GLOBAL COMPLEX DYNAMICS OF THE HYPERBOLIC TANGENT OF THE LOGARITHM OF ONE PLUS THE SQUARE OF THE HARDY Z FUNCTION

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ABSTRACT. The function $Y(t) = \tanh(\ln(1 + Z(t)^2))$ is studied

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1. Introduction

Let $Y(t)$ be the hyperbolic tangent of the logarithm[1] of $1+Z(t)^2$

$$\begin{aligned} Y(t) &= \tanh(\ln(1+Z(t)^2)) \\ &= \frac{(1+Z(t)^2)^2-1}{(1+Z(t)^2)^2+1} \end{aligned} \tag{1}$$

where $Z(t)=e^{i\vartheta(t)}\zeta\left(\frac{1}{2}+it\right)$ is the Hardy Z function, $\vartheta(t)$ is the Riemann-Siegel theta function

$$\vartheta(t) = -\frac{i}{2}\left(\ln\Gamma\left(\frac{1}{4}+\frac{it}{2}\right)-\ln\Gamma\left(\frac{1}{4}-\frac{it}{2}\right)\right)-\frac{\ln(\pi)\,t}{2} \tag{2}$$

$\ln\Gamma(t)$ is the principle branch of the logarithm of the Γ function

$$\Gamma(t)=(t-1)! = \int_0^\infty x^{t-1}\,e^{-x}\mathrm{d}x \forall \mathrm{Re}(t)>0 \tag{3}$$

and $\zeta(t)$ is the Riemann zeta function

$$\begin{aligned} \zeta(s) &= \sum_{n=1}^{\infty}\frac{1}{n^s}\forall s>1 \\ &= \frac{1}{1-2^{1-s}}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^s}\forall s>0 \end{aligned} \tag{4}$$

1. See Appendix 6.4

Furthermore, Let the locus of points where the real part of $Y(t)$ has an inverse $a\in[-1,1]$ be denoted[2] as

$$L_a=\{(t,s)\colon \mathrm{Re}(Y(t+i s)=a)\} \tag{5}$$

and the locus of points where the imaginary part of $Y(t)$ has an inverse $a\in[-1,1]$ be denoted as

$$R_a=\{(t,s)\colon \mathrm{Im}(Y(t+i s)=a)\} \tag{6}$$

The sets L_a and R_a are not simply connected since L_a forms an island around each root, so let us define the restriction of L_a to the simply-connected component containing n-th root, y_n, of $Z(t)$ on the real line, of which an infinity are known to exist.

$$L_a^n=\{L_{_a}\colon L\subset \mathrm{SimplyConnectedComponent}(y_n)\} \tag{7}$$

The sets R_a are not simply connected since they are punctured by an infinite set of regions L_a^n containing the roots of Z.

1.1. Holomorphic “Analytic” Functions and Harmonic Conjugates,.

Because of the Cauchy-Riemann theorem we know that L and R are harmonic conjugate of each other, which means that their angle of intersection, then they do intersect, is always 90 degrees. They are related via the Hilbert transform and it's inverse. This fact is a direct consequence of complex differentiability and the related fact that holomorphic functions satisfy the Cauchy-Riemann equations.

1.1.1. *The Hilbert Transform.*

Also closely related is the Hilbert transform and holomorphic conjugates. [3]

1.2. The Advantages of Studying $Y(t)$ instead of $Z(t)$ or $\zeta(t)$?.

The Hardy Z function has the property that it is known, independently of the Riemann hypothesis, that $Z(t)\in\mathbb{R}\forall t\in\mathbb{R}$, that is, Z is real-valued when t is real.

If one looks at the curves where the real and imaginary parts of $Z(t)$ vanish independently, we find that the boundaries of both sets extend to infinity and do not cross orthogonally, rather, they meet at infinity; as can be seen in Figure 1.

If we look at the curves where $\mathrm{Re}(Y)$ and $\mathrm{Im}(Y)$ vanish we see that they are orthogonal at their intersection points where they intersect at 90 degree angles, at 1 point at the root on the real axis for a total of 5 intersection points.

The hyperbolic tangent serves to bound the range of the function in the interval $[-1,-1]$ and this is important because $Z(t)$ is known to grow without bound as t increases. It also has the property that 0 is a fixed-point of tanh, that is $\tanh(0)=0$, so that roots of Z are roots of Y.

2. The notation $\{x\colon$ logical predicate$\}$ can be read as ″the set of all points x such that the logical predicate is true″

3. Thanks to Petr Blaschke for pointing out that these curves are related via harmonic conjugation and not circle inversion as I had earlier conjectured.

Ultimately, the feature that the functional $Y(t)$ has over the 'simpler' (as some are apt to perceive) functions $Z(t)$, $\zeta(t)$, $\xi(t)$, $\eta(t)$,etc,$\ldots$ is that the corresponding Riemann surface of Y has a topology which features closed lemniscate curves around each root, which allows the constructs of the Riemann mapping theorem and the Helly's selection theorem on tight measures to be invoked since measures on the surface of Y do not escape to infinite like measures on the surfaces of the Hardy Z, Riemann ζ(zeta), Riemann ξ(xi), or Dirichlet η(eta) functions.

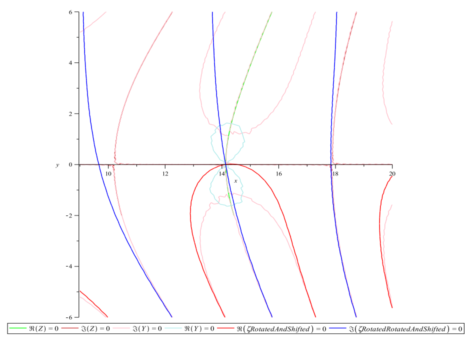

Figure 1. Curves where the real and imaginary parts of $\zeta\left(\frac{1}{2} + y + i\,x\right)$, $Z(x + i\,y)$, and $Y(x + i\,y)$vanish independently. TODO: The jaggedness of the curves is due to imperfections in the rendering process of Maple and is not an inherently mathematical property of the curves which are smooth. Rerender.

1.3. The Connected Components L_n of $L = \{(t, s)\colon \mathrm{Re}(Y(t + i\,s) = 0)\}$.

Let L_n be the maximal connected component that contains the point y_n where y_n is the imaginary part of the n-th Riemann zero $\zeta\left(\frac{1}{2} + i\,y_n\right) = 0$ ordered by increasing y_n of which there are known to be an infinity on the line, independent of the Riemann hypothesis[4]. Let ℓ_n represent the length of the boundary ∂L_n (*perimeter*) of L_n, which is also known as the arc-length of the curve L_n

$$\ell_n = \oint_{\partial L_n} 1 \mathrm{d}t \tag{8}$$

The integrand 1 is just the constant indicator function which takes the constant value 1 along the boundary ∂L_n. In this way, L is the union of its simply-connected maximal components $L = \bigcup_{n=0}^{\infty} L_n$.

To calculate the value of the contour integral (8) , the following algorithm is proposed:

1. a. Choose parameter n, the zero of the index;

1. b. Choose parameter $h > 0$, the small delta with which the integral will be calculated

4. TODO: link to reference or say something else about this?

2. Let $x_0 = y_n$ and $\mathrm{len} = 0$

3. Determine angle θ_m between 0 and 2π where the real part vanishes and the derivative at the vanishing point is positive(negative?)[5]

$$\theta_m = \{a\colon \mathrm{Re}(Y(t = x_{m-1} + h\, e^{ia}))) = 0\colon Y'(t) > 0, \mathrm{Im}(t) > 0\} \tag{9}$$

when iterating from $x_{(m-1)}$ to x_m a step size of h and angle $\theta_m(h)$ given by the circle with radius h centered at z defined by the formula

$$\mathrm{circle}(z, r, \theta) = z + r e^{i\theta} = z + r(\cos(\theta) + i\sin(\theta)) \tag{10}$$

where angle parameter θ varies from 0 to 2π.

4. Set

$$x_m = x_{m-1} + h\, e^{i\theta_m} \tag{11}$$

and

$$\mathrm{len} = \mathrm{len} + h \tag{12}$$

then m represents the index of the m-th iterate of the Newton iteration $\theta_m(h) = \lim_{k\to\infty} N_{\theta_m}(a_k; h)$ defined below.

5. If $x_m = y_n$ then terminate because the loop around the lobe has reached its beginning point, and $\ell_n = 2\,\mathrm{len}$ will be its length;

else set

$$m = m + 1 \tag{13}$$

and goto Step 3.

Note 1. The idea of this iteration procedure is to start at a root of $Y(t)$ on the real axis, and then trace out a path along the curve where $\mathrm{Re}(Y(t)) = 0$ going in a clockwise direction in the upper half-plane. The length of the entire boundary of the connected component will be twice the value accumulated along the contour integral, due to complex conjugate symmetry $Y(t) = \overline{Y(\bar{t})}$

1.3.1. *Integration Along a Curve: A Newton Iteration for the Angle.*

In order to find an explicit expression for the angle $\theta_m(h)$ in Formula (9) we can use Newton's method

$$N_{\theta_m}(a_k; h) = a_{k-1} - \frac{Y(x_m + h\, e^{i a_{k-1}})}{\frac{\mathrm{d}}{\mathrm{d}a} Y(x_m + h\, e^{ia})_{|a = a_{k-1}}} \tag{14}$$

where the initial angle $a_0 = \theta_{m-1}$, the angle at the last iteration if not at the origin, or $\frac{3\pi}{2}$ (270°) if starting at the origin so that we trace a path along the curve in a positive clockwise direction moving first into the upper-left quadrant . Let the angle at the m-th step of the iteration be defined as the limit

$$\theta_m = \theta_m(h) = \lim_{k\to\infty} N_{\theta_m}(a_k; h) \tag{15}$$

5. There will be 4 points around any given point where the real and imaginary parts of the function vanishes; due to complex conjugate symmetry. On a point where only the real or imaginary part vanishes, there will be 2 points, "forward and backward". At the root, the movements are "forward up", "forward down", "backward up", and "backward down".

which is dependent on h, but the when the dependence is not written as $\theta_m(h)$ it is still implied. To calculate $\frac{\mathrm{d}}{\mathrm{d}a}Y(x_m+he^{ia})_{|a=a_{k-1}}$ firstly differentiate $Y(t)$ to get

$$\frac{\mathrm{d}}{\mathrm{d}t}Y(t)=\frac{\mathrm{d}}{\mathrm{d}t}\tanh(\ln(1+Z(t)^2))=8\frac{(1+Z(t)^2)Z(t)}{(Z(t)^4+2Z(t)^2+2)^2}\frac{\mathrm{d}}{\mathrm{d}t}Z(t) \tag{16}$$

then it can be seen that the derivative of $Y(t)$ with respect to the angle a on the circle of radius h centered at the point t is given by

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}a}Y(t+h\,e^{ia}) &= \frac{\mathrm{d}}{\mathrm{d}a}\tanh(\ln(1+Z(t+h\,e^{ia})^2))\\ &= \frac{8(1+Y(t)^2)Y(t)}{(Y(t)^4+2Y(t)^2+2)^2}\frac{\mathrm{d}}{\mathrm{d}t}Y(t)he^{i\pi a}\\ &= \frac{8(1+Y(t)^2)Y(t)}{(Y(t)^4+2Y(t)^2+2)^2}\frac{8(1+Z(t)^2)Z(t)}{(Z(t)^4+2Z(t)^2+2)^2}\frac{\mathrm{d}}{\mathrm{d}t}Z(t)he^{ia}\\ &= i8\pi he^{ia}\,Z(t)\frac{(Z(t)+i)\,(Z(t)-i)}{(Z(t)^2+1+i)^2\,(Z(t)^2+1-i)^2}\frac{\mathrm{d}}{\mathrm{d}t}Z(t)\end{aligned} \tag{17}$$

and therefore Formula (14) is expressed as

$$N_{\theta_m}(t,a_k;h) = a_{k-1}-\mathrm{Re}\left(\frac{Y(t+h\,e^{ia_{k-1}})}{h\,i\,8\pi\,\mathrm{e}^{ia_{k-1}}\,Z(t)\frac{(Z(t)+i)\,(Z(t)-i)}{(Z(t)^2+1+i)^2\,(Z(t)^2+1-i)^2}\frac{\mathrm{d}}{\mathrm{d}t}Z(t)}\right) \tag{18}$$

with the change-of-variables $t=x_m+he^{ia_{k-1}}$ such that the iterates a_k converge to θ_m as $k\to\infty$.

1.3.2. *A Banach Space From the Cauchy Sequence.*

TODO: define the Banache spaces formed by the Newton sequences

1.4. **Zeros of Y(t).**

1.4.1. *Zeros of $Y(t)$ where $Z(t)=0$.*

All zeros of $Z(t)$ are zeros of $Y(t)$, since, when $Z(t)=0$ the formula for Y reads

$$\tanh(\ln(1+0^2))=\tanh(\ln(1))=\tanh(0)=0 \tag{19}$$

1.4.2. *Zeros of Y(t) where $Z(t)=\pm i\sqrt{2}$.*

There are additional points where $Y(t)=0$ and $Z(t)=\pm i\sqrt{2}$ since

$$\tanh(\ln(1+(-i\sqrt{2})^2))=\tanh(\ln(1+(i\sqrt{2})^2))=\tanh(\ln(1+(-2)))=\tanh(\ln(-1))=\tanh(i\pi)=0 \tag{20}$$

because of the identities

$$\ln(-1)=i\pi \tag{21}$$

and

$$(-i\sqrt{2})^2=(i\sqrt{2})^2=-2 \tag{22}$$

The points where $Z(t) = \pm i\sqrt{2}$ are near the top or bottom of the lemniscate curve where $Y(t) = 0$ since this point is the intersection of the curves of vanishing real and imaginary part.

$$r_{1,\text{topPOC}} = \{Z(t) = i\sqrt{2}: t \text{ near 1st root}\} = 14.33046626 + 1.612371374\, i \tag{23}$$

$$r_{1,\text{bottomPOC}} = \{Z(t) = -i\sqrt{2}: t \text{ near 1st root}\} = 14.33046626 - 1.612371374\, i \tag{24}$$

1.4.3. *The Isolated Points Where $Y(t) = -1$ When $Z(t) = \pm i$.*

When $Z(t) = \pm i$ the value of $Y(t) = -1$ since

$$\lim_{t \to \pm i} \tanh(\ln(1 + Z(t)^2)) = -1 \tag{25}$$

$$r_{1,\text{topInt}} = \{Z(t) = i: t \text{ near 1st root}\} = 14.24593979 + 1.192627391\, i \tag{26}$$

$$r_{2,\text{bottomInt}} = \{Z(t) = -i: t \text{ near 1st root}\} = 14.24593979 - 1.192627391\, i \tag{27}$$

Conjecture 2. *The points t where $Y(t) = -1$ are always within one of the simply-connected maximal components L_n. That is, $\{t: Y(t) = -1\} \subset L \forall t \in \mathbb{C}^\infty$*

Conjecture 3. The real part of Y is negative inside L. That is, $\mathrm{Re}(Y(t)) < 0 \forall t \subset L$

Corollary 4. *The real part of Y is positive outside L. That is,* $\mathrm{Re}(Y(t)) > 0 \forall t \not\subset L$

1.4.4. *Singularities Where $\lim_{s \to t} \frac{1}{Y(s)} = 0$ And $\lim_{s \to t} Y(s) = \infty$.*

These points are singularities of the imaginary part of Y on the zero curve of the real part of Y.

$$\lim_{t \to r^+_{1,\text{eitherPole}}} \mathrm{Im}(Y(H_n(t))) = \infty \neq \lim_{t \to r^-_{1,\text{eitherPole}}} \mathrm{Im}(Y(H_n(t))) = -\infty \tag{28}$$

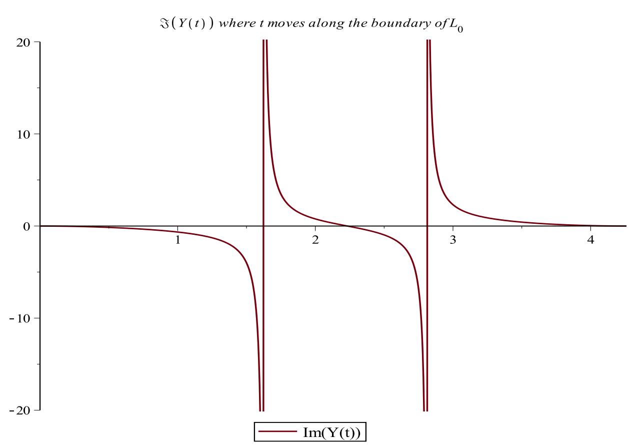

Figure 2. $\mathrm{Im}(Y(\partial L_0(t)))$ where $\partial L_0(t)$ is the implicit function where the real part of Y vanishes around y_0. The point y_0 is represented by the far left and right part of the figure; the point where the value is 0 in the middle is the root where $Z(t) = \pm\sqrt{2}i$ and the poles on the left and right are the poles where the imaginary part diverges when tracing out the path where the real part vanishes..

$$r_{1,\mathrm{topLeftPole}} = 13.78236061 + 1.421843162\, I \tag{29}$$

$$r_{1,\mathrm{topRightPole}} = 14.73435872 + 1.235291930\, I \tag{30}$$

$$r_{1,\mathrm{bottomLeftPole}} = 13.78236061 - 1.421843162\, I \tag{31}$$

$$r_{1,\mathrm{bottomRightPole}} = 14.73435872 - 1.235291930\, I \tag{32}$$

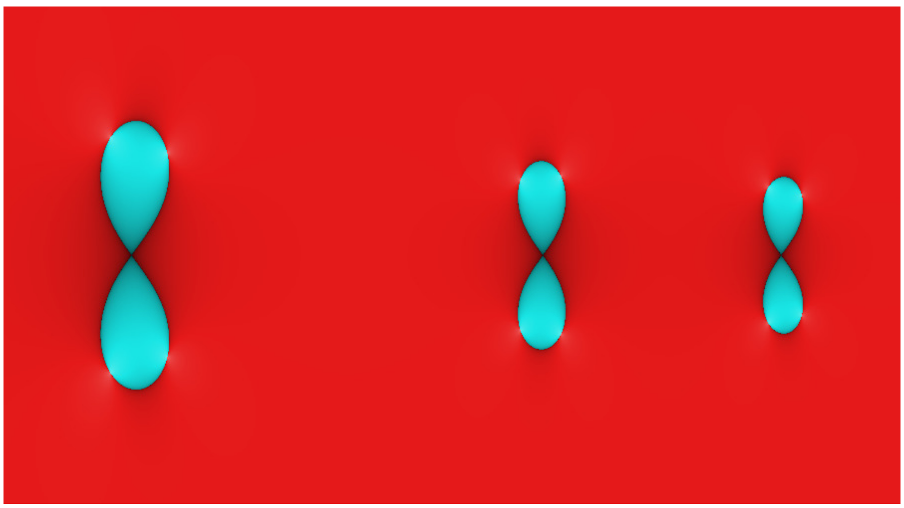

Figure 3. plot of the real part $\mathrm{Re}(Y(t+is))$ where $t = 12...27$ and $s = -3...3$. Red=Positive, Blue=Negative

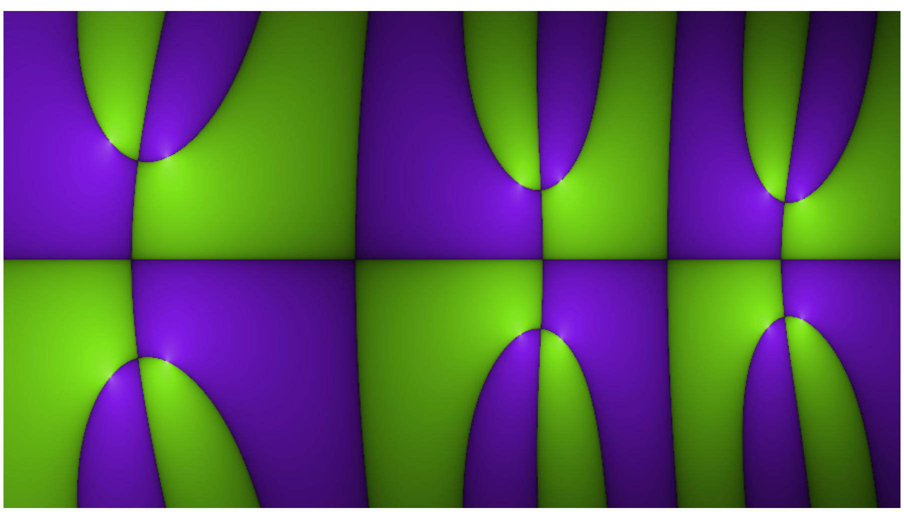

Figure 4. plot of the imaginary part $\mathrm{Im}(Y(t+is))$ where $t = 12...27$ and $s = -3...3$

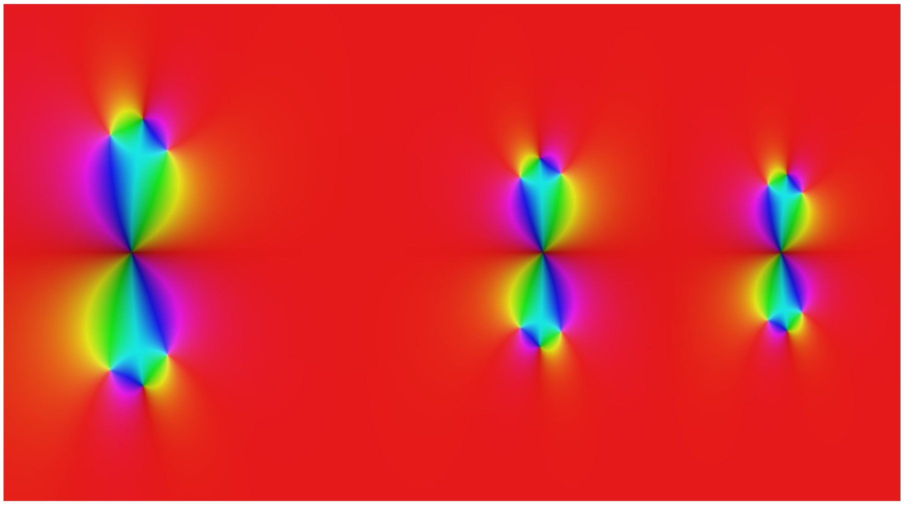

Figure 5. plot of $Y(t+is)$ where $t=12...27$ and $s=-3...3$.

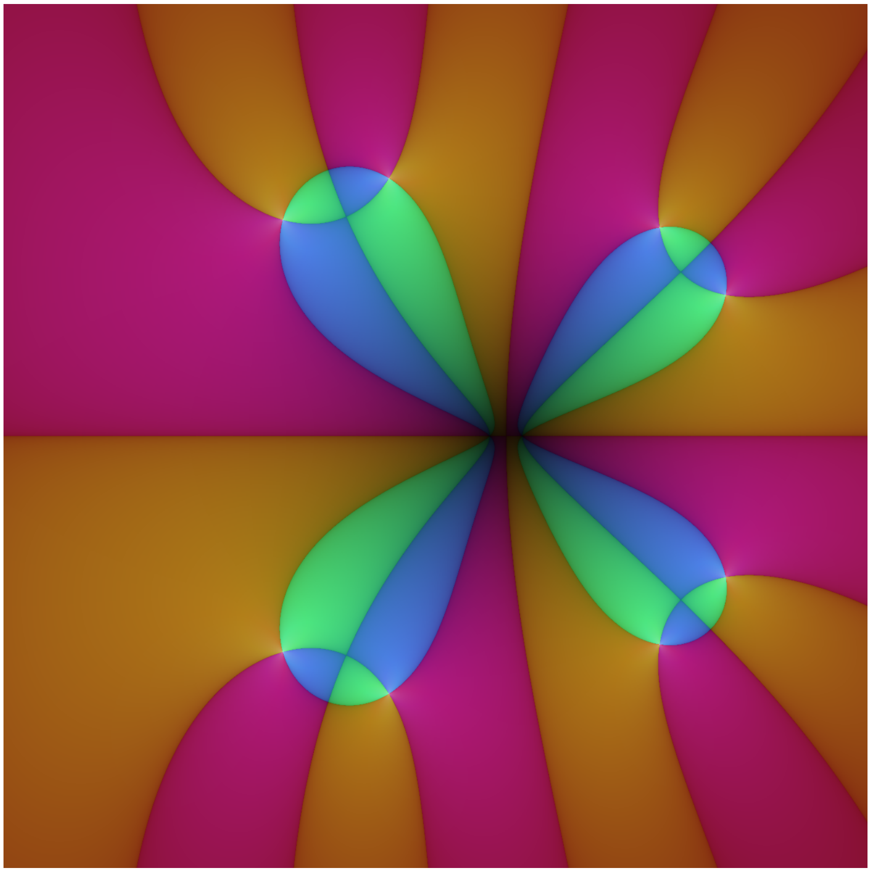

Figure 6. Composite image of real and imaginary parts of $Y(t)$ from $t=7004.5-0.5i$ to $7005.5+0.5i$ near the 1st Lehmer pair

2. Complex Dynamics

2.1. Fixed-Point Classification.

Theorem 5. *Leau-Fatou Flower Theorem*

If the origin is a fixed point of multiplicity $n+1 \geqslant 2$ then there exists n disjoint attracting petals U_i and n disjoint repelling petals $U_i^{'}$ so that the union of these $2n$ petals, together with the origin itself, forms a neighborhood N_0 of the origin. These petals alternate with each other so that each U_i intersects only $U_i^{'}$ and $U_{i-1}^{'}$ (where $U_0^{'}$ is to be identified with $U_n^{'}$).

2.2. Siegel Discs: Linearization and Lie Groups.

Theorem 6. *The* ***Riemann mapping theorem*** *states that if U is a non-empty simply connected open subset of the complex number plane* $\boldsymbol{C}$ *which is not all of* $\boldsymbol{C}$*, then there exists a biholomorphic mapping f (i.e. a bijective holomorphic mapping whose inverse is also holomorphic) from U onto the open unit disk*

$$D = \{z \in C : |z| < 1\}.$$

This mapping is known as a ***Riemann mapping****.*

2.2.1. *Linearization: Lie Groups → Lie Algebra.*

Theorem 7. ***(Koenig's Linearisation Theorem)***

Let $f: U \to \mathbb{C}$ be a holomorphic function where $f(0) = 0$ and $\dot{f}(0) = \lambda$. If the multiplier $|\lambda| \neq 1$ then f has an indifferent fixed-point in the neighborhood of 0 and is thus ***linearizable****.*

The process of going from a Lie group to a Lie algebra is known as **linearization**. The inverse of that, going from a Lie algebra to a Lie group is known as **exponentiation**.

The Riemann mapping theorem gaurantees the existence of a homeomorphism between any closed region on the complex plane and the unit disk. This unit disk can be mapped onto another closed region. These mappings should constitute the Siegel discs. Kawahira's reformulation of the Riemann hypotheis states it as as the nonexistence of Siegel discs in a certain region and therefore this approach is the inverse of that in the sense that the elucidation of the way the Siegel discs are structured will prove the rotation numbers of the Siegel discs around the roots are irrational geometric invariants and therefore the Riemann zeros are irrational as welll. [5]

Definition 8. ***A Siegel disc*** *is a* `connected component` *in the Fatou set where the dynamics is* `analytically conjugate` *to an irrational rotation. In other words, a Fatou component U is said to be a Siegel disc if there exists an analytic homeomorphism*

$$\varphi(f(\varphi^{-1}(z))) = z e^{2i\pi\alpha} \exists \alpha \in \mathbb{R} \backslash \mathbb{Q} \tag{33}$$

Let us define Cupid's arrow

$$g(t) = t + i\sqrt{1-t^2} \tag{34}$$

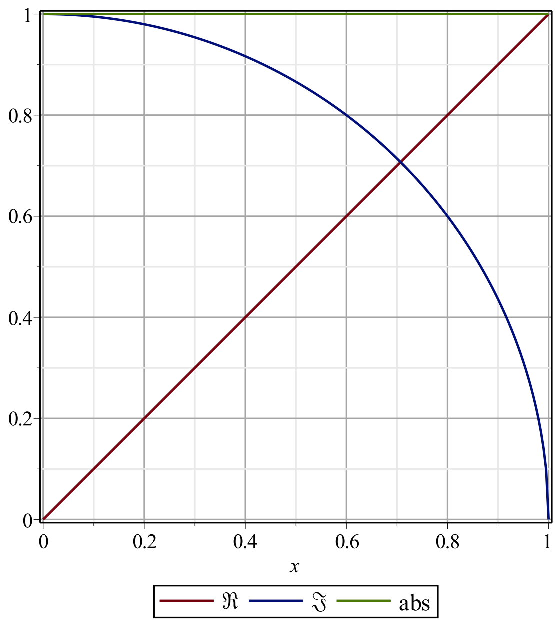

Figure 7. Cupid's arrow. $g(t)$

3. Conformal Mappings and Incompressible Fluid Flow

3.1. Navier-Stokes.

In [3, 1.3.6] the relation of conformal mappings (which are synonymous with holomorphic functions) to incompressible fluid flow hints at applicibility to the Navier-Stokes equation [8].

The 3 attributes of the mathematical model of incompressable fluid flow considred here are:

- *incompressiblity which implies constant density*
- *irrotational means that the curl is zero*
- *no viscosity*

Definition 9. *Curl*

The curl *is a* `vector operator` *that describes the infinitesimal circulation of a vector field in* $\mathbb{R}^3$*. The curl at a point in the field is represented by a vector whose length and direction denote the magnitude and axis of the maximum circulation. The curl of a field is formally defined as the* `circulation density` *at each point of the field.*

A vector field whose curl is zero is called `irrotational`*. The curl is a form of differentiation for vector fields. The corresponding form of the fundamental theorem of calculus is* Stokes' theorem*, which relates the* surface integral *of the curl of a vector field to the* line integral *of the vector field around the boundary curve.*

The alternative terminology rotation or rotational and alternative notations rot F or the cross product with the del (nabla) operator $\nabla \times F$ *are sometimes used for curl F. The ISO/IEC 80000-2 standard recommends the use of the rot notation in boldface as opposed to the curl notation.*

Unlike the gradient and divergence, curl does not generalize as simply to other dimensions; some generalizations are possible, but ***only in three dimensions*** *is* `the geometrically defined curl of a vector field again a vector field`*. This is a phenomenon similar to the 3-dimensional cross product, and the connection is reflected in the notation* $\nabla\times$ *for the curl.*

The name "curl" was first suggested by James Clerk Maxwell in 1871 but the concept was apparently first used in the construction of an optical field theory by James MacCullagh in 1839.

Definition 10. *Viscosity*

(Physics) A property possessed by a viscous fluid, being a resistance to the forces causing a fluid to flow, caused by interactions between the elements of the fluid and between the fluid and the walls of the conduit through which it moves; also, a measure of such a property.

4. Compact 3-Surfaces

4.1. A Family of Banach Spaces Around Each Root.

Not only is there a closed curve defined by $\mathrm{Re}(Y(t))=0$ around each root , there is densely defined family of closed curves defined by $\mathrm{Re}(Y(t))=a$ where $a\in[0,1]$, the curve shrinks to a point as a goes from $0\rightarrow-1$.

TODO: define the Banach spaces to which the Newton sequences above correspond.

4.2. Continuous Shape Transformations Homeomorhisms and Homotopies.

A homeomorphism is a way of mapping two spaces into each other without any tearing or gluing together. “No tearing” implies continuity, and “no gluing” implies bijectivity. A bijective continuous function on a compact Hausdorff spaces is automatically a homeomorphism (that is, the inverse is automatically continuous).

A homotopy, on the other hand, is also a deformation, but need not respect the “no gluing” condition. For instance, the closed unit interval $[0,1]$ is homotopic to a point. In constructing this homotopy, one does not “tear” the interval, but one does “contract it continuously". Each lobe of each $L_n(s)$ will form a closed curve which is continuous everywhere except a single point on the real line. The Riemann mapping theorem does not depend on continuity of the curves.

4.2.1. *Interior and Exterior Projections.*

The Riemann mapping theorem allows us to construct a mapping from the interior of any closed surface in the complex plane to its exterior and vice versa. The interior of L_1 will include only 1 root to which it is the connected component of, y_1. The mapping will allow us to express any point on Y outside L_1 in a different coordinate system relative to the manifold $L_1(s)$ will will necessarily have roots lying at some projection of every other root that is NOT itself. These mappings could be studied in isolation, or they could possibly be nested in one another. That is, if $Y_1=Y\setminus L_1$ then let us find in $Y_2=Y_1\setminus L_2$, $Y_k=Y_{k-1}\setminus L_k$ so that Y_k would contain projections of all roots $Y_l\forall l>k$ coming after it and be orthogonal to all the roots coming before it at $Y_j\forall j<k$.

5. Yang-Mills Mass Gap and Quantum Field Theory

5.1. Non-Pertubative Quantization of Yang-Mills Theory.

Hermann Weyl was one of the first to conceive of combining general relativity with the laws of electromagnetism. [4, 1.8 Weyl's Gauge Theory of Electromagnetism]

In [6] the Yang-Mills mass-gap 'problem' is reformulated as the existence of a probability measure on an infinite–dimensional space of gauge equivalence classes of connections on $\mathbb{R}^3$. A formally self–adjoint expression for the quantized Yang–Mills Hamiltonian as an operator on the corresponding Lebesgue $\mathbb{L}^2$–space exists and when the Yang–Mills field is associated to the Abelian group U(1) a probability measure which depends on two real parameters $m>0$ and $c\neq 0$ yields a non–standard quantization of the Hamiltonian of the electromagnetic field, and the associated probability measure is Gaussian. The corresponding quantized Hamiltonian is a self–adjoint operator in a Fock space the spectrum of which is $\{0\} \cup \left[\frac{n}{2},\infty\right)$, i.e. it has a gap.

Proposition 11. *[6, Proposition 1] Let $\mathcal{D}$ be the space of compactly supported K-connections on $\mathbb{R}^3$ and $\mathcal{K}$ be the group of compactly supported gauge transformations. Then*

i. *The space $\mathcal{D}$ is an infinite dimensional Riemannian manifold whose natural embeddding $T\mathcal{D}\hookrightarrow T^*\mathcal{D}$ is defined by the metric*

$$\langle E,F\rangle = -\int_{\mathbb{R}^3}(E\wedge,*F)\forall E,F\in T_A\mathcal{D}$$

and whose tangent bundle $T\mathcal{D}$ is a Poisson manifold induced by the canonical symplectic structure of $T^*\mathcal{D}$.

ii. *The gauge action $\mathcal{K}\times\mathcal{D}\to\mathcal{D}$ preserves the Riemannian metric and gives rise to a Hamiltonian group action $\mathcal{K}*T\mathcal{D}\to T\mathcal{D}$ with the moment map*
$\mu\colon T\mathcal{D}\to\Omega_c^0(\mathbb{R}^3,\mathfrak{k})$
$\mu(E,A) = \mathrm{div}_A E \qquad \forall(E,A)\in T\mathcal{D}\cong\mathcal{D}\times\mathcal{D}$

iii. *Gauge group action is free and reduced phase-space is a smooth manifold.*

I shall try to prove that ψ in this paper has as its natural and perhaps only choice up to equivalence, some function derived from Y, as the eggs on this surface are actually 3-surfaces defining this infinite set of probability measures on $\mathbb{R}^3$. This function is big enough to contain an infinite of universes.

Theorem 12. ***Voronin* Universality**. *Any non-vanishing analytic function can be* `approximated uniformly` *by certain purely imaginary shifts of the zeta function in the critical strip.*

The meaning of the word 'approximated' might as well just as be the real-deal, because it will be seen later how the cardinality of the continuum in $\mathbb{R}^4=(t,x,y,z)=\mathbb{R}\bigcup\mathbb{R}^3$ spacetime has emerged.

6. Appendix

6.1. Lie Groups and Algebras.

See [2, 1.4 and 3.3.4]

6.2. Riemann Surfaces.

6.2.1. *Uniformization Theorem.*

In [1, Theorem 1.1] the Uniformization Theorem is stated.

6.2.2. *Conformal Automorphism Groups.*

Any pair of Riemann surfaces S and T which are conformally isomorphic iff there is a homeomorphism from S onto T which is holomorphic relative to the local uniformizing parameters. Synonyms for 'conformally isomorphic' are 'conformal isomorphism', 'biholomorphic', and 'biholomorphism'. When $T = S$ the biholomorphism it is called a 'conformal automorphism.

For any Riemann surface S the notation $\mathcal{G}(S)$ will be used to denote the group consisting of all conformal automorphisms of S.

6.3. Clifford Algebras.

See [7]

6.4. The Rational Function tanh(ln(x)).

The hyperbolic tangent of the natural logarithm can be expressed as

$$\tanh(\ln(x)) = \frac{x^2 - 1}{x^2 + 1} \tag{35}$$

which has a pair of inverse branches given by

$$\{x \colon \tanh(\ln(x)) = y\} = \pm \frac{\sqrt{-(y+1)(y-1)}}{y - 1} \tag{36}$$

The singular points of $\tanh(\ln(x))$ are $\pm i$, that is,

$$\lim_{y \to \pm i} \tanh(\ln(y)) = \infty \tag{37}$$

The residue of $\tanh(\ln(x))$ at the point $\pm i$ is $\pm i$, that is

$$\mathrm{Res}(\tanh(\ln(x)), x = i) = i \tag{38}$$

and

$$\mathrm{Res}(\tanh(\ln(x)), x = -i) = -i \tag{39}$$

7. Is $Y(t)$ the Tao?

Lao Tzu(老子) is the reputed author of the Tao Te Ching who lived in China some around the time 500 B.C. and held a position as an archive keeper in some small province.

7.1. Verse 41.

The **path** *into* the **light** SEEMS **dark**,
the **path forward** SEEMS to *go* **back**,
the **direct path** SEEMS **long**,
true **power** SEEMS **weak**,

true **purity** SEEMS **tarnished,**
true **steadfastness** SEEMS **changeable**,
true **clarity** SEEMS **obscure**,
the `greatest` **art** SEEMS **unsophisticated**,
the `greatest` **love** SEEMS **indifferent**,
the `greatest` **wisdom** SEEMS **childish**.
The Tao is *nowhere* to be *found*.
Yet it *nourishes* and *completes* all things."

7.2. Is Wu-Wei Synonymous With The Principle of Least Action?.

Wu wei (無為), literally "non-action" or "not acting", is a central concept of the *Tao Te Ching*. The concept of *wu wei* is multifaceted, and reflected in the words' multiple meanings, even in English translation; it can mean "not doing anything", "not forcing", "not acting" in the theatrical sense, "creating nothingness", "acting spontaneously", and "flowing with the moment".[47]

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