EML3005 Fall 2021 Project

Group 09: Nick Hansen, Chris Crouch, Nick Bernardoni

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Final Report: Basketball Hoop Raiser

Section 1.0: Need

There is a need to raise a basketball net in a timely manner.

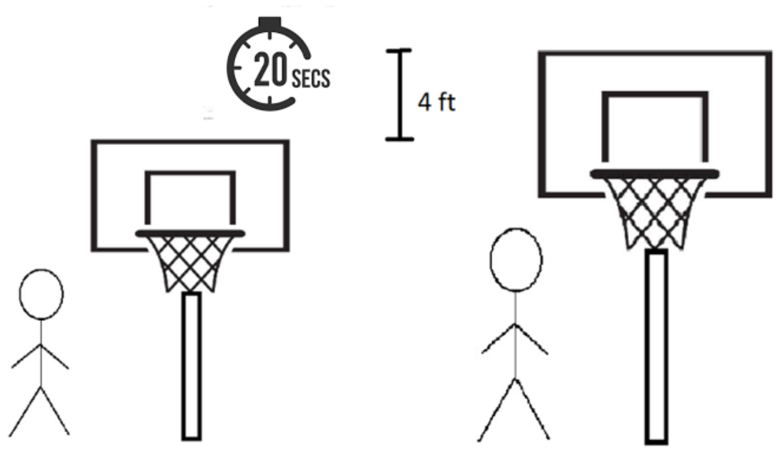


Figure 1: Scenario

Section 2.0 Specs.

Figure 2: Free body diagram of system output

The basketball hoop is being used my multiple teams of different ages that need the goal height to be different. The hoop will need to be raised from its lowest height of h_{min} = 6 ft to its highest at h_{max} = 10 ft. The system will have a self-locking mechanism once the raising is completed. The basketball goal will be a 48-inch-wide steel frame backboard with a mounting bracket [1, 2]. A 6 ft steel pole is attached to the mounting bracket that will slide along the main beam in the ground [3]. The gross weight of the backboard and hoop assembly that needs to be moved is F_O = 75 lbf. The average time for a basketball hoop to be raised from its proper position to the ceiling in current gyms is 55 seconds, however, since the goal is moving less than half of the distance (from 6ft to 10ft), t = 20 seconds is sufficient [4]. If the teams have a 10-minute warm-up time, raising the goal for 20 seconds would only take up a small fraction of that time (3.3%), which could be used for non-shooting practice. The assembly needs to be raised by 4 ft over 20 seconds, which gives a speed of v_O = 0.2 ft/sec = 2.4 in/sec. This movement will be powered by an electric motor at the input.

$$\left(75\ lbf * 0.2\frac{ft}{s}\right) * \frac{1\ Hp}{550\ lb\frac{ft}{s}} = \ 0.027\ Hp$$

To produce this output, the motor will need to produce H_O = 0.027 Hp.

Section 3.0 Concept.

The backboard and hoop assembly will be mounted on a steel pole that is supported by the mounted pole. The support pole will be able to slide along the mounted pole as it's pushed up by the motor. The motor will drive a gear system that ends on a rack and pinion, where the rack is mounted on the support pole. This will push it upwards to the needed height. Once the system gets to its highest point and the motor is turned off, a self-locking mechanism will keep the system in place.

3.1 Overview

The system will have 3 stages, the worm gear, a spur gear, and a rack and pinion. Worm Gear Stage 1 will be between gear 1 and gear 2, which will have a ratio G_1 of 30 and an estimated efficiency of η_{12} of 0.65. Gear 2 drives the countershaft which drives the next spur gear stage between gears 3 and 4. G_{34} is 4 and operates an efficient of $\eta_{34} = 0.85$ [5]. The 4th stage of rack and pinion consists of gear 4 driving the rack that is mounted on the backboard assembly with a G_{45} of 1 and efficiency of $\eta_{45} = 0.85$.

SolidWorks Concept Images:

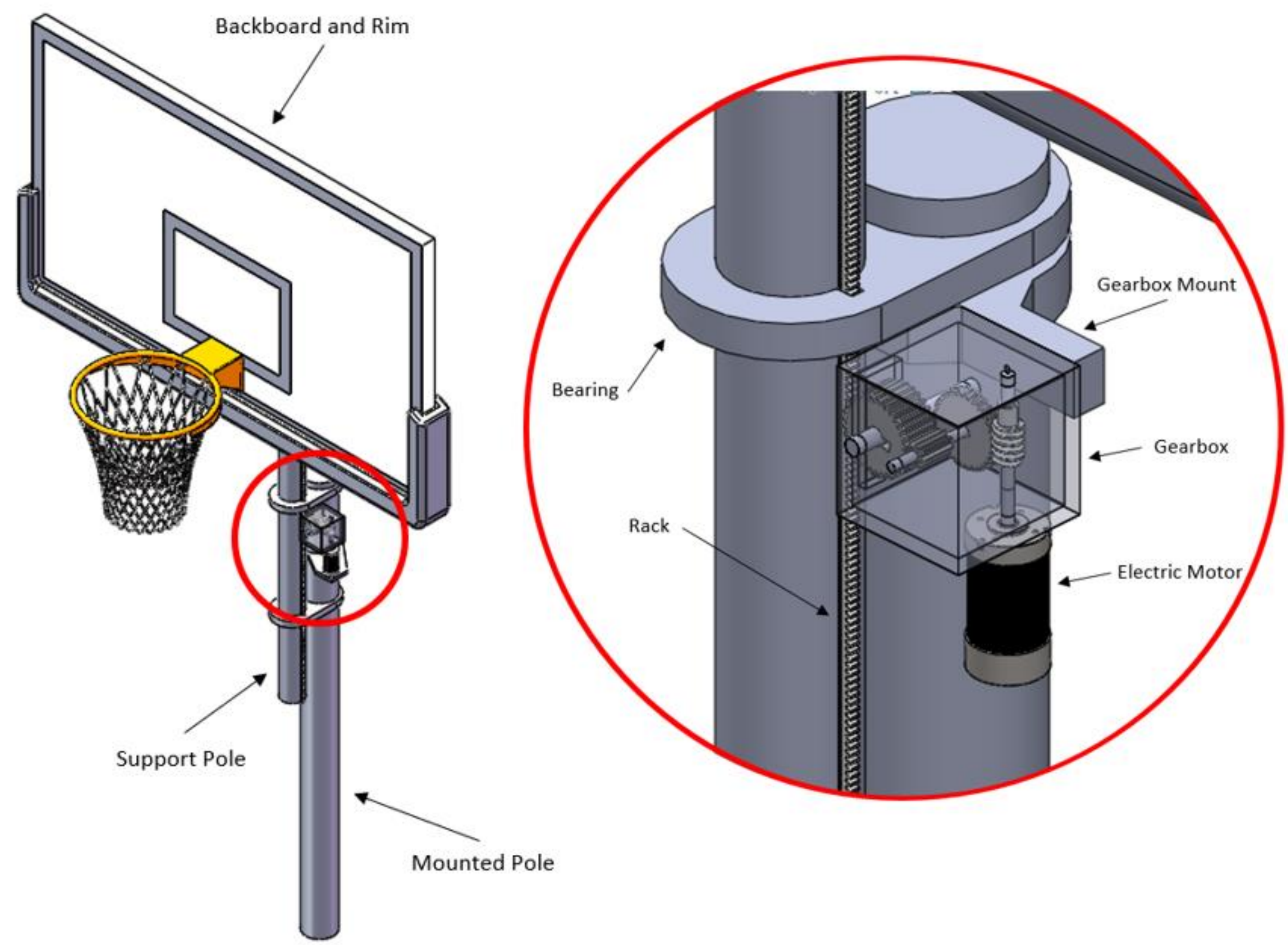


Figure 3: Concept Presentation

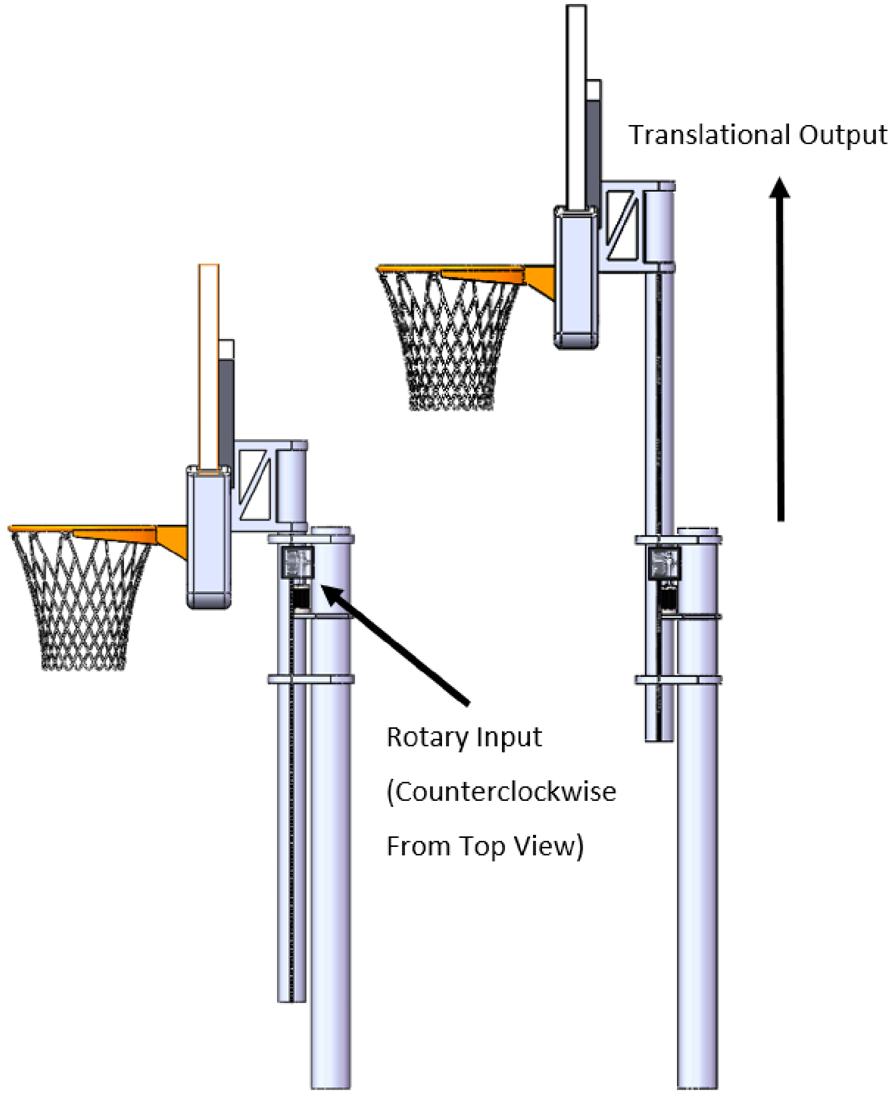


Figure 4: Motion

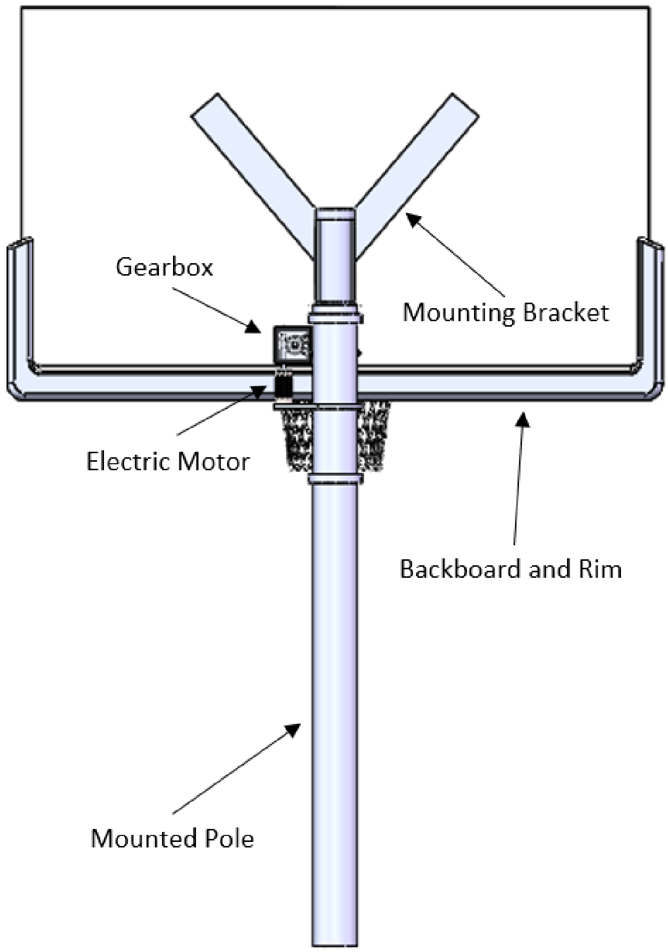


Figure 5: Back View

Section 3.2

The input of power is a supplied by brushless electric DC motor. This motor is a rotary input which will turn a worm gear. The input worm gear will need to spin at n_1 = 2750.1 RPM with an input torque of T_1 = 1.317 lbf-in. The first stage is a worm to worm gear reduction. The worm gear is situated on a countershaft that spins at $n_2 = n_3$ = 91.67 RPM and a torque of $T_2 = T_3$ = 25.69 lbf-in. At the end of the countershaft, a spur gear turns another spur gear on a parallel shaft. This gear is reduced to a speed of n_4 = 22.92 RPM with a T_4 = 87.35 lbf-in torque. Finally, that gear acts as a pinion to translate a rack vertically, at a speed of v_O = 2.4 in/s and an output load of F_O = 75 lbf.

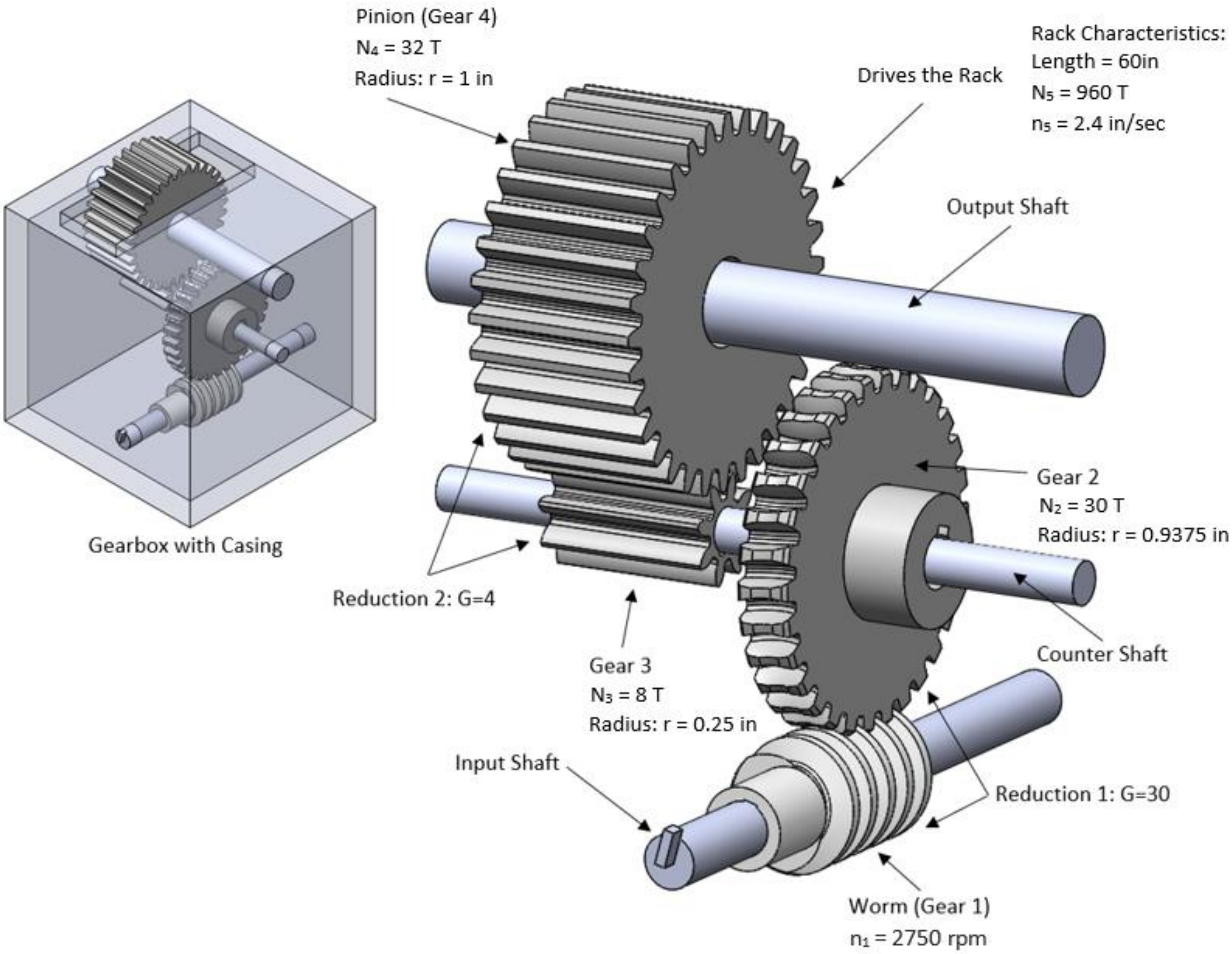


Figure 6: Gear system driven by the motor

Section 4.0 Analysis.

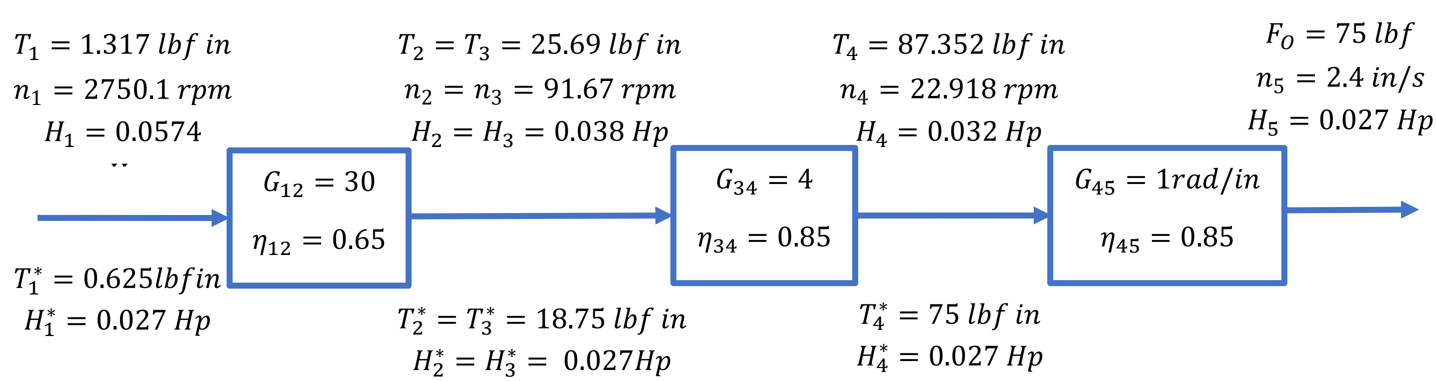
Overview

The analysis section of the report will cover the calculations to determine if different parts of the design will meet the target specs. It will dive into the formulas and thought processes behind different design decisions and provide SolidWorks simulations and studies to verify these calculations and gain insight on a realistic representation. The analysis section will focus on the entirety of the system, each stage within the system, and designing certain elements within the gearbox to maximize safety.

4.1 Power Flow

The process for calculating all the different specifications for the system started with the output. Since the force that the backboard would push down and its speed was known, working backwards through each stage was the strategy. To start off, we knew what each stage consisted of, worm gear, spur gear, or rack and pinion, which allowed us to estimate an efficiency based on the stage type. Efficiencies for worm gear were 0.65, spur gear were 0.85 and rack and pinion were 0.85 [5]. Next, gear ratios were needed from each stage which were found by guessing and checking common values for the stage types and computing the necessary input torque and speed to check if the values were logical and matched with motors rated values. From this process a system wide gear ratio G_s = 120 rad/in was chosen with stage gear ratios as mentioned above. The necessary angular velocity was then found to be $\omega_i = G_{sys} * V_o = (120\frac{rad}{in}) * (2.4\,in/s) = 288\,rad/s$. Then, the necessary rating of the motor was $n_i = \left(288\frac{rad}{s}\right)\left(\frac{1rev}{2pi}\right)\left(\frac{60s}{1min}\right) \approx 2750.1\frac{rev}{mn}$. Based on the efficiencies of the stages, the overall efficiency is efficiency was $\eta_s = 0.65 * 0.85 * 0.85 = 0.47$. Therefore, the overall mechanical advantage was $m_s = 0.47 * 120 = 56.4$. The necessary power of the motor was $H = \frac{0.027}{0.47} = 0.574\,Hp = 42\,Watts$. The motor chosen has a higher rated power of 92 Watts = 0.123 Hp, with a higher rated speed of *n* = 4000 rpm so it can run at n_i = 2750 rpm for an output time of t_O = 20 seconds.

Power Flow Diagram



A SolidWorks motion study of the full system was performed to simulate the power flow through the system. The motion study utilized solid body contact within each stage, including the rack and pinion, to simulate the required motor torque and velocity output. Figures 8 and 9 were created through the results of the SolidWorks motion study. All the data was run through a filter and the final torque and velocity was calculated from the average of that list. The motor torque and output linear velocity were used to estimate the efficiency of the system along with the gear ratio.

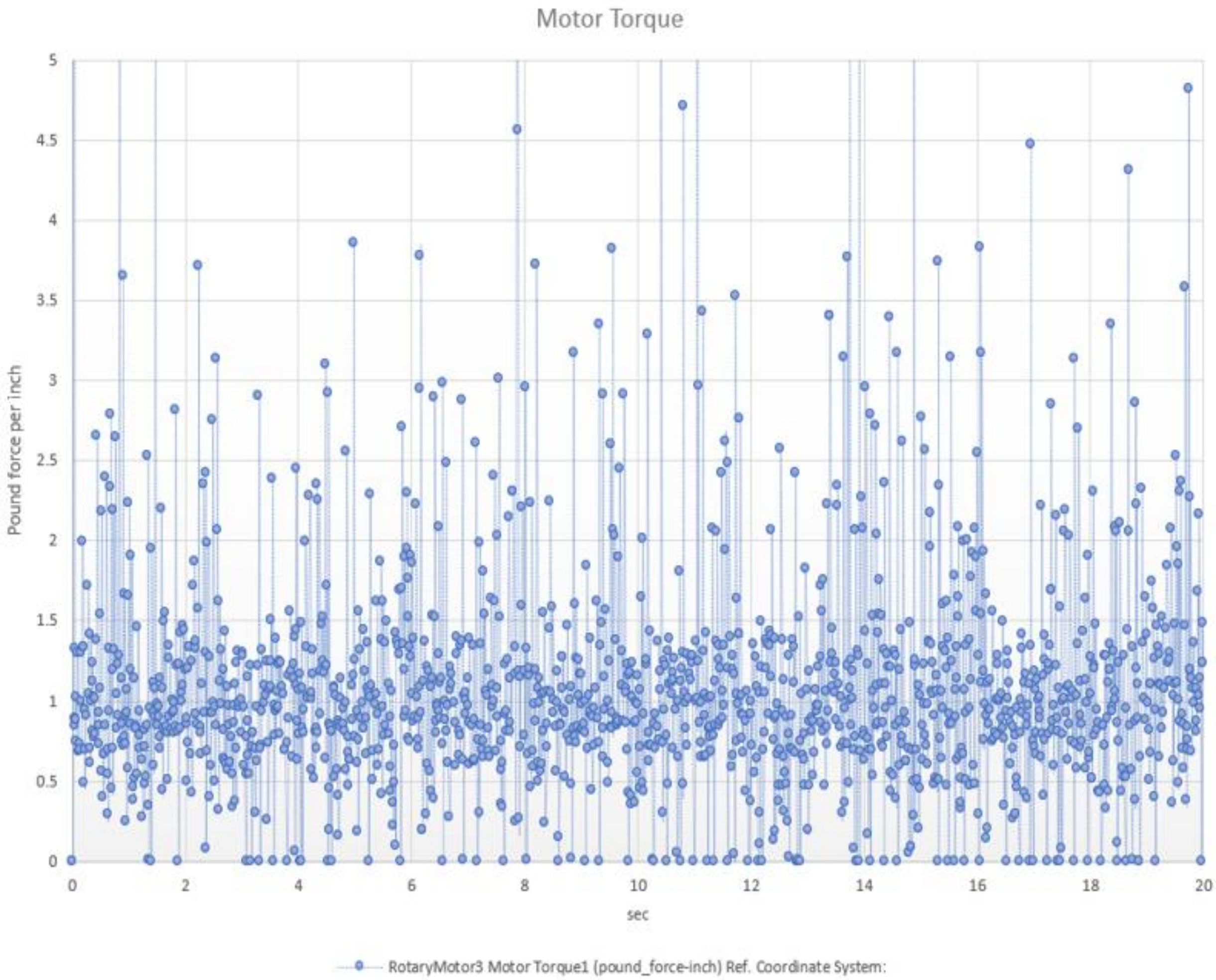


Figure 7: SolidWorks Motor Torque Results

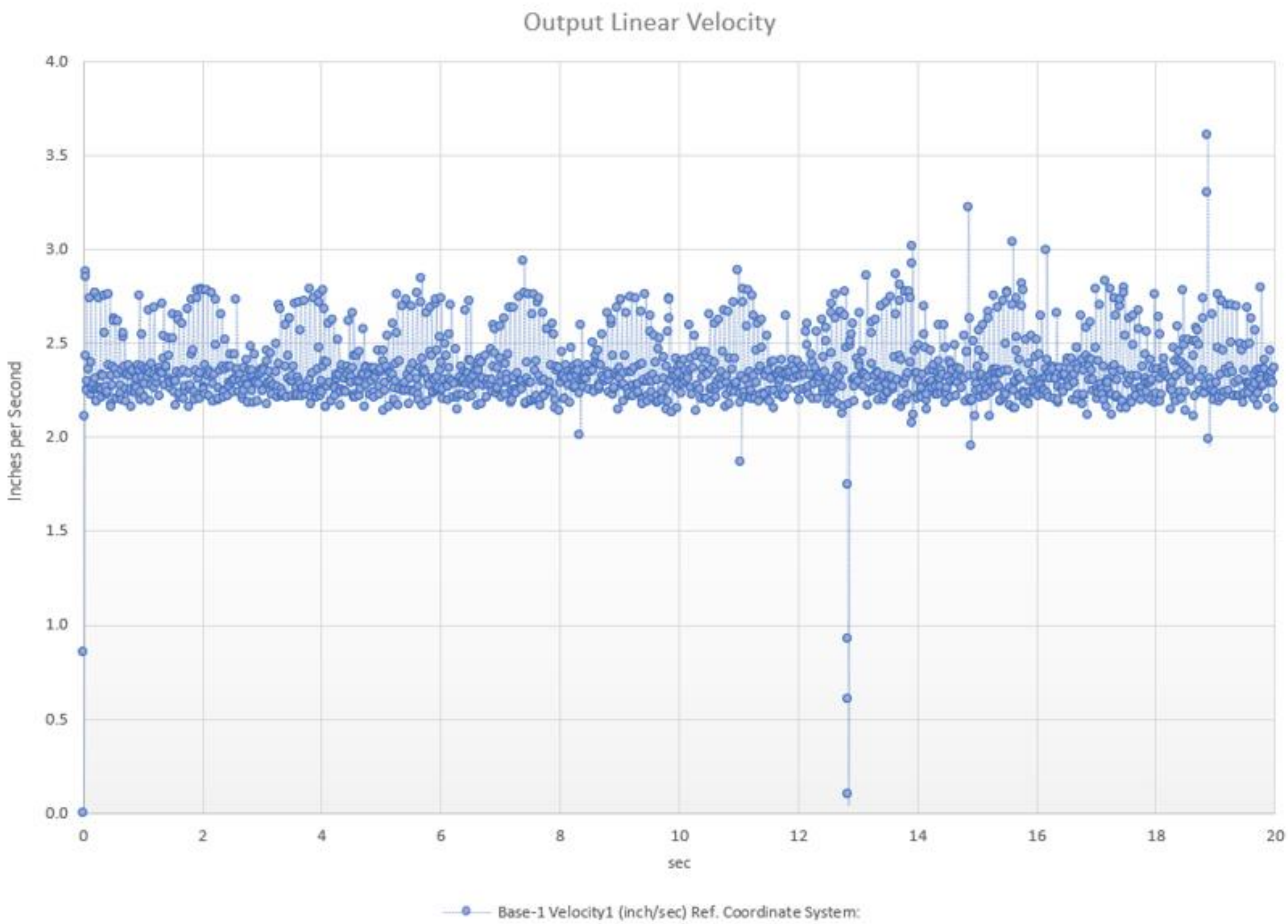


Figure 8: SolidWorks Output Linear Velocity Results

From the study, the input torque was found to be T_1 = 1.372 lbf-in and the output speed of the rack/basketball hoop was V_O = 2.35 in/s. The overall gear ratio of the system was calculated to be $G_{sys} = \frac{\omega_i}{V_o} = \frac{288\, rad/s}{2.35\, in/s} = 122.463\, rad/in$

and the system efficiency was found to be $\eta_{sys} = \frac{F_0}{T_i} = \frac{75\, lbf}{1.372\, lbf*in} = 0.546\ or\ 54.6\%$. Figure 9 represents the overall results from the motion study.

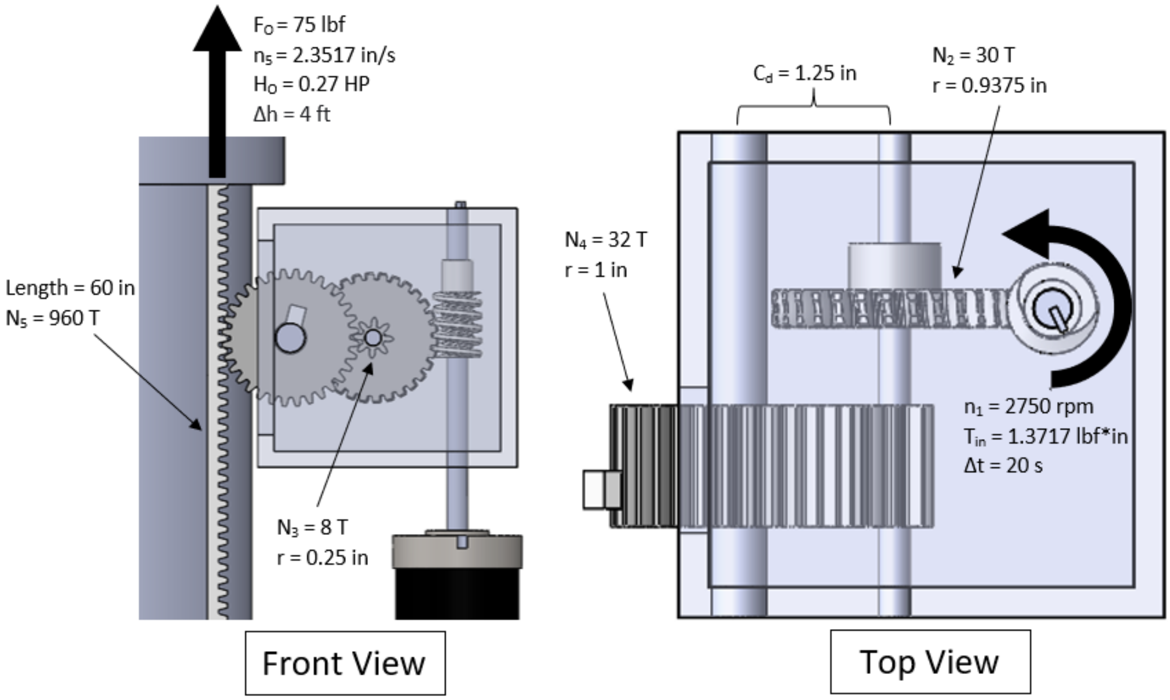


Figure 9: Results of SolidWorks Motion Analysis on the Full System

Theory vs. SolidWorks

	Gear Ratio (rad/in)		Efficiency	
	Theoretical	SolidWorks	Theoretical	SolidWorks
Entire System	120	122.463	0.47	0.547

Table 1: Theoretical Vs. SolidWorks on Full System

The relative difference between the theory and the SolidWorks simulation is 2.032% for the gear ratio. For the system's efficiency, the relative difference is 15.143%.

The efficiency was higher than expected, likely due to conservative estimates of individual gear efficiencies. The gear ratio was also slightly higher, possibly due to the spacing of the gears in SolidWorks to prevent contact errors. The input torque was slightly higher, and the output speed of the rack was slightly lower than calculated. All values were like the calculated counterparts. One margin of error that may have impacted the values was the input speed used. Rather than an input speed of 2750.1 rpm, a rounded input speed of 2750 rpm was used.

4.2 Stage by Stage Details

Stage 1 is the biggest reduction in the system is $G_{12} = \frac{N_2}{N_1} = \frac{30\ T}{1\ T} = 30$, which features the input motor driving a worm which then drives a worm gear. An estimated efficiency for a worm-to-worm gear stage is 0.7 and to add extra safety it was lowered to 0.65. The face width of the worm gear is F_G = 0.63 inches to match distance tangent to the worm pitch circle between its points of intersection with the addendum circle [5]. The input and output shaft of this stage are perpendicular to each other because of the nature of a worm gear, which allows the design to fit into more of a compact cubic shape on the side of the basketball hoop. The stage 1 output shaft is the countershaft of the entire system and what drives gear 3 in stage 2.

Stage 2 consists of a two spur gears that have a ratio of $G_{34} = \frac{N_4}{N_3} = \frac{32\,T}{8\,T} = 4$. This is an intermediary stage to further reduce the system. Similarly, to stage 1, the efficiency was estimated and lowered to be 0.85 [5]. The face width of each gear was found using equation (14-2) [5], which uses the Lewis Bending Equation for the tooth of the gears. The tooth load, W^t, was equivalent to the torque T on each gear divided by the radius of the gear. The gears are formed from 1030 hot rolled steel, which has a yield strength of σ_y = 30 kpsi, and with a factor of safety of n_d = 3 assumed for the bending, the allowable stress was σ_{all} = 10 kpsi. The torque on Gear 3 was T_3 = 25.69 lbf-in and its radius was r_3 = 0.25 in. With diametral pitch of 16 T/in and Lewis form factor Y = 0.144 for a 14.5° pressure angle from the curve of the set of factors [6], the face width of Gear 3 was found as

$$F_{S3} = \frac{W^t P}{Y\sigma_{all}} = \frac{T_3 P}{Y r_3 \sigma_{all}} = \frac{(25.69\,lbf - in)(16\,T/in)}{(0.144)(0.25\,in)(10000\,lbf/in^2)} = 1.141\,in$$

which was rounded to F_{s3} = 1.15 in. Similarly, the face width of Gear 4 was calculated using a form factor of Y = 0.322 [6], torque T_4 = 87.352 lbf-in, and radius of r_4 = 1 in.

$$F_{S4} = \frac{T_4 P}{Y r_4 \sigma_{all}} = \frac{(87.352\,lbf - in)(16\,T/in)}{(0.322)(1\,in)(10000\,lbf/in^2)} = 0.434\,in.$$

This was rounded to find the face width of Gear 4 as F_{s4} = 0.45 in.

Gear 4 is the output of this stage but also drives the rack of stage 3, with a gear ratio of 1 rad/in, which raises the basketball hoop. Stage 3 is a rack and pinion which is the final stage and has an efficiency of 0.85. The system produces an output of 0.027 Hp that is needed to drive the basketball hoop upwards at 2.4 in/s.

The original motion study in SolidWorks was used to determine the stage-by-stage gear ratios by calculating the angular velocity of gear 2 and 4 (pinion). Note: the worm (gear 1) has a known rpm, gear 3 rotates on the same shaft as gear 2, and the rack's linear velocity was previously solved for in figure 8. The findings of the angular velocities of gear 2 and 4 are shown below in figures 10 and 11.

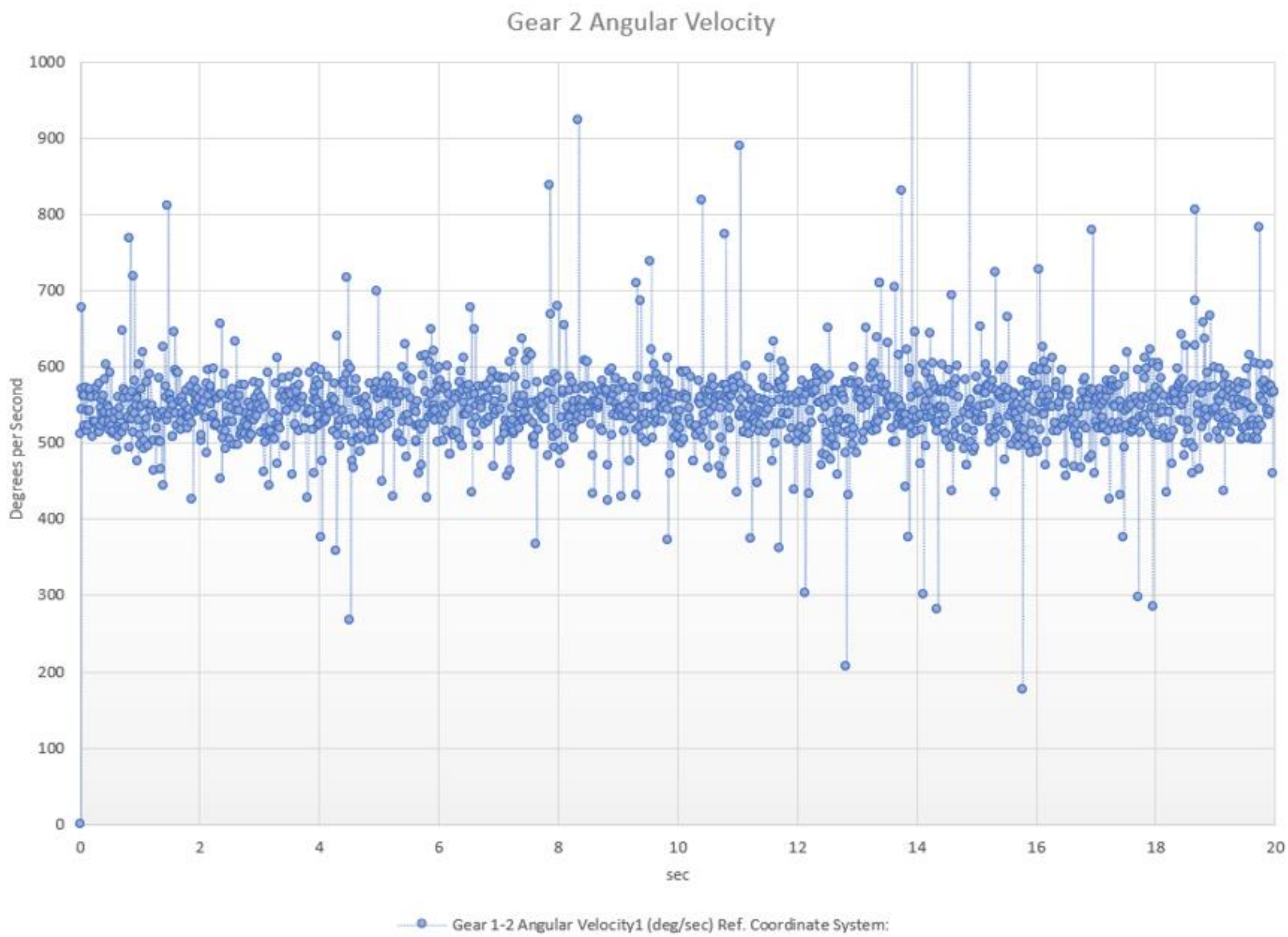


Figure 10: SolidWorks Gear 2 Angular Velocity Results

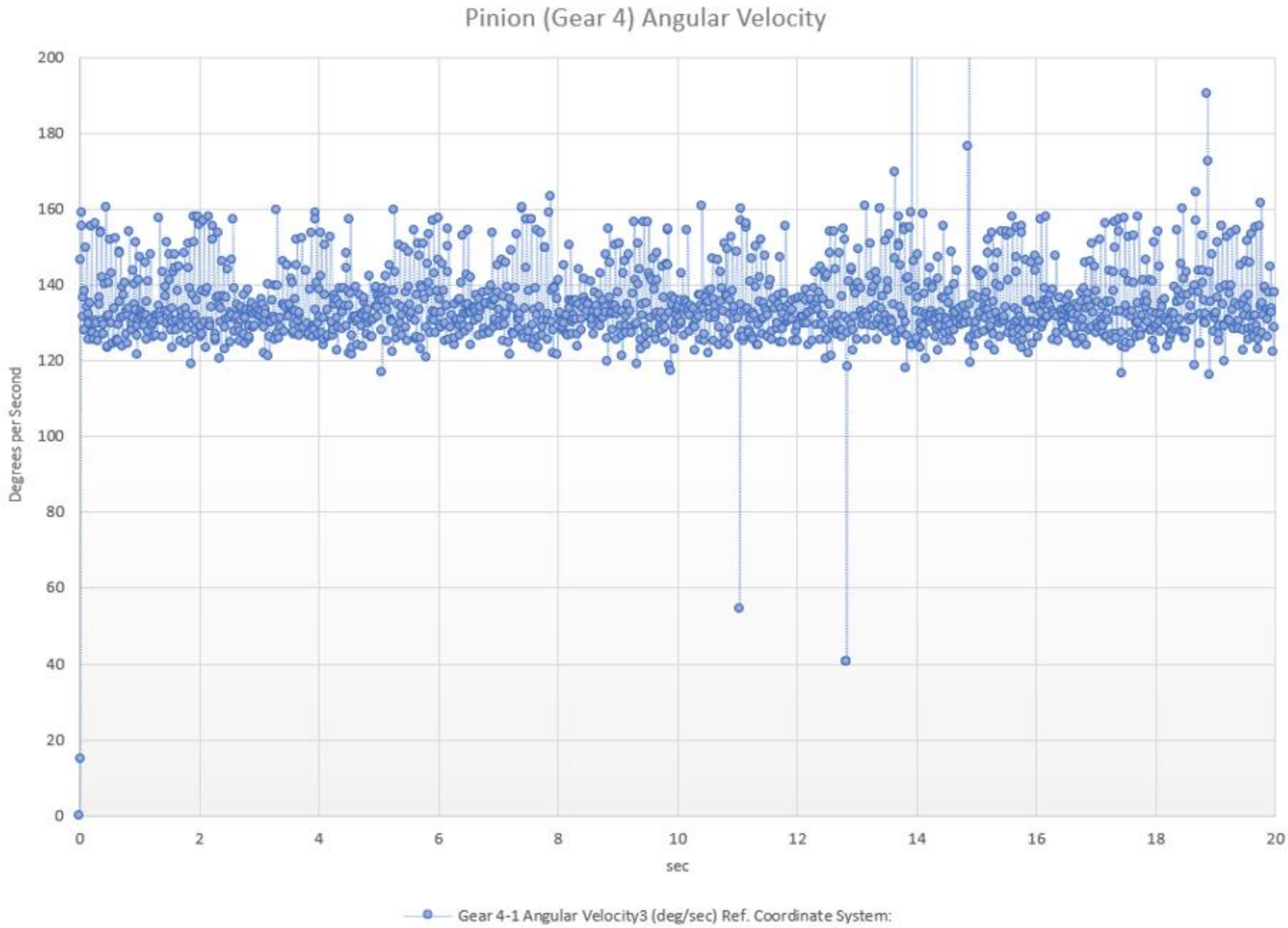


Figure 11: SolidWorks Gear 4 Angular Velocity Results

The process to calculate the angular velocities was similar to the previous section. The filtered data was averaged to determine a singular value. The study found that gear 2's angular velocity was n_2 = 91.23 rpm and gear 4's angular velocity was n_4 = 22.48 rpm. The simulated stage by stage gear ratios are as follows:

$$G_{12} = \frac{\omega_1}{\omega_2} = \frac{2750\, rpm}{91.23\, rpm} = 30.141$$

$$G_{34} = \frac{\omega_3}{\omega_4} = \frac{91.23\, rpm}{22.48\, rpm} = 4.058$$

$$G_{45} = \frac{\omega_4}{V_5} = \frac{22.48\, rpm}{2.35\, in/s} * \frac{2\pi}{60} \frac{\frac{rad}{rev}}{\frac{s}{min}} = 1.000889\, rad/in$$

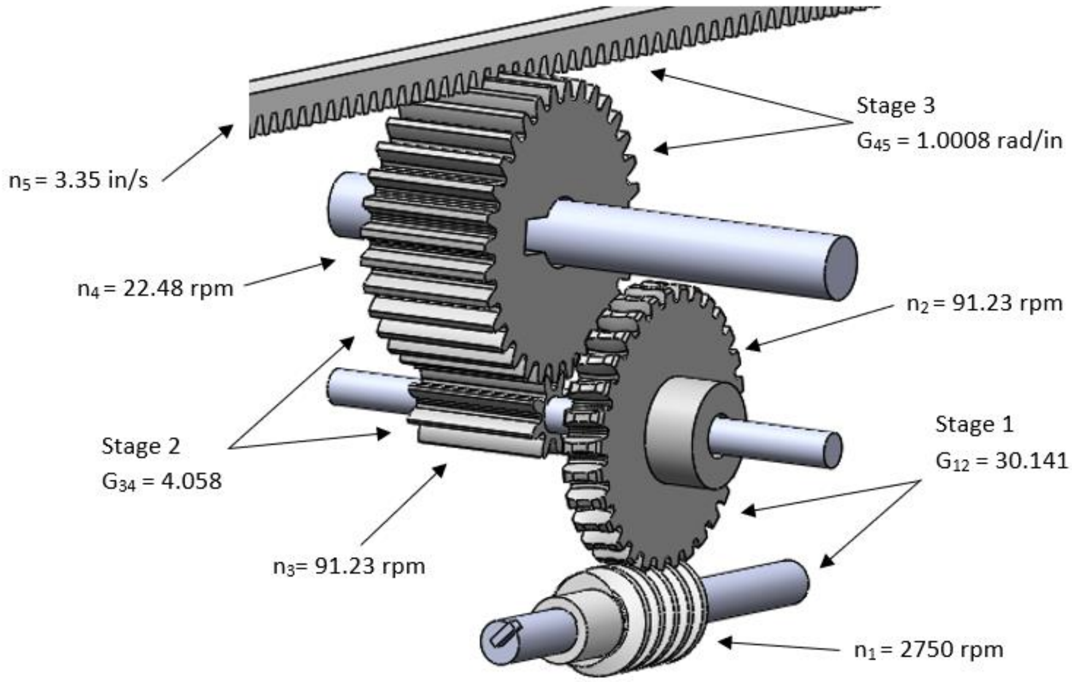


Figure 12: Results of SolidWorks Motion Analysis on each Stage

Theory vs. SolidWorks

	Theoretical Gear Ratio	SolidWorks Gear Ratio
Stage 1 (worm to worm gear)	30	30.141
Stage 2 (spur gear to pinion)	4	4.058
Stage 3 (pinion to rack)	1	1.000889 rad/in

Table 2: Theoretical Vs. SolidWorks for each Stage

The relative differences between the theoretical and the SolidWorks simulation are listed below.

Stage 1 Relative Difference: 0.469%

Stage 2 Relative Difference: 1.44%

Stage 3 Relative Difference: 0.089%

As you can tell from the table and the relative differences, the gear ratios for each stage in the simulated results were very similar to the theoretical calculations.

4.3 Bearings

In order to choose bearings for each of the shafts, the first step is to calculate the loads it will need to support. This is done by finding the reaction forces at the center of each gear that is on the shaft due to the force interaction between the gears. In figure 8 we can see the reaction forces from gears 2 and 3 labeled as F_{b2} and F_{b3} respectively. A bearing will be placed at points A and C on the shaft. These values were calculated using force and moment equilibrium. All reaction forces from the center of each gear are labeled as F_{bx} where x is the gear number, and each bearing load is labeled as F_x where x is the letter designated to the bearing.

[Insert equations for force calculations]

The countershaft is shown below with all forces acting on it. The shaft extends father than shown in the figure but the ends are trimmed in order to show the points and which the bearing forces will act.

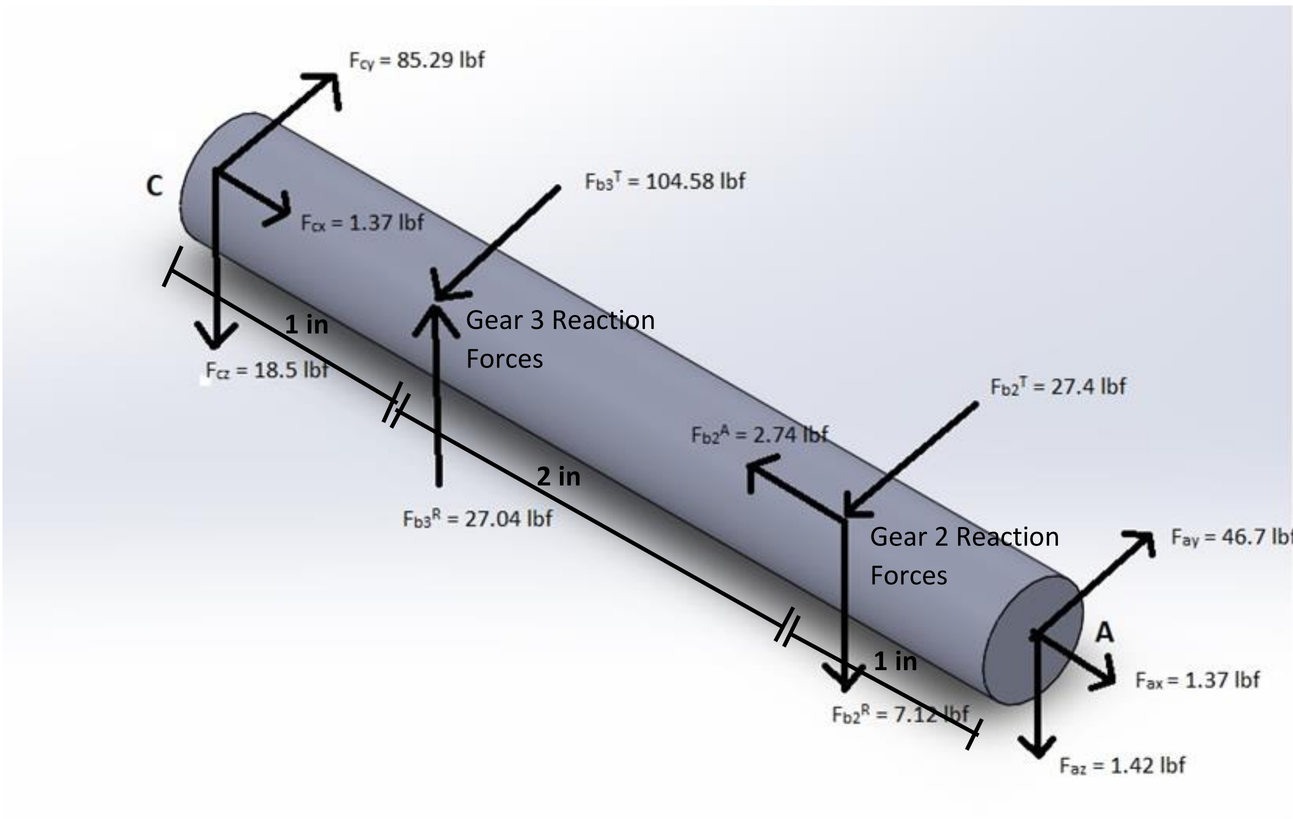


Figure 13: Counter shaft free body diagram

For points A and C we can calculate the combined radial and axial load to find the load the bearing will need to support.

$$F_{rA} = \sqrt{1.42^2 + 46.96^2} = 46.71\ lbf$$

$$F_{rC} = \sqrt{18.5^2 + 85.29^2} = 87.29\ lbf$$

There is also an axial load of F_a = 1.37 lbf, a bearing selected will be able to support axial load but since the value is negligible it will not be included in calculations. We will use the larger value of F_{rC} = 87.29 lbf for bearing selection. Since the radial load is fair, a cylindrical roller bearing will be used. Now a C_{10} values needs to be calculated using $n_3 = n_2$ = 91.67 rpm, $\mathcal{L}_D$, F_{rC}, a = 10/3 for roller bearings, and $L_R = 10^6$. The estimated life is estimated by saying the device will be used 20 times per week, for 52 weeks a year for 10 years, and 1 minute per use including the lowering time, which gives a $\mathcal{L}_D$ = 173 hours.

$$C_{10} = F_D(\frac{\mathcal{L}_D n_D 60}{L_R})^{\frac{3}{10}} = 97.29 * \frac{(173 * 91.67 * 60)^{\frac{3}{10}}}{10^6} = 85.99\ lbf = 0.382\ kN$$

The lowest rated bearing is the NJ202ECP cylindrical roller bearing which has C_{10} = 12.5 kN and bore d = 15 mm[7]. This will be used for the counter shaft.

The following bearings for the input and output shafts will use similar methods. The input shaft is shown below

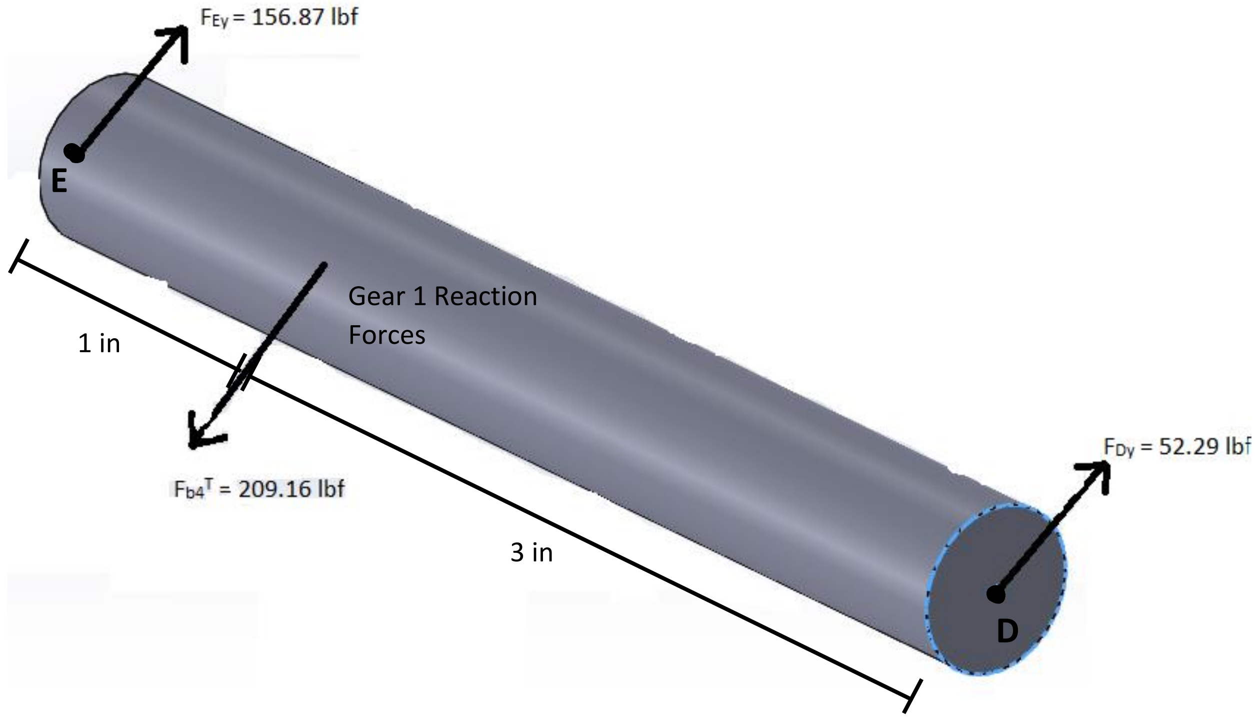


Figure 14: Output shaft free body diagram

Since there is only one reaction force at each of the bearings, the radial load used will be the greater of the two F_{rE} = 156.87 lbf and the axial load is zero. A cylindrical roller bearing can be used since there is no axial force present. The rotational speed of n_4 = 22.91 rpm, $\mathcal{L}_D = 173$ hours, F_{rC}, a = 10/3 for roller bearing, and $L_R = 10^6$

$$C_{10} = F_D\left(\frac{\mathcal{L}_D n_D 60}{L_R}\right)^{\frac{3}{10}} = 156.87 * \frac{(173 * 22.91 * 60)^{\frac{3}{10}}}{10^6} = 101.51\ lbf = 0.453\ kN$$

The lowest rated cylindrical roller bearing is the same one used for the countershaft. NJ202ECP is rated for C_{10} = 12.5 kN and bore d = 15 mm[7].

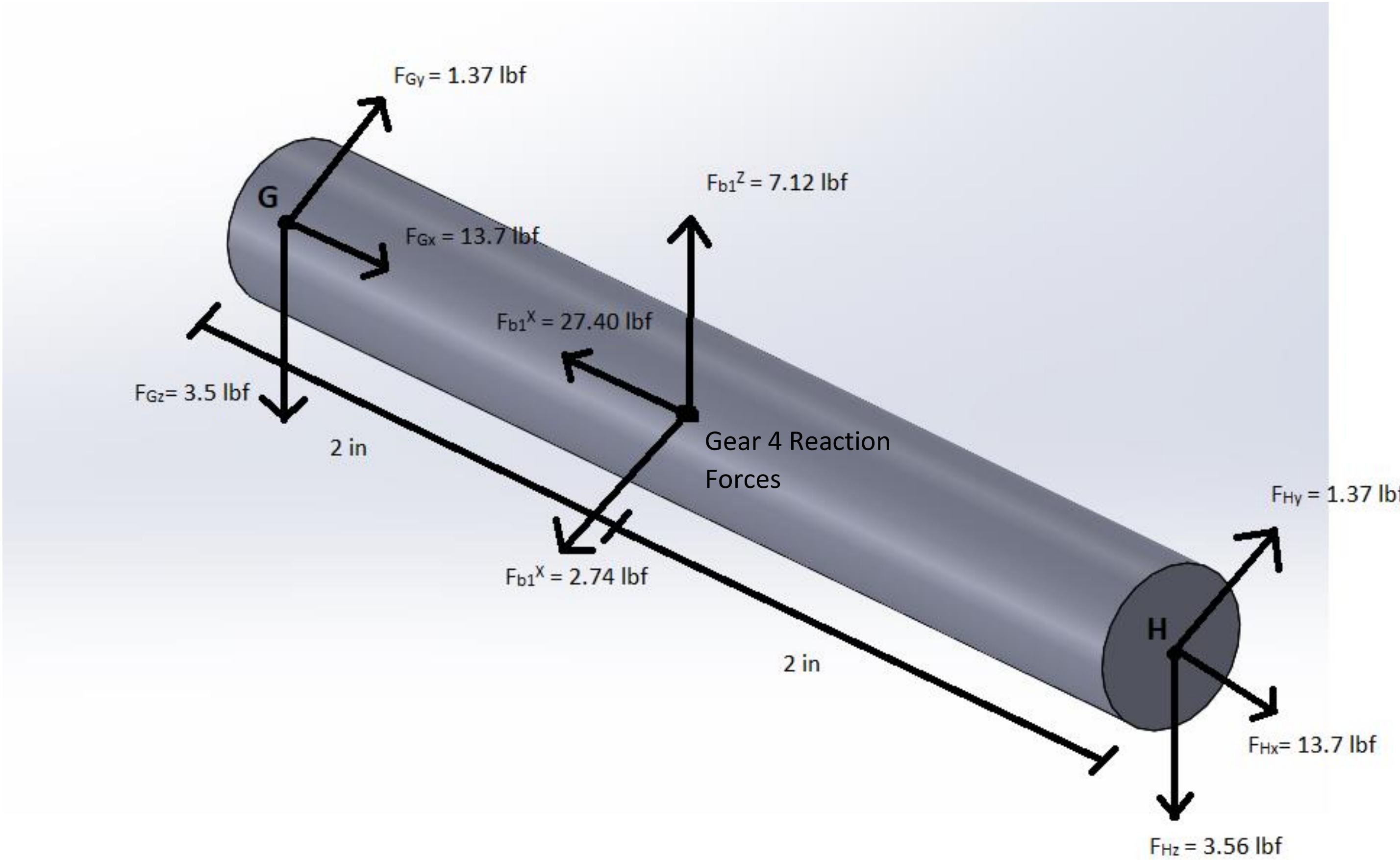


Figure 15: Input shaft free body diagram

Calculating the reaction forces at points G and H are shown below.

$$F_{rG} = \sqrt{1.37^2 + 3.56^2} = 3.75\ lbf$$

$$F_{rH} = \sqrt{1.37^2 + 3.56^2} = 3.75\ lbf$$

There is also a non-negligible axial force of F_a = 13.7 lbf, so the bearing will have to support radial and axial load. The radial and axial load will be combined into F_e. First an initial angular contact bearing is selected from table 11-2 with a C_0 = 2.21 kN [5]. From this a value for e can be determined from table 11-1,

$$\frac{F_a}{C_0} = \frac{13.7\ lbf * 4.4482\frac{N}{lbf} * \frac{1N}{1000kN}}{2.21\ kN} = 0.0278$$

Then the following condition is checked against e = 0.22 value from table 11-1, where V = 1 because the inner ring of the bearing is rotating,

$$e \geq \frac{F_a}{VF_r} = \frac{13.7}{1 * 3.75} = 3.65$$

Since 3.65 is greater than e = 0.22, we use the secondary values of X_2 = 0.56 and Y_2 = 1.99 in the following equation,

$$F_e = X_2VF_r + Y_2F_a = 0.56 * 1 * 3.75 + 1.99 * 13.7 = 29.363\ lbf = .131\ kN$$

Now solving for C_{10} with the adapted load F_e, n_1 = 2750 rpm, $\mathcal{L}_D$ = 173 hours, F_{rC}, a = 3 for ball bearing, and $L_R = 10^6$

$$C_{10} = F_e(\frac{\mathcal{L}_D n_D 60}{L_R})^{\frac{3}{10}} = 29.363 * \frac{(173 * 2750 * 60)^{\frac{1}{3}}}{10^6} = 89.74\ lbf = .40\ kN$$

With the C_{10} value an angular contact ball bearing with a rated load higher than 0.4 kN is selected. The 7200 BECBP is selected which is the lowest rated angular contact ball bearing with C_{10} = 7.02 kN and a bore d = 10 mm [7].

Shaft	Bearing	C_{10} (kN)	Bore d (inches)	Outer D (inches)
Counter Shaft	NJ202ECP	12.5	0.59	1.38
Output Shaft	NJ202ECP	12.5	0.59	1.38
Input Shaft	7200 BECBP	7.02	0.39	1.18

4.4 Fastening

The gearbox is fastened to the raising apparatus via an edge flange which is on the mounted pole. Two bolts are inserted through the gearbox into the flange.

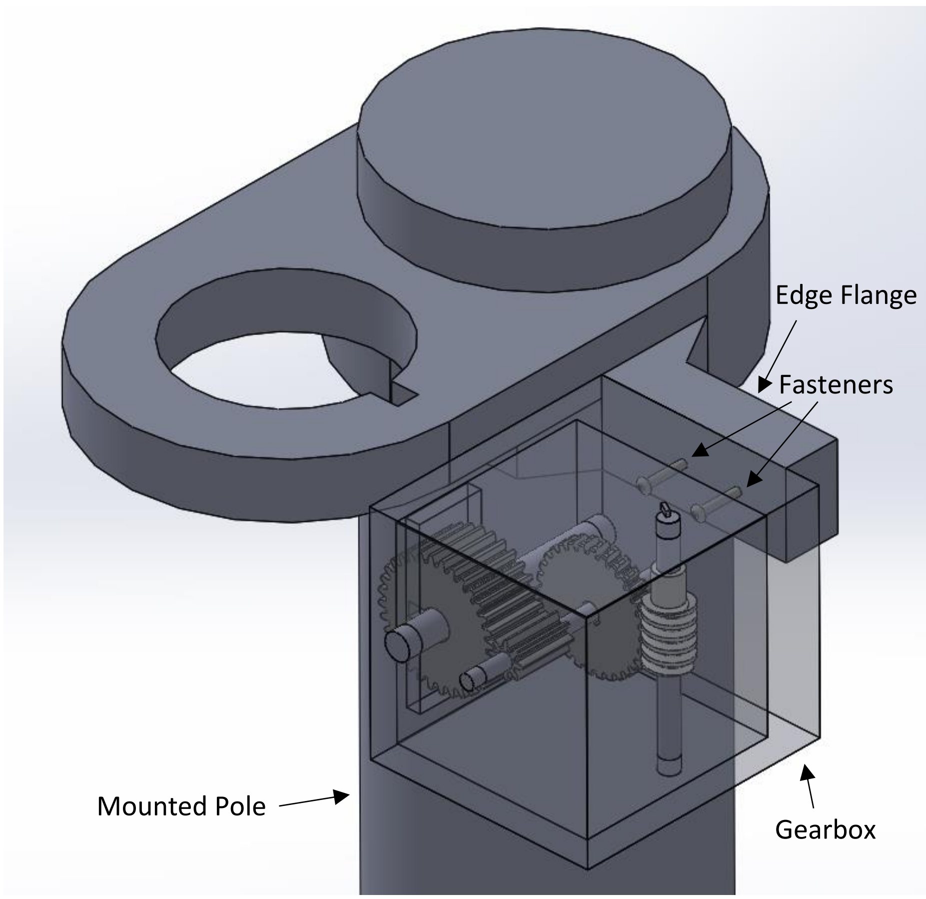


Figure 16: Gearbox mounted to flange with fasteners. (Support pole removed for better view).

An approximate size of the necessary bolts to withstand the loads on from the gearbox was found, and standard size bolts were selected based on the approximation. The loads on the gearbox-flange joint were found using the radial and tangential loads on the pinion. Due to the positioning of the fasteners, the tangential load provided both a tensile and slip load on the joints. This meant that the tensile load was P = 104.58 lbf due to the case acting as a lever and the fulcrum being the edge of the case. The slip load was W = 104.58 lbf as well. Since there are two bolts, the tensile and slip loads were halved per bolt for the calculations. The size for the Tensile-Stress Area, A_t, was calculated using the factors of safety for yielding, loading, joint separation, and slip. For sizing of the bolts, the largest A_t from the calculations was referenced for the size selection of the bolts. For each factor of safety, a factor of $n_P = n_L = n_O = n_S = 3$ was assumed as well as a Grade of 5 for the bolts. This meant that the minimum proof of strength, S_p, was 85 kpsi. Since the bolts will function as permanent fasteners, the equivalent preload was represented by $F_i = 0.9S_pA_t$. A suitable fraction of the load carried by the bolt was assumed as C = 0.20. Since relatively low loads are withstood by the gearbox and flange bracket, both will be constructed out of aluminum. For the slip factor of safety, the coefficient of friction was necessary to consider. For an aluminum-aluminum static connection, the coefficient of friction was assumed to be $\mu = 1.05$ [8]. Using each of the equations for the four factors of safety, the tensile-stress area was calculated [5].

$$A_t = \frac{CP/2n_P}{S_p - 0.9S_pn_P} = \frac{0.20 * 104.58lbf/2 * 3}{85000psi - 0.9 * 85000kpsi * 3} = -0.00022\ in^2$$

$$A_t = \frac{10CP/2n_L}{S_p} = \frac{10 * 0.20 * 104.58lbf/2 * 3}{85000psi} = 0.00369\ in^2$$

$$A_t = \frac{10(1-C)Pn_O}{9 * S_p} = \frac{10(1-0.20) * 104.58lbf/2 * 3}{9 * 85000psi} = 0.01476\ in^2$$

$$A_t = \frac{(1-C)P/2 - \frac{Wn_S}{2\mu}}{0.95S_p} = \frac{(1-0.20) * 104.58lbf/2 - \frac{104.58lbf * 3}{2 * 1.05}}{0.95 * 85000psi} = -0.00133\ in^2$$

Based on the largest tensile-stress area, bolts with at least $A_t = 0.01476\ in^2$ were needed, meaning at least size UNC-10 or UNF-10 were needed. The width of the gearbox wall is w = 0.75 in, and the width of the flange is 1 in. This means that the bolt will need to be at least 1.75 inches long. For a reasonably high length to diameter ratio, a UNC-10 bolt was acceptable, meaning the nominal major diameter was d = 0.1900 in and $A_t = 0.0175\ in^2$. From Table A-32 [5], the washer thickness for a #10 fastener is 0.049 in. Therefore, the length, *l*, of all material between the bolt and its nut is *l* = 1.8 in. A regular hexagonal nut will be used on the end of the bolts. The smallest value for the nut from Table A-31 [5] was assumed, with a height of H = 7/32 in. The length of the bolts was found using the relation $L > l + H = 1.8in + 7/32in$, therefore $L > 2.02\ in$. For the thread length, the relation $L_T = 2d + 1/4in$ was used, therefore $L_T = 2 * 0.138\ in\ + 0.25\ in\ =\ 0.526\ in$. From Table A-17 [5], the standard length for the bolts was selected as 2.25 in. Therefore, the final bolt selections were $10 - 32UNC\ X\ 2\frac{1}{4}\ in$.

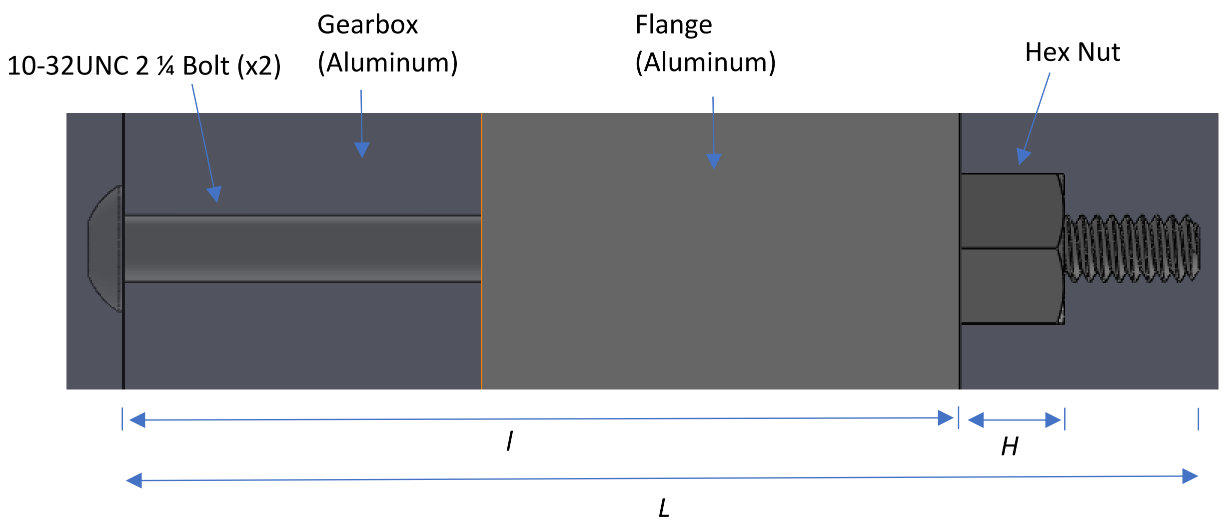


Figure 17: Bolt through members with hex nut.

To check that the bolts were valid for use under the loads, the actual factors of safety were calculated along with the fraction of the load carried by the bolts, C. The C value was found using the relation $C = \frac{P_b}{P} = \frac{k_b}{k_b+k_m}$, where kb is the stiffness of the bolt and km is the stiffness of the members. Using Young's modulus $E =\ 28Mpsi$ for steel [9] for the bolt the relations for the area of the unthreaded portion $A_d = \frac{\pi d^2}{4} = 0.0284\ in^2$, length of unthreaded portion in grip $l_d = L - L_T = 1.724\ in$, and length of threaded portion in grip $l_t = l - l_d = 0.076\ in$, the bolt stiffness was found.

$$k_b = \frac{EA_dA_t}{A_dl_t + A_tl_d} = 430457\ lbf/in$$

The member stiffness was calculated using Young's modulus E = 10.4 MPsi for aluminum [9] since both members (gearbox and flange bracket) are 7075T6 aluminum.

$$k_m = \frac{0.5774\pi Ed}{2ln(5\frac{0.5774l + 0.5d}{0.5774l + 2.5d})} = 1357199\ lbf/in$$

Therefore, the actual bolt load ratio was found to be

$$C = \frac{k_b}{k_b + k_m} = \frac{430457\ lbf/in}{430457\ lbf/in + 1357199\ lbf/in} = 0.241$$

which is means the bolt only takes 24.1% of the external load, which is close to the estimated 20% and is an acceptable value. The updated factors of safety were then calculated based on the new C value.

$$n_P = \frac{S_pA_t}{CP/2 + 0.9S_pA_t} = 1.10$$

$$n_L = \frac{S_pA_t - 0.9S_pA_t}{CP/2} = 11.80$$

$$n_O = \frac{0.9S_pA_t}{(1-C)P/2} = 106.2$$

$$n_S = \frac{\mu|(1-C)P/2 - S_pA_t|}{|W|/2} = 29.1$$

All the factors of safety were above 1, which means the fasteners were valid for use. The yielding was lower than anticipated but still acceptable. The loading, joint separation, and slip factors were quite high, meaning the joint will highly withstand the force and be unlikely to separate or dislocate.

4.5 Structure

The safety of the gearbox will be analyzed for this section. The gearbox is made from 7075T6 aluminum. There are 6 locations which exhibit a bearing load due to the radial bearing load over the projected area. As calculated in the bearing section, the magnitude of the bearing load for each location is A = 46.71 lbf, C = 87.29 lbf, D= 52.29 lbf, E = 156.87 lbf, G = 3.75 lbf, and H = 3.75 lbf. The magnitude of each bearing load acts on the casing on an angle between the designated, positive y and z axis (same coordinate system as bearing analysis). A theoretical analysis on the critical point of the casing will be conducted, and therefore the factor of safety can be calculated.

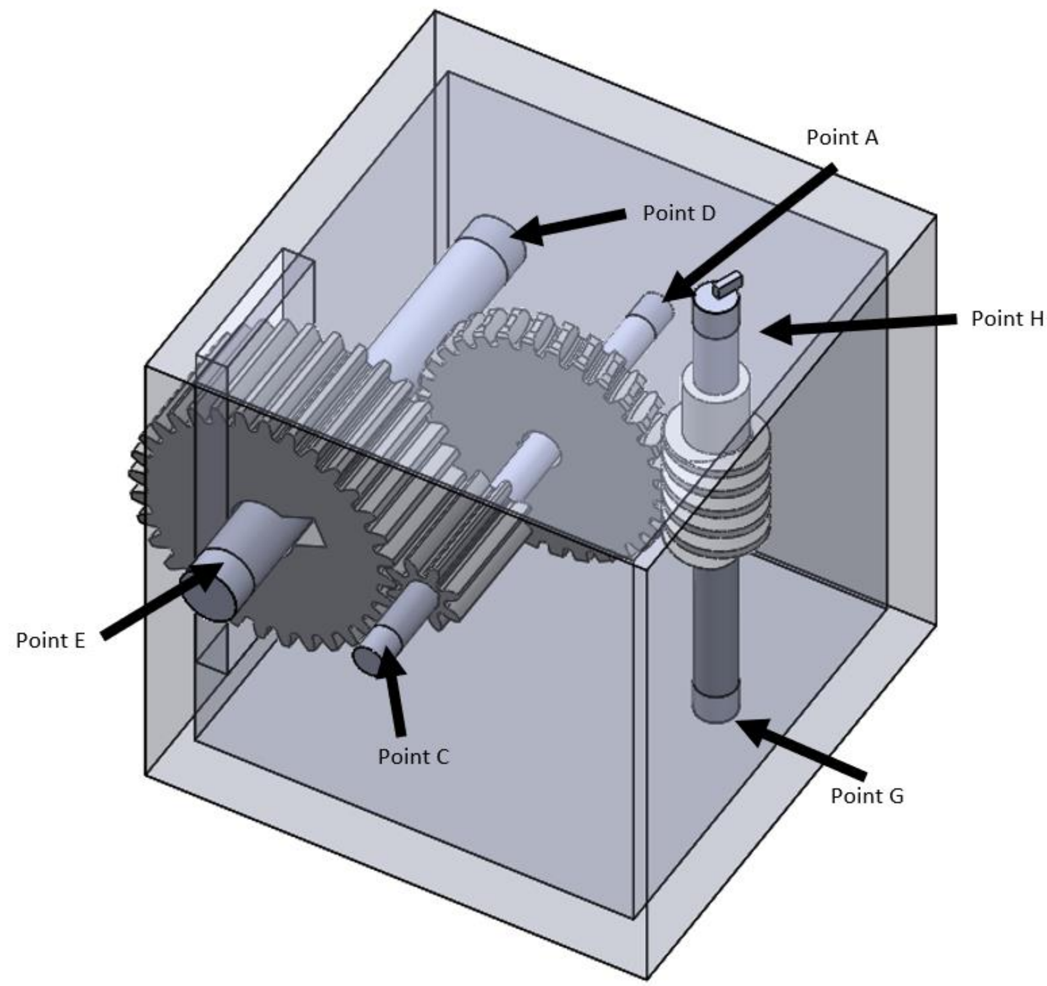


Figure 18: Locations of Stress and Shear on Gearbox

The critical location due to the bearing loads is located at E since it has the largest value. Since there are no moments, torques, and axial loads that act on the casing, the only force that will be present is a transverse shear. The transverse shear stress at location E is $\tau_{trv} = \frac{16\, F_y}{3\pi d^2} = \frac{16\;(156.87\; lbf)}{3\pi (0.37\; in)^2} = 2{,}173.965\; psi$.

To next step to calculate the principle stresses is to determine where the max stress at the critical location is situated. The formula used to determine them is below.

Location	σ_x	σ_y	σ_z	τ_{xy}	τ_{yz}	τ_{zx}
Bottom	$-\sigma_b - \sigma_n$	0	0	0	0	$-\tau_{tor}$
Top	$+\sigma_b - \sigma_n$	0	0	0	0	$+\tau_{tor}$
Front side	$-\sigma_n$	0	0	$-\tau_{tor} - \tau_{trv}$	0	0
Back side	$-\sigma_n$	0	0	$+\tau_{tor} - \tau_{trv}$	0	0

As you can tell, the max stress on the casing is going to be equal at the front and backside of the bearing since there is only a transverse shear.

To find σ_a and σ_b through the following equation will be utilized: xy: $\sigma_A, \sigma_B = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$

$$\sigma_a = +\sqrt{(2{,}173.965\ psi)^2} = 2{,}173.965\ psi$$

$$\sigma_b = -2{,}173.965\ psi$$

Therefore, σ_1 = $2{,}173.965\ psi$, σ_2 = 0 psi, and σ_3 = $-2{,}173.965\ psi$

The max principle shear stress is $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{2{,}173.965\ psi-(-2{,}173.965\ psi)}{2} = 2{,}173.965\ psi$

The casing is made from 7075T6 aluminum which has a yield strength of 120 kpsi. The factor of safety for location E is calculated through the following equation.

$$\sigma_{nmss} = \frac{Sy}{\sigma 1 - \sigma 3} = \frac{120{,}000\ psi}{2{,}173.965\ psi-(-2{,}173.965\ psi)} = 27.559$$

Clearly with a factor of safety this high, the casing's material will stand the bearing loads during the lifting of the basketball hoop.

SolidWorks Static Study

A SolidWorks static study was conducted to determine the simulated max shear stress on the gearbox casing. The casing was fixed through "fixed geometry" on both bolt holes, as well as a slider fixture (with exact geometry as the edge flange) which applies a normal force to keep the gearbox in place, however the bolting is responsible for the rotational resistance. Bearing forces were applied in each of the locations listed in Figure 18 along with their corresponding values. The results of the study are below.

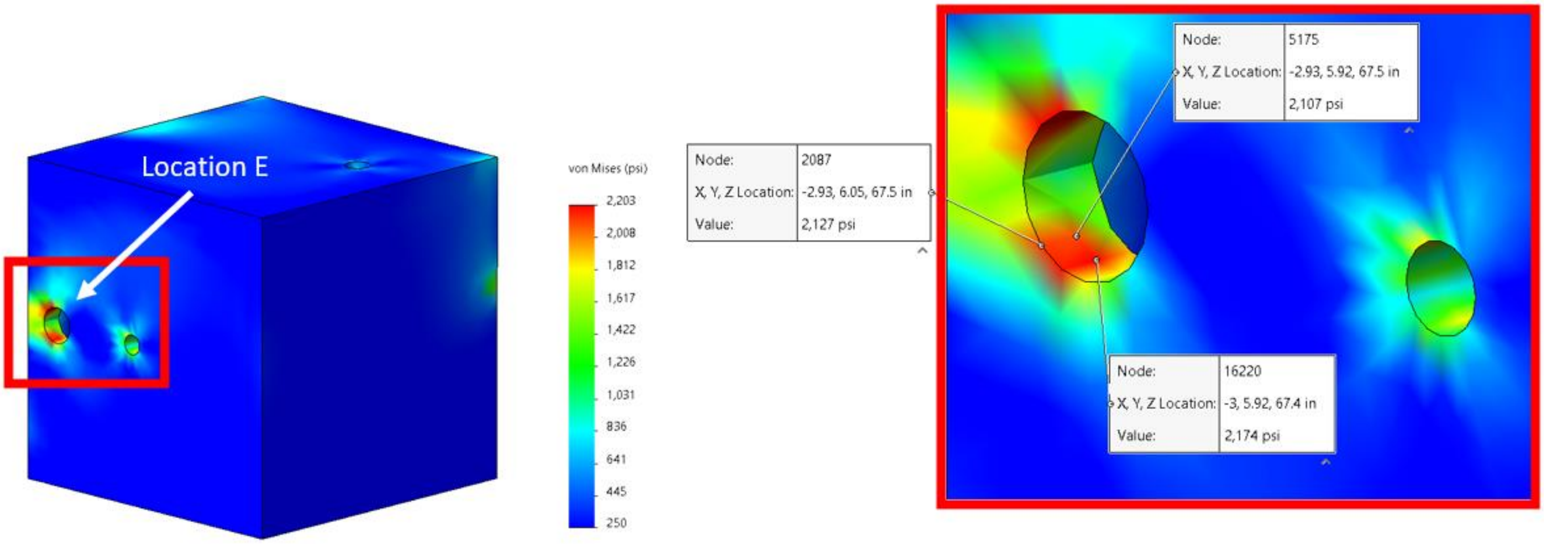


Figure 19: Results of the Static Study on Gearbox in SolidWorks

Max shear stress from SolidWorks autoscaling plot: $\tau_{max} = 2{,}203\ psi$

Max shear stress from probing: $\tau_{max} = 2{,}174\ psi$

Due to the autoscaling max shear stress being more reliable than probing, the simulated factor of safety becomes

$\sigma_{nmss} = \frac{Sy}{2\ \tau_{max}} = \frac{120{,}000\ psi}{2*(2{,}203\ psi)} = 27.24$

Theory Vs. SolidWorks

The static study within SolidWorks confirmed that the critical location of the gearbox was in location E, which is located on the output shaft on the opposite side the gearbox is fixed. After probing, it is evident that the simulated max shear stress is quite similar to that of the theoretical even though some of the probes had values that were smaller. The auto-scaling plot that SolidWorks offers set the max simulated shear stress to roughly 2203 psi. Being more accurate than probing, the relative difference between the theoretical shear stress and the simulated shear stress is 1.32672%. This means that one can be confident in the theoretical results and the decision to make the casing 7075T6 aluminum.

4.6 Shafting

The most complex shaft of the system is the countershaft, it is under the load of two separate gears that both need to be axially locked. The gears are supported by shoulders and held in place by retaining rings. The gears then transmit their torque through keyways into the shaft. The gear and bearing specifications were defined above and are displayed below along with reaction forces. A force and moment equilibrium was performed on the shaft to find reaction forces at points A and D.

$F_{By} = 27.04$ lbf, $F_{Bz} = 104.58$ lbf

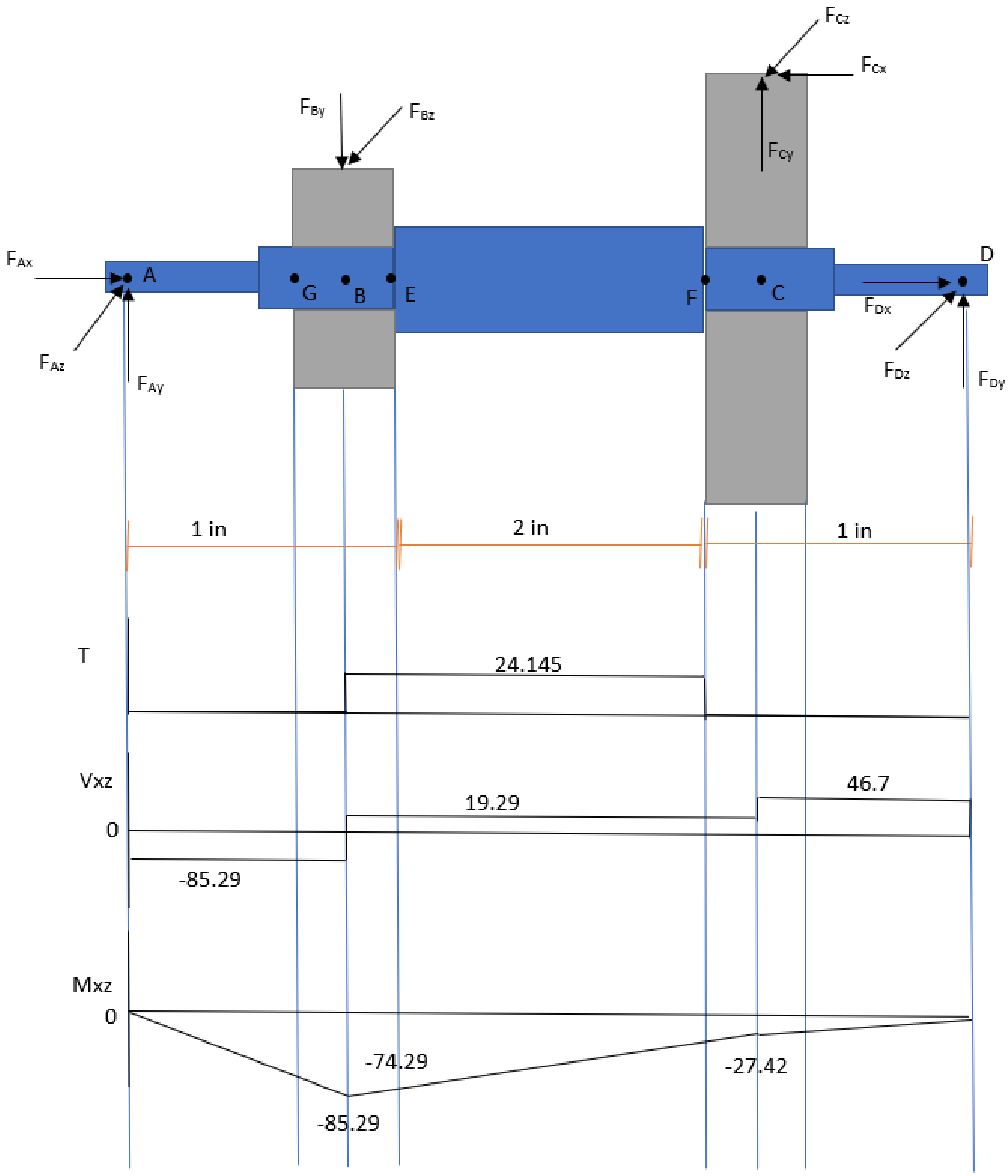
$F_{Cz} = 27.50$ lbf, $F_{Cx} = 2.74$ lbf, $F_{Cy} = 7.12$ lbf

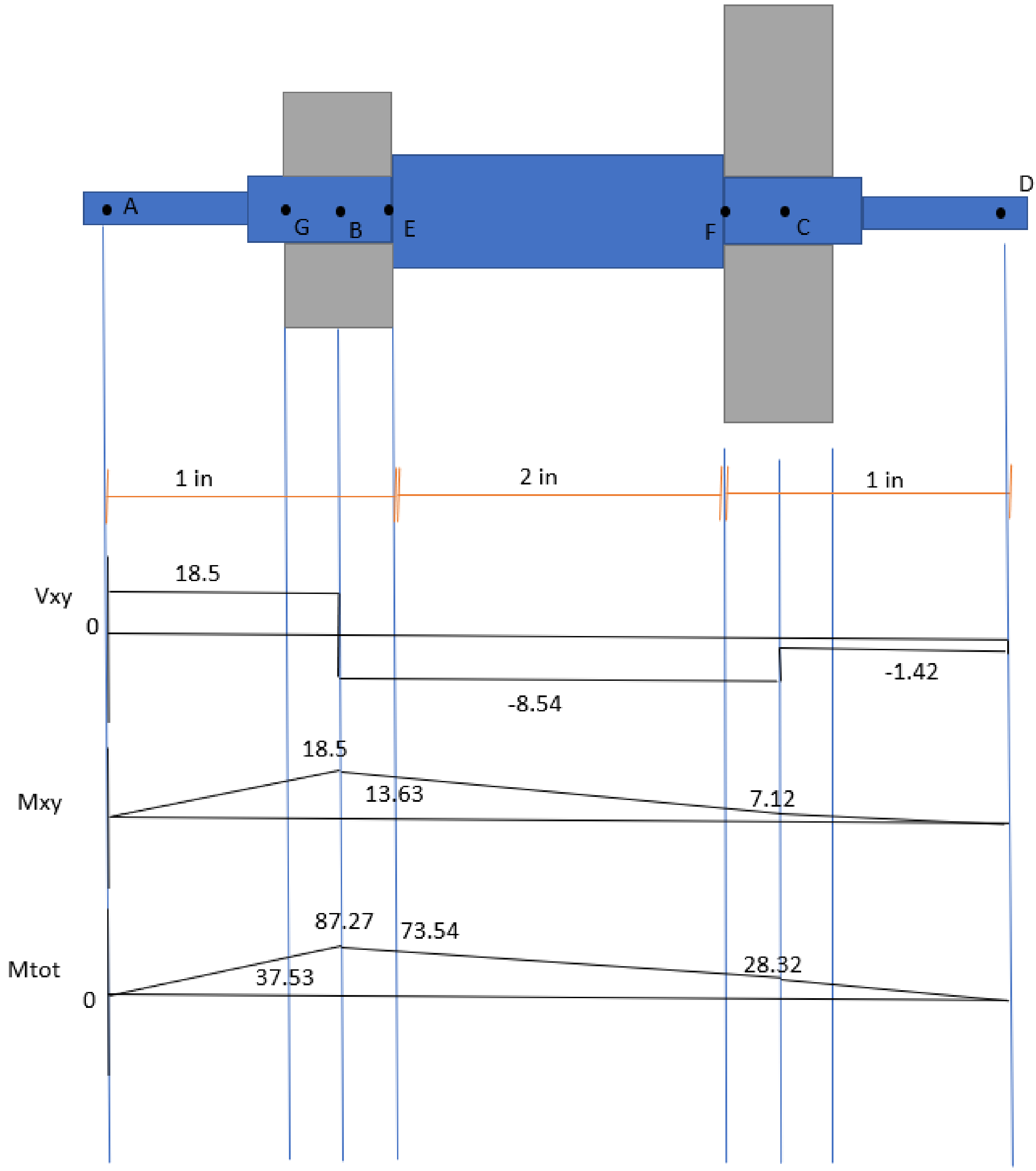
Reaction forces at A and D

$F_{Ax} = 1.37$ lbf, $F_{Ay} = 18.5$ lbf, $F_{Az} = 85.29$ lbf

$F_{Dx} = 1.37$ lbf, $F_{Dy} = 1.42$ lbf, $F_{Dz} = 46.7$ lbf

From these values shear and bending moment diagrams are generated for the X-Z and Z-Y planes. From these a total moment diagram can be shown to find values for calculations on the shaft. From face widths of gears 2 and 3, the intermediary values at possible critical points G, E, F, and C can be calculated.





First, the most likely critical point which would have the highest bending moment at an intersection of different diameters will be examined. This would be point E as it is the highest bending moment of M_a = 73.54 lbf in at a shoulder along with the torque T_m = 25.145 lbf in. Estimates for some variables will be made for initial guesses and then adjusted later on. Estimating k_t = 1.7, k_{ts} = 1.5. A material must also be selected, since the loads involved are low we will choose the weakest steal 1006 CD with an S_y = 41 Kpsi and S_{ut} = 48 kpsi. The steel is cold rolled so a = 2.7 and b = -0.265,

$$k_a = a(S_{ut})^b = 2.7\ (48)^{-0.265} = 0.9679$$

K_b is estimated to 0.9,

$$S_e = \frac{k_a k_b S_{ut}}{2} = 20.91\ kpsi$$

A design factor of 2.0 is desired, but with the small loads the safety factors will probably be very large. The diameter of section containing points G and E is calculated,

$$d = (16 * \frac{n}{\pi}\left(\frac{2k_f M_a}{S_e} + \frac{\sqrt{3}k_{fs}T_m}{S_{ut}}\right))^{\frac{1}{3}} = 16 * \frac{2}{\pi}\left(\frac{2 * 1.7 * 73.54}{20.91} + \frac{\sqrt{3} * 1.* 25.14}{48}\right)^{\frac{1}{3}} = 2.168\ in$$

Rounding to the nearest standard value, d = 2.25 in. Using $\frac{D}{d}$ = 1.2, D = 2.7 which gets rounded to nearest standard value of 2.75 so $\frac{D}{d}$ = 1.22. r = $\frac{3}{32}$in so $\frac{r}{d} = 0.0417$. Using tables 15-8 and 15-9 gives k_t = 2.25, k_{ts} = 1.72. Using r and S_{ut}, figures 6-20 and 6-21 give q = 0.7, q_s = 0.75. k_f and k_{fs} can now be solved for,

$$k_f = 1 + q(k_t + 1) = 1.875$$

$$k_{fs} = 1 + q_s(k_{ts} + 1) = 1.54$$

From these K_b and S_e can be redefined,

$$k_b = 0.91(d)^{-0.157} = 0.801$$

$$S_e = \frac{K_b K_a S_{ut}}{2} = 18.61\ kpsi$$

With all these values solved for, the stresses and then safety factors can be derived,

$$\sigma_a' = \frac{32k_f M_a}{\pi d^3} = 32 * 1.875 * \frac{73.54}{\pi * 2.25^3} = 123.30\ lbf/in^2$$

$$\sigma_m' = \frac{\sqrt{3}k_{fs}T_m}{\pi d^3} = \sqrt{3} * 16 * 1.54 * \frac{25.145}{\pi * 2.25^3} = 29.98\ lbf/in^2$$

$$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{1}{137.9}$$

$$\frac{1}{n_y} = \frac{\sigma_a'}{S_y} + \frac{\sigma_m'}{S_y} = \frac{1}{267}$$

The factors of safety n_f = 137.9 and n_y = 267, which are more than ample to support the loads present. The groove at point G for the retaining ring for gear 3 is another possible critical point that isn't show on the bending moment diagrams. Its safety is tested by using k_f = k_t = 5 and M_a = 37.53 from the diagram.

$$n_f = \frac{S_e}{\sigma_a'} = \frac{18610 * \pi * 2.25^3}{32 * 5 * 37.53} = 110$$

Clearly the safety factors are extremely high even though the weakest steel was selected. The loads present are no where near dangerous values for the materials being used. Below are the diameters of each section of the shaft.

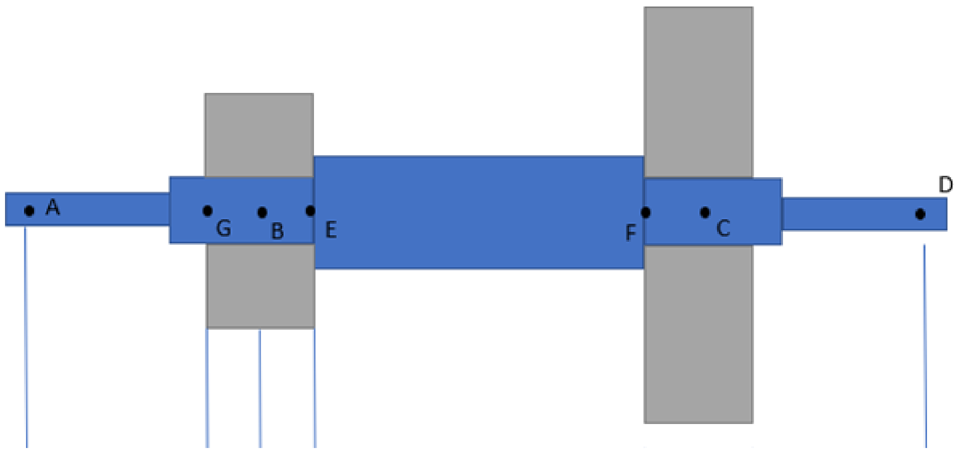


Figure 20: Shaft view

Material: 1006 CD Steel	
Section	Diameter (inches)
A	1.38
GBE	2.25
EF	2.75
FC	2.25
D	1.38

5.0 Evaluation

There was a need to raise a basketball hoop in a timely manner, which was then elaborated with specifications for the time limit and height change. The hoop needed to be raised from 6ft to 10 ft in 20 seconds. To accomplish this, a gear system was designed and combined with a motor having a load and speed rating above the necessary values was selected. The system included 3 different stages to transfer the rotational input to the motor into translational output of the basketball hoop. The motor and gear system performed in SolidWorks as calculated in theory and had enough power to move the basketball hoop from a height of 6ft to 10 ft in 20 seconds.

The bearings, fastening, shafting, and structure must also be designed to support the structure and keep costs low. There are 3 different bearing pairs that need to support loads caused by the gear system. The axial and radial loads were calculated for each pair and the bearings selected. They each had high safety factors showing they are more than capable of supporting the forces.

To connect the gear box to the mounted pole, two fasteners were inserted directly into the edge of the gearbox and through a flange attached to the pole. At these joints, tensile and slip loads were present, P = W = 104.58 lbf, both caused by the tangential force from the rack and pinion. For both fasteners, 10-32UNC 2 ¼ screws were selected. The factors of safety for yielding, loading, joint separation, and slip were all greater than 1, therefore the screws were acceptable for use.

The counter shaft had 3 different dimensions which were determined from the bearings used and evaluating an assumed critical point which had high loads. The critical point had a safety factor above 100 using 1006 CD Steel which is one of the weakest steels listed. This is a place where an even weaker and cheaper material could have been used. Since the safety factors were so high, this shaft has been designed well and will not fail.

The casing of the gearbox under the stress of the bearing loads was analyzed to calculate if the 7075T6 aluminum, with a yield strength of 120 kpsi, would be safe. The critical location on the gearbox was at point E, where the highest bearing load of $156.87\ lbf$ was located. The theoretical calculated max shear stress at the location was $2{,}174\ psi$. After a SolidWorks static study, a maximum shear stress of $2{,}203\ psi$ was found. The relative difference between the theory and simulation was 1.32672% and the factor of safety, using the greater shear stress, was 27.24. The 7075T6 aluminum is considered safe proven by the theoretical analysis, as well as the SolidWorks static study.

Overall, each section of the design meets the necessary requirement while also trying to minimize the strength and amount of material. Safety factors and comparisons between theory and simulation were used to determine each part and dimension. Since all the different sections exceeded or met desired safety factors, it can be said that this design meets requirements.

Resources:

[1] Basketball Goal (34 lbs):
https://www.amazon.com/Lifetime-73729-Shatter-Proof-Backboard/dp/B00023RCAG/ref=asc_df_B00023RCAG/?tag=hyprod-20&linkCode=df0&hvadid=241958812525&hvpos=&hvnetw=g&hvrand=15602511940387648573&hvpone=&hvptwo=&hvqmt=&hvdev=c&hvdvcmdl=&hvlocint=&hvlocphy=9011700&hvtargid=pla-616041649582&psc=1

[2] Mounting Bracket (13.22 lbs): https://www.amazon.com/Lifetime-9594-Universal-Mounting-Kit/dp/B0007U5C2O/ref=pd_bxgy_img_1/136-1166305-8013238?pd_rd_w=dR2cz&pf_rd_p=c64372fa-c41c-422e-990d-9e034f73989b&pf_rd_r=EPM0MMT6W2NHSM45NM6G&pd_rd_r=f2b4f910-b0be-4b42-bab9-74294d022d98&pd_rd_wg=JbNgk&pd_rd_i=B0007U5C2O&psc=1

[3] Pole Mount (27.04 lbs): https://www.industrialtube.com/catalog-product/?product_sku=3501

[4] Basketball Hoop Lowering Time (20 seconds):
https://www.youtube.com/watch?v=f_CqLVaWwVo

[5] Shigley's Mechanical Engineering Design 11th Edition, R.G. Budynas, J. K. Nisbett

[6] Lewis Form Factor: https://www.engineersedge.com/gears/lewis-factor.htm

[7] Bearing Selection and CAD Modeling: https://www.skf.com/us

[8] Coefficient of Friction https://www.engineeringtoolbox.com/friction-coefficients-d_778.html

[9] Young's Modulus https://www.pro-bolt.com/material-guide/