Probability of Recession

William Chiu

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Summary

Forecast the probability of a recession in the next 6 months using the spread between 10Y CMT and Effective Federal Funds Rate and its transformations:

- 1. Lags
- 2. Adstock
- 3. Moving average

Extract Historical Data

Refer to this vignette for FRED data access.

```
library(tidyverse)
library(lubridate)
library(fredr)
library(car)
library(MLmetrics)
library(caret)
library(pdp)
library(gridExtra)
library(mboost)
library(gbm)
library(randomForest)
library(glmnet)
library(gtsummary)
randSeed <- 1983
startTestDate <- "1978-01-01"
startTrainDate <- "1988-01-01"
```

```
series_id <- c("FEDFUNDS", "GS10", "USREC")

full_data <- map_dfr(series_id, function(x) {
   fredr(
       series_id = x,
       observation_start = as.Date("1950-01-01"),
       observation_end = as.Date("2023-12-01")
   )
})</pre>
```

Pivot Wider

Calculate Features/Predictors

Calculate Adstock

The adstock transformation is an auto-regressive transformation of a time series. The transformation takes into account past values of the time series. The intuition is that past values of the time series has a contemporaneous effect on the outcome.

$$AdStock(x_t) = x_t + \theta AdStock(x_{t-1})$$
$$0 < \theta < 1$$

where

The parameters cannot be estimated easily with least squares or logistic regression. Instead, we assume a range of potential values.

```
filter=0.91,
                                    method="recursive") ,
       adstk92 = ~stats::filter(.x,
                                    filter=0.92,
                                    method="recursive"),
       adstk93 = ~stats::filter(.x,
                                    filter=0.93,
                                    method="recursive"),
       adstk94 = ~stats::filter(.x,
                                    filter=0.94,
                                    method="recursive"),
       adstk95 = ~stats::filter(.x,
                                    filter=0.95,
                                    method="recursive"),
       adstk99 = ~stats::filter(.x,
                                    filter=0.99,
                                    method="recursive")
))) %>%
mutate(constant=1)
```

Calculate Moving Average

```
ma_fun <- function(k_param){</pre>
  rep(1/k_param, k_param)
full_data_wide_features_adstock <- full_data_wide_features_adstock %%
  arrange(date) %>%
    mutate(across(
    .cols=c(SPRD_10YCMT_FEDFUNDS, D_SPRD),
    .fns=list(
         ma2m = ~stats::filter(.x,
                                       filter=ma_fun(2),
                                     method="convolution",
                                     sides=1),
         ma3m = ~stats::filter(.x,
                                       filter=ma_fun(3),
                                     method="convolution",
                                     sides=1),
         ma6m = ~stats::filter(.x,
                                       filter=ma_fun(6),
                                     method="convolution",
                                     sides=1),
         ma9m = ~stats::filter(.x,
                                       filter=ma_fun(9),
                                     method="convolution",
                                     sides=1),
         ma12m = ~stats::filter(.x,
                                       filter=ma_fun(12),
                                     method="convolution",
                                     sides=1)
 )))
```

Remove the last 12 months of historical data

Since the NBER often dates recessions after they have already occurred (and sometimes ended), remove the last 12 months of historical data from both the training and test data sets.

```
recent_data <- tail(full_data_wide_features_adstock, 12)
train_test <- head(full_data_wide_features_adstock, -12) %>%
    drop_na()
```

Recession in next 6 months

Split Train/Test

Characteristic	N = 402
date	1988-01-01 to 2021-06-01
USREC	36 (9.0%)
GS10	4.48 (2.63, 6.22)
FEDFUNDS	$2.62\ (0.35,\ 5.28)$
SPRD_10YCMT_FEDFUNDS	$1.48 \ (0.53, \ 2.55)$
D_SPRD	-0.02 (-0.16, 0.14)
SPRD_10YCMT_FEDFUNDS_lag1	$1.50 \ (0.53, \ 2.55)$
SPRD_10YCMT_FEDFUNDS_lag3	$1.50 \ (0.53, \ 2.55)$
SPRD_10YCMT_FEDFUNDS_lag6	$1.53\ (0.53,\ 2.55)$
SPRD_10YCMT_FEDFUNDS_lag9	$1.55 \ (0.53, \ 2.55)$
SPRD_10YCMT_FEDFUNDS_lag12	$1.55 \ (0.53, \ 2.55)$
D_SPRD_lag1	-0.02 (-0.16, 0.14)
D_SPRD_lag3	-0.02 (-0.16, 0.14)
D_SPRD_lag6	-0.02 (-0.16, 0.14)
D_SPRD_lag9	-0.02 (-0.16, 0.15)
D_SPRD_lag12	-0.02 (-0.16, 0.15)
SPRD_10YCMT_FEDFUNDS_adstk85	10 (4, 17)
SPRD_10YCMT_FEDFUNDS_adstk91	17(7, 26)
SPRD_10YCMT_FEDFUNDS_adstk92	19 (8, 29)
SPRD_10YCMT_FEDFUNDS_adstk93	22 (9, 32)
SPRD_10YCMT_FEDFUNDS_adstk94	$26\ (12,37)$
SPRD_10YCMT_FEDFUNDS_adstk95	$31\ (15,\ 44)$
SPRD_10YCMT_FEDFUNDS_adstk99	$137\ (112,\ 161)$
D_SPRD_adstk85	-0.05 (-0.47, 0.42)
D_SPRD_adstk91	-0.08 (-0.62, 0.50)
D_SPRD_adstk92	-0.10 (-0.64, 0.49)
D_SPRD_adstk93	$-0.10 \ (-0.68, \ 0.51)$
D_SPRD_adstk94	$-0.10 \ (-0.69, \ 0.59)$
D_SPRD_adstk95	-0.13 (-0.72, 0.60)
D_SPRD_adstk99	0.01 (-0.79, 1.19)
constant	402 (100%)
SPRD_10YCMT_FEDFUNDS_ma2m	$1.49 \ (0.50, \ 2.54)$
SPRD_10YCMT_FEDFUNDS_ma3m	$1.50 \ (0.49, \ 2.56)$
SPRD_10YCMT_FEDFUNDS_ma6m	$1.51 \ (0.50, 2.59)$
SPRD_10YCMT_FEDFUNDS_ma9m	$1.49 \ (0.49, \ 2.62)$
SPRD_10YCMT_FEDFUNDS_ma12m	1.49 (0.51, 2.60)
D_SPRD_ma2m	-0.03 (-0.12, 0.11)
D_SPRD_ma3m	-0.02 (-0.12, 0.10)
D_SPRD_ma6m	-0.01 (-0.10, 0.08)
D_SPRD_ma9m	-0.01 (-0.09, 0.07)
D_SPRD_ma12m	-0.01 (-0.08, 0.08)
FUTREC	56 (14%)

Remove stale data from test set

Exclude historical data prior to 1978-01-01 because the economy changed dramatically (due to computational innovation).

<pre>summary(test_data\$date)</pre>	

Min. 1st Qu. Median Mean 3rd Qu. Max.

```
## "1978-01-01" "1980-06-23" "1982-12-16" "1982-12-16" "1985-06-08" "1987-12-01"

test_data <- test_data %>%
    filter(date >= startTestDate)

summary(test_data$date)

## Min. 1st Qu. Median Mean 3rd Qu. Max.
## "1978-01-01" "1980-06-23" "1982-12-16" "1982-12-16" "1985-06-08" "1987-12-01"
```

Setup Parallel Processing

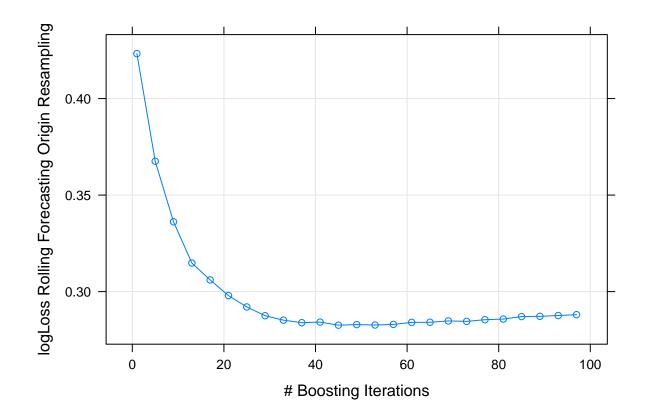
```
library(doParallel)

cl <- makePSOCKcluster(3)
registerDoParallel(cl)</pre>
```

Cross-Validation Framework

```
fcstHorizon <- 6
initWindow <- 120
param_skip <- fcstHorizon - 1</pre>
if(initWindow < 100){</pre>
  stop("Too few observations.")
}
fitControl_oneSE <- trainControl(method = "timeslice",</pre>
                            initialWindow=initWindow,
                            horizon=fcstHorizon,
                            fixedWindow=FALSE,
                            skip=param_skip,
                            ## Estimate class probabilities
                            classProbs = TRUE,
                            ## Evaluate performance using
                            ## the following function
                            summaryFunction = mnLogLoss,
                            selectionFunction="oneSE")
fitControl_best <- trainControl(method = "timeslice",</pre>
                            initialWindow=initWindow,
                            horizon=fcstHorizon,
                            fixedWindow=FALSE,
                            skip=param_skip,
                            ## Estimate class probabilities
                            classProbs = TRUE,
                            ## Evaluate performance using
                            ## the following function
                            summaryFunction = mnLogLoss,
                            selectionFunction="best")
```

Gradient Boosting for Additive Models

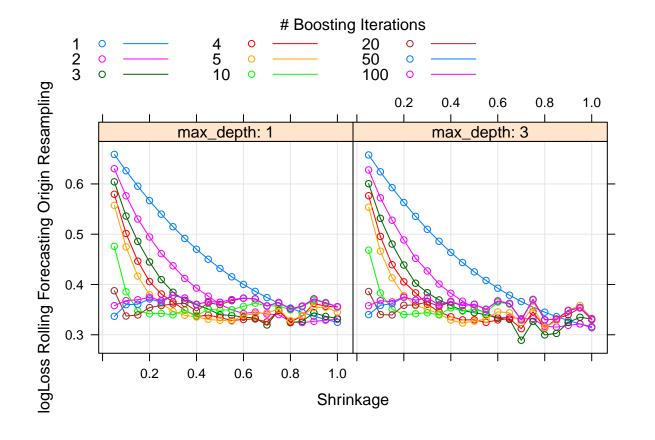


```
gam_mod$bestTune
```

```
## mstop prune
## 3 9 no
```

eXtreme Gradient Boosting Trees

```
grid_xgb <- expand.grid(nrounds=c(1,2,3,4,5,10,20,</pre>
                                    50,100),
                         max_depth=c(1,3),
                         eta = seq(0.05, 1, 0.05),
                         gamma=0,
                         colsample_bytree=1,
                         min_child_weight=10,
                         subsample=1
set.seed(randSeed)
xgb_mod <- train(</pre>
  FUTREC ~ . - date - USREC - constant,
  data = train_yes_no,
  method = "xgbTree",
  trControl = fitControl_oneSE,
  metric = "logLoss",
  tuneGrid = grid_xgb,
  objective = "binary:logistic"
plot(xgb_mod)
```



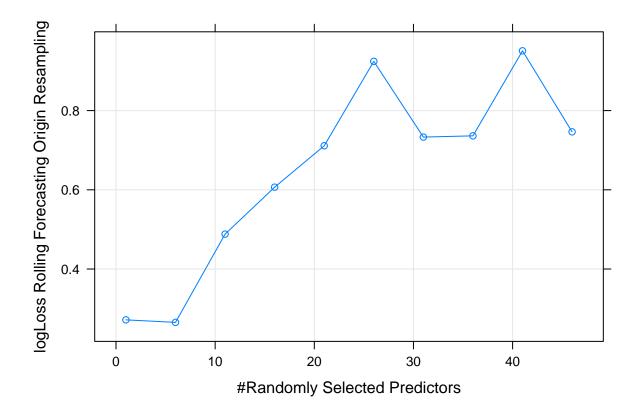
```
xgb_mod$bestTune
```

Random Forest

```
grid_rf <- data.frame(mtry=seq.int(1,50,5))

set.seed(randSeed)

rf_mod <- train(
  FUTREC ~ . - date - USREC - constant,
  data = train_yes_no,
  method = "rf",
  trControl = fitControl_oneSE,
  metric = "logLoss",
  tuneGrid = grid_rf,
  importance = TRUE
)</pre>
```



rf_mod\$bestTune

```
## mtry
## 1 1
```

Stepwise Regression

The glmStepAIC method uses the glm() function from the stats package. The documentation for glm() says:

For binomial and quasibinomial families the response can also be specified as a factor (when the first level denotes failure and all others success) or as a two-column matrix with the columns giving the numbers of successes and failures.

However, for most methods (that do not invoke glm()) in train, the first level denotes the success (the opposite of glm()). This behavior causes the coefficient signs to flip. Be highly suspicious when interpreting coefficients from models that are fit using train.

```
set.seed(randSeed)

stepwise_mod <- train(
  FUTREC ~ . - date - USREC - constant,
  data = train_yes_no,
  method = "glmStepAIC",
  trControl = fitControl_oneSE,
  metric = "logLoss",
  tuneLength = 10,
  family = binomial,
  trace = 0,
  k = 10*log(nrow(train_yes_no)),
  direction = "forward"
)</pre>
```

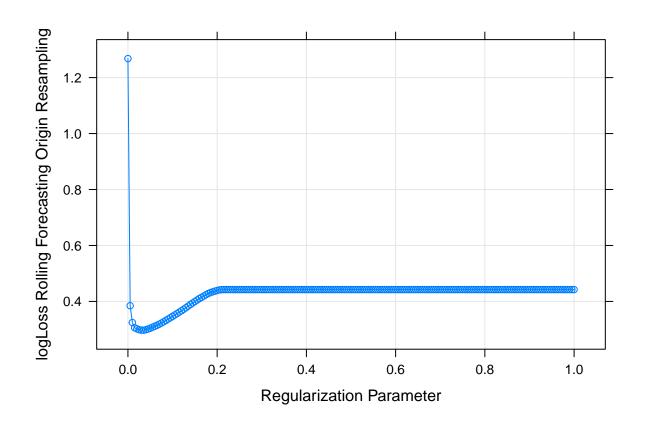
Elastic Net (Lasso)

```
grid_glmnet <- expand.grid(
   alpha = 1,
   lambda = seq(0, 1, 0.005)
)

set.seed(randSeed)

glmnet_mod <- train(
   FUTREC ~ . - date - USREC - constant,
   data = train_yes_no,
   method = "glmnet",
   trControl = fitControl_best,
   metric = "logLoss",
   tuneGrid = grid_glmnet,</pre>
```

```
family = "binomial"
)
plot(glmnet_mod)
```



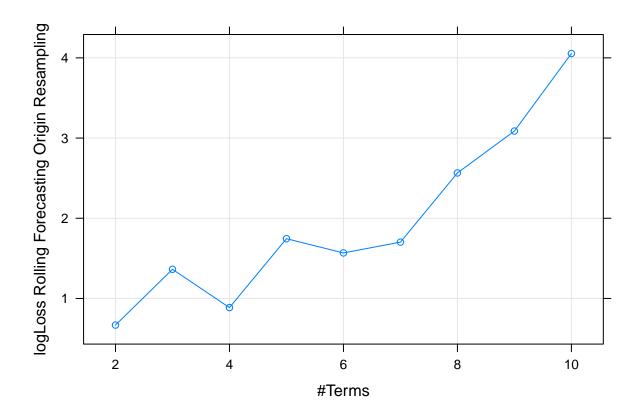
glmnet_mod\$bestTune

```
## alpha lambda
## 8 1 0.035
```

Multivariate Adaptive Regression Splines

```
metric = "logLoss",
  tuneGrid = grid_mars,
  glm = list(family = binomial)
)

plot(earth_mod)
```



```
\verb| earth_mod\$bestTune| \\
```

```
## nprune degree
## 1 2 1
```

Null Model: Intercept-only Model

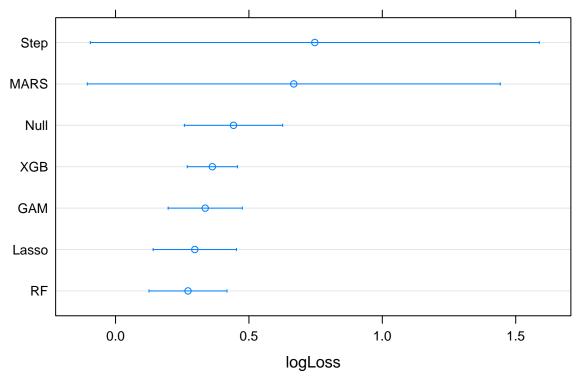
```
set.seed(randSeed)
null_mod <- train(
  FUTREC ~ constant,
  data = train_yes_no,
  method = "glm",
  trControl = fitControl_best,
  metric = "logLoss",</pre>
```

```
family = binomial
)
```

Compare Models

```
## Models: XGB, GAM, RF, Step, Lasso, MARS, Null
## Number of resamples: 47
##
## logLoss
##
                 Min.
                           1st Qu.
                                         Median
                                                     Mean
                                                             3rd Qu.
## XGB
         2.073761e-01 2.135106e-01 0.2326050186 0.3629273 0.24120808 1.654720
         7.048603e-02 8.717637e-02 0.0991593395 0.3361715 0.25521995 1.662805
## GAM
         2.003341e-03 9.896432e-03 0.0347085730 0.2716410 0.24694403 2.075257
## RF
## Step 9.992007e-16 6.449674e-12 0.0005685938 0.7469361 0.03272406 17.269388
## Lasso 5.080772e-04 1.057790e-02 0.0352911181 0.2971916 0.19291705 2.105329
## MARS 9.992007e-16 4.664113e-03 0.0326306288 0.6680720 0.05272320 17.269388
## Null 9.461598e-02 1.424865e-01 0.1607379851 0.4423188 0.19785505 2.277267
##
        NA's
## XGB
            0
## GAM
            0
## RF
            0
## Step
            0
## Lasso
## MARS
            0
## Null
            0
```

```
dotplot(resamps, metric = "logLoss", conf.level=0.95)
```

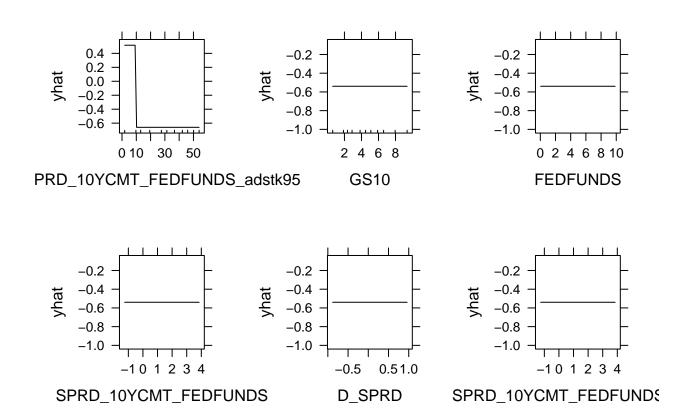


Confidence Level: 0.95

Explore XGB Model

```
variable
                                 Overall
GS10
                                     0
                                     0
FEDFUNDS
SPRD_10YCMT_FEDFUNDS
                                     0
                                     0
D SPRD
SPRD 10YCMT FEDFUNDS lag1
                                     0
SPRD\_10YCMT\_FEDFUNDS\_lag3
                                     0
SPRD_10YCMT_FEDFUNDS_lag6
                                     0
                                     0
SPRD_10YCMT_FEDFUNDS_lag9
SPRD\_10YCMT\_FEDFUNDS\_lag12
                                     0
D SPRD lag1
                                     0
D_SPRD_lag3
                                     0
D SPRD lag6
                                     0
D_SPRD_{lag9}
                                     0
D_SPRD_lag12
                                     0
SPRD\_10YCMT\_FEDFUNDS\_adstk85
                                     0
SPRD 10YCMT FEDFUNDS adstk91
                                     0
SPRD\_10YCMT\_FEDFUNDS\_adstk92
                                     0
SPRD\_10YCMT\_FEDFUNDS\_adstk93
                                     0
                                     0
SPRD\_10YCMT\_FEDFUNDS\_adstk94
SPRD\_10YCMT\_FEDFUNDS\_adstk99
                                     0
                                     0
D SPRD adstk85
D\_SPRD\_adstk91
                                     0
                                     0
D_SPRD_adstk92
D SPRD adstk93
                                     0
D_SPRD_adstk94
                                     0
D_SPRD_adstk95
                                     0
D SPRD adstk99
                                     0
SPRD_10YCMT_FEDFUNDS_ma2m
                                     0
SPRD\_10YCMT\_FEDFUNDS\_ma3m
                                     0
SPRD\_10YCMT\_FEDFUNDS\_ma6m
                                     0
SPRD 10YCMT FEDFUNDS ma9m
                                     0
SPRD 10YCMT FEDFUNDS ma12m
                                     0
D SPRD ma2m
                                     0
D_SPRD_ma3m
                                     0
D SPRD ma6m
                                     0
D\_SPRD\_ma9m
                                     0
D_SPRD_ma12m
                                     0
```

```
chull = TRUE
  )
pdp.top4 <- partial(xgb_mod,</pre>
    pred.var = df_imp$variable[4],
    plot = TRUE,
    chull = TRUE
  )
pdp.top5 <- partial(xgb_mod,</pre>
    pred.var = df_imp$variable[5],
    plot = TRUE,
    chull = TRUE
  )
pdp.top6 <- partial(xgb_mod,</pre>
    pred.var = df_imp$variable[6],
    plot = TRUE,
    chull = TRUE
  )
grid.arrange(pdp.top1, pdp.top2, pdp.top3,
              pdp.top4, pdp.top5, pdp.top6, ncol = 3)
```



Peeking

Peeking means we use the insights from the automated models to choose variables in subsequent models. This is technically cheating and causes the cross-validation errors to be artificially low. This is addressed in the test set which does not have peeking bias.

Logistic Regression (with peeking)

As mentioned early, train and glm treat the reference level differently for binary outcomes. Hence, the coefficients are flipped when training a logistic regression inside train.

```
logit_mod <- train(
  top1_fmla,
  data = train_yes_no,
  method = "glm",
  trControl = fitControl_oneSE,
  metric = "logLoss",
  family=binomial
)</pre>
```

```
##
## Call:
## NULL
##
## Deviance Residuals:
       Min
                  1Q
                        Median
                                       3Q
                                                Max
            0.01565
                       0.05180
                                0.21858
## -2.87348
                                            1.44220
##
## Coefficients:
##
                               Estimate Std. Error z value Pr(>|z|)
                                             0.4762 -4.963 6.92e-07 ***
## (Intercept)
                                 -2.3634
## SPRD_10YCMT_FEDFUNDS_adstk95
                                0.2294
                                             0.0347
                                                      6.610 3.84e-11 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## (Dispersion parameter for binomial family taken to be 1)
##
##
      Null deviance: 324.57 on 401 degrees of freedom
```

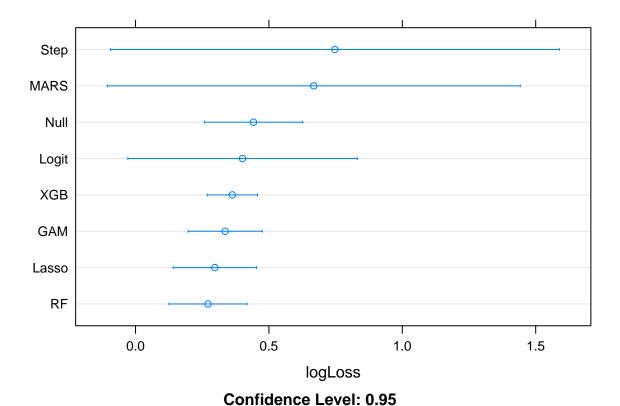
```
## Residual deviance: 170.31 on 400 degrees of freedom
## AIC: 174.31
##
## Number of Fisher Scoring iterations: 8
```

Compare Models

CV errors for models with peeking are misleadingly low. This will be addressed with a test set.

```
##
## Call:
## summary.resamples(object = resamps)
## Models: XGB, GAM, RF, Step, Lasso, MARS, Null, Logit
## Number of resamples: 47
##
## logLoss
                                         Median
                                                              3rd Qu.
##
                 Min.
                           1st Qu.
                                                      Mean
                                                                           Max.
## XGB
         2.073761e-01 2.135106e-01 0.2326050186 0.3629273 0.24120808
                                                                       1.654720
         7.048603e-02 8.717637e-02 0.0991593395 0.3361715 0.25521995
## GAM
         2.003341e-03 9.896432e-03 0.0347085730 0.2716410 0.24694403
                                                                       2.075257
## Step 9.992007e-16 6.449674e-12 0.0005685938 0.7469361 0.03272406 17.269388
## Lasso 5.080772e-04 1.057790e-02 0.0352911181 0.2971916 0.19291705
                                                                       2.105329
## MARS 9.992007e-16 4.664113e-03 0.0326306288 0.6680720 0.05272320 17.269388
## Null 9.461598e-02 1.424865e-01 0.1607379851 0.4423188 0.19785505 2.277267
## Logit 9.992007e-16 1.149218e-04 0.0010131744 0.4010993 0.05443663 8.604311
         NA's
##
## XGB
## GAM
            0
## RF
## Step
            0
## Lasso
            0
## MARS
            0
## Null
            0
## Logit
            0
```

```
dotplot(resamps, metric = "logLoss", conf.level=0.95)
```



Test Set Performance

```
perf <-
  function(lst_mods,
           f_metric = caTools::colAUC,
           metricname = "ROC-AUC",
           dat=test_data,
           response="FUTREC") {
    lst_preds <- map(</pre>
      .x = lst_mods,
      .f = function(x) {
        if (class(x)[1] != "train") {
          predict(x, newdata = dat, type = "response")
        } else
            predict(x, newdata = dat, type = "prob")[, "yes"]
      }
    map_dfr(lst_preds, function(x) {
      f_metric(x, dat[,response, drop=TRUE])
    }) %>%
```

```
pivot_longer(everything(), names_to = "model", values_to = metricname)
}

perf(mymods, caTools::colAUC, "ROC-AUC") %>%
    arrange(desc(`ROC-AUC`)) %>%
    knitr::kable()
```

model	ROC-AUC
Step	0.9634233
Logit	0.9634233
GAM	0.9620028
Lasso	0.9502841
MARS	0.9318182
RF	0.9137074
XGB	0.8863636
Null	0.5000000

```
perf(mymods, MLmetrics::LogLoss, "LogLoss") %>%
  arrange(LogLoss) %>%
  knitr::kable()
```

model	LogLoss
MARS	0.2855183
GAM	0.3427243
RF	0.3481919
Lasso	0.3692023
XGB	0.4383928
Null	0.6356365
Step	0.6795398
Logit	0.6795398

Probability of Recession (the 12 most recent months)

```
curr_data <- recent_data

curr_data$date

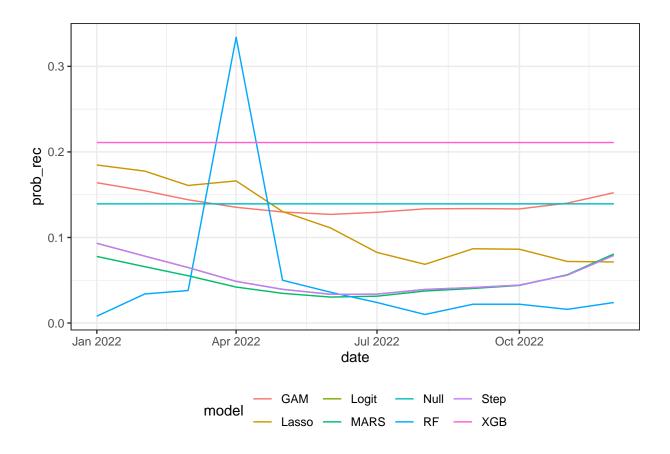
## [1] "2022-01-01" "2022-02-01" "2022-03-01" "2022-04-01" "2022-05-01"

## [6] "2022-06-01" "2022-07-01" "2022-08-01" "2022-09-01" "2022-10-01"

## [11] "2022-11-01" "2022-12-01"

score_fun <- function(mods, dat) {
    output <- map_dfc(.x = mods, .f = function(x) {
        if(class(x)[1] != "train") {
            predict(x, newdata = dat, type = "response")
        } else(
            predict(x, newdata = dat, type = "prob")[,"yes"]</pre>
```

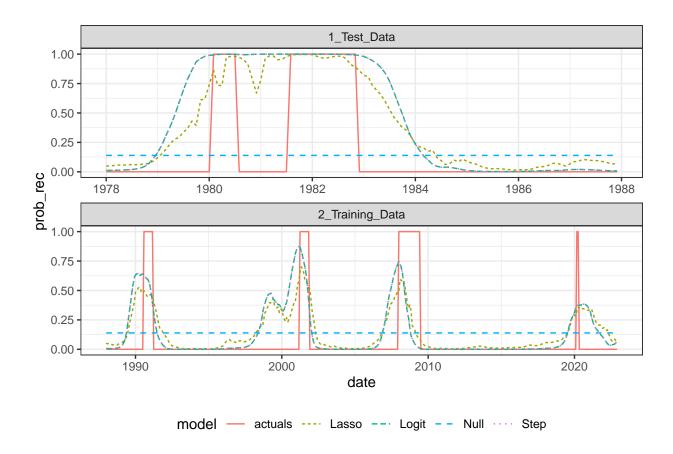
date	model	prob_rec
2022-10-01	XGB	0.2109551
2022-10-01	GAM	0.1333660
2022-10-01	RF	0.0220000
2022-10-01	Step	0.0443194
2022-10-01	Lasso	0.0862626
2022-10-01	MARS	0.0439631
2022-10-01	Null	0.1393035
2022-10-01	Logit	0.0443194
2022-11-01	XGB	0.2109551
2022-11-01	GAM	0.1401590
2022-11-01	RF	0.0160000
2022-11-01	Step	0.0560137
2022-11-01	Lasso	0.0719518
2022-11-01	MARS	0.0565218
2022-11-01	Null	0.1393035
2022-11-01	Logit	0.0560137
2022-12-01	XGB	0.2109551
2022-12-01	GAM	0.1523327
2022-12-01	RF	0.0240000
2022-12-01	Step	0.0790733
2022-12-01	Lasso	0.0714112
2022-12-01	MARS	0.0807284
2022-12-01	Null	0.1393035
2022-12-01	Logit	0.0790733

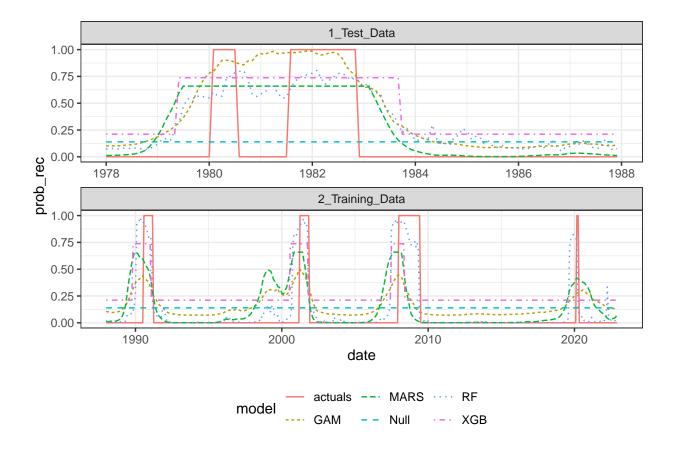


Backtesting

```
full_data_bktst <- full_data_wide_features_adstock %>%
  filter(date >= startTestDate)
bkst_fun <- function(mods, dat) {</pre>
  output <- map_dfc(.x = mods, .f = function(x) {</pre>
    if(class(x)[1] != "train"){
      predict(x, newdata = dat, type = "response")
    } else(
       predict(x, newdata = dat, type = "prob")[,"yes"]
    )
  })
  output$date <- dat$date</pre>
  output <- output%>%
    pivot_longer(-date, names_to = "model",
                 values_to = "prob_rec")
  return(output)
}
```

```
df_plot <- bkst_fun(mymods, full_data_bktst)</pre>
actuals <- full_data_bktst %>%
  mutate(model="actuals") %>%
  select(date, model, prob_rec=USREC)
df_plot_final <- bind_rows(df_plot, actuals)</pre>
end_test_date <- max(test_data$date)</pre>
df_plot_final <- df_plot_final %>%
  mutate(epoc = case_when(date <= end_test_date ~ "1_Test_Data",</pre>
                           TRUE ~ "2_Training_Data")
  )
df_plot_logit_scam <- df_plot_final %>%
  filter(model %in% c('actuals', 'Null',
                       'Logit', 'Step', 'Lasso',
                       'LogitKnot'))
df_plot_knots_gbm <- df_plot_final %>%
  filter(model %in% c('actuals', 'Null',
                       'XGB', 'RF',
                       'GAM',
                       'MARS'))
```





stopCluster(cl)