> restart;

>

> Theta := t→− I (lnGAMMA(1/4 + I t/2) − lnGAMMA(1/4 − I t/2))/2 − ln(π) t/2

$$\Theta := t \mapsto -\frac{\mathrm{I}\cdot\left(\mathrm{lnGAMMA}\left(\frac{1}{4}+\frac{\mathrm{I}\cdot t}{2}\right)-\mathrm{lnGAMMA}\left(\frac{1}{4}-\frac{\mathrm{I}\cdot t}{2}\right)\right)}{2}-\frac{\ln(\pi)\cdot t}{2} \quad (1)$$

>

> R := (t, s)→exp((Theta(t) − Theta(s))^2)

$$R := (t, s) \mapsto e^{(\Theta(t)-\Theta(s))^2} \quad (2)$$

>

> lambda := unapply(simplify(sqrt((4 k + 1)/Pi)·pochhammer(k + 1, −1/2)^2), k)

$$\lambda := k \mapsto \frac{\sqrt{4\cdot k+1}}{\sqrt{\pi}\cdot\mathrm{pochhammer}\left(k+\frac{1}{2}, \frac{1}{2}\right)^2} \quad (3)$$

> evalf([seq(simplify(sqrt(lambda(k))/lambda(k)), k = 0 ..56)])

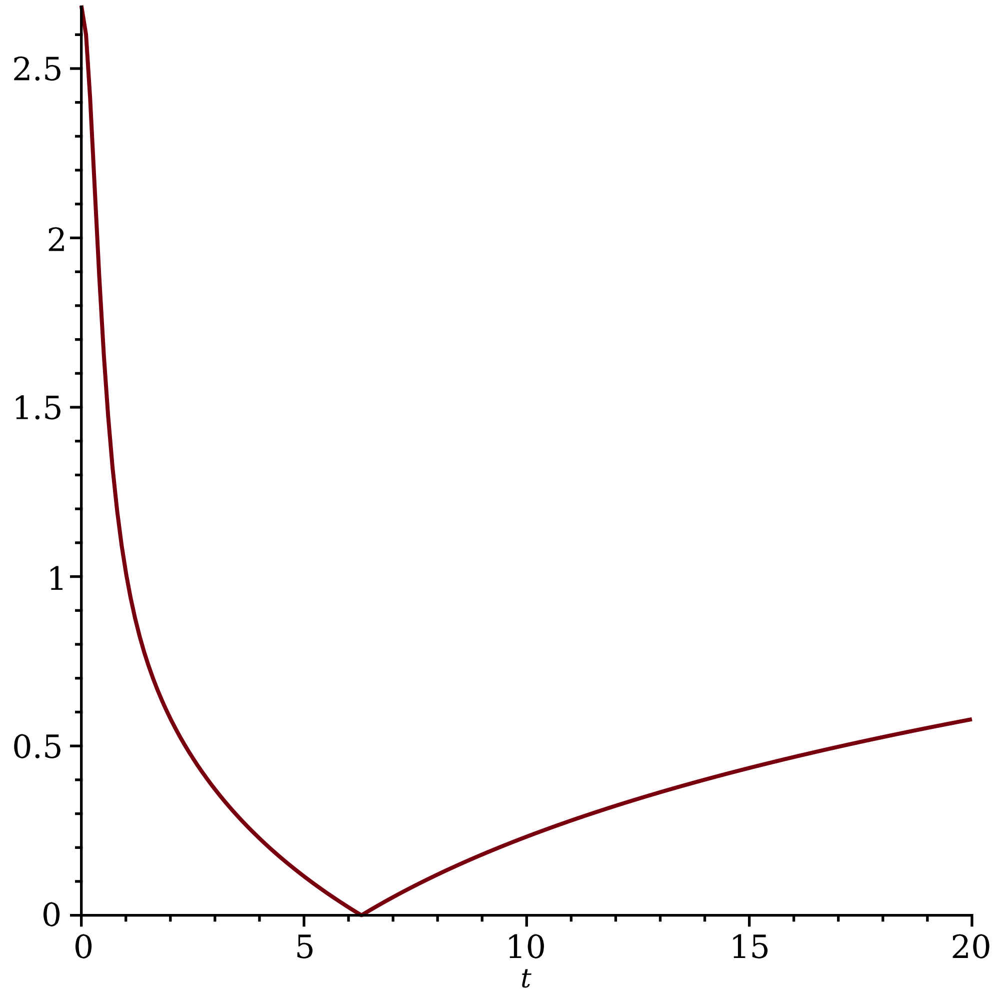
[0.7511255444, 1.004615851, 1.156433428, 1.265833754, 1.352826889, 1.425795120, 1.489071345, 1.545203532, 1.595825610, 1.642052613, 1.684681977, 1.724305255, 1.761374342, 1.796242871, 1.829193200, 1.860454658, 1.890216259, 1.918635774, 1.945846385, 1.971961620, 1.997079112, 2.021283478, 2.044648587, 2.067239331, 2.089113051, 2.110320690, 2.130907721, 2.150914927, 2.170379029, 2.189333226, 2.207807628, 2.225829650, 2.243424320, 2.260614557, 2.277421405, 2.293864240, 2.309960941, 2.325728043, 2.341180877, 2.356333675, 2.371199691, 2.385791276, 2.400119965, 2.414196554, 2.428031151, 2.441633247, 2.455011759, 2.468175078, 2.481131111, 2.493887316, 2.506450742, 2.518828050, 2.531025551, 2.543049223, 2.554904737, 2.566597482, 2.578132576] (4)

>

> simplify(sqrt(limit(simplify(−diff(exp((Theta(t) − Theta(s))^2)/2, t, s)), s = t)))

$$-\frac{1}{4}\left(\mathrm{csgn}\left(2\ln(\pi)-\Psi\left(\frac{1}{4}-\frac{\mathrm{I}\,t}{2}\right)-\Psi\left(\frac{1}{4}+\frac{\mathrm{I}\,t}{2}\right)\right)\left(-2\ln(\pi)+\Psi\left(\frac{1}{4}-\frac{\mathrm{I}\,t}{2}\right)+\Psi\left(\frac{1}{4}+\frac{\mathrm{I}\,t}{2}\right)\right)\right) \quad (5)$$

> $plot(\mathbf{(5)}, t = 0..20)$



> $Digits := 20 : ThetaMin := fsolve(\mathrm{Theta}'(t) = 0, t = 0..8)$

$$ThetaMin := 6.2898359888369027797 \qquad (6)$$

> $evalf\left(\left(1 - \frac{2 \cdot \mathrm{Pi}}{ThetaMin}\right) \cdot 100\right) + \text{"percent"};$

$$0.10573696464669982218 + \text{"percent"} \qquad (7)$$

> $MinThetaValue := simplify(\mathrm{Theta}(ThetaMin))$

$$MinThetaValue := -3.5309728290166074379 \qquad (8)$$

> $MinThetaValue \cdot 2$

$$-7.0619456580332148758 \qquad (9)$$

> $is(ThetaMin > 2 \cdot \mathrm{Pi})$

$$true \qquad (10)$$

> $solve\left(\mathrm{sqrt}(x) = \frac{\mathrm{sqrt}(2)}{2}\right)$

$$\frac{1}{2} \qquad (11)$$

> $simplify(\mathrm{Theta}'(ThetaMin))$

$$-5. \times 10^{-20} \qquad (12)$$

> $identify(-3.5309728290166074379-$

$$-3.5309728290166074379 \tag{13}$$

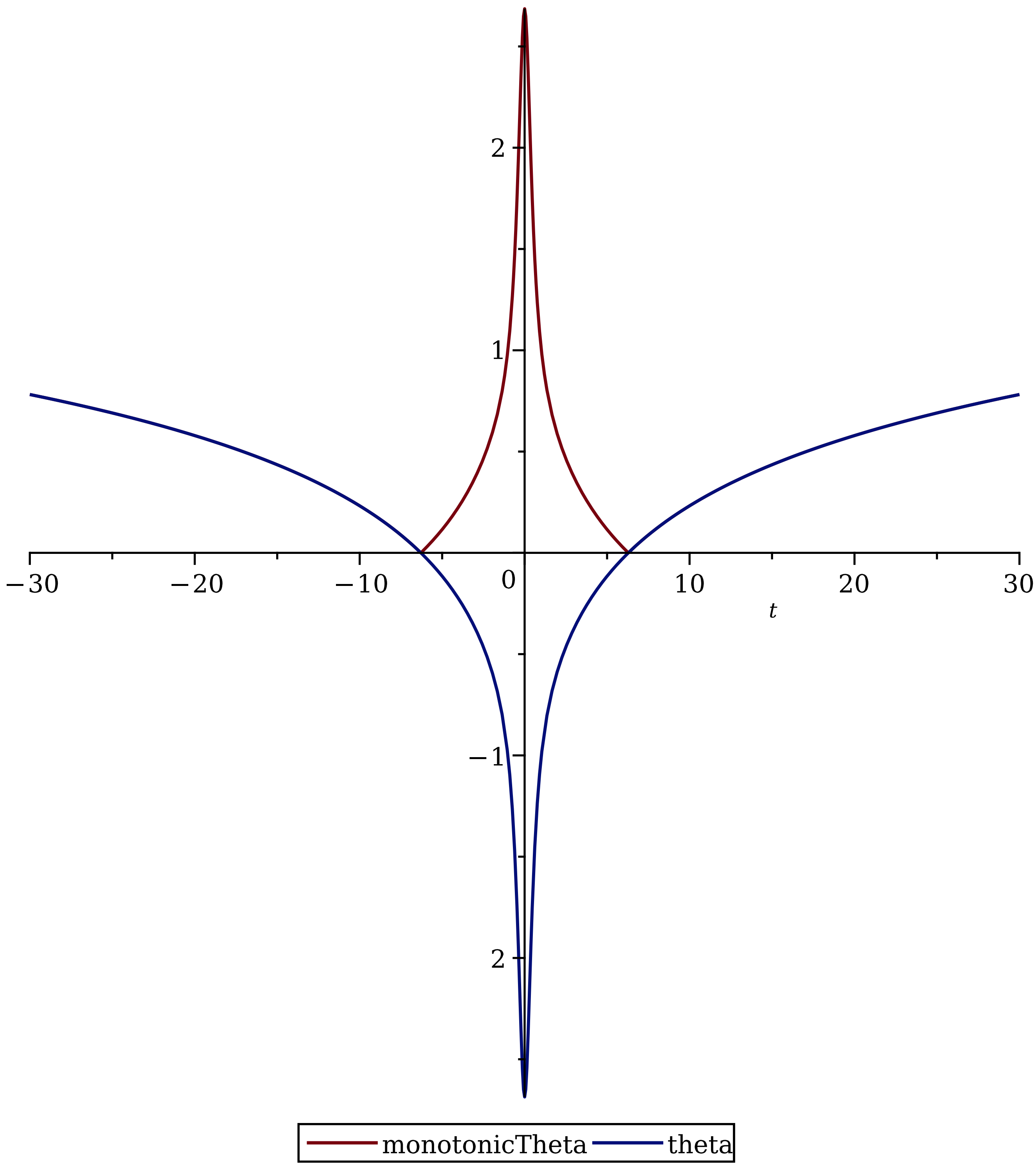
> $monotoneTheta := unapply(simplify(convert(piecewise(-ThetaMin < t < ThetaMin, MinThetaValue - (\mathrm{Theta}(t) - MinThetaValue), t < -ThetaMin, 2\cdot MinThetaValue - (-\mathrm{Theta}(t) - 2\cdot MinThetaValue), \mathrm{Theta}(t)), \mathrm{Heaviside})), t);$

$$\begin{aligned} monotoneTheta := t \mapsto {} & (0.50000000000000000000\cdot \mathrm{I} + \mathrm{I}\cdot \mathrm{Heaviside}(t \\ & - 6.2898359888369027797) - 1.\cdot \mathrm{I}\cdot \mathrm{Heaviside}(6.2898359888369027797 \\ & + t))\cdot \mathrm{lnGAMMA}\left(\frac{1}{4} - \frac{\mathrm{I}\cdot t}{2}\right) + (-0.50000000000000000000\cdot \mathrm{I} - 1.\cdot \mathrm{I} \\ & \cdot \mathrm{Heaviside}(t - 6.2898359888369027797) + \mathrm{I} \\ & \cdot \mathrm{Heaviside}(6.2898359888369027797 + t))\cdot \mathrm{lnGAMMA}\left(\frac{1}{4} + \frac{\mathrm{I}\cdot t}{2}\right) \\ & + (7.0619456580332148762 + 1.1447298858494001742\cdot t) \\ & \cdot \mathrm{Heaviside}(6.2898359888369027797 + t) + (7.0619456580332148758 \\ & - 1.1447298858494001742\cdot t)\cdot \mathrm{Heaviside}(t - 6.2898359888369027797) \\ & - 14.123891316066429752 - 0.57236494292470008710\cdot t \end{aligned} \tag{14}$$

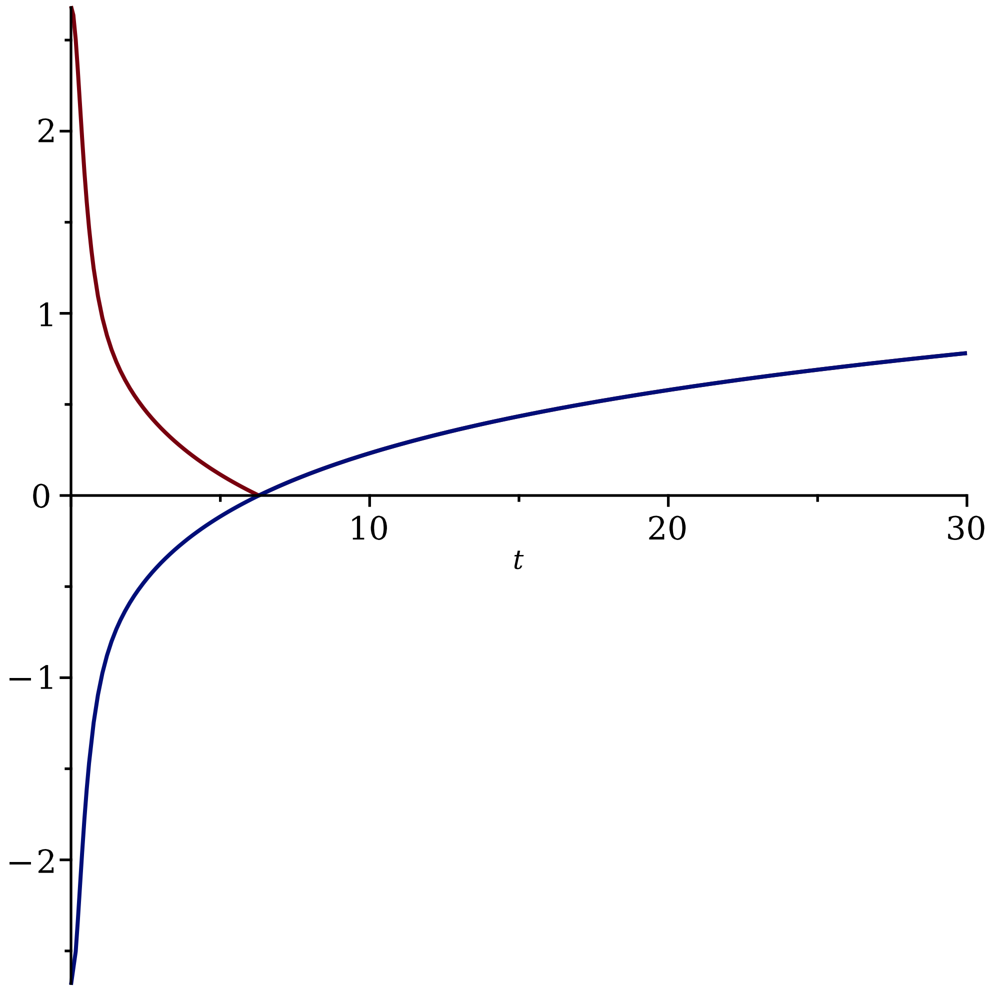
> $monotoneTheta(0)$

$$-7.0619456580332148758 + 0.\,\mathrm{I} \tag{15}$$

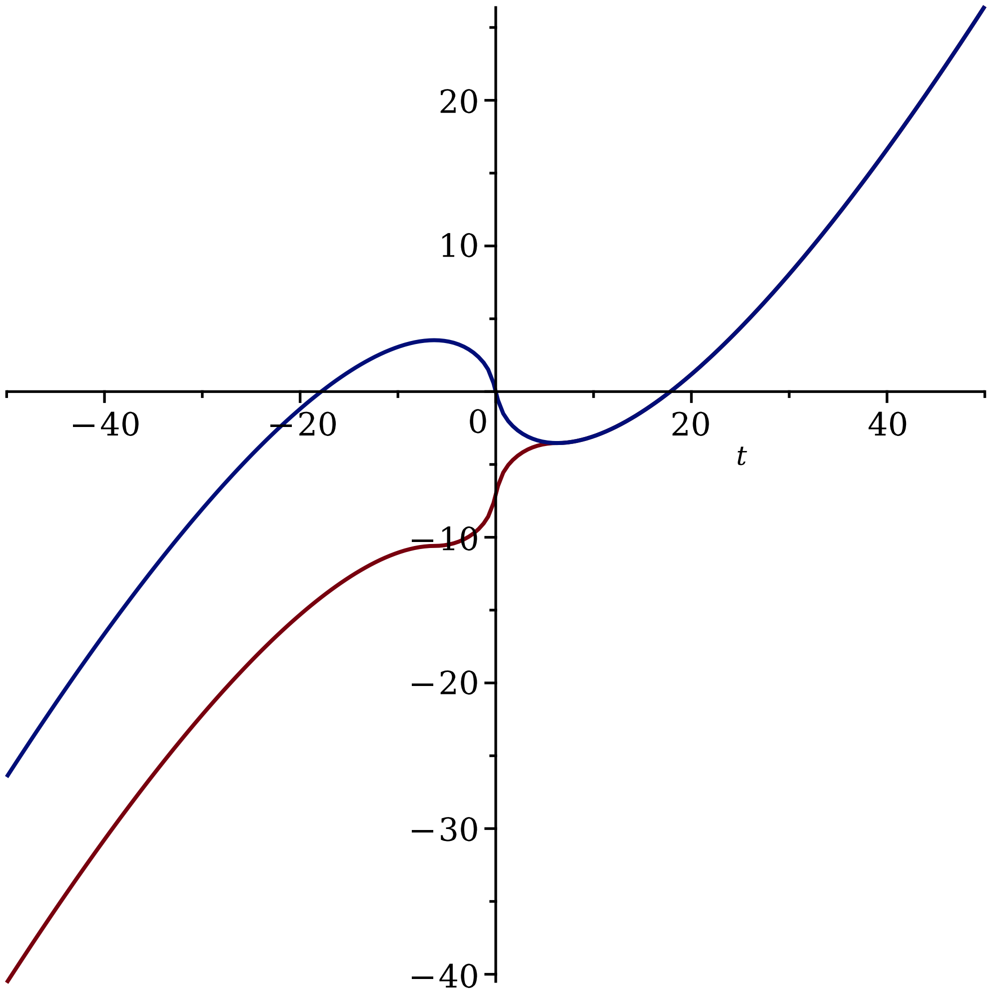
> $plot([monotoneTheta'(t), \mathrm{Theta}'(t)], t = -30..30, legend = [\text{"monotonicTheta"}, \text{"theta"}])$



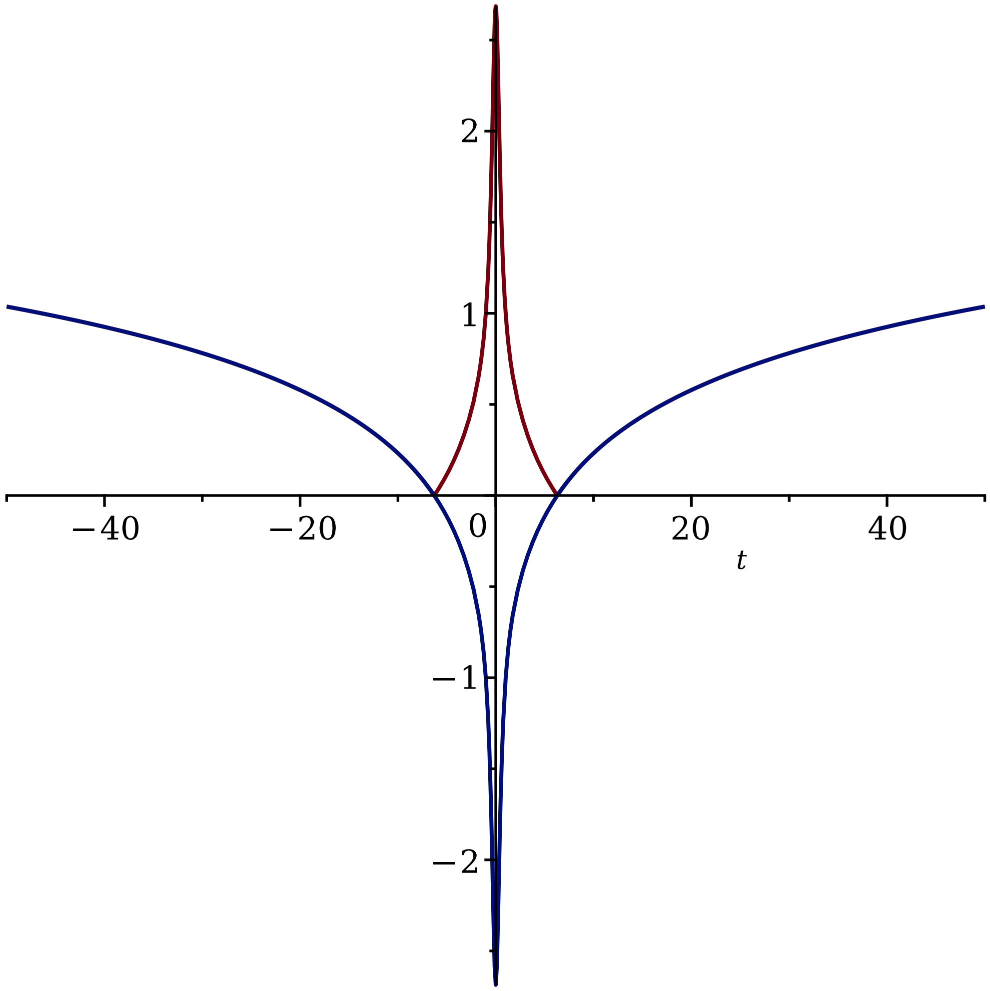
> $plot([monotoneTheta'(t), \mathrm{Theta}'(t)], t = 0..30)$



> $plot([monotoneTheta(t), \mathrm{Theta}(t)], t = -50..50)$



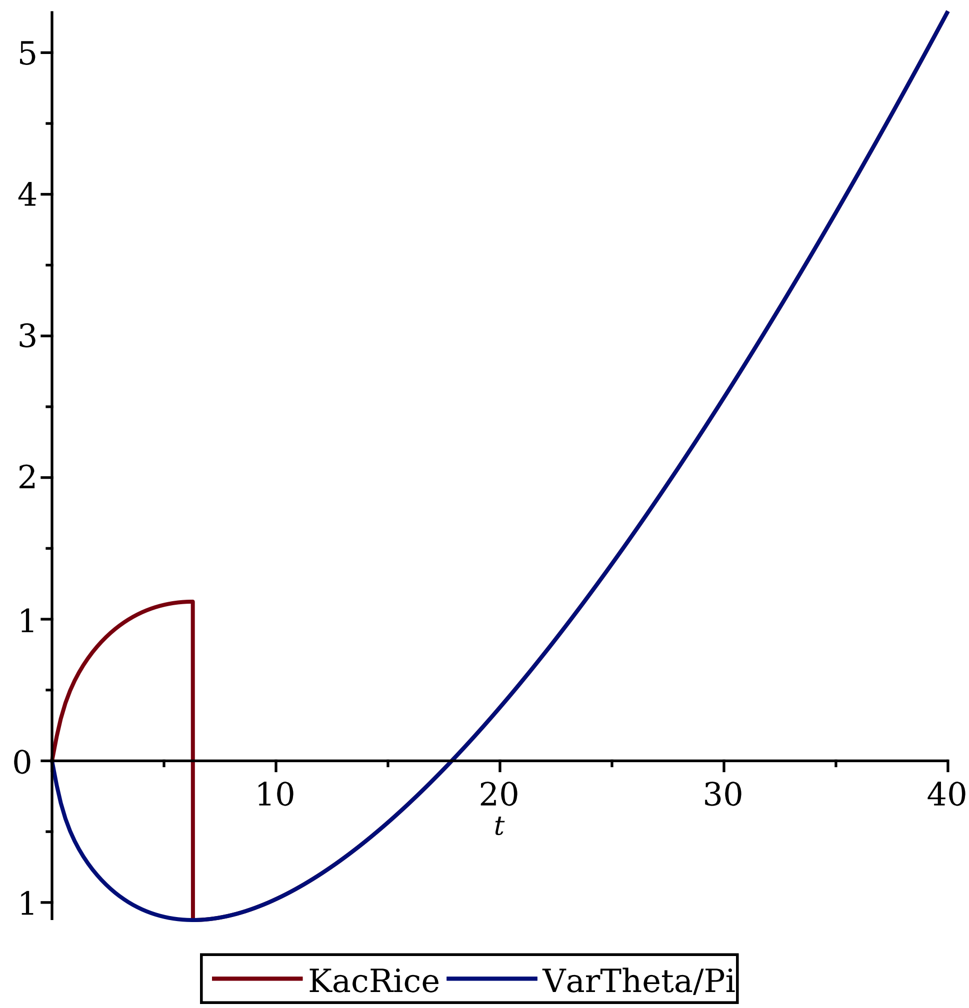
> $plot([monotoneTheta'(t), \mathrm{Theta}'(t)], t=-50..50)$



> $meanZeroCountingFunction := unapply\left(\frac{simplify(int(\mathbf{(5)}, t=0..t))}{\mathrm{Pi}}, t\right);$

$$meanZeroCountingFunction := t \mapsto -\frac{1}{2\cdot\pi}\left(\mathrm{csgn}\left(2\cdot\ln(\pi) - \Psi\left(\frac{1}{4} - \frac{\mathrm{I}\cdot t}{2}\right) - \Psi\left(\frac{1}{4} + \frac{\mathrm{I}\cdot t}{2}\right)\right)\cdot\left(\mathrm{I}\cdot\Psi\left(-1, \frac{1}{4} - \frac{\mathrm{I}\cdot t}{2}\right) - \mathrm{I}\cdot\Psi\left(-1, \frac{1}{4} + \frac{\mathrm{I}\cdot t}{2}\right) - \ln(\pi)\cdot t\right)\right) \tag{16}$$

> $plot\left(\left[meanZeroCountingFunction(t), \frac{\mathrm{Theta}(t)}{\mathrm{Pi}}\right], t=0..40, legend = [\text{"KacRice"}, \text{"VarTheta/Pi"}]\right)$



>

>

> $\dfrac{\mathrm{sqrt}\left(\mathit{limit}\left(\dfrac{\mathit{simplify}(-\mathrm{BesselJ}''(0,x))}{\mathrm{BesselJ}(0,0)}, x=0\right)\right)}{\mathrm{Pi}}$; *evalf*(%)

$$\frac{\sqrt{2}}{2\pi}$$

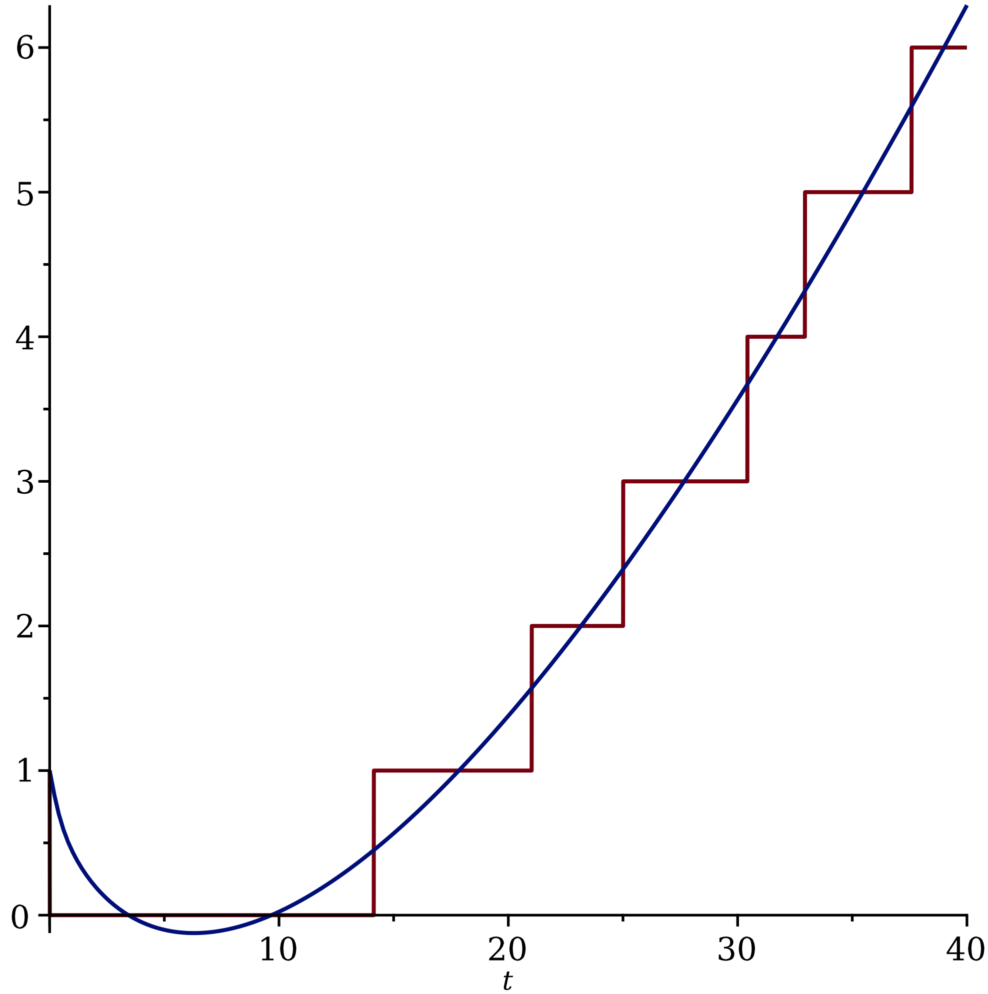
$$0.2250790790 \tag{17}$$

>

> $\mathit{approximateZeroCount} := \mathit{unapply}\left(\dfrac{\mathrm{Theta}(t)}{\mathrm{Pi}} + 1, t\right)$:

> $\mathit{zeroCount} := \mathit{unapply}\left(\dfrac{\mathrm{Theta}(t)}{\mathrm{Pi}} + 1 + \dfrac{-\frac{\mathrm{I}}{2}\left(\ln\left(\zeta\left(\frac{1}{2} + t\,\mathrm{I}\right)\right) - \ln\left(\zeta\left(\frac{1}{2} - \mathrm{I}\,t\right)\right)\right)}{\mathrm{Pi}}, t\right)$:

> $plot\left(\left[zeroCount(t), \frac{\mathrm{Theta}(t)}{\mathrm{Pi}} + 1\right], t = 0..40\right)$



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