1 Proof of Non-redundant Parameter Space Construction

Theorem 1. *Given a likelihood function $L(\theta)$ where $\theta \in \mathbb{R}^n$, the transformation $\theta_{\text{reduced}} = V_k^T \theta$ constructed from the Fisher Information Matrix (FIM) eigenvectors corresponding to non-zero eigenvalues yields a non-redundant parameter space where $k < n$ if and only if there were redundancies in the original parameterization.*

Proof. Let F be the Fisher Information Matrix with elements:

$$F_{ij} = -\mathbb{E}\left[\frac{\partial^2}{\partial \theta_i \partial \theta_j} \log L(\theta)\right]$$

Since F is symmetric and positive semidefinite, it has eigendecomposition $F = V \Lambda V^T$ where:

- $\Lambda = \text{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \ldots, \lambda_n)$ where $\lambda_i \geq 0$
- $V = [v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ \cdots \ v_n]$ with orthonormal eigenvectors

The rank k of F equals the number of non-zero eigenvalues[1]. When $k < n$, the model is parameter redundant with deficiency $d = n - k$[5].

Let $V_k = [v_1 \ v_2 \ v_3 \ \cdots \ v_k]$ contain only the eigenvectors corresponding to non-zero eigenvalues. The transformation $\theta_{\text{reduced}} = V_k^T \theta$ then:

- Projects onto a space of dimension $k < n$ if and only if redundancies existed
- Contains only independent parameters since each basis vector corresponds to a unique non-zero eigenvalue
- Preserves all information in the likelihood since zero eigenvalues indicate directions of flat likelihood[2]

Therefore, θ_{reduced} provides a minimal, non-redundant parameterization. □