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Errata: Instantaneous Portfolio theory

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The derivation of the multivariate Lévy measure for the multivariate variance gamma model was in error in the paper entitled Instantaneous Portfolio Theory.

Let $g(t)$ be a gamma process with unit mean rate and variance rate ν. Let $\beta(t)$ be a multivariate Brownian motion with mean rate μ and covariance matrix Σ.

The multivariate Variance Gamma model is the process $X(t) = \beta(g(t))$, and

$$X(t) = \mu g(t) + W(g(t))$$

where W is a correlated Brownian motion with covariance matrix Σ.

PROPOSITION 1 *The multivariate Lévy density $m(x)$ for the process $X(t)$, is given by*

$$m(x) = \frac{2\exp\left(\mu^T\Sigma^{-1}x\right)}{\nu\,(2\pi)^{n/2}\sqrt{|\Sigma|}}\left(\sqrt{\mu^T\Sigma^{-1}\mu + \frac{2}{\nu}}\right)^{\frac{n}{2}}\left(\sqrt{x^T\Sigma^{-1}x}\right)^{-n/2} \times K_{n/2}\left(\sqrt{\left(\mu^T\Sigma^{-1}\mu + \frac{2}{\nu}\right)x^T\Sigma^{-1}x}\right),$$

where $K_{n/2}$ is the modified Bessel function of the second kind of order $n/2$.

Proof The process $g(t)$ is a Gamma process with Lévy measure

$$k(\mathrm{d}y) = \frac{\mathrm{e}^{-\frac{y}{\nu}}}{\nu y}\,\mathrm{d}y.$$

The multivariate Lévy density for the process $X(t)$ is given by Sato (1999) theorem 30.1 as

$$\begin{aligned} m(x) &= \int_0^\infty \frac{1}{\nu}\frac{\mathrm{e}^{-\frac{y}{\nu}}}{y}\frac{1}{(2\pi)^{n/2}\sqrt{|\Sigma y|}} \times \exp\left(-\frac{(x-\mu y)^T\Sigma^{-1}(x-\mu y)}{2y}\right)\mathrm{d}y \\ &= \frac{1}{\nu\,(2\pi)^{n/2}\sqrt{|\Sigma|}}\int_0^\infty \frac{1}{\sqrt{y^{2+n}}}\mathrm{e}^{-\frac{y}{\nu}} \times \exp\left(-\frac{x^T\Sigma^{-1}x}{2y} + \mu^T\Sigma^{-1}x - \frac{\mu^T\Sigma^{-1}\mu}{2}y\right)\mathrm{d}y \\ &= \frac{\exp\left(\mu^T\Sigma^{-1}x\right)}{\nu\,(2\pi)^{n/2}\sqrt{|\Sigma|}}\int_0^\infty y^{-n/2-1} \times \exp\left(-\frac{x^T\Sigma^{-1}x}{2y} - \frac{\mu^T\Sigma^{-1}\mu + \frac{2}{\nu}}{2}y\right)\mathrm{d}y \end{aligned}$$

On substituting

$$t = \mu^T\Sigma^{-1}\mu + \frac{2}{\nu},$$

we find that

$$m(x) = \frac{\exp\left(\mu^T\Sigma^{-1}x\right)}{\nu\,(2\pi)^{n/2}\sqrt{|\Sigma|}}\left(\frac{2}{\mu^T\Sigma^{-1}\mu + \frac{2}{\nu}}\right)^{-n/2}\int_0^\infty t^{-n/2-1} \times \exp\left(-t - \frac{\left(\mu^T\Sigma^{-1}\mu + \frac{2}{\nu}\right)x^T\Sigma^{-1}x}{4t}\right)\mathrm{d}t.$$

We now use the result

$$K_a(z) = \frac{1}{2}\left(\frac{z}{2}\right)^a\int_0^\infty \exp\left(-\left(t + \frac{z^2}{4t}\right)\right)t^{-a-1}\,\mathrm{d}t$$

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$$z = \sqrt{\left(\mu^T \Sigma^{-1} \mu + \frac{2}{\nu}\right) x^T \Sigma^{-1} x}$$
$$a = n/2$$

to get

$$m(x) = \frac{2 \exp\left(\mu^T \Sigma^{-1} x\right)}{\nu \left(2\pi\right)^{n/2} \sqrt{|\Sigma|}} \left(\frac{2}{\mu^T \Sigma^{-1} \mu + \frac{2}{\nu}}\right)^{-n/2}$$
$$\times \left(\frac{2}{\sqrt{\left(\mu^T \Sigma^{-1} \mu + \frac{2}{\nu}\right) x^T \Sigma^{-1} x}}\right)^{\frac{n}{2}}$$
$$\times K_{n/2}\left(\sqrt{\left(\mu^T \Sigma^{-1} \mu + \frac{2}{\nu}\right) x^T \Sigma^{-1} x}\right)$$
$$= \frac{2 \exp\left(\mu^T \Sigma^{-1} x\right)}{\nu \left(2\pi\right)^{n/2} \sqrt{|\Sigma|}} \left(\sqrt{\mu^T \Sigma^{-1} \mu + \frac{2}{\nu}}\right)^{\frac{n}{2}} \left(\sqrt{x^T \Sigma^{-1} x}\right)^{-n/2}$$
$$\times K_{n/2}\left(\sqrt{\left(\mu^T \Sigma^{-1} \mu + \frac{2}{\nu}\right) x^T \Sigma^{-1} x}\right).$$

■

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No potential conflict of interest was reported by the author(s).

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