perplexity

so it either wanders off or stays near where it started

No - the 0-1 property is not about **spatial wandering** but about **statistical behaviors** of sample paths.

For stochastic processes, invariant sets describe **classes of sample path behaviors**, not spatial regions. The search results confirm: "various sample path properties of self-similar processes with ergodic scaling transformations can be shown to hold with probability zero or one" [1] [2].

Examples of invariant sets for stochastic processes:

- $A_1 = \{\text{paths where } \lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} X_i = \mu\}$
- $A_2 = \{\text{paths that visit every neighborhood infinitely often}\}$
- $A_3 = \{\text{paths where sample variance converges to true variance}\}$

The binary property means: Either almost all sample paths exhibit the behavior (measure 1) or almost no sample paths exhibit it (measure 0).

Concrete example from search results [3]: Mix two Poisson processes with rates 1 and 2. With probability p, sample paths behave like rate-1 Poisson; with probability $1-p$, like rate-2 Poisson. This creates **non-ergodic** behavior because sample paths split into distinct statistical classes.

In ergodic systems: Such splitting cannot occur. The invariant set "paths exhibiting rate-1 behavior" must have measure 0 or 1, not some intermediate value like p.

The process values still wander randomly - what's constrained is whether **entire classes of long-term statistical behaviors** occur with probability 0 or 1. The search results emphasize: "an ergodic process is one where its statistical properties, like variance, can be deduced from a sufficiently long sample" [4].

Ergodicity eliminates statistical **heterogeneity** among sample paths, not spatial wandering.

⁂

1. https://projecteuclid.org/journals/osaka-journal-of-mathematics/volume-26/issue-1/Sample-path-properties-of-ergodic-self-similar-processes/ojm/1200781336.pdf
2. https://ocw.mit.edu/courses/6-441-information-theory-spring-2016/099eee31611a15cbb951bd427a89428f_MIT6_441S16_chapter_8.pdf
3. http://www.columbia.edu/~ks20/6712-14/6712-14-Notes-Ergodic.pdf

4. https://dsp.stackexchange.com/questions/1167/what-is-the-distinction-between-ergodic-and-stationary