perplexity

## Pre-Envelope and Spectral Function Relations

### Stationary Case: Complex Pre-Envelope ζ(t)

The pre-envelope ζ(t) for the stationary process x(t) is:

$$\zeta(t) = x(t) + i\hat{x}(t)$$

where x̂(t) is the quadrature process (Hilbert transform).

### Spectral Representation of Pre-Envelope

For absolutely continuous spectral measures:

$$\zeta(t) = \int_0^\infty e^{i\lambda t}[U'(\lambda) + iV'(\lambda)]\, d\lambda$$

where the complex spectral density function is:
$$W'(\lambda) = U'(\lambda) + iV'(\lambda)$$

### Inverse Relations for Pre-Envelope

$$W'(\lambda) = U'(\lambda) + iV'(\lambda) = \frac{1}{\pi}\int_{-\infty}^{\infty} \zeta(t)e^{-i\lambda t}\, dt$$

Separating real and imaginary parts:
$$U'(\lambda) = \frac{1}{\pi}\mathrm{Re}\left[\int_{-\infty}^{\infty} \zeta(t)e^{-i\lambda t}\, dt\right]$$
$$V'(\lambda) = \frac{1}{\pi}\mathrm{Im}\left[\int_{-\infty}^{\infty} \zeta(t)e^{-i\lambda t}\, dt\right]$$

### Oscillatory Case: Complex Process Z(t)

For the oscillatory process Z(t):

$$Z(t) = Y(t) + i\hat{Y}(t)$$

### Oscillatory Spectral Representation

$$Z(t) = \int_0^\infty e^{i\lambda t}A(t,\lambda)[U'(\lambda) + iV'(\lambda)]\, d\lambda$$

where A(t,λ) = a(t,λ) + iβ(t,λ) is the complex gain function.

### Inverse for Oscillatory Pre-Envelope

$$A(t,\lambda)[U'(\lambda) + iV'(\lambda)] = \frac{1}{\pi}\int_{-\infty}^{\infty} Z(t)e^{-i\lambda t}\, dt$$

### Envelope Relations

The envelopes are:

- **Stationary**: $R(t) = |\zeta(t)| = \sqrt{x^2(t) + \hat{x}^2(t)}$
- **Oscillatory**: $R(t) = |Z(t)| = \sqrt{Y^2(t) + \hat{Y}^2(t)}$

The spectral functions $U'(\lambda)$ and $V'(\lambda)$ encode the frequency domain structure of the complex pre-envelope processes.