Quantitative Finance

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/rquf20

Continuous-Time Asset Pricing Theory

Philip Protter

To cite this article: Philip Protter (2022) Continuous-Time Asset Pricing Theory, Quantitative Finance, 22:5, 813-815, DOI: 10.1080/14697688.2022.2063756

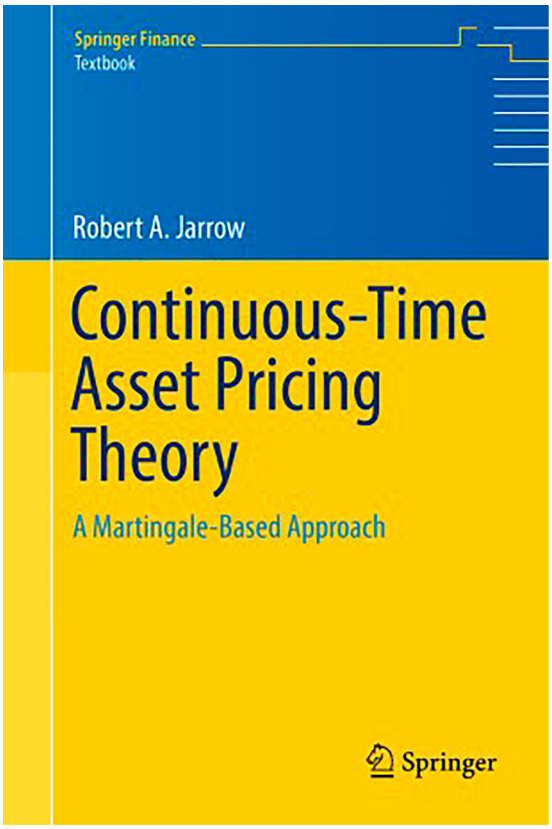
To link to this article: https://doi.org/10.1080/14697688.2022.2063756

Published online: 04 May 2022.

Quantitative Finance, 2022
Vol. 22, No. 5, 813–815, https://doi.org/10.1080/14697688.2022.2063756

Routledge
Taylor & Francis Group

Book review



Continuous-Time Asset Pricing Theory, by Robert A. Jarrow, 2nd ed., Springer (2021). E-book. ISBN 978-3-030-74409-0.

The subject of the mathematical modeling of Asset Prices is a relatively new subject, at least as regards mathematically oriented subjects. The modern approach can be traced back to Louis Bachlier's PhD thesis, written under the eyes of no less a thinker than Henri Poincaré (yes, *that* Poincaré!). Bachelier published his thesis in 1900, which should have ushered in a revolution in thought for the twentieth century, but alas, it was largely ignored. Bachelier was black-balled by the powerful probabilist Paul Lévy and ended up spending his career in Besancon, a charming little town near the Swiss border. However, in France, intellectual life revolved around Paris, and back then, it was quite arduous to travel from Besancon to Paris, and it took a long time, even by train. Although Kolmogorov himself made a point of seeing Bachelier and referencing his work, he and his work were largely ignored by the community.

Until, that is, the great statistician L. J. Savage alerted the Nobel Laureate Paul Samuelson to the work of Bachelier. That began a renaissance of Math Finance, centered at MIT, and around the work of Samuelson himself, combined with H.P. McKean, Jr. (who connected the analysis of American options to free boundary problems in partial differential equations). MIT has smart graduate students, and some of the best worked with Samuelson, especially F. Black, M. Scholes, and R. Merton, who created the Black-Scholes model for giving a fair price to call options in the early 1970s (1973 the first of their papers appeared).

The scene changed then to Stanford, where J.M. Harrison and S. Pliska wrote a seminal paper relating fair pricing to an absence of arbitrage, using martingale theory and stochastic integration, albeit on a finite probability space. This was improved by their colleague D. Kreps, who extended the theory to infinite probability spaces. A beautiful simplification of the complicated analysis of Kreps was finally, definitively, achieved in the work of F. Delbaen and W. Schachermayer in the 1990s.

Since then, there have been quite a few texts pedagogically treating the subject. A sampling might be those of K. Back, M. Baxter & A. Rennie, T. Bielecki & M. Rutkowski, T. Bjork, R. Cont & P. Tankov, F. Delbaen & W. Schachermayer, D. Duffie, H. Föllmer & A. Schied, the three of M. Jeanblanc, M. Yor, & M. Chesney, I. Karatzas & S. Shreve, D. Lamberton & B. Lapeyre, M. Musiela & M. Rutkowski, C. Skiadis, and D. Sondermann.

In view of this impressive list of texts that already exist, one might well ask: Why this one by Jarrow, and why now? The answer is concise:

Bubbles.

In my opinion, for what it is worth, financial bubbles are one of the important phenomena of the twenty-first century. It behooves us, as academics, to understand this phenomenon. Bubbles are, of course, as old as stock markets themselves. The first known bubble was that of Tulipomania, which occurred in Amsterdam in the seventeenth century (circa 1630s). It is the first bubble of the modern era. After Tulipomania, in the eighteenth century, John Law advised the Banque Royale (Paris, 1716–1720) to finance the crown's war debts by selling off notes giving rights to the gold yet to be discovered in the Louisiana territories. When no gold was found, the bubble collapsed, leading to an economic catastrophe, and helped to create the French distrust of banks, which lasted almost 100 years. Not to be outdone by the French, the South Sea Company of London (1711–1720) sold the rights to the gold pillaged from the Inca and Aztec civilizations in Mexico and South America, neglecting the small detail that the Spanish controlled such trade and had command of the high seas at the time. As this was eventually realized by the British public, the bubble collapsed.

The real king of bubbles, however, is the United States. A quick adumbration is as follows:

- The 1816 crash was due to real estate speculation
- With the construction of the spectacular Erie Canal connecting New York to Chicago via the Great Lakes and through inland waterways, 'irrational exuberance' (in the words of Alan Greenspan) led to the Crash of 1837
- Not having the learned its lesson in 1837, 'irrational exuberance' due to the construction of the railroad system within the U.S. led to The Panic of 1873
- The Wall Street panic of 1907, where the market fell by 50%, helped to solidify the fame of J.P. Morgan, who (as legend has it) stepped into the fray and ended the panic by announcing he would buy everything. It also had some good effects, as its aftermath created the atmosphere that led to the creation and development of the Federal Reserve in 1913, via the Glass-Owen bill.
- And of course, the mother of all bubbles began with Florida land speculation, as people would buy swampland that was touted as beautiful waterfront property; this then segued into massive stock market speculation, ending with The Great Crash of 1929.
- The marvel of 'junk bond financing' led to the fame of Michael Milken, as well as the movie *Wall Street* and the major stock market crash of 1987.
- While it did not occur in the U.S., we need to mention the Japanese housing bubble, circa 1970 to 1989, which upon bursting led to Japan's 'lost decade', one of a stagnant economy and 'zombie' banks.
- Back to the U.S. next, where speculation due to the commercial promise of the internet led to the 'dot-com' crash, from March 11th, 2000 to October 9th, 2002. Many of the internet dot-coms were listed on the Nasdaq Composite index, and it lost 78% of its value as it fell from 5046.86 to 1114.11; a truly dramatic crash.
- Finally, we are all familiar with the recent US housing bubble tied to subprime mortgages, and the creation of many three-letter acronym financial products, such as ABS, CDO, CDS, and even CDO^2. It is worth noting that the crash of 2007/2008, along with the one of 1929, escaped the economic borders of North America and thrust much of the world into economic depression.

Given this rather spectacular history of financial bubbles, not to mention the many bubbles in individual stock prices, it is perhaps surprising that nary a mention of the subject appears in the panoply of books that precede the text of Jarrow. This may be due in part to the controversy that surrounds even the idea of a financial bubble: Many noteworthy scholars (even Nobel Laureates) think the idea of a financial bubble is a fiction. After all, it goes against the ideology that markets are rational and always know best. Ideology is a powerful force, and it is not to be underestimated! Even worse, since the idea of how to model a bubble revolves around the concept of a strict local martingale, and such processes do not have a good analog in discrete time, many people think that is reason enough to dismiss the theory of bubbles.

In my view, admittedly hopelessly biased, kudos to Bob Jarrow for including treatment of bubbles in his book, and moreover, without any special mention, as if it is the most natural thing in the world to do so. That alone is a reason to read this book. There are other reasons, however, as Jarrow treats many of the modern subjects from the financial world, not just asset pricing, but—notably—credit risk, and in particular, reduced form credit risk. He integrates disparate sub-topics of the subject into one coherent, and if I may indulge myself, beautiful narrative.

This brings us to the key question that motivated this review. What is new in the Second Edition? Well. The First and Second Fundamental Theorems, are still buried deep inside the book (Chapter 17, circa page 350), which is unfortunate, but the content of Chapter 17 has broadened, as well as in Chapters 1–5, 9, 14, and 16. In Chapter 17 in particular, to quote the author, 'the static model ... is expanded to include a discussion of the fundamental theorems of asset pricing'. Jarrow's approach is taken from the wonderful book of Föllmer and Schied, which is a shame: He could have used this occasion to include the fundamental theorems for the continuous (and not the discrete) models. After all, one needs continuous time models to discuss financial bubbles! This is, however, nothing but a quibble. The book is a monumental

triumph, and the Second Edition has not made it worse, but better!

Finally, we have a modern, pedagogic treatise of the most important topics in the subject of financial mathematics.

Funding

This work was supported in part by NSF [grant number DMS-2106433].

Philip Protter
Statistics Department, Columbia University,
New York, NY, USA

Philip Protter is a Professor of Statistics at Columbia University in the City of New York. He is the author/co-author of three books and way too many research papers. He's given lots of talks in North and South America, all over Europe, North Africa, sub-Saharan Africa, Israel, Singapore, and Japan. He works in Probability Theory, with an emphasis in Mathematical Finance. He's a Fellow of the IMS, the AAAS, and Sigma Xi.