Definition

A **current** T of degree k on a manifold M is a continuous linear functional on the space of compactly supported smooth differential forms of complementary degree $n-k$ (where n is the dimension of the manifold). This is expressed as:

$$T(\omega) \tag{1}$$

for a compactly supported smooth $(n-k)$-form ω on M.

Properties and Operations

1. **Boundary of a Current:** If T is a k-current, its boundary ∂T is a $(k-1)$-current defined by:

$$\partial T(\omega) = T(d\omega) \tag{2}$$

   where $d\omega$ is the exterior derivative of ω.

2. **Pushforward and Pullback:** Given a smooth map $f: M \to N$, the pushforward of a current can be defined from M to N. The pullback, however, is generally not well-defined for currents.

3. **Integration of Currents:** A n-dimensional current T on M can be integrated over M, represented by:

$$\int_M T \tag{3}$$

Examples of Currents

1. **Dirac Delta Current:** For a point p in M, the Dirac delta current δ_p of degree n acts on a n-form ω by:

$$\delta_p(\omega) = \omega(p) \tag{4}$$

2. **Integration Along a Submanifold:** Let S be a k-dimensional oriented smooth submanifold of M. The current $[S]$ associated with S acts on a $(n-k)$-form ω as:

$$[S](\omega) = \int_S \omega \tag{5}$$

Application in Equidistribution

Currents can be used to study the asymptotic distribution of zeros of holomorphic sections in Hermitian vector bundles, particularly how these zeros distribute themselves across the manifold, often converging to a certain limiting current.