SIR model with shielded and isolated population

Req 550 Syria Team

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1 Model

$$\dot{S} = -\frac{\beta}{N}ES + \alpha * N - \mu S, \tag{1}$$
$$\dot{S_S} = -\frac{\beta_S}{N}E_S S_S - \frac{\beta_{S,ext}}{N}ES_S, \tag{2}$$
$$\dot{E} = +\frac{\beta}{N}ES - \gamma_E E - \mu E, \tag{3}$$
$$\dot{E_S} = +\frac{\beta_S}{N}E_S S_S + \frac{\beta_{S,ext}}{N}ES_S - \gamma_E E_S - \mu E_S, \tag{4}$$
$$\dot{I} = +\gamma_E(E + E_S) - \gamma_I I - \mu I, \tag{5}$$
$$\dot{R} = \gamma_I I - \mu R. \tag{6}$$

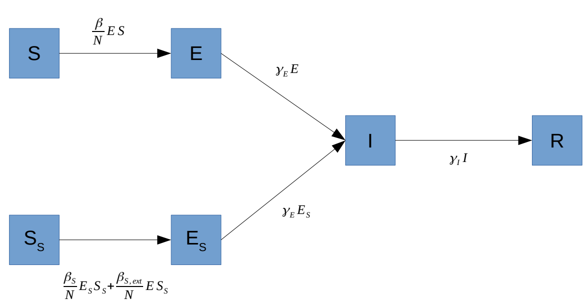

Figure 1: Flow chart of the model.

2 Notation

In the model, S represents the population susceptible to be infected by the disease. E represents undetected infectious individuals (note that this differs from other models where E models exposed non-infectious). The subscript S distinguishes the general population and the shielded population. Finally, I is the number of detected cases, and R the number of individuals that had the disease (in the present formulation, this includes recovered and death). See Figure 1 for an schematic representation of the model.

3 Parameters

The parameter β controls the interaction between individuals. Similarly β_S controls the interaction between individuals within the shielded population, whilst $\beta_{S,ext}$ controls the interaction between the shielded population and the rest of the camp. γ_E is (one over) the average time to detect infectious individuals. γ_I is (one over) the average time from detection to recovery.

The birth rate α is included only in the general population. Natural (non COVID related) death rate μ is included in all compartments.

4 Rationale

We want to model the dynamics of two interacting populations, the general population of the camp, and the shielded population. We make the following assumptions:

1. The two populations are well mixed (i.e., within each population, everybody interacts with everybody).

2. The confirmed cases (I) are isolated from the rest of the population.

3. The shielded population has limited contact with the infectious individuals in the general population ($\beta_{S,ext}$).

5 Example

Consider a camp with a population of 2000, and assume 10% of the population will be shielded. Assume that initially there is 1 infected individual in the general population.

Assuming that infected are detected as soon as they exhibit symptoms, we consider $\gamma_E = 0.1$ (10 days) and $\gamma_I = 0.1$. Note that this values can be estimated more accurately, but they are of the right order.

Assume $\beta = \beta_S = 0.5$, based on the estimates for the Rohingya camps, and assuming that within the shielded population individuals interact freely.

We take $\beta_{S,ext} = \beta/10$, to account for the fact that approximately 10% of the shielded population will have contact with the rest of the camp.

Birth and death rates are taken from the overall Syria rates, as estimated by J. Villers.

With this parameters, the model exhibits the dynamics represented in Figure 2. Note that after about 100 days the solution is mostly stable and the shielded population remains constant and uninfected but in this scenario most of the general population will have the disease at the same time.

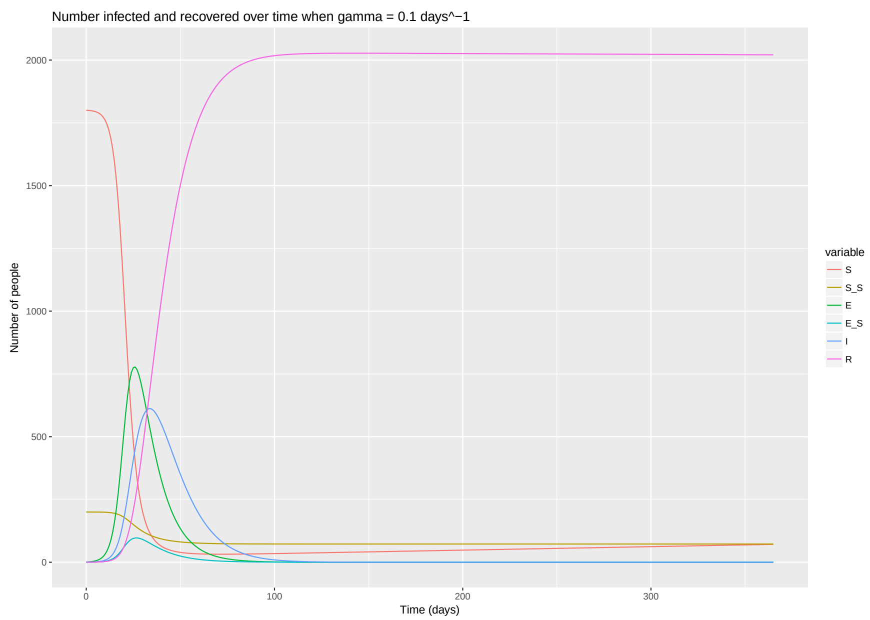

Figure 2: Solution with parameters described in the text.

6 TO DO

To avoid double counting deaths, we have to split the R compartment to distinguish deaths from recovered individuals.

Natural death rate in shielded population, flux of individuals from general to shielded pop.

Account for asymptomatic individuals, that will remain infectious and in contact with the population. Fraction of asymptomatic cases can be estimated from other countries.