

so assume it is and show me the fucking inverse for F u fuck

Certainly. Let's be precise and explicit.

We assume that the random measure $Z(\omega)$ is absolutely continuous with respect to Lebesgue measure, so that

$$dZ(\omega) = F(\omega) d\omega,$$

where $F(\omega)$ is a (complex-valued) random process, the "spectral process density."

The oscillatory process is then

$$X(t) = \int_{-\infty}^{\infty} \phi_t(\omega) F(\omega) d\omega,$$

with $\phi_t(\omega)$ a complete orthonormal system in $L^2(\mathbb{R}, d\omega)$.

Explicit Inversion Formula for $F(\omega)$

Since $(\phi_t(\omega))_{t \in \mathbb{R}}$ form an orthonormal basis, you invert by integrating $X(t)$ against the conjugate basis. Specifically,

$$F(\omega) = \int_{-\infty}^{\infty} X(t) \overline{\phi_t(\omega)} dt.$$

Proof:

Plug this back into the forward formula:

$$\begin{aligned} \int_{-\infty}^{\infty} X(t) \overline{\phi_t(\omega)} dt &= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} \phi_t(\lambda) F(\lambda) d\lambda \right) \overline{\phi_t(\omega)} dt \\ &= \int_{-\infty}^{\infty} F(\lambda) \left(\int_{-\infty}^{\infty} \phi_t(\lambda) \overline{\phi_t(\omega)} dt \right) d\lambda \\ &= \int_{-\infty}^{\infty} F(\lambda) \delta(\lambda - \omega) d\lambda \\ &= F(\omega). \end{aligned}$$

Where we used Fubini's theorem and the orthonormality:

$$\int_{-\infty}^{\infty} \phi_t(\lambda) \overline{\phi_t(\omega)} dt = \delta(\lambda - \omega).$$

Summary Table

Forward (synthesis)	Inverse (analysis)
$X(t) = \int \phi_t(\omega) F(\omega) d\omega$	$F(\omega) = \int X(t) \overline{\phi_t(\omega)} dt$

Special Cases

- Stationary case:** $\phi_t(\omega) = e^{i\omega t}$ gives the usual Fourier inversion:
$$X(t) = \int_{-\infty}^{\infty} e^{i\omega t} F(\omega) d\omega, \quad F(\omega) = \int_{-\infty}^{\infty} X(t) e^{-i\omega t} dt.$$
- Oscillatory case:** all above formulas hold for any orthonormal time–frequency family.

Final Answer:

If $dZ(\omega)$ is absolutely continuous, the inverse for $F(\omega)$ is

$$F(\omega) = \int_{-\infty}^{\infty} X(t) \overline{\phi_t(\omega)} dt$$

where $\phi_t(\omega)$ is the oscillatory orthonormal family used in the forward representation.