Theorem 1. *For any rational function* $f(t-s) = \frac{P(t-s)}{Q(t-s)}$*, where* P *and* Q *are polynomials, there exist rational functions* $g(t)$ *and* $g(s)$ *such that* $f(t-s) = g(t)\, g(s)$*.*

Proof. Let $P(t-s) = c_p \prod_{i=1}^{n} (t-s-\alpha_i)$ and $Q(t-s) = c_q \prod_{j=1}^{m} (t-s-\beta_j)$ be the complete factorizations over $\mathbb{C}$. Define:

$$g(t) = \sqrt{\frac{c_p}{c_q}}\, \frac{\prod_{i=1}^{n} (t-\alpha_i)}{\prod_{j=1}^{m} (t-\beta_j)}$$

Then:

$$\begin{aligned}
g(t)\, g(s) &= \frac{c_p}{c_q}\, \frac{\prod_{i=1}^{n} (t-\alpha_i)}{\prod_{j=1}^{m} (t-\beta_j)} \cdot \frac{\prod_{i=1}^{n} (s-\alpha_i)}{\prod_{j=1}^{m} (s-\beta_j)}\\
&= \frac{c_p}{c_q}\, \frac{\prod_{i=1}^{n} (t-\alpha_i)\,(s-\alpha_i)}{\prod_{j=1}^{m} (t-\beta_j)\,(s-\beta_j)}\\
&= \frac{c_p \prod_{i=1}^{n} ((t-s)-\alpha_i)}{c_q \prod_{j=1}^{m} ((t-s)-\beta_j)}\\
&= f(t-s)
\end{aligned}$$

For complex roots, we pair each α_i or β_j with its complex conjugate in the factorization of $g(t)$. This ensures that the product $(t-\alpha_i)\,(t-\bar{\alpha_i})$ results in a quadratic polynomial with real coefficients, making $g(t)$ a real-valued function. □