Inverse Formulas for Absolutely Continuous Case

When U(λ) and V(λ) are absolutely continuous, the inverse formulas are:

Derivative Functions

$$U'(\lambda) = \frac{2}{\pi} \int_0^\infty X(t) \cos(\lambda t) \, dt$$

$$V'(\lambda) = \frac{2}{\pi} \int_0^\infty X(t) \sin(\lambda t) \, dt$$

Integrated Forms

$$U(\lambda) = \int_0^\lambda U'(\mu) \, d\mu = \frac{2}{\pi} \int_0^\lambda \left[\int_0^\infty X(t) \cos(\mu t) \, dt\right] d\mu$$

$$V(\lambda) = \int_0^\lambda V'(\mu) \, d\mu = \frac{2}{\pi} \int_0^\lambda \left[\int_0^\infty X(t) \sin(\mu t) \, dt\right] d\mu$$

Interchanging Integration Order

$$U(\lambda) = \frac{2}{\pi} \int_0^\infty X(t) \left[\int_0^\lambda \cos(\mu t) \, d\mu\right] dt = \frac{2}{\pi} \int_0^\infty X(t) \frac{\sin(\lambda t)}{t} \, dt$$

$$V(\lambda) = \frac{2}{\pi} \int_0^\infty X(t) \left[\int_0^\lambda \sin(\mu t) \, d\mu\right] dt = \frac{2}{\pi} \int_0^\infty X(t) \frac{1-\cos(\lambda t)}{t} \, dt$$

These are the Fourier cosine and sine transform inversions for the absolutely continuous spectral measures.