

A cannot be random. And this paper On prediction of oscillatory processes seems to indicate that All oscillatory processes are modulated stationary processes. Can you show me where either my understanding is wrong or the paper is wrong? Question mark.

Let's carefully address your question in light of **the Mandrekar paper** and the broader Priestley theory, focusing precisely on your two main points:

1. Can $a_t(u)$ (or $A(t)$ in operator form) be random?
2. Are all oscillatory processes just modulated stationary processes?

1. Can $a_t(u)$ be random?

Mandrekar's formalism (in your attached text) and the original Priestley definition **take $a_t(u)$ as deterministic functions**. In these frameworks, the stochasticity comes exclusively from the orthogonal increment process Z , not from $a_t(u)$.

- In Priestley's 1965 JRSSB paper and in Mandrekar's exposition, $x_t = \int e^{itu} a_t(u) Z(du)$ with **deterministic** $a_t(u)$ that encodes the time-varying amplitude for each frequency.
- The randomness is in Z , whose increments are orthogonal (for Gaussian processes, this is a Brownian motion in the spectral domain).
- If $a_t(u)$ were random, one would have to specify how it is correlated with Z , which complicates both the spectral representation and the covariance structure.

So: **You are correct**-for the classic definition, $a_t(u)$ is not random.

2. Are all oscillatory processes just modulated stationary processes?

Let's clarify the definitions:

Stationary Process (Spectral Form):

$$y_t = \int e^{itu} Z(du)$$

where Z is an orthogonal increment process determined by the stationary process's spectral measure.

Oscillatory Process (Priestley/Mandrekar):

$$x_t = \int e^{itu} a_t(u) Z(du)$$

with deterministic $a_t(u)$.

Modulated Stationary Process (in the strictest sense):

$$x_t = f(t)y_t = f(t) \int e^{itu} Z(du)$$

which is a *special case* of the oscillatory process where $a_t(u) = f(t)$.

But **Priestley's oscillatory class is richer** because $a_t(u)$ can vary with both t and u . This enables much more general types of time-frequency modulation-**not just multiplication by a function of time**.

What does the Mandrekar paper say?

Mandrekar proves that any oscillatory process (in Priestley's sense) can be written as

$$x(t) = A(t)y(t)$$

where $A(t)$ is a *bounded linear operator* acting on the stationary process $y(t)$, *not* just a multiplication by a function of t .

- In the spectral representation, $A(t) = \int a_t(u) E(du)$, where E is the spectral measure. This is, in general, an **operator-valued modulation** rather than pointwise multiplication.

Thus, **Mandrekar's result is that oscillatory processes are "deformed" stationary processes under time-dependent operators**-which is more general than just multiplication by a scalar function.

Intuitive Example:

- If $a_t(u) = f(t)$, this reduces to scalar modulation.
- If $a_t(u) = \exp(i u (\theta(t) - t))$, this corresponds to time-warping the stationary process (see your earlier derivation).
- If $a_t(u)$ is any function of both t and u , you can, for instance, filter different frequencies differently at each time.

3. Summary Table

	Form of $a_t(u)$	Interpretation
Stationary	1	No modulation
Modulated Stationary	$f(t)$	Multiply entire process by $f(t)$

	Form of $a_t(u)$	Interpretation
Oscillatory (Priestley/Mandrekar)	$a_t(u)$ any measurable, deterministic function	Time-frequency modulation (filtering, warping, etc.)

All modulated stationary processes are oscillatory, but not all oscillatory processes are simply modulated stationary processes.

4. Where Might Confusion Arise?

- If you *restrict* "modulation" to mean "multiplication by a deterministic function of time", *not* all oscillatory processes have this form.
- But if you *generalize* "modulation" to mean "application of a time-dependent linear operator (possibly defined by multiplication in the spectral domain)", then **yes**, every oscillatory process is a modulated stationary one *in this operator sense*.

Mandrekar's point is that every oscillatory process is a deformed (operator-modulated) stationary process, but not merely a scalar-modulated one.

5. Contextual Clarification

In statistical time series, "modulated" often means "scalar-multiplied". In functional analysis (the Hilbert space approach), "modulated" can mean **applying any time-dependent bounded linear operator**. This operator can act differently on different frequency components, as in the oscillatory representation.

Final Answer

You are correct: In Priestley's and Mandrekar's definitions, $a_t(u)$ is deterministic, not random.

You are also correct that not all oscillatory processes are just scalar-modulated stationary processes.

But Mandrekar proves that all oscillatory processes are operator-modulated stationary processes ("deformed stationary curves"), with the operator defined by spectral multiplication with $a_t(u)$.

Neither you nor the paper is wrong: it's only a matter of **how broadly you define "modulated"**. In the operator-theoretic Hilbert space sense, all oscillatory processes are "modulated stationary processes", but not all are simply multiplied by a scalar function of time.

If you have a specific example you think is not covered by the operator modulation interpretation, share it-and we can dig deeper!

