
## Number of resamples: 20
##
## logLoss
##
                         1st Qu.
                                     Median
                 Min.
                                                  Mean
                                                         3rd Qu.
                                                                      Max. NA's
## XGB
         9.192479e-02 0.11526830 0.13046527 0.3986430 0.4869654 1.703535
         4.987642e-02 0.08323740 0.12319739 0.4190890 0.4005984 2.122464
## GAM
         7.040866e-03 0.06073099 0.24665857 0.9677976 0.6102503 10.913992
## Step 2.022540e-07 0.05006111 0.23849694 1.5420435 0.5442494 17.817623
                                                                              0
## Lasso 1.422861e-02 0.07611252 0.14990590 0.4398014 0.4424810 2.242503
                                                                              0
## MARS 7.981488e-05 0.05497337 0.07126270 1.3066759 0.6221208 17.817623
                                                                              0
## Null 9.011199e-02 0.13169314 0.16115082 0.5175254 0.2956142 2.193465
                                                                              0
## Logit 5.135720e-05 0.03049939 0.09049806 0.4547867 0.4109226 4.244014
                                                                              0
```



map_dfr(lst_preds, function(x) {

}) %>%

f_metric(x, dat[,response, drop=TRUE])

Test Set Performance

```
pivot_longer(everything(), names_to = "model", values_to = metricname)
}

perf(mymods, caTools::colAUC, "ROC-AUC") %>%
    arrange(desc(`ROC-AUC`)) %>%
    knitr::kable()
```

model	ROC-AUC
XGB	0.9308781
GAM	0.9275417
Lasso	0.9206350
Logit	0.9058277
MARS	0.8478348
Step	0.8170368
RF	0.7931440
Null	0.5000000

```
perf(mymods, MLmetrics::LogLoss, "LogLoss") %>%
  arrange(LogLoss) %>%
  knitr::kable()
```

model	LogLoss
XGB	0.3282365
Lasso	0.3619716
Logit	0.3764121
GAM	0.4657387
RF	0.5407493
Null	0.5885335
Step	0.7092285
MARS	1.0010820

Probability of Recession (Most Recent Trading Day)

```
curr_data <- tail(full_data_wide_features_adstock, 1)

curr_data$date

## [1] "2022-11-23"

score_fum <- function(mods, dat) {
  output <- map_dfc(.x = mods, .f = function(x) {
    if(class(x)[1] != "train"){
      predict(x, newdata = dat, type = "response")
    } else(
      predict(x, newdata = dat, type = "prob")[,"yes"]

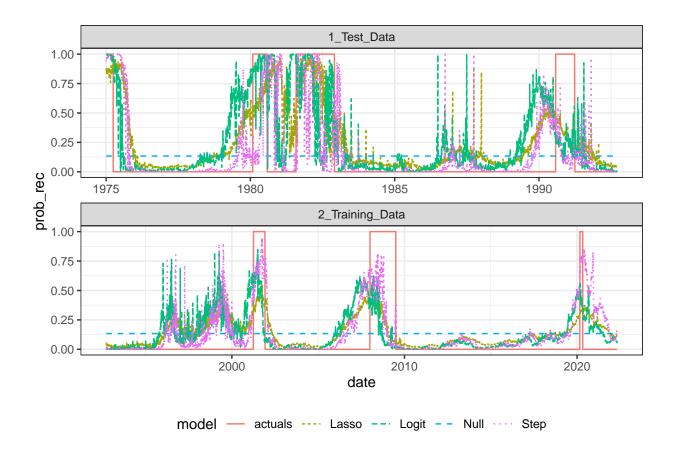
}</pre>
```

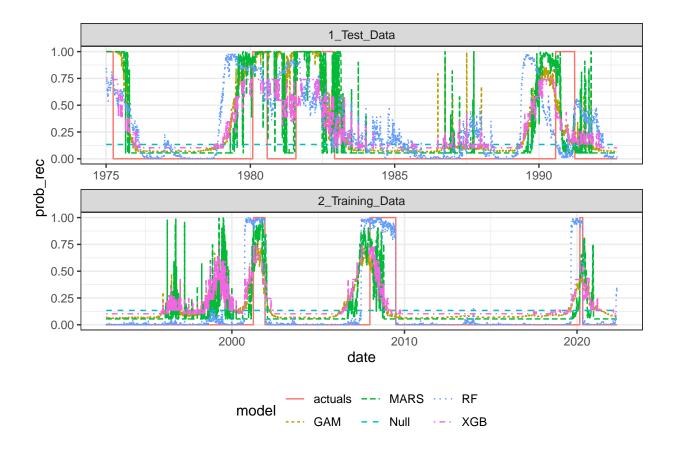
model	prob_rec
XGB	10.27%
GAM	7.57%
RF	1.20%
Step	1.61%
Lasso	8.66%
MARS	5.46%
Null	13.40%
Logit	3.29%

Backtesting

```
full_data_bktst <- full_data_wide %>%
  filter(date >= startTestDate)
bkst_fun <- function(mods, dat) {</pre>
  output <- map_dfc(.x = mods, .f = function(x) {</pre>
    if(class(x)[1] != "train"){
      predict(x, newdata = dat, type = "response")
    } else(
       predict(x, newdata = dat, type = "prob")[,"yes"]
    )
  })
  output$date <- dat$date</pre>
  output <- output%>%
    pivot_longer(-date, names_to = "model",
                 values_to = "prob_rec")
 return(output)
df_plot <- bkst_fun(mymods, full_data_bktst)</pre>
```

```
actuals <- full_data_bktst %>%
  mutate(model="actuals") %>%
  select(date, model, prob_rec=USREC)
df_plot_final <- bind_rows(df_plot, actuals)</pre>
end_test_date <- max(test_data$date)</pre>
df_plot_final <- df_plot_final %>%
  mutate(epoc = case_when(date <= end_test_date ~ "1_Test_Data",</pre>
                           TRUE ~ "2_Training_Data")
  )
df_plot_logit_scam <- df_plot_final %>%
  filter(model %in% c('actuals', 'Null',
                       'Logit', 'Step', 'Lasso',
                       'LogitKnot'))
df_plot_knots_gbm <- df_plot_final %>%
  filter(model %in% c('actuals', 'Null',
                       'XGB', 'RF',
                       'GAM',
                       'MARS'))
```





stopCluster(cl)