# Hilbert paradox of the Grand Hotel

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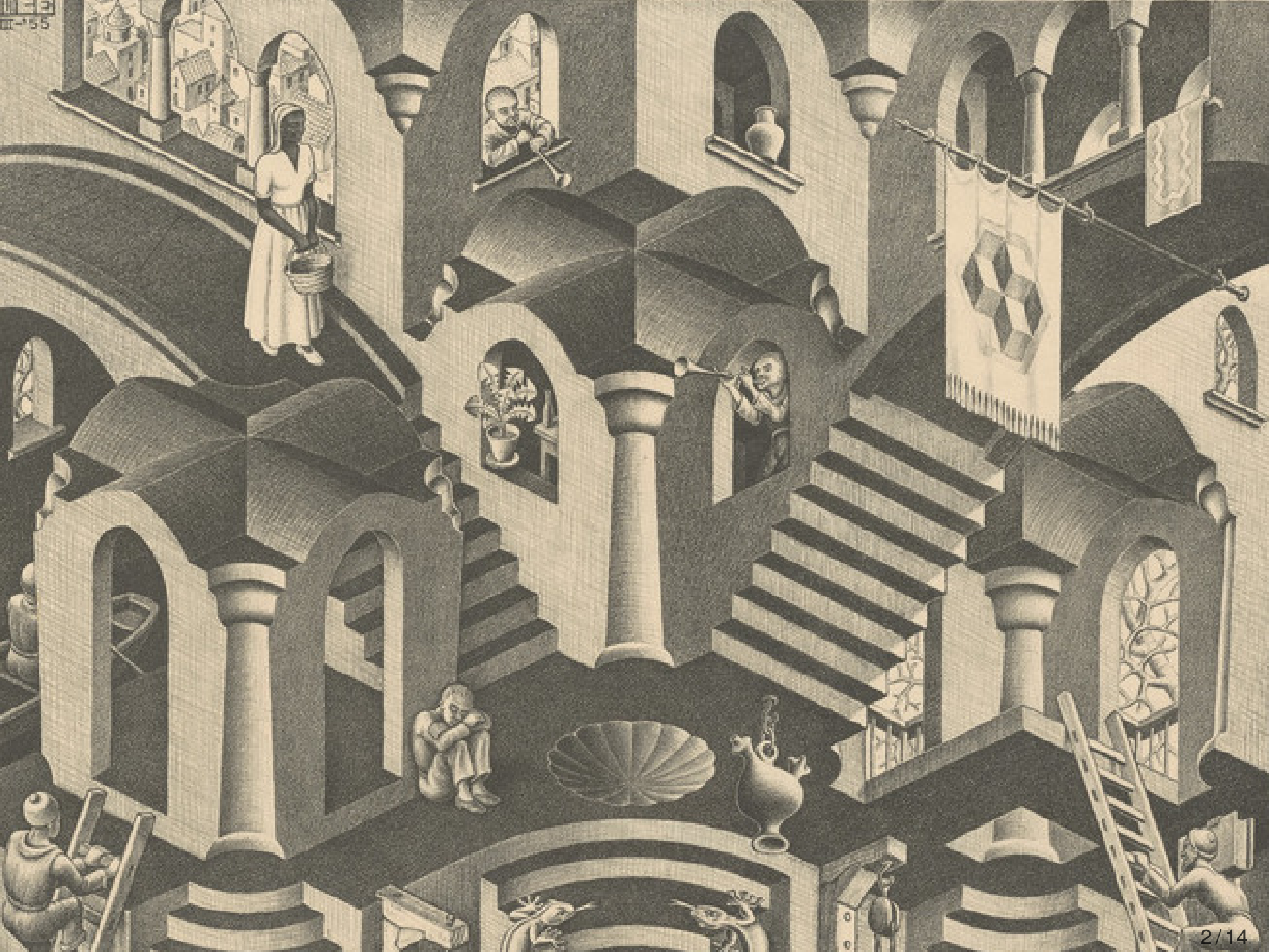
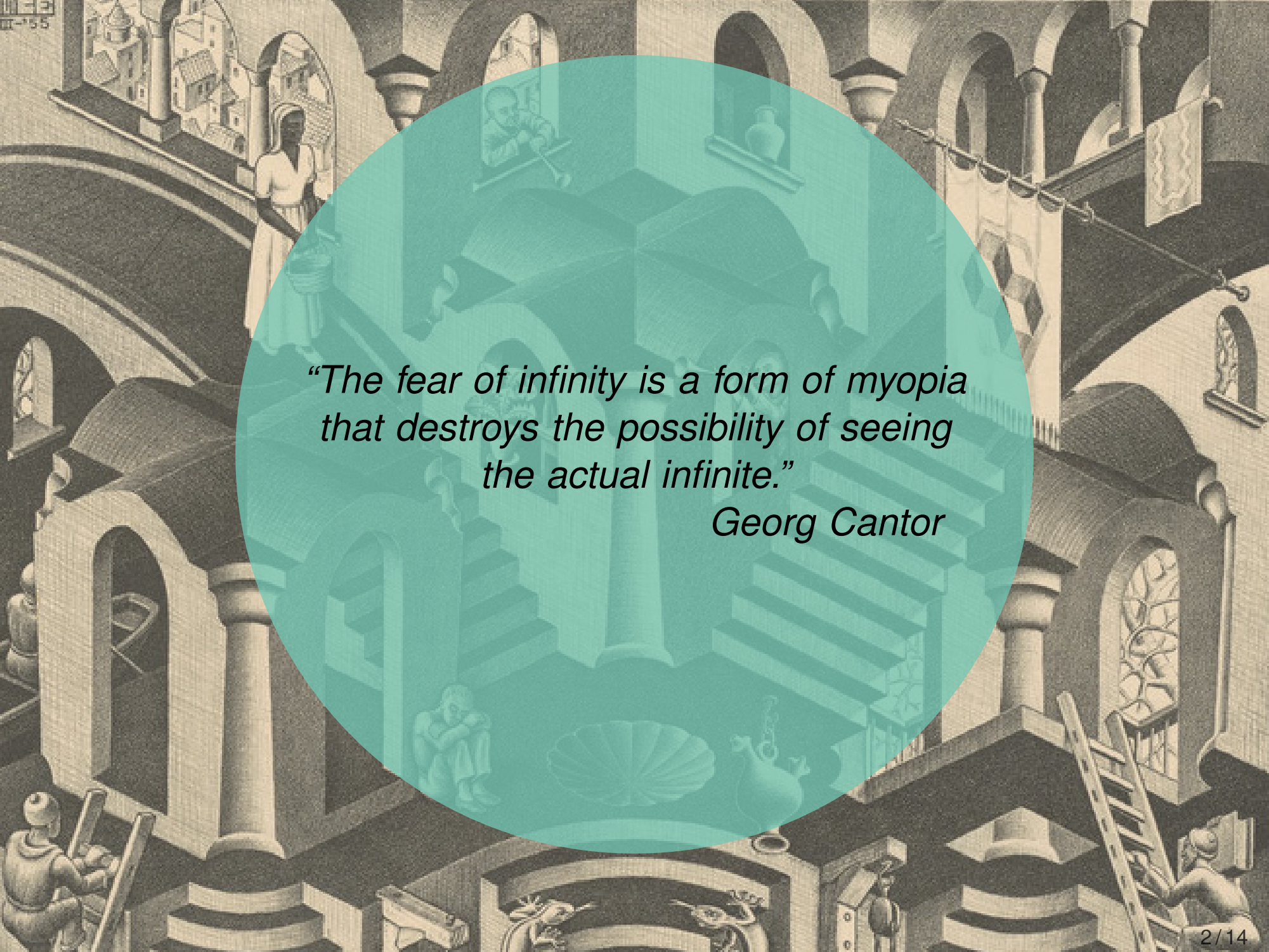
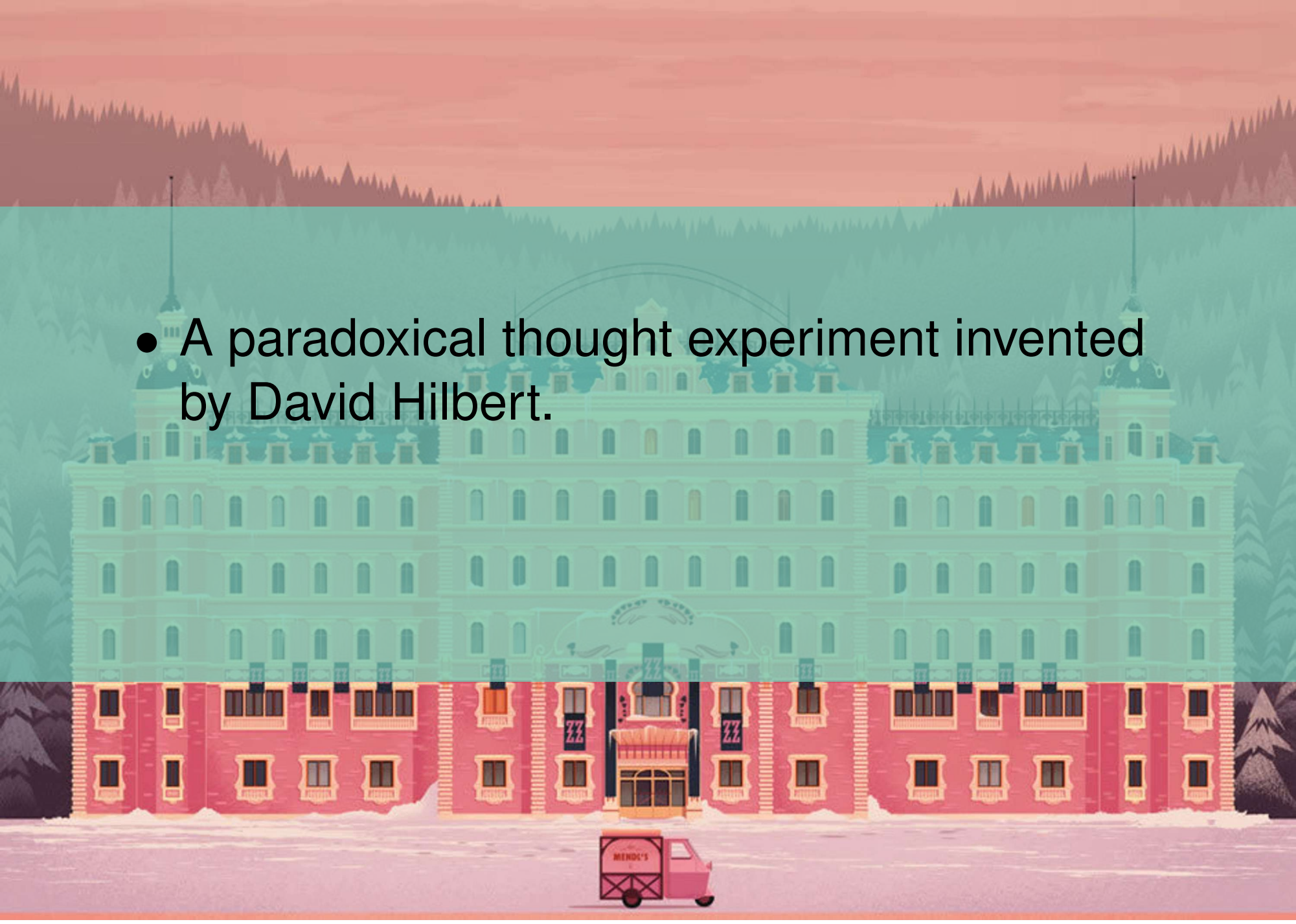


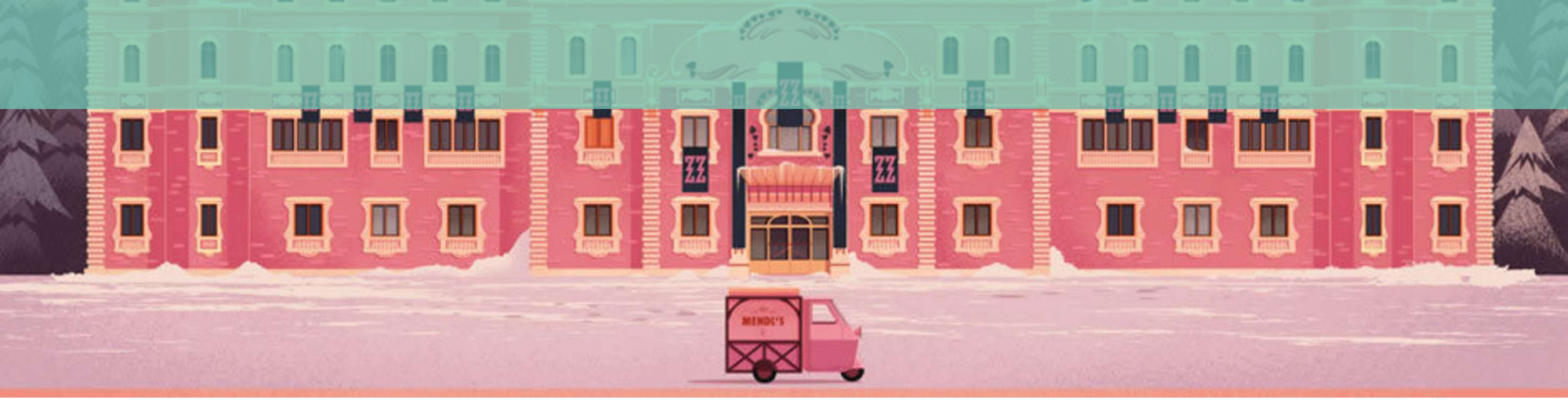

“The fear of infinity is a form of myopia
that destroys the possibility of seeing
the actual infinite.”
Georg Cantor

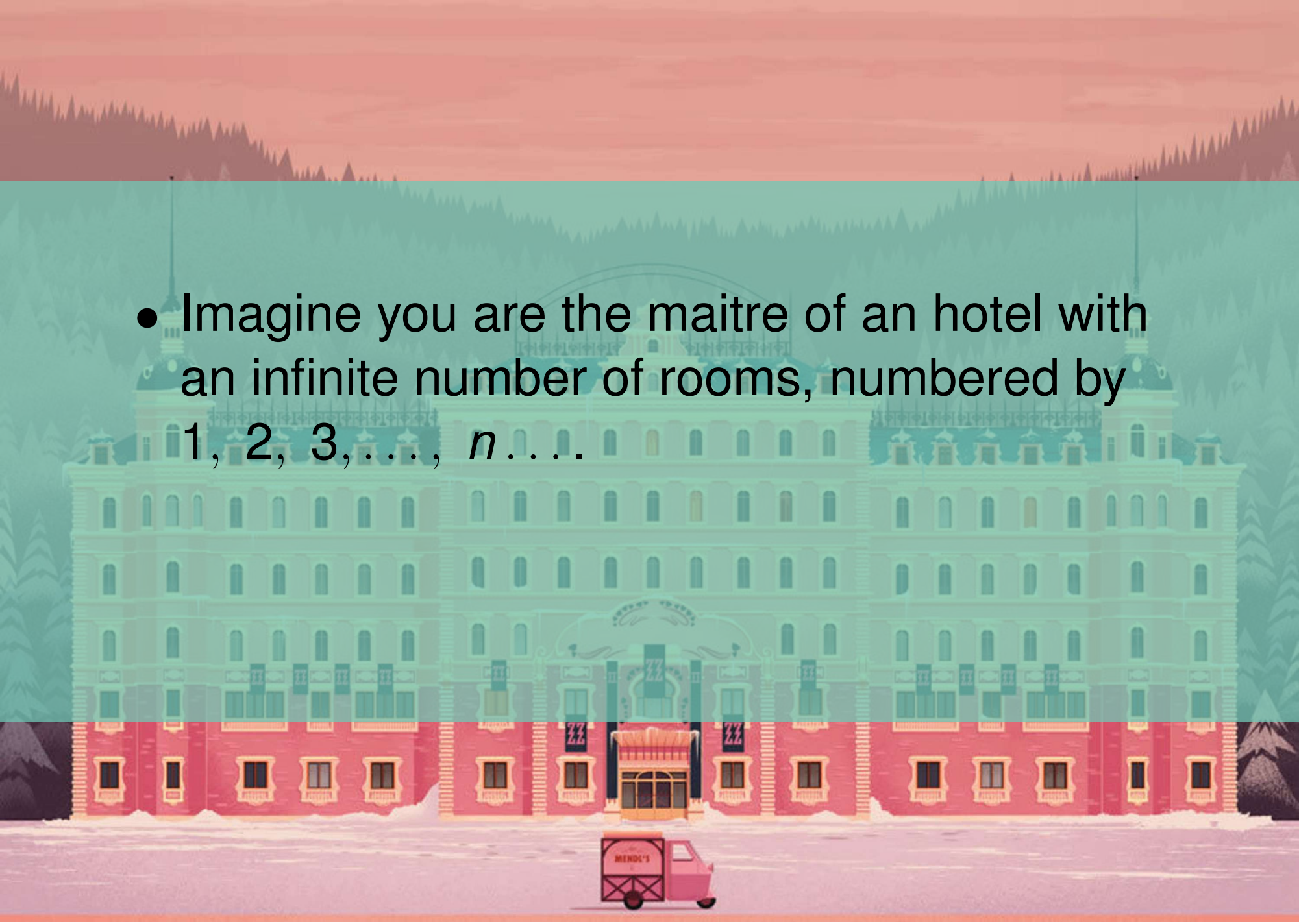
# WELCOME TO THE GRAND HILBERT HOTEL

- A paradoxical thought experiment invented by David Hilbert.

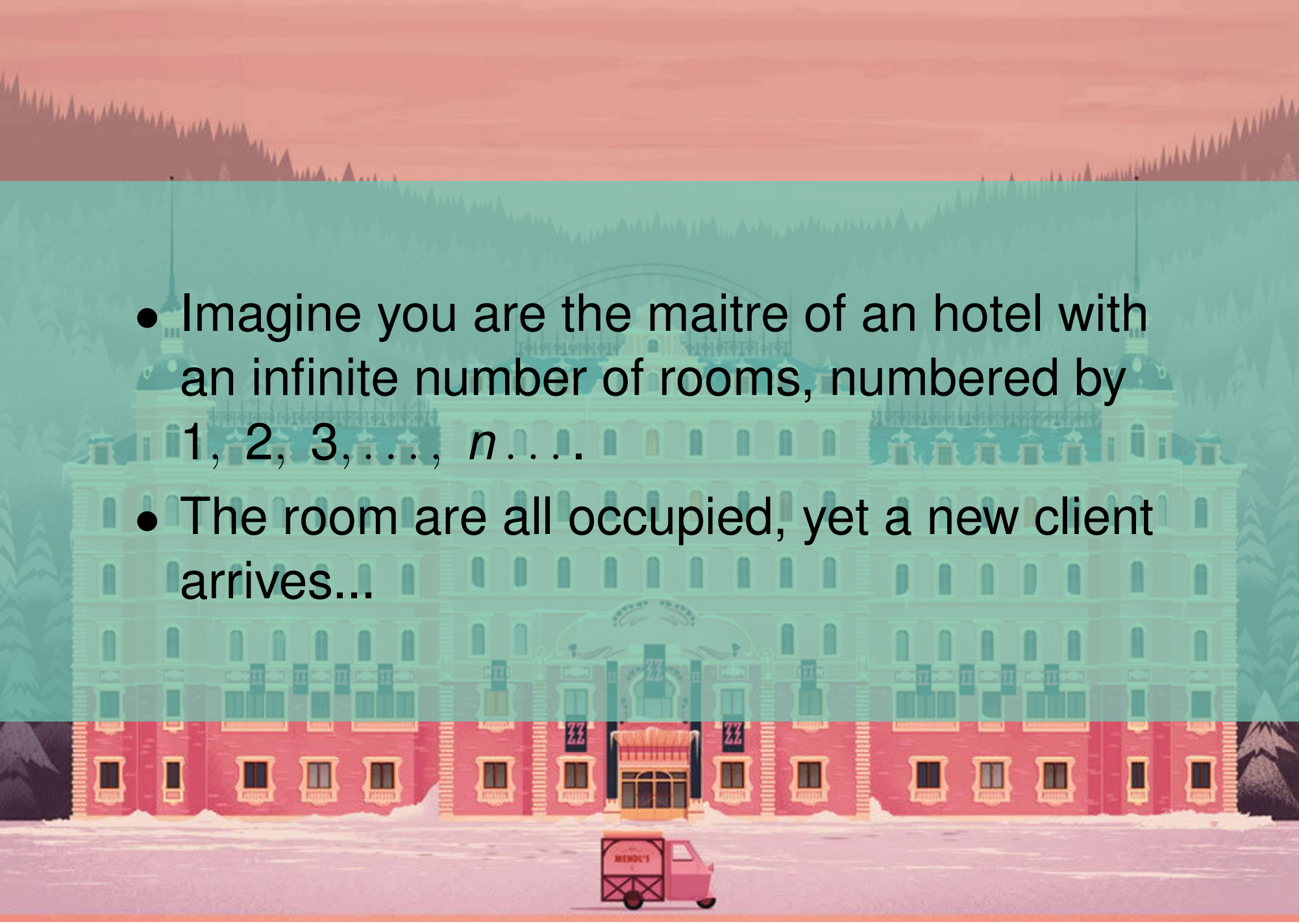


- A paradoxical thought experiment invented by David Hilbert.
- Its purpose is to show the counterintuitive, yet beautiful nature of infinity.





- Imagine you are the maitre of an hotel with an infinite number of rooms, numbered by $1, 2, 3, \ldots, n \ldots$.



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- The room are all occupied, yet a new client arrives... Can you find a room for them?

- Easy peasy!

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- Starting from the first room, you ask the person occupying room $n$ to move to room $n + 1$, and so on and so forth...

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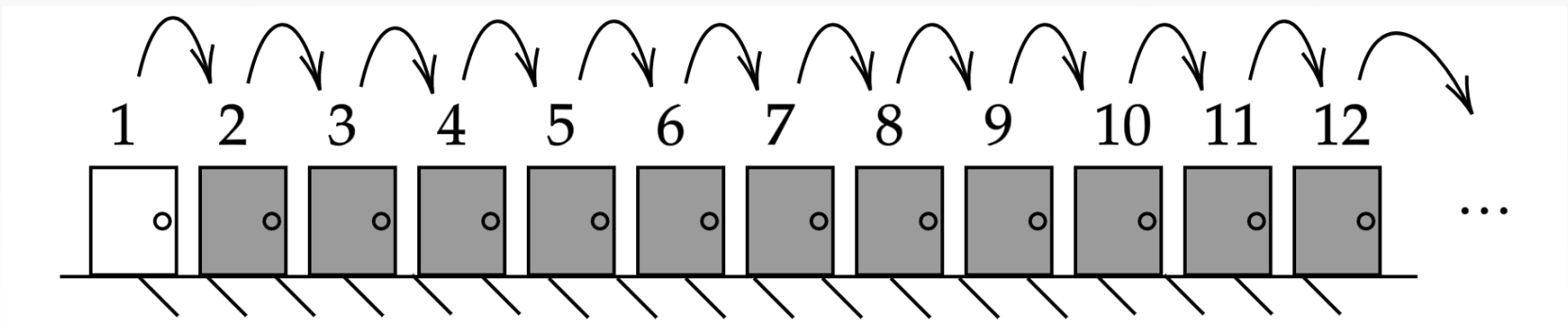
- At the end, room 1 is free for your new avventor!



- Now, a train with infinite seats arrives a the hotel.

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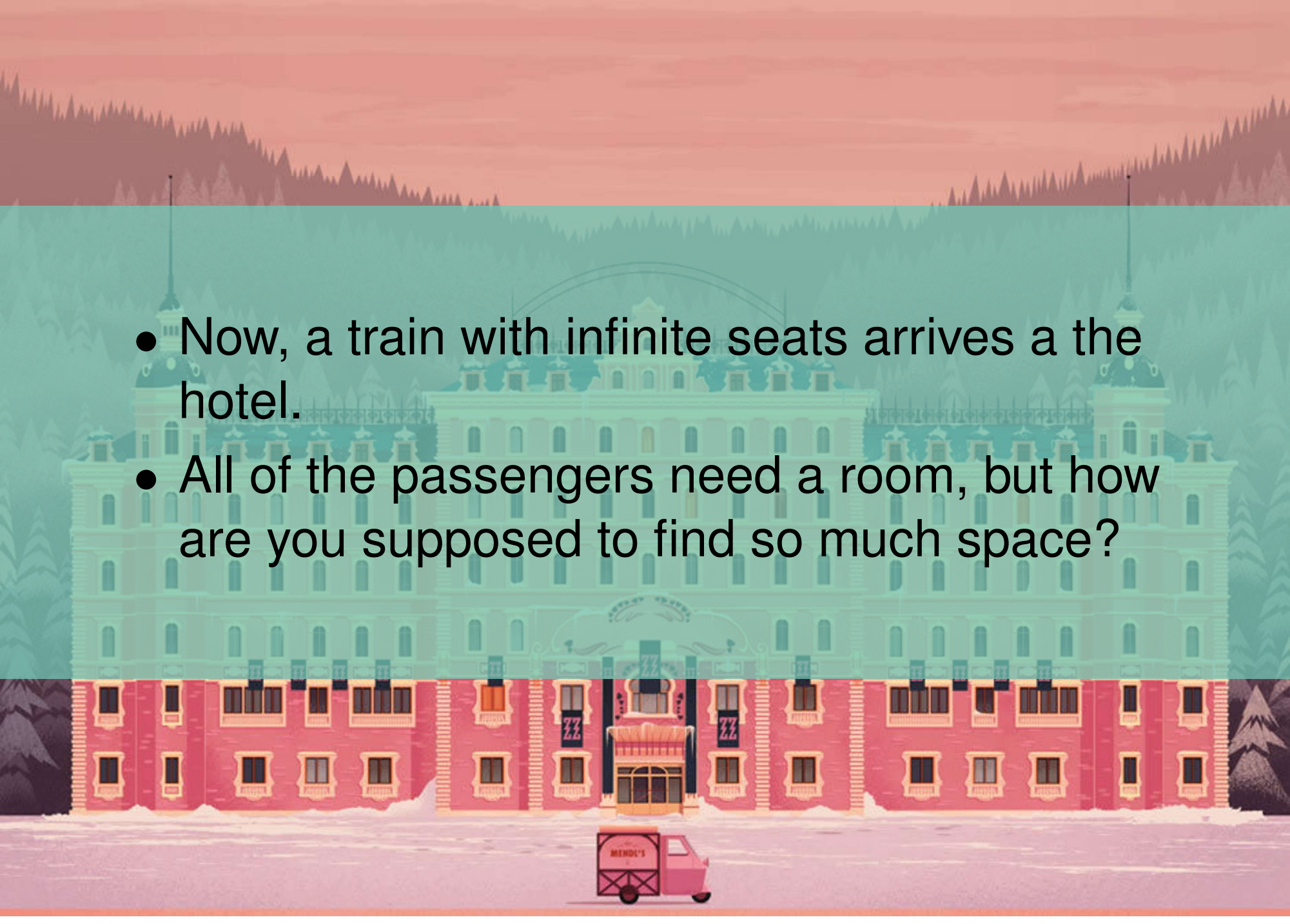
- All of the passengers need a room, but how are you supposed to find so much space?



- Once again, not so hard!

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- For each $n$, you move the person in room $n$ to room $2n$.

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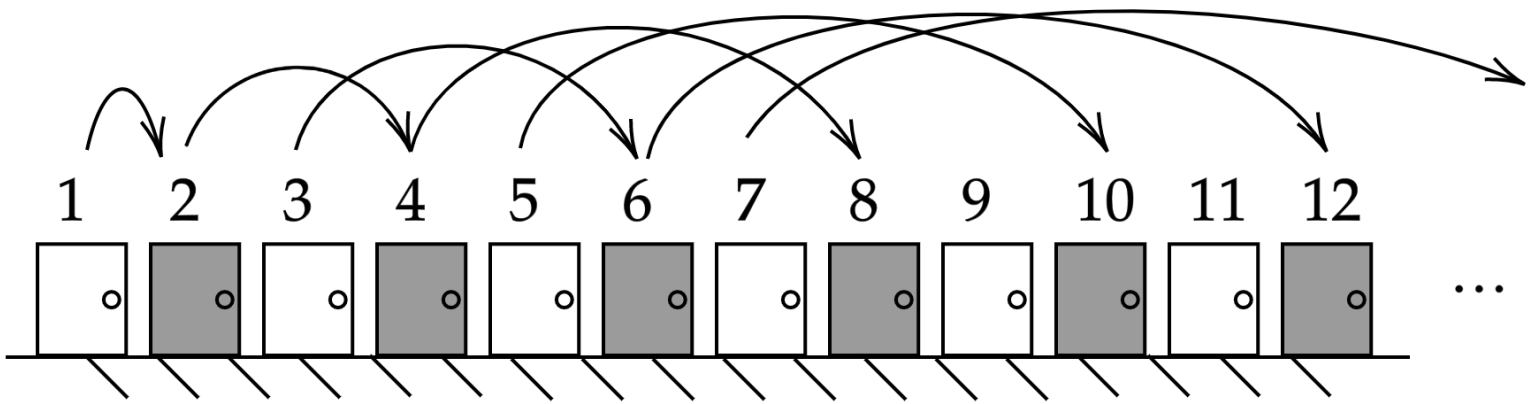
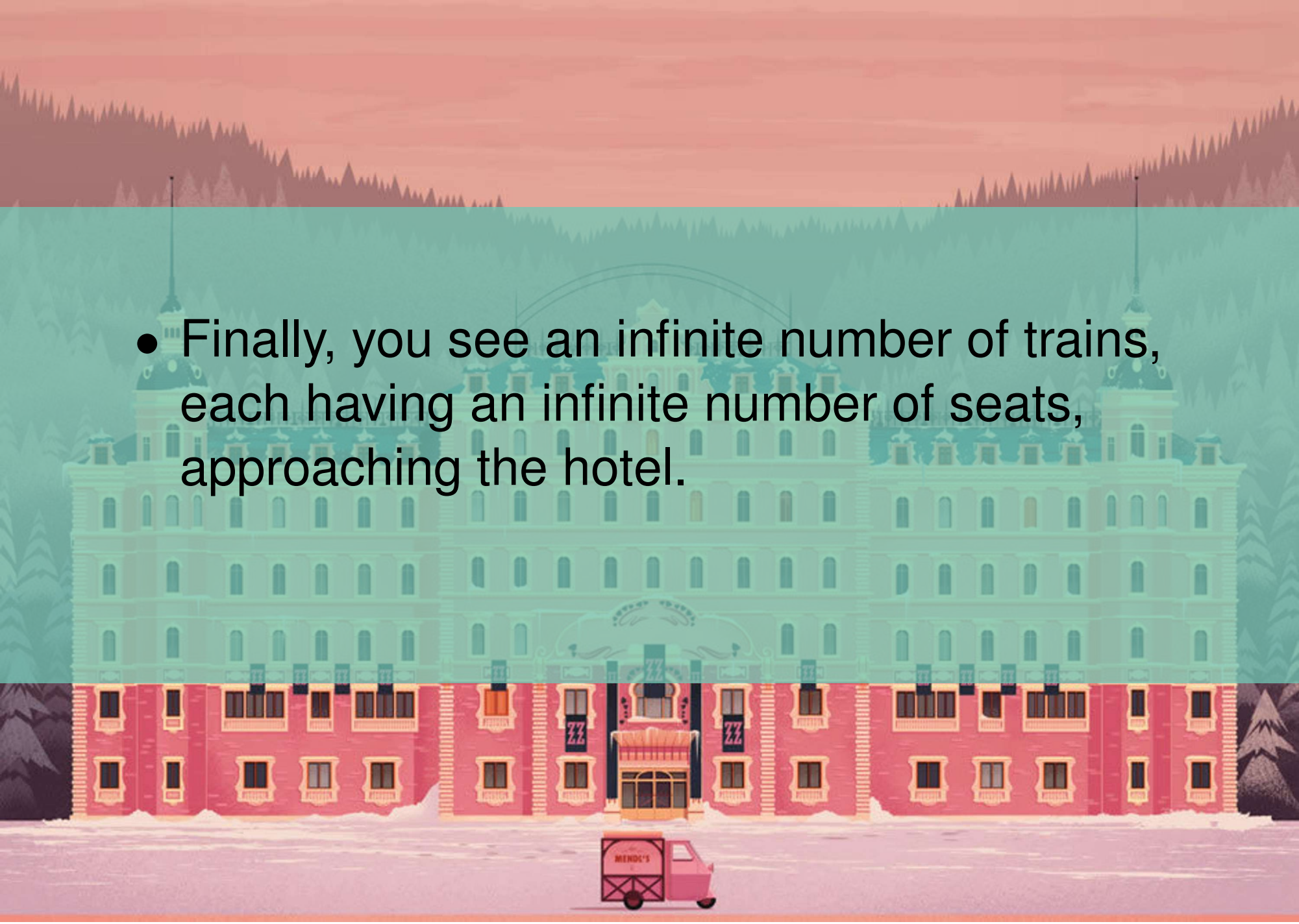
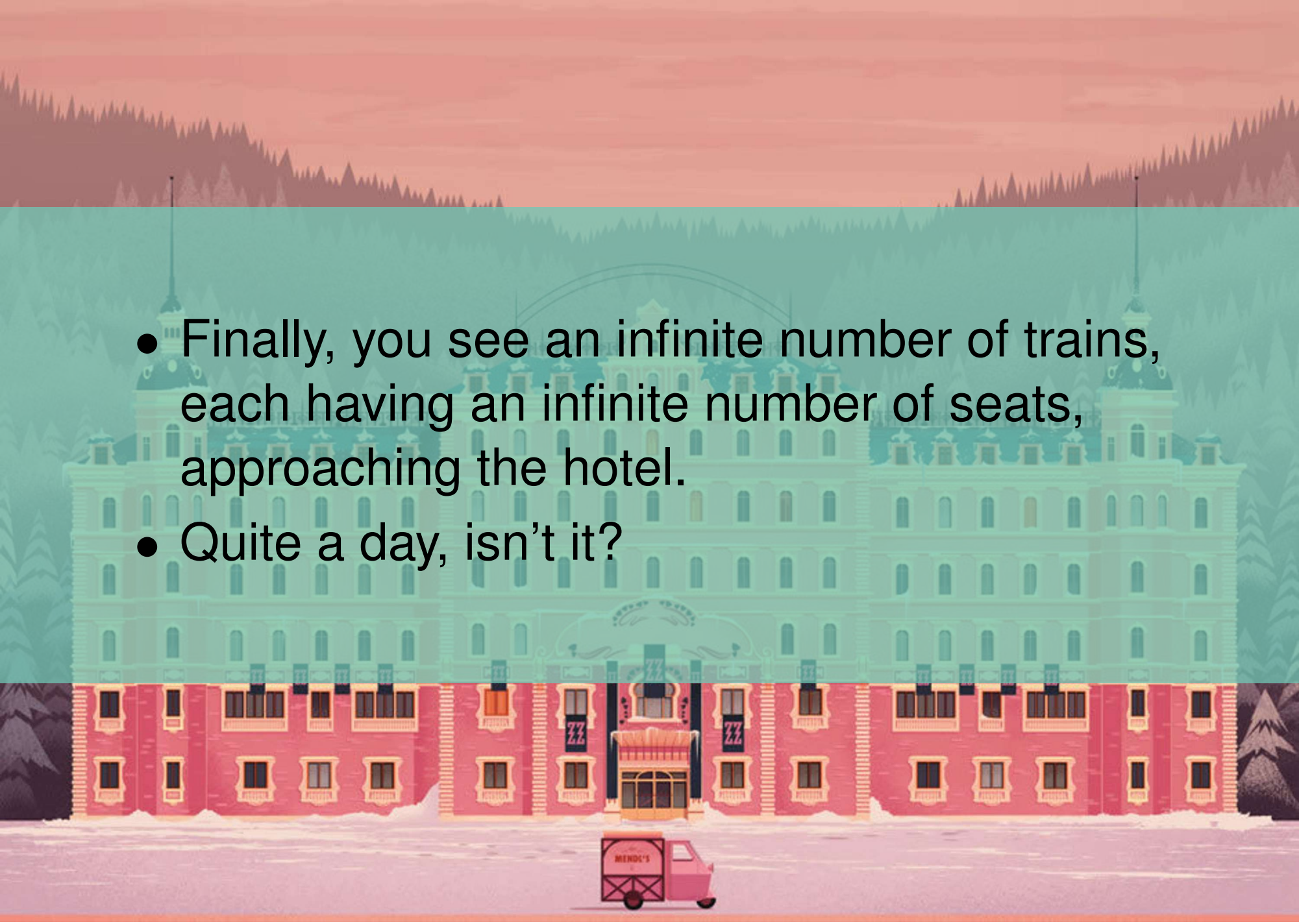
- Now, all odd numbered rooms are free for the passengers.



- Finally, you see an infinite number of trains, each having an infinite number of seats, approaching the hotel.





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- Quite a day, isn't it?

- This time, you really need to take your time. But then, you remember that there exists infinite prime numbers.

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- You take the person seated in the $m-$th seat of the $n-$th train, and you place them in the room $p_n^m$, where $p_n$ represent the $n-$th prime number.

- You did it! Everyone has a room, and everyone is satisfied.

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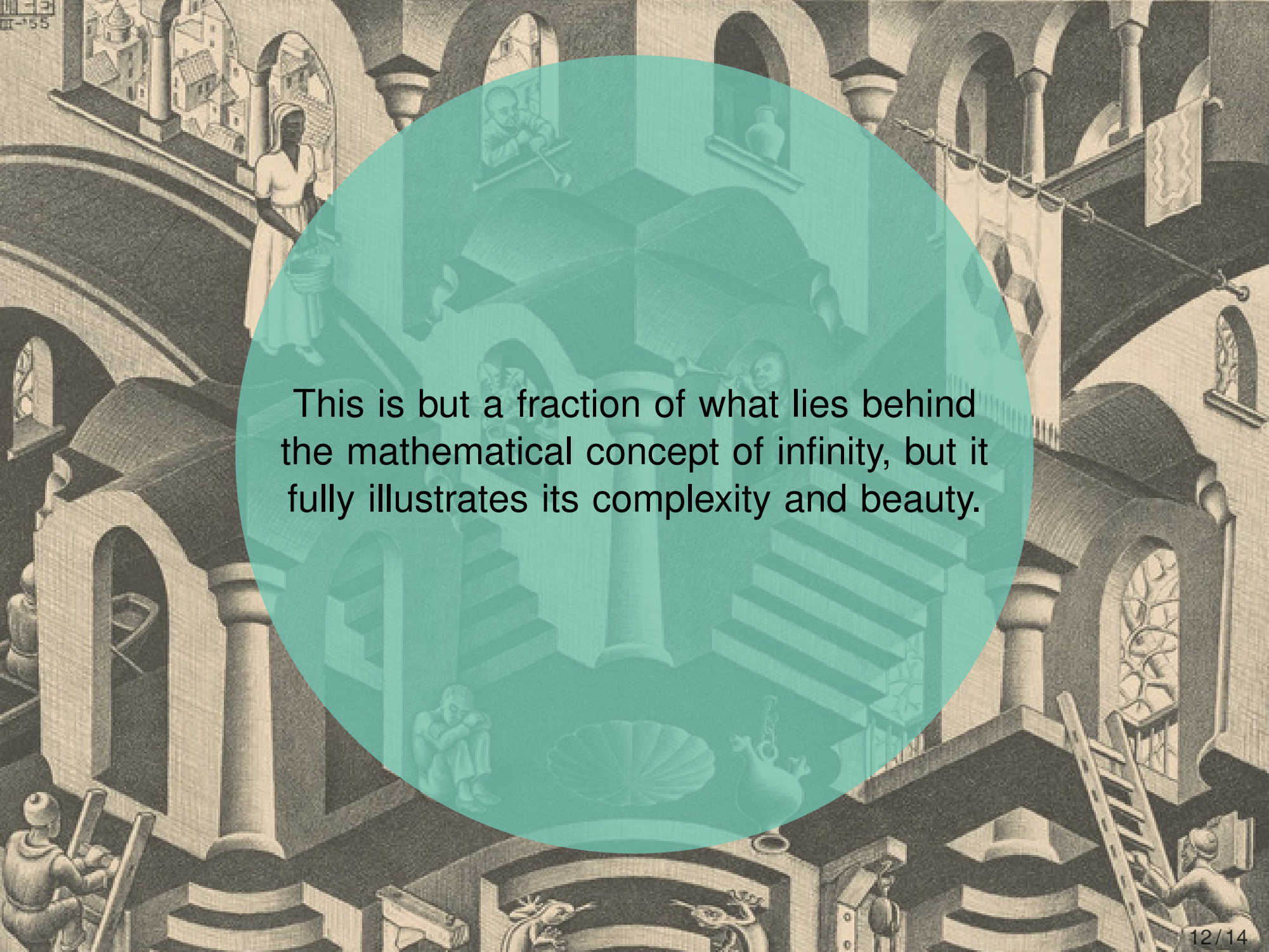
- But... wait a minute! Many rooms are now empty!
- This infinite hotels sure are strange...



This is but a fraction of what lies behind the mathematical concept of infinity, but it fully illustrates its complexity and beauty.


HOPE YOU ENJOYED
YOUR STAY!

# SOURCES

digitalcommonwealth.org
the-dots.com
jpmacmanus.me.