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AN EQUIVALENCE OF TWO MASS GENERATION MECHANISMS FOR GAUGE FIELDS

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Two mass generation mechanisms for gauge theories are studied. It is proved that in the Abelian case the topological mass generation mechanism introduced in Refs. 4, 12 and 15 is equivalent to the mass generation mechanism defined in Refs. 5 and 20 with the help of "localization" of a nonlocal gauge invariant action. In the non-Abelian case the former mechanism is known to generate a unitary renormalizable quantum field theory, describing a massive vector field.

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1. Introduction

In the last two decades several mass generation mechanisms for non-Abelian gauge fields were suggested (see Ref. 14 for discussion of these mechanisms in three-dimensional case). In the framework of perturbation theory these mechanisms are expected to provide a new theoretical background for describing the electroweak sector of the Standard Model in such a way that the unobserved Higgs boson does not appear in the physical spectrum. On the other hand the problem of finding mass terms in gauge theories is motivated by nonperturbative Quantum Chromodynamics.

In this paper we study two mass generation mechanisms for non-Abelian gauge theories introduced in Refs. 4, 12, 15 and Refs. 5, 20 in the case of four-dimensional space–time. In Refs. 5 and 20 a new classical gauge invariant nonlocal Lagrangian generating a local quantum field theory was constructed. This phenomenon is similar to that for the Faddeev–Popov determinant in case of the quantized Yang–Mills field. Recall that being *a priori* nonlocal quantity the Faddeev–Popov determinant can be made local by introducing additional anticommuting ghost fields and applying a formula for Gaussian integrals over Grassmann variables. Similarly, in case of the nonlocal Lagrangian suggested in Refs. 5 and 20, one can introduce extra ghost fields, both bosonic and fermionic, and make the expression for the generating function of the Green functions local using tricks with Gaussian integrals. In the local expression for the Green functions, the "localized" Lagrangian containing extra ghost fields should be used instead of the original one. It was shown in Refs. 5, 6 and 21 that the corresponding "localized" Lagrangian containing extra ghost fields is renormalizable. When the coupling constant vanishes the nonlocal Lagrangian, in a certain gauge, is reduced to that of several copies of the massive vector field.

In fact, the "localized" Lagrangian describes the gauge field A coupled to an antisymmetric $(2,0)$-type tensor potential Φ via the topological term $\mathrm{tr}(*\Phi \wedge F)$ with a coupling constant m of mass dimension one, F being the curvature of A, and $*$ is the Hodge star operator (here and below we assume that the tensor fields take values in a compact Lie algebra $\mathfrak{g}$, and tr is an invariant scalar product in that Lie algebra). The "localized" Lagrangian also contains a gauge invariant kinetic term for Φ and gauge invariant kinetic terms for the fermionic ghost fields.

In this paper, we show that there are some hidden symmetries for the corresponding Abelian "localized" Lagrangian, and using these symmetries one can define a physical sector of the theory in a consistent way. In the physical sector, the Abelian "localized" Lagrangian describes $\mathfrak{g}$-valued massive vector field.

There is another mass generation mechanism for non-Abelian gauge fields for which the corresponding Lagrangian is constructed with the help of an antisymmetric $\mathfrak{g}$-valued $(2,0)$-type tensor potential B coupled to the gauge field A via the topological term $\mathrm{tr}(B \wedge F)$. This mechanism was suggested in Refs. 4, 12 and 15. Using BRST cohomology technique, one can prove that the corresponding non-Abelian gauge field theory is unitary and renormalizable (see Refs. 12, 16 and 17). In the physical sector, the theory describes the massive $\mathfrak{g}$-valued vector field. So, there are some similarities between constructions suggested in Refs. 4, 12, 15 and Refs. 5, 20.

In this paper, we prove that in the Abelian case the massive gauge theories constructed in Refs. 4, 12, 15 and Refs. 5, 20 are equivalent. Besides of the gauge symmetry, the action for the Abelian theory defined in Refs. 4, 12 and 15 has also a vector symmetry, and the Abelian version of the action introduced in Refs. 5 and 20 is a gauge fixed version of the former one, with respect to the vector symmetry. So, in both cases, the physical sector can be described with the help of the BRST cohomology corresponding to the gauge and the vector symmetries.

This paper is organized as follows. In Sec. 2 we recall the main construction of Refs. 5 and 20 and fix the notation used throughout of the paper. In Sec. 3 the Hamiltonian formulation for the non-Abelian theory suggested in Refs. 5 and 20 is introduced. Then we study the corresponding unperturbed Abelian theory in Sec. 4. In particular, we find a canonical form for the corresponding unperturbed quadratic Hamiltonian and study the symmetries of this Hamiltonian. It turns out that there are some hidden first-class constraints for the unperturbed Hamiltonian.

These constraints allow to reduce the number of physical degrees of freedom. In Sec. 5 we quantize the unperturbed system and show that one can define a physical sector for the quantized theory. The physical sector describes the quantized $\mathfrak{g}$-valued massive vector field. In Sec. 6 we compare the actions defined in Refs. 4, 12, 15 and Refs. 5, 20 in the Abelian case. We prove that the Abelian version of the action introduced in Refs. 5 and 20 is a gauge fixed version of the Abelian action defined in Refs. 4, 12 and 15. Following Refs. 12 and 16, we also define the BRST cohomology which can be used to describe the physical sector for both theories.

2. Recollection

In this section we recall the definition of the action introduced in Refs. 5 and 20 for describing non-Abelian massive gauge fields. First, we fix the notation as in Ref. 13. Let G be a compact simple Lie group, $\mathfrak{g}$ its Lie algebra with the commutator denoted by $[\cdot\,,\cdot]$. We fix a nondegenerate invariant under the adjoint action scalar product on $\mathfrak{g}$ denoted by tr (for instance, one can take the trace of the composition of the elements of $\mathfrak{g}$ acting in the adjoint representation). Let t^a, $a = 1, \dots, \dim \mathfrak{g}$ be a linear basis of $\mathfrak{g}$ normalized in such a way that $\mathrm{tr}(t^a t^b) = -\frac{1}{2}\delta^{ab}$.

We denote by A_μ the $\mathfrak{g}$-valued gauge field (connection on the Minkowski space),

$$A_\mu = A^a_\mu t^a \,.$$

Let D_μ be the associated covariant derivative,

$$D_\mu = \partial_\mu - \mathrm{g} A_\mu \,,$$

where g is a coupling constant, and $F_{\mu\nu}$ the strength tensor (curvature) of A_μ,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{g}[A_\mu, A_\nu] \,.$$

We shall also need a covariant d'Alambert operator $\Box_A$ associated to the gauge field A_μ,

$$\Box_A = D_\mu D^\mu \,.$$

The covariant d'Alambert operator can be applied to any tensor field defined on the Minkowski space and taking values in a representation space of the Lie algebra $\mathfrak{g}$, the $\mathfrak{g}$-valued gauge field A_μ acts on the tensor field according to that representation.

Finally, recall that the gauge group of G-valued functions $g(x)$ defined on the Minkowski space acts on the gauge field A_μ by

$$A_\mu \mapsto \frac{1}{\mathrm{g}}\,(\partial_\mu g)g^{-1} + gA_\mu g^{-1} \,. \tag{1}$$

The corresponding transformation laws for the covariant derivative and the strength tensor are

$$D_\mu \mapsto gD_\mu g^{-1} \,, \tag{2}$$

$$F_{\mu\nu} \mapsto gF_{\mu\nu}g^{-1} \,. \tag{3}$$

Formula (2) implies that the covariant d'Alambert operator is transformed under gauge action (1) as follows:

$$\Box_A \mapsto g\Box_A g^{-1}\,. \tag{4}$$

In the last formula it is assumed that the gauge group acts on tensor fields according to the representation of the group G induced by that of the Lie algebra $\mathfrak{g}$.

The "localized" action for the massive gauge field introduced in Refs. 5 and 20 can be defined by the formula

$$S = \int \mathrm{tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{8}(\Box_A\Phi_{\mu\nu})\Phi^{\mu\nu} + \frac{m}{2}\Phi_{\mu\nu}F^{\mu\nu} - 2i\sum_{i=1}^{3}\bar{\eta}_i(\Box_A\eta_i)\right)d^4x\,. \tag{5}$$

Here $\Phi_{\mu\nu}$ is a skew-symmetric $(2,0)$-type tensor field in the adjoint representation of $\mathfrak{g}$; η_i, $\bar{\eta}_i$, $i = 1,2,3$ are pairs of anticommuting scalar ghost fields in the adjoint representation of $\mathfrak{g}$; they satisfy the following reality conditions: $\eta_i^* = \eta_i$, $\bar{\eta}_i^* = \bar{\eta}_i$. In formula (5) g should be regarded as a coupling constant and m is a mass parameter. From (2)–(4) it follows that action (5) is invariant under gauge transformations (1).

To define the Green functions corresponding to the gauge invariant action S, we have to add to action (5) another term S^{gf}, containing a gauge fixing condition and the corresponding Faddeev–Popov operator. As it was observed in Ref. 20, the most convenient choice of S^{gf} is

$$S^{\mathrm{gf}} = \int \mathrm{tr}\left(\partial^\mu A_\mu \partial^\nu A_\nu + m^2\partial^\mu A_\mu(\Box_F^{-1}\partial^\nu A_\nu) - 2i\bar{\eta}(\partial^\mu D_\mu \eta)\right)d^4x\,,$$

where $\Box_F^{-1}$ is the operator inverse to d'Alambert operator with radiation boundary conditions, η, $\bar{\eta}$ is a pair of anticommuting scalar ghost fields in the adjoint representation of $\mathfrak{g}$; they satisfy the following reality conditions: $\eta^* = \eta$, $\bar{\eta}^* = \bar{\eta}$.

The total action $S' = S + S^{\mathrm{gf}}$, that should be used in the definition of the Green functions, takes the form

$$\begin{aligned} S' = \int \mathrm{tr}\Bigg(&\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{8}(\Box_A\Phi_{\mu\nu})\Phi^{\mu\nu} + \frac{m}{2}\Phi_{\mu\nu}F^{\mu\nu} - 2i\sum_{i=1}^{3}\bar{\eta}_i(\Box_A\eta_i) \\ &+ \partial^\mu A_\mu\partial^\nu A_\nu + m^2\partial^\mu A_\mu(\Box_F^{-1}\partial^\nu A_\nu) - 2i\bar{\eta}(\partial^\mu D_\mu\eta)\Bigg)d^4x\,. \end{aligned} \tag{6}$$

The generating function $Z(J, I, \xi, \bar{\xi}, \xi^i, \bar{\xi^i})$ of the Green functions corresponding to (6) is

$$\begin{aligned} Z(J, I, \xi, \bar{\xi}, \xi^i, \bar{\xi^i}) = \int &\mathcal{D}(A_\mu)\mathcal{D}(\Phi_{\mu\nu})\mathcal{D}(\eta)\mathcal{D}(\bar{\eta})\prod_{i=1}^{3}\mathcal{D}(\eta_i)\mathcal{D}(\bar{\eta}_i) \\ &\times \exp\left\{i\int \mathrm{tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{8}(\Box_A\Phi_{\mu\nu})\Phi^{\mu\nu} + \frac{m}{2}\Phi_{\mu\nu}F^{\mu\nu}\right.\right. \end{aligned}$$

$$- 2J^\mu A_\mu - I^{\mu\nu}\Phi_{\mu\nu} - 2\bar{\xi}\eta - 2\xi\bar{\eta} - 2\bar{\xi}^i\eta_i - 2\xi^i\bar{\eta}_i - 2i\bar{\eta}(\partial^\mu D_\mu\eta)$$
$$- 2i\sum_{i=1}^{3}\bar{\eta}_i(\Box_A\eta_i) + \partial^\mu A_\mu\partial^\nu A_\nu + m^2\partial^\mu A_\mu(\Box_F^{-1}\partial^\nu A_\nu)\Bigg)d^4x\Bigg\}\,, \qquad (7)$$

where J^μ, $I^{\mu\nu}$, ξ, $\bar{\xi}$, ξ^i, $\bar{\xi}^i$, $i = 1, 2, 3$ are the sources for the fields A_μ, $\Phi_{\mu\nu}$, η, $\bar{\eta}$, η_i, $\bar{\eta}_i$, $i = 1, 2, 3$, respectively.

Now consider the expression for the generating function $Z(J) = Z(J, 0, 0, 0, 0, 0)$ via a Feynman path integral,

$$Z(J) = \int \mathcal{D}(A_\mu)\mathcal{D}(\Phi_{\mu\nu})\mathcal{D}(\eta)\mathcal{D}(\bar{\eta})\prod_{i=1}^{3}\mathcal{D}(\eta_i)\mathcal{D}(\bar{\eta}_i)$$
$$\times \exp\Bigg\{i\int \mathrm{tr}\Bigg(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{8}(\Box_A\Phi_{\mu\nu})\Phi^{\mu\nu}$$
$$+ \frac{m}{2}\Phi_{\mu\nu}F^{\mu\nu} - 2J^\mu A_\mu - 2i\bar{\eta}(\partial^\mu D_\mu\eta)$$
$$- 2i\sum_{i=1}^{3}\bar{\eta}_i(\Box_A\eta_i) + \partial^\mu A_\mu\partial^\nu A_\nu + m^2\partial^\mu A_\mu(\Box_F^{-1}\partial^\nu A_\nu)\Bigg)d^4x\Bigg\}\,. \qquad (8)$$

Observe that in the right-hand side (r.h.s.) of formula (8) all the integrals over the ghost fields are Gaussian. The Gaussian integrals can be explicitly evaluated (see Refs. 5 and 20 for details). This yields

$$Z(J) = \int \mathcal{D}(A_\mu)\mathcal{D}(\eta)\mathcal{D}(\bar{\eta})\exp\Bigg\{i\int\Bigg[\mathrm{tr}\Bigg(\frac{1}{2}\,F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}(\Box_A^{-1}F_{\mu\nu})F^{\mu\nu}$$
$$- 2J^\mu A_\mu + \partial^\mu A_\mu\partial^\nu A_\nu + m^2\partial^\mu A_\mu(\Box_F^{-1}\partial^\nu A_\nu)\Bigg)\Bigg]d^4x\Bigg\}\det(\partial^\mu D_\mu)\,. \qquad (9)$$

The r.h.s. of (9) looks like the generating function of the Green functions for the Yang–Mills theory with an extra nonlocal term, $\frac{m^2}{2}\,\mathrm{tr}\,\big((\Box_A^{-1}F_{\mu\nu})F^{\mu\nu}\big)$, and in a generalized gauge (see Ref. 8, Chapter 3, Sec. 3). The gauge invariant action S_m that appears in formula (9),

$$S_m = \int d^4x\,\mathrm{tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}(\Box_A^{-1}F_{\mu\nu})F^{\mu\nu}\right), \qquad (10)$$

is not local. But the generating function $Z(J)$ for this action is equal to that for local action (5). This phenomenon was observed in Refs. 5 and 20.

In case when the coupling constant g vanishes action (10) takes the form

$$S_m^0 = \int d^4x\,\mathrm{tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}(\Box^{-1}F_{\mu\nu})F^{\mu\nu}\right), \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \qquad (11)$$

After imposing the Lorentz gauge fixing condition $\partial^\mu A_\mu = 0$, action (11) coincides with the action of the $\mathfrak{g}$-valued massive vector field (see Ref. 20 for details),

$$S_m^0|_{\partial^\mu A_\mu=0} = \int d^4x \,\mathrm{tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - m^2 A_\mu A^\mu\right), \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,.$$

3. Hamiltonian Formulation

In this section, we study the dynamical properties of the system described by the Lagrangian L that appears in formula (6),

$$L = \int \mathrm{tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{8}(\Box_A \Phi_{\mu\nu})\Phi^{\mu\nu} + \frac{m}{2}\Phi_{\mu\nu}F^{\mu\nu} - 2i\sum_{i=1}^{3}\bar{\eta}_i(\Box_A \eta_i)\right)d^3x \,. \quad (12)$$

We start with the Hamiltonian formulation for the dynamical system generated by Lagrangian (12). For the needs of the Hamiltonian formulation, we split the coordinates on the Minkowski space into spatial and time components, $x = (x^0, x^1, x^2, x^3) = (t, \mathbf{x})$, $\mathbf{x} = (x^1, x^2, x^3)$. We shall also write $d^3x = dx^1\, dx^2\, dx^3$, and $\cdot$ will stand for the scalar product in three-dimensional Euclidean space or Minkowski space. For any $\mathfrak{g}$-valued quantity X, the superscript a will indicate the ath component of X, $X = X^a t^a$; the Latin indexes will always take values $1, 2, 3$, i.e. $i, j, k = 1, 2, 3$; and summations over all repeated indexes will be assumed.

In order to find the Hamiltonian formulation for the system associated to the Lagrangian L, we rewrite L in the following form:

$$L = \int \left(p_i^a \partial_0 A_i^a + \sum_{\mu<\nu} p^{\mu\nu a}\partial_0 \Phi_{\mu\nu}^a + (\partial_0 \eta_i^a)\bar{\rho}_i^a + (\partial_0 \bar{\eta}_i^a)\rho_i^a - h + A_0^a C^a\right)d^3x \,, \quad (13)$$

where the quantities $p_i = p_i^a t^a$, $i = 1, 2, 3$ and $p^{\mu\nu} = p^{\mu\nu a}t^a$ are introduced as follows:

$$p_i = F_{0i} + \frac{m}{2}\Phi_{0i}\,, \quad p^{\mu\nu} = -\frac{1}{4}D_0\Phi^{\mu\nu}\,, \quad \rho_i = -iD_0\eta_i\,, \quad \bar{\rho}_i = iD_0\bar{\eta}_i\,, \quad (14)$$

and the functions $h(x)$ and $C(x) = C^a(x)t^a$ are defined by

$$\begin{aligned} h = \frac{1}{2}&\left(\left(p_i^a - \frac{m}{2}\Phi_{0i}^a\right)\left(p_i^a - \frac{m}{2}\Phi_{0i}^a\right) + \sum_{i<j} F_{ij}^a F_{ij}^a\right) \\ &- 2\sum_{i<j} p^{ij^a} p^{ij^a} + 2p^{0i^a}p^{0i^a} \\ &+ \frac{1}{8}\left(-\sum_{i<j}(D_k\Phi_{ij})^a(D_k\Phi_{ij})^a + (D_k\Phi_{0i})^a(D_k\Phi_{0i})^a\right) \\ &+ \frac{m}{2}\sum_{i<j}\Phi_{ij}^a F_{ij}^a + i\big(\bar{\rho}_i^a\rho_i^a + (D_k\bar{\eta}_i)^a(D_k\eta_i)^a\big)\,, \end{aligned} \quad (15)$$

$$C = D_i p_i + \sum_{\mu<\nu}[p^{\mu\nu}, \Phi_{\mu\nu}] + [\rho_i, \bar{\eta}_i]_+ + [\bar{\rho}_i, \eta_i]_+ \,. \tag{16}$$

In the formulas above and thereafter, $[\cdot\,,\cdot]_+$ stands for the anticommutator.

From formulas (13), (15) and (16), we deduce that the dynamical system described by Lagrangian (12) is a generalized Hamiltonian system with Hamiltonian $H = \int h d^3x$, the pairs (A_i^a, p_i^a), $(\Phi_{\mu\nu}^a, p^{\mu\nu a})$, $(\eta_i^a, \bar{\rho}_i^a)$, $(\bar{\eta}_i^a, \rho_i^a)$, for $i = 1, 2, 3$, $\mu < \nu$, $a = 1, \ldots, \dim \mathfrak{g}$ are canonical conjugate coordinates and momenta on the phase space Γ of our system, A_0^a are Lagrange multipliers, and C^a are constraints generating the gauge action on the phase space.

The (super-)Poisson structure on Γ has the standard form,

$$\begin{aligned}
\{A_i^a(\mathbf{x}), p_j^b(\mathbf{y})\} &= -\delta_{ij}\delta^{ab}\delta(\mathbf{x}-\mathbf{y})\,,\\
\{\Phi_{\mu\nu}^a(\mathbf{x}), p^{\gamma\delta\, b}(\mathbf{y})\} &= -\delta_\mu^\gamma\delta_\nu^\delta\delta^{ab}\delta(\mathbf{x}-\mathbf{y})\,,\\
\{\eta_i^a(\mathbf{x}), \bar{\rho}_j^b(\mathbf{y})\}_+ &= \delta_{ij}\delta^{ab}\delta(\mathbf{x}-\mathbf{y})\,,\\
\{\bar{\eta}_i^a(\mathbf{x}), \rho_j^b(\mathbf{y})\}_+ &= \delta_{ij}\delta^{ab}\delta(\mathbf{x}-\mathbf{y})\,,
\end{aligned} \tag{17}$$

and all the other (super-)Poisson brackets of the canonical variables vanish.

One can also show that the constraints C^a have the following Poisson brackets:

$$\{C^a(\mathbf{x}), C^b(\mathbf{y})\} = \mathrm{g}\; C_c^{ab} C^c(\mathbf{x})\delta(\mathbf{x}-\mathbf{y})\,, \tag{18}$$

where C_c^{ab} are the structure constants of the Lie algebra $\mathfrak{g}$, $[t^a, t^b] = C_c^{ab} t^c$.

Moreover, for any a, the Poisson bracket of the Hamiltonian H and of the constraint C^a vanish,

$$\{H, C^a\} = 0\,. \tag{19}$$

Formulas (18) and (19) imply that the constraints C^a are of the first class. Therefore, according to the general theory of constrained Hamiltonian systems (see Ref. 8, Chapter 3, Sec. 2), the generalized Hamiltonian system with first-class constraints described above is equivalent to the associated usual Hamiltonian system defined on the reduced phase space Γ^*. The description of Γ^* presented below is similar to that in case of the Yang–Mills field, and we refer the reader to Ref. 8, Chapter 3, Sec. 2 for technical details.

Recall that, in order to explicitly describe the reduced space, one needs to impose additional subsidiary (gauge fixing) conditions on the canonical variables. In the Hamiltonian formulation, the most convenient gauge fixing condition is the Coulomb condition,

$$\partial_i A_i = 0\,. \tag{20}$$

This condition is admissible in the sense that the determinant of the matrix of Poisson brackets of the components of the constraint C and of the components of subsidiary condition (20) does not vanish.

The realization of the reduced space Γ^* associated to subsidiary condition (20) is a Poisson submanifold in Γ defined by the following equations:

$$\partial_i A_i = 0\,, \quad C = D_i p_i + \sum_{\mu<\nu}[p^{\mu\nu}, \Phi_{\mu\nu}] + [\rho_i, \bar{\eta}_i]_+ + [\bar{\rho}_i, \eta_i]_+ = 0\,, \tag{21}$$

and the Hamiltonian of the associated Hamiltonian system on Γ^* is simply the restriction of the original Hamiltonian H to Γ^*.

The first equation in (21) suggests that it is natural to use the transversal components of the field with spatial components A_i, $i = 1, 2, 3$, their conjugate momenta and the variables $(\Phi^a_{\mu\nu}, p^{\mu\nu\, a})$, $(\eta^a_i, \bar{\rho}^a_i)$, $(\bar{\eta}^a_i, \rho^a_i)$ as canonical coordinates on the reduced space Γ^*.

Indeed, let $e^i(\mathbf{k})$, $i = 1, 2$ be two arbitrary orthonormal vectors such that $e^i(\mathbf{k}) \cdot \mathbf{k} = \mathbf{0}$ and $e^1(-\mathbf{k}) = e^2(\mathbf{k})$. Let $u^1(\mathbf{k}) = \frac{e^1(\mathbf{k})+e^2(\mathbf{k})}{\sqrt{2}}$ and $u^2(\mathbf{k}) = i\frac{e^1(\mathbf{k})-e^2(\mathbf{k})}{\sqrt{2}}$ be the complex linear combinations of e^1 and e^1 satisfying the property $u^i(\mathbf{k}) = {u^i}^*(-\mathbf{k})$. Then the real coordinates

$$ {A^i}^a(\mathbf{x}) = \frac{1}{(2\pi)^3} \int e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} u^i_j(\mathbf{k}) A^a_j(\mathbf{y}) d^3k\, d^3y\,, \quad i = 1, 2 \tag{22}$$

have the following conjugate momenta:

$$ {p^i}^a(\mathbf{x}) = \frac{1}{(2\pi)^3} \int e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} u^i_j(\mathbf{k}) p^a_j(\mathbf{y}) d^3k\, d^3y\,, \quad i = 1, 2\,, \tag{23}$$

i.e.

$$\{{A^i}^a(\mathbf{x}), {p^j}^b(\mathbf{y})\} = -\delta^{ij}\delta^{ab}\delta(\mathbf{x}-\mathbf{y})\,.$$

The longitudinal component $A_\|$ of the vector field with spatial components A_i, $i = 1, 2, 3$,

$$A^a_\|(\mathbf{x}) = \frac{i}{(2\pi)^3} \int e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \frac{k_j}{|\mathbf{k}|} A^a_j(\mathbf{y}) d^3k\, d^3y\,, \tag{24}$$

vanishes on Γ^* while using the second equation in (21), and the fact that subsidiary condition (20) is admissible, the momenta $p^a_\|$ conjugate to the coordinates $A^a_\|$,

$$p^a_\|(\mathbf{x}) = -\frac{i}{(2\pi)^3} \int e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \frac{k_j}{|\mathbf{k}|} p^a_j(\mathbf{y}) d^3k\, d^3y\,, \tag{25}$$

can be expressed on the reduced space via the canonical variables $({A^i}^a, {p^i}^a)$ $(\Phi^a_{\mu\nu}, p^{\mu\nu\, a})$, $(\eta^a_i, \bar{\rho}^a_i)$, $(\bar{\eta}^a_i, \rho^a_i)$ introduced on Γ^* above. The canonical variables on Γ^* are the true dynamical variables for the system described by Lagrangian (12).

4. The Classical Unperturbed System

In Secs. 4–6 we assume that the mass parameter m is not equal to zero.

In this section, we investigate the classical unperturbed system for which the coupling constant g vanishes, and the corresponding equations of motion become linear. For the unperturbed system the Lagrangian is given by the following formula

$$L_0 = \int \mathrm{tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{8}(\Box\Phi_{\mu\nu})\Phi^{\mu\nu} + \frac{m}{2}\Phi_{\mu\nu}F^{\mu\nu} - 2i\sum_{i=1}^{3}\bar{\eta}_i(\Box\eta_i)\right)d^3x\,, \quad (26)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,.$$

In view of the discussion in Sec. 2, we expect that in a physical sector the unperturbed system describes the massive vector field with values in the Lie algebra $\mathfrak{g}$.

For convenience, we introduce the following notation for the components $\Phi^a_{\mu\nu}$ of the ghost field $\Phi_{\mu\nu}$ and for their conjugate momenta $p^{\mu\nu\,a}$:

$$G^a_k = \frac{1}{2}\varepsilon_{ijk}\Phi^a_{ij}\,, \qquad \phi^a_k = \Phi^a_{0k}\,,$$

$$P^a_k = \frac{1}{2}\varepsilon_{ijk}p^{ij\,a}\,, \qquad \pi^a_k = p^{0k\,a}\,,$$

and define $\mathfrak{g}$-valued vector fields A, p, G, P, ϕ, π on $\mathbb{R}^3$. By definition, these vector fields have spatial components A_i, p_i, G_i, P_i, ϕ_i, π_i, respectively. We also write E for the $\mathfrak{g}$–valued vector field on $\mathbb{R}^3$, with components

$$E^a_k = \frac{1}{2}\varepsilon_{ijk}F^{ij\,a}\,.$$

Let H_0 be the free Hamiltonian corresponding to H, i.e. H_0 is obtained from H by putting $\mathrm{g} = 0$. Using the new notation, one can rewrite H_0 in the following form:

$$\begin{aligned} H_0 = \int d^3x\bigg\{&\frac{1}{2}\left(\left(p^a - \frac{m}{2}\phi^a\right)\cdot\left(p^a - \frac{m}{2}\phi^a\right) + E^a\cdot E^a\right) \\ &- 2P^a\cdot P^a + 2\pi^a\cdot\pi^a + \frac{1}{8}(G^a\cdot\triangle G^a - \phi^a\cdot\triangle\phi^a) \\ &+ \frac{m}{2}G^a\cdot E^a + i\big(\bar{\rho}^a_i\rho^a_i + (D_k\bar{\eta}_i)^a(D_k\eta_i)^a\big)\bigg\}\,, \end{aligned} \quad (27)$$

where $\triangle = \partial_i\partial_i$ is the Laplace operator.

For the unperturbed system the constraint $C = 0$ is reduced to

$$C = \partial_i p_i = 0\,, \quad (28)$$

and the subsidiary condition remains the same,

$$\partial_i A_i = 0\,. \quad (29)$$

Clearly, the reduced space Γ^*_0 associated to constraints (28) and subsidiary conditions (29) is isomorphic to Γ^*.

To describe the dynamics generated by Hamiltonian (27) on Γ^*_0, we shall use the canonical coordinates on Γ^* introduced in Sec. 3. One can further simplify the study of the equations of motion generated by H_0 on the reduced phase space Γ_0^* by introducing the longitudinal and the transversal components of the coordinates and of the momenta. The longitudinal components $G_\parallel$ and $\phi_\parallel$ of G and ϕ are defined by formulas similar to (24) with A replaced by G and ϕ, respectively, and the longitudinal components $P_\parallel$ and $\pi_\parallel$ of P and π, which are the conjugate momenta to $G_\parallel$ and $\phi_\parallel$, are introduced by formulas similar to (25). By definition, the transversal component $A_\perp$ of A is equal to $A - \mathrm{grad}\triangle^{-1}\partial_i A_i$,

$$A_\perp = A - \mathrm{grad}\triangle^{-1}\partial_i A_i\,, \tag{30}$$

and the transversal components of other bosonic canonical variables are defined by formulas similar to (30).

Now the restriction of H_0 to the reduced space Γ_0^* can be represented as follows:

$$\begin{aligned} H_0 = \int d^3x \Big\{ &\frac{1}{2}\Big(\Big(p_\perp^a - \frac{m}{2}\phi_\perp^a\Big)\cdot\Big(p_\perp^a - \frac{m}{2}\phi_\perp^a\Big) + E^a\cdot E^a\Big) \\ &- 2P_\perp^a\cdot P_\perp^a + 2\pi_\perp^a\cdot\pi_\perp^a + \frac{1}{8}(G_\perp^a\cdot\triangle G_\perp^a - \phi_\perp^a\cdot\triangle\phi_\perp^a) \\ &+ \frac{m}{2}G_\perp^a\cdot E^a - 2P_\parallel^a P_\parallel^a + \frac{1}{8}G_\parallel^a\triangle G_\parallel^a + 2\pi_\parallel^a\pi_\parallel^a \\ &+ \frac{1}{8}\phi_\parallel^a(-\triangle + m^2)\phi_\parallel^a + i(\bar{\rho}_i^a\rho_i^a + (D_k\bar{\eta}_i)^a(D_k\eta_i)^a)\Big\}\,. \end{aligned} \tag{31}$$

Note that in the expression above the transversal and the longitudinal components are completely separated.

To investigate the dynamics generated by Hamiltonian (31), we observe that the expression in the r.h.s. of (31) is quadratic in canonical variables. Therefore, according to the general theory of normal forms for quadratic Hamiltonians (see Ref. 3, App. 6), Hamiltonian (31) can be reduced to a canonical form by a linear symplectic transformation.

Indeed, if we introduce new variables $\bar{A}_\perp$, $\bar{p}_\perp$, q_1, r_1, q_2, r_2,

$$\begin{aligned} r_1 &= \sqrt{2}\Big(P_\perp + \frac{1}{4}\,\mathrm{curl}\,\phi_\perp\Big)\,, \\ q_1 &= \sqrt{2}\frac{3m^2 - 2\triangle}{8m^2}G_\perp - \sqrt{2}\frac{m^2 - 2\triangle}{4m\triangle}\,\mathrm{curl}\,A_\perp + \sqrt{2}\frac{m^2 + 2\triangle}{2m^2\triangle}\,\mathrm{curl}\,\pi_\perp\,, \\ r_2 &= \sqrt{2}\Big(\pi_\perp + \frac{1}{4}\,\mathrm{curl}\,G_\perp - \frac{1}{2}mA_\perp\Big)\,, \\ q_2 &= \frac{\sqrt{2}}{2}\,\frac{2\triangle - m^2}{4m^2}\,\phi_\perp + \frac{\sqrt{2}}{m}p_\perp + \frac{\sqrt{2}}{2}\,\frac{m^2 - 2\triangle}{m^2\triangle}\,\mathrm{curl}\,P_\perp\,, \\ \bar{p}_\perp &= p_\perp - \frac{2}{m}\,\mathrm{curl}\,P_\perp + \frac{\triangle - m^2}{2m}\phi_\perp\,, \\ \bar{A}_\perp &= \frac{1}{2m}\,\mathrm{curl}\,G_\perp + \frac{2}{m}\pi_\perp\,, \end{aligned} \tag{32}$$

then the pairs of their components, $\left(\bar{A}^{i\,a}, \bar{p}^{i\,a}\right)$, $\left(q_1^{i\,a}, r_1^{i\,a}\right)$, $\left(q_2^{i\,a}, r_2^{i\,a}\right)$, defined by formulas similar to (22), (23), and the pairs $\left(\phi_{\|}^a, \pi_{\|}^a\right)$, $\left(G_{\|}^a, P_{\|}^a\right)$, $(\eta_i^a, \bar{\rho}_i^a)$, $(\bar{\eta}_i^a, \rho_i^a)$ are canonical conjugate coordinates and momenta on the reduced phase space Γ_0^*. Moreover, in terms of the new variables the Hamiltonian H_0 takes the canonical form

$$\begin{aligned} H_0 = \int d^3x \bigg\{ & \frac{1}{2}\big(\bar{p}_\perp^a \cdot \bar{p}_\perp^a + \bar{A}_\perp^a \cdot (-\triangle + m^2)\bar{A}_\perp^a\big) - \frac{1}{2}(r_1^a \cdot r_1^a + r_2^a \cdot r_2^a) \\ & + r_1^a \cdot \operatorname{curl} q_2^a - r_2^a \cdot \operatorname{curl} q_1^a + 2\pi_{\|}^a \pi_{\|}^a + \frac{1}{8}\phi_{\|}^a(-\triangle + m^2)\phi_{\|}^a \\ & - 2P_{\|}^a P_{\|}^a + \frac{1}{8}G_{\|}^a \triangle G_{\|}^a + i\big(\bar{\rho}_i^a \rho_i^a + (D_k\bar{\eta}_i)^a (D_k\eta_i)^a\big)\bigg\}. \end{aligned} \tag{33}$$

Note that Hamiltonian (33) and the momenta r_1 and r_2 have the following Poisson brackets:

$$\{H_0, r_1\} = \operatorname{curl} r_2\,, \qquad \{H_0, r_2\} = -\operatorname{curl} r_1\,, \tag{34}$$

and hence r_1 and r_2 can be regarded as first-class constraints. Therefore, recalling the general scheme of constrained reduction (see Ref. 8, Chapter 3, Sec. 2), one can further reduce the effective number of degrees of freedom using first-class constraints

$$r_1 = 0\,, \qquad r_2 = 0\,. \tag{35}$$

Since r_1, r_2 are the momenta conjugate to coordinates q_1 and q_2, the subsidiary conditions

$$q_1 = 0\,, \qquad q_2 = 0 \tag{36}$$

are admissible for constraints (35), i.e. the determinant of the matrix of Poisson brackets of the components of the constraints and of the components of the subsidiary conditions does not vanish. The reduced space Γ_0^{**} associated to constraints (35) and subsidiary conditions (36) is defined by the following equations in Γ_0^*:

$$r_1 = 0\,, \quad r_2 = 0\,, \quad q_1 = 0\,, \quad q_2 = 0\,,$$

and the components $\bar{A}^{i\,a}$, $\bar{p}^{i\,a}$ of the transversal parts $\bar{A}_\perp$, $\bar{p}_\perp$, the longitudinal components $G_{\|}^a$, $P_{\|}^a$, $\phi_{\|}^a$ $\pi_{\|}^a$ and η_i^a, $\bar{\rho}_i^a$, $\bar{\eta}_i^a$, ρ_i^a are canonical variables on Γ_0^{**}. We denote by H_0^r the Hamiltonian H_0 restricted to Γ_0^{**}, $H_0^r = H_0|_{\Gamma_0^{**}}$,

$$\begin{aligned} H_0^r = \int d^3x \bigg\{ & \frac{1}{2}\big(\bar{p}_\perp^a \cdot \bar{p}_\perp^a + \bar{A}_\perp^a \cdot (-\triangle + m^2)\bar{A}_\perp^a\big) \\ & + 2\pi_{\|}^a \pi_{\|}^a + \frac{1}{8}\phi_{\|}^a(-\triangle + m^2)\phi_{\|}^a \\ & - 2P_{\|}^a P_{\|}^a + \frac{1}{8}G_{\|}^a \triangle G_{\|}^a + i(\bar{\rho}_i^a \rho_i^a + (D_k\bar{\eta}_i)^a (D_k\eta_i)^a)\bigg\}. \end{aligned} \tag{37}$$

The equations of motion generated by Hamiltonian (37) read

$$\Box G_\parallel = 0\,, \quad \Box\eta_i = 0\,, \quad \Box\bar{\eta}_i = 0\,, \tag{38}$$

$$(\Box + m^2)\phi_\parallel = 0\,, \tag{39}$$

$$(\Box + m^2)\bar{A}_\perp = 0\,. \tag{40}$$

Therefore the Hamiltonian H_0^r effectively describes propagation of the two massive transversal components of the field $\bar{A}_\perp$, the massive longitudinal component of the field ϕ, the massless longitudinal component of G and the massless fermions η_i^a, $\bar{\eta}_i^a$.

For the purposes of quantization, it is natural to introduce the holomorphic representation for these fields,

$$\begin{aligned}
\bar{A}_\perp^a(\mathbf{x}) &= \frac{1}{(2\pi)^{\frac{3}{2}}}\int \frac{d^3k}{\sqrt{2(\mathbf{k}^2+m^2)^{\frac{1}{2}}}} \\
&\quad\times \sum_{i=1,2}\left(b_i^a(\mathbf{k})e^i(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + b_i^{a*}(\mathbf{k})e^i(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\right), \\
\bar{p}_\perp^a(\mathbf{x}) &= \frac{i}{(2\pi)^{\frac{3}{2}}}\int d^3k\sqrt{\frac{(\mathbf{k}^2+m^2)^{\frac{1}{2}}}{2}} \\
&\quad\times \sum_{i=1,2}\left(-b_i^a(\mathbf{k})e^i(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + b_i^{a*}(\mathbf{k})e^i(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\right),
\end{aligned} \tag{41}$$

$$\begin{aligned}
\eta_i^a(\mathbf{x}) &= -\frac{1}{(2\pi)^{\frac{3}{2}}}\int \frac{d^3k}{\sqrt{2|\mathbf{k}|}}\left(c_i^a(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}} + c_i^{a*}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}\right), \\
\bar{\eta}_i^a(\mathbf{x}) &= \frac{i}{(2\pi)^{\frac{3}{2}}}\int d^3k\frac{d^3k}{\sqrt{2|\mathbf{k}|}}\left(\bar{c}_i^a(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}} - \bar{c}_i^{a*}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}\right), \\
\rho_i^a(\mathbf{x}) &= \frac{1}{(2\pi)^{\frac{3}{2}}}\int d^3k\sqrt{\frac{|\mathbf{k}|}{2}}\left(c_i^a(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}} - c_i^{a*}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}\right), \\
\bar{\rho}_i^a(\mathbf{x}) &= \frac{i}{(2\pi)^{\frac{3}{2}}}\int d^3k\sqrt{\frac{|\mathbf{k}|}{2}}\left(\bar{c}_i^a(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}} + \bar{c}_i^{a*}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}\right),
\end{aligned} \tag{42}$$

$$\begin{aligned}
G_\parallel^a(\mathbf{x}) &= \frac{2}{(2\pi)^{\frac{3}{2}}}\int \frac{d^3k}{\sqrt{2|\mathbf{k}|}}\left(a^a(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}} + a^{a*}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}\right), \\
P_\parallel^a(\mathbf{x}) &= \frac{i}{2(2\pi)^{\frac{3}{2}}}\int d^3k\sqrt{\frac{|\mathbf{k}|}{2}}\left(-a^a(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}} + a^{a*}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}\right),
\end{aligned} \tag{43}$$

$$
\begin{aligned}
\phi_{\|}^{a}(\mathbf{x}) &= \frac{2}{(2\pi)^{\frac{3}{2}}} \int \frac{d^3k}{\sqrt{2(\mathbf{k}^2+m^2)^{\frac{1}{2}}}} \left(b^a(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + b^{a*}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\right), \\
\pi_{\|}^{a}(\mathbf{x}) &= \frac{i}{2(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{\frac{(\mathbf{k}^2+m^2)^{\frac{1}{2}}}{2}} \left(-b^a(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + b^{a*}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\right).
\end{aligned} \tag{44}
$$

The new complex coordinates b_i^a, b_i^{a*}, c_i^a, $\bar{c}_i^{a*}$, $\bar{c}_i^a$, c_i^{a*}, a^a, a^{a*}, b^a, b^{a*} have the standard (super-)Poisson brackets,

$$
\begin{aligned}
\left\{a^a(\mathbf{k}), a^{b*}(\mathbf{k}')\right\} &= i\delta^{ab}\delta(\mathbf{k}-\mathbf{k}'), \\
\left\{b_i^a(\mathbf{k}), b_j^{b*}(\mathbf{k}')\right\} &= i\delta_{ij}\delta^{ab}\delta(\mathbf{k}-\mathbf{k}'), \\
\left\{b^a(\mathbf{k}), b^{b*}(\mathbf{k}')\right\} &= i\delta^{ab}\delta(\mathbf{k}-\mathbf{k}'), \\
\left\{c_i^a(\mathbf{k}), \bar{c}_j^{b*}(\mathbf{k}')\right\}_+ &= i\delta_{ij}\delta^{ab}\delta(\mathbf{k}-\mathbf{k}'), \\
\left\{\bar{c}_i^a(\mathbf{k}), c_j^{b*}(\mathbf{k}')\right\}_+ &= i\delta_{ij}\delta^{ab}\delta(\mathbf{k}-\mathbf{k}').
\end{aligned} \tag{45}
$$

One can express Hamiltonian (37) in terms of the holomorphic coordinates,

$$
\begin{aligned}
H_0^r = &\int d^3k(\mathbf{k}^2+m^2)^{\frac{1}{2}}(b^{a*}b^a + b_i^{a*}b_i^a) \\
&- \int d^3k|\mathbf{k}|(\bar{c}_i^{a*}c_i^a - \bar{c}_i^a c_i^{a*} + a^{a*}a^a).
\end{aligned} \tag{46}
$$

Now Hamiltonian (37) can be naturally split into two parts,

$$
H_0^r = H_0^+ + H_0^- . \tag{47}
$$

The first one,

$$
H_0^+ = \int d^3k(\mathbf{k}^2+m^2)^{\frac{1}{2}}(b^{a*}b^a + b_i^{a*}b_i^a), \tag{48}
$$

corresponding to the first line in formula (37), describes propagation of the two transversal massive components of the gauge field $\bar{A}_\perp$ and the massive longitudinal component of the field ϕ. According to formula (48), they propagate with positive energy (positive energy sector). These three components can be regarded as three independent components of one massive vector field with values in the Lie algebra $\mathfrak{g}$ (see Ref. 13, Sec. 3-2-3 for the description of the dynamics of the massive vector field). We conclude that in the positive energy sector we have obtained the desired result: the Hamiltonian H_0^+ describes the massive vector field with values in the Lie algebra $\mathfrak{g}$.

The second part H_0^-,

$$
H_0^- = -\int d^3k|\mathbf{k}|(\bar{c}_i^{a*}c_i^a - \bar{c}_i^a c_i^{a*} + a^{a*}a^a), \tag{49}
$$

corresponding to the last line in (37), describes the massless fields (negative energy sector). Note that according to formula (49) the massless fields indeed propagate with negative energy (for fermions it is true in the quantum case).

An important property of decomposition (47) is that the positive and the negative energy sectors are Poincaré invariant.

Indeed, consider the coordinate transformation which is inverse to (32),

$$\begin{aligned}
A_\perp &= \bar{A}_\perp - \frac{\sqrt{2}}{m} r_2 \,, \\
p_\perp &= -\frac{m^2 - 2\triangle}{2\sqrt{2}\triangle} \operatorname{curl} r_1 + \frac{m\sqrt{2}}{2} q_2 \,, \\
G_\perp &= \sqrt{2} q_1 - \frac{2\operatorname{curl}}{m} \bar{A}_\perp - \frac{-2\triangle + m^2}{\sqrt{2} m^2 \triangle} \operatorname{curl} r_2 \,, \\
P_\perp &= -\frac{\sqrt{2}}{4} \operatorname{curl} q_2 + \frac{1}{2m} \operatorname{curl} \bar{p}_\perp + \frac{3m^2 - 2\triangle}{4\sqrt{2} m^2} r_1 \,, \\
\phi_\perp &= \sqrt{2} q_2 - \frac{2}{m} \bar{p}_\perp - \frac{m^2 + 2\triangle}{m^2 \sqrt{2}\triangle} \operatorname{curl} r_1 \,, \\
\pi_\perp &= -\frac{m^2 - 2\triangle}{4\sqrt{2} m^2} r_2 - \frac{\sqrt{2}}{4} \operatorname{curl} q_1 + \frac{m^2 - \triangle}{2m} \bar{A}_\perp \,.
\end{aligned} \tag{50}$$

The transformation induced by (50) on the reduced space Γ_0^{**} has the form

$$\begin{aligned}
A_\perp &= \bar{A}_\perp \,, & p_\perp &= 0 \,, \\
G_\perp &= -\frac{2\operatorname{curl}}{m} \bar{A}_\perp \,, & P_\perp &= \frac{1}{2m} \operatorname{curl} \bar{p}_\perp \,, \\
\phi_\perp &= -\frac{2}{m} \bar{p}_\perp \,, & \pi_\perp &= \frac{m^2 - \triangle}{2m} \bar{A}_\perp \,.
\end{aligned} \tag{51}$$

Note that the equations of motion generated by Hamiltonian H_0^r imply that $\bar{p}_\perp = \frac{\partial \bar{A}_\perp}{\partial t}$.

Now observe that the positive energy solutions to equations of motion (38)–(40) are the only solutions which satisfy the Poincaré invariant condition

$$d\Phi = 0 \,, \tag{52}$$

where $\Phi = \Phi_{\mu\nu}\, dx^\mu \wedge dx^\nu$ is the differential form with components $\Phi_{\mu\nu}$, and d is the exterior differential defined on the Minkowski space. Indeed, condition (52) takes the following form in components:

$$-\partial_0 G + \operatorname{curl} \phi = 0 \,, \qquad \partial_i G_i = 0 \,. \tag{53}$$

Using formulas (51) and recalling Eqs. (39) and (40), one checks directly that for the positive energy solutions conditions (53) are satisfied. Therefore the positive energy sector is Poincaré invariant.

The condition dual to (52) with respect to the scalar product $\langle\cdot\,,\cdot\rangle$ of $\mathfrak{g}$-valued forms on the Minkowski space,

$$\langle \Phi, \Psi \rangle = \int d^4x \operatorname{tr}(\Phi \wedge *\Psi) \,, \tag{54}$$

is

$$d^*\Phi = 0\,, \tag{55}$$

where $*$ is the Hodge star operator associated to the standard metric on the Minkowski space, and d^* is the operator conjugate to d with respect to scalar product (54).

Condition (55) takes the following form in components:

$$\partial_0\phi = -\operatorname{curl} G\,, \qquad \partial_i\phi_i = 0\,. \tag{56}$$

Using the definition of the component $G_\|$ and reconstructing the vector field $\widehat{G}_\|$ on $\mathbb{R}^3$, which corresponds to the solution $G_\|$, by the formula

$$\hat{G}_\| = -\operatorname{grad}\frac{1}{\sqrt{|-\triangle|}}G_\|\,,$$

one immediately obtains that $\widehat{G}_\|$ is the only solution to equation of motions (38)–(40) that obeys Poincaré invariant condition (56). Finally, observe that the fermionic part of Lagrangian (26) is obviously Poincaré invariant. Therefore the negative energy sector is Poincaré invariant as well. The unwanted negative energy sector can be easily split off in the quantum case.

5. Quantization

We start by discussing quantization procedure for the unperturbed system defined on the phase space Γ_0^*. To construct the quantized unperturbed system, we shall use the following coordinates on Γ_0^*: the holomorphic coordinates c_i^a, $\bar{c}_i^{a*}$, $\bar{c}_i^a$, c_i^{a*}, a^a, a^{a*}, b^a, b^{a*}, b_i^a, b_i^{a*}, and the components $q_1^{i\,a}$, $r_1^{i\,a}$, $q_2^{i\,a}$, $r_2^{i\,a}$ of q_1, r_1, q_2, r_2. After quantization, these variables become operators obeying the standard (super-) commutation relations,

$$\begin{aligned}
\left[\mathbf{a}^a(\mathbf{k}), \mathbf{a}^{b*}(\mathbf{k}')\right] &= \delta^{ab}\delta(\mathbf{k}-\mathbf{k}')\,,\\
\left[\mathbf{b}^a(\mathbf{k}), \mathbf{b}^{b*}(\mathbf{k}')\right] &= \delta^{ab}\delta(\mathbf{k}-\mathbf{k}')\,,\\
\left[\mathbf{b}_i^a(\mathbf{k}), \mathbf{b}_j^{b*}(\mathbf{k}')\right] &= \delta_{ij}\delta^{ab}\delta(\mathbf{k}-\mathbf{k}')\,,\\
\left[\mathbf{c}_i^a(\mathbf{k}), \bar{\mathbf{c}}_j^{b*}(\mathbf{k}')\right]_+ &= \delta_{ij}\delta^{ab}\delta(\mathbf{k}-\mathbf{k}')\,,\\
\left[\bar{\mathbf{c}}_i^a(\mathbf{k}), \mathbf{c}_j^{b*}(\mathbf{k}')\right]_+ &= \delta_{ij}\delta^{ab}\delta(\mathbf{k}-\mathbf{k}')\,,\\
\left[\mathbf{q}_1^{i\,a}(\mathbf{x}), \mathbf{r}_1^{j\,b}(\mathbf{y})\right] &= i\delta^{ij}\delta^{ab}\delta(\mathbf{x}-\mathbf{y})\,,\\
\left[\mathbf{q}_2^{i\,a}(\mathbf{x}), \mathbf{r}_2^{j\,b}(\mathbf{y})\right] &= i\delta^{ij}\delta^{ab}\delta(\mathbf{x}-\mathbf{y})\,.
\end{aligned} \tag{57}$$

We shall use the standard coordinate representation $\mathcal{H}_Q$ for the operators $\mathbf{q}_1^{i\,a}$, $\mathbf{r}_1^{i\,a}$, $\mathbf{q}_2^{i\,a}$, $\mathbf{r}_2^{i\,a}$. This representation is diagonal for $\mathbf{q}_1^{i\,a}$ and for $\mathbf{q}_2^{i\,a}$. The operators $\mathbf{c}_i^{a*}$, $\bar{\mathbf{c}}_i^{a*}$, $\mathbf{c}_i^a$, $\bar{\mathbf{c}}_i^a$, $\mathbf{a}^a$, $\mathbf{a}^{a*}$, $\mathbf{b}^a$, $\mathbf{b}^{a*}$, $\mathbf{b}_i^a$, $\mathbf{b}_i^{a*}$ act as usual in the fermionic and in the bosonic Fock spaces, respectively; all the operators with superscript $*$ being regarded as creation operators. We denote the Hilbert space tensor product of the fermionic and of the bosonic Fock spaces by $\mathcal{H}_F$.

We shall also denote by $\mathbf{q}_1$, $\mathbf{r}_1$, $\mathbf{q}_2$, $\mathbf{r}_2$ the vector valued operator quantities with components $\mathbf{q}_1^{i\,a}$, $\mathbf{r}_1^{i\,a}$, $\mathbf{q}_2^{i\,a}$, $\mathbf{r}_2^{i\,a}$.

Let $\mathcal{H}$ be the Hilbert space tensor product of the coordinate representation space $\mathcal{H}_Q$ and of the Fock space $\mathcal{H}_F$, $\mathcal{H} = \mathcal{H}_Q \otimes \mathcal{H}_F$. $\mathcal{H}$ is the space of states for the quantized system associated to Hamiltonian (33).

We overemphasize that the quantized Hamiltonian H_0 is a self-adjoint operator $\mathbf{H}_0$ acting in the Hilbert space $\mathcal{H}$ equipped with a positive definite sesquilinear scalar product. But the quantization procedure described above does not guarantee that the energy spectrum of $\mathbf{H}_0$ belongs to the positive semiaxis! The negative energy states have, of course, no physical meaning.

Now recall that actually we need to quantize the system associated to the reduced Hamiltonian (37). According to Dirac's quantum constraint reduction scheme, the quantized Hamiltonian (37) acts in the space $\mathcal{H}^0_{\rm red}$, which can be obtained from $\mathcal{H}$ by imposing the constraints $\mathbf{r}_1$ and $\mathbf{r}_2$,

$$\mathcal{H}^0_{\rm red} = \{|v\rangle \in \mathcal{H} :\ \mathbf{r}_1^{i\,a}|v\rangle = 0,\ \mathbf{r}_2^{i\,a}|v\rangle = 0\}\,. \tag{58}$$

Note that by construction $\mathcal{H}^0_{\rm red} \simeq \mathcal{H}_F$.

Now we can define the space of the *physical* states $\mathcal{H}^0_{\rm phys}$ for the reduced Hamiltonian (37) by removing the unwanted Poincaré invariant negative energy sector. As we observed in the end of the last section, the Poincaré invariant negative energy sector for the reduced free Hamiltonian H_0^r contains all the fermions and the longitudinal component $G_\parallel$ of the spatial part of the field $\Phi_{\mu\nu}$. Therefore, in view of formulas (42), (43) and (57), $\mathcal{H}^0_{\rm phys}$ can be naturally defined as the subspace of $\mathcal{H}^0_{\rm red}$ which does not contain states with excitations created by the operators $\mathbf{c}_i^{a\,*}$, $\bar{\mathbf{c}}_i^{a\,*}$, $\mathbf{a}^{a\,*}$. In other words

$$\mathcal{H}^0_{\rm phys} = \{|v\rangle \in \mathcal{H}^0_{\rm red} :\ \mathbf{c}_i^a|v\rangle = \bar{\mathbf{c}}_i^a|v\rangle = \mathbf{a}^a|v\rangle = 0\}\,. \tag{59}$$

Note that the space of the physical states $\mathcal{H}^0_{\rm phys}$ can also be described with the help of the quantized Hamiltonian H_0^-. Following our convention, we denote the quantized Hamiltonian H_0^- by $\mathbf{H}_0^-$. From formula (49) it immediately follows that

$$\mathcal{H}^0_{\rm phys} = \{|v\rangle \in \mathcal{H}^0_{\rm red} :\ \mathbf{H}_0^-|v\rangle = 0\}\,. \tag{60}$$

Since the negative energy sector is Poincaré invariant the operator $\mathbf{H}_0^-$ commutes with the Hamiltonian $\mathbf{H}_0$,

$$[\mathbf{H}_0^-, \mathbf{H}_0] = 0\,. \tag{61}$$

At the classical level this can be seen directly from definitions (46) and (49). Since H_0^- does not depend on the canonical variables $q_1^{i\,a}$, $r_1^{i\,a}$, $q_2^{i\,a}$, $r_2^{i\,a}$, we also have

$$[\mathbf{r}_1^{i\,a}, \mathbf{H}_0^-] = [\mathbf{r}_2^{i\,a}, \mathbf{H}_0^-] = 0\,.$$

The last formula together with (61) implies that $\mathbf{r}_{1,2}$ and $\mathbf{H}_0^-$ can be regarded as a system of first class quantum constraints, and description (58), (60) of the space of physical states is a realization of Dirac's quantum constraint reduction scheme.

From the definition of the space $\mathcal{H}$, the description (59) of the physical subspace and the definition of the coordinate representation for the operators ${\mathbf{q}_1^i}^a$, ${\mathbf{r}_1^i}^a$, ${\mathbf{q}_2^i}^a$, ${\mathbf{r}_2^i}^a$, it follows that $\mathcal{H}^0_{\mathrm{phys}}$ is simply the bosonic Fock space for the operators $\mathbf{b}^a$, $\mathbf{b}^{a*}$, $\mathbf{b}_i^a$, $\mathbf{b}_i^{a*}$, all the operators with superscript $*$ being regarded as creation operators.

The quantized Hamiltonian $\mathbf{H}_0$ restricted to $\mathcal{H}^0_{\mathrm{phys}}$ is the quantization of H_0^+,

$$\mathbf{H}_0^+ = \int d^3k(\mathbf{k}^2 + m^2)^{\frac{1}{2}}(\mathbf{b}^{a*}\mathbf{b}^a + \mathbf{b}_i^{a*}\mathbf{b}_i^a)\,.$$

Therefore the quantized Hamiltonian $\mathbf{H}_0$ restricted to $\mathcal{H}^0_{\mathrm{phys}}$ can be identified with that of the quantized massive $\mathfrak{g}$-valued vector field.

6. Another Massive Non-Abelian Theory

In this section we study the relation of the theory with unperturbed Lagrangian (26) and the Abelian version of the theory suggested in Refs. 4, 12 and 15 for describing massive gauge fields. We show that these two massive theories are equivalent in the physical sector. Note that in the non-Abelian case the Lagrangian introduced in Refs. 4, 12 and 15 generates a unitary renormalizable quantum field theory describing the $\mathfrak{g}$-valued massive vector field only.

First we recall the definition of the gauge invariant action introduced in Refs. 4, 12 and 15. We use the same notation for gauge fields as in the previous sections. Let $B_{\mu\nu}$ and C_μ are the $(2,0)$-type skew-symmetric tensor field and vector field, respectively, with values in the adjoint representation of $\mathfrak{g}$. The action defined in Refs. 4, 12 and 15 can be written in the following form:

$$W = \int d^4x \,\mathrm{tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{6}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \frac{m}{2}\epsilon^{\mu\nu\rho\lambda}B_{\mu\nu}F_{\rho\lambda}\right), \tag{62}$$

where m ia a mass parameter,

$$H_{\mu\lambda\nu} = D_{[\mu}B'_{\lambda\nu]} = D_{[\mu}B_{\lambda\nu]} + \mathrm{g}[F_{[\mu\lambda}, C_{\nu]}]\,, \quad B'_{\lambda\nu} = B_{\lambda\nu} - D_{[\lambda}C_{\nu]}\,,$$

and for the lower indexes square brackets always mean antisymmetrization, e.g.

$$D_{[\mu}B'_{\lambda\nu]} = D_\mu B'_{\lambda\nu} + D_\lambda B'_{\nu\mu} + D_\nu B'_{\lambda\mu}\,, \quad D_{[\lambda}C_{\nu]} = D_\lambda C_\nu - D_\nu C_\lambda\,.$$

$\epsilon^{\mu\nu\rho\lambda}$ is the absolutely antisymmetric tensor of rank four such that $\epsilon^{0123} = 1$.

The action (62) is invariant under the gauge transformations

$$A_\mu \mapsto \frac{1}{\mathrm{g}}(\partial_\mu g)g^{-1} + gA_\mu g^{-1}\,, \quad B_{\mu\nu} \mapsto gB_{\mu\nu}g^{-1}\,, \quad C_\mu \mapsto gC_\mu g^{-1} \tag{63}$$

and under vector transformations

$$A_\mu \mapsto A_\mu\,, \quad B_{\mu\nu} \mapsto B_{\mu\nu} + D_{[\lambda}\Lambda_{\nu]}\,, \quad C_\mu \mapsto C_\mu + \Lambda_\mu\,, \tag{64}$$

where Λ_μ is an arbitrary vector field with values in the adjoint representation of the gauge group.

In particular, definition (62) and formulas (64) imply that the field C_μ is not dynamical and can be removed by transformations (64) (see Refs. 4, 10, 12 and 15 for more detailed discussion of this phenomenon). In Ref. 10, Sec. IV, it is also shown that action (62) describes a massive $\mathfrak{g}$-valued vector field.

A precise analysis of the reduced phase space in the framework of Hamiltonian reduction can be found in Refs. 10 and 18. Actually, due to the presence of the non-dynamical vector field C_μ, the explicit description of the reduced phase associated to action (62) is much more complicated than in the corresponding Abelian case, i.e. when the coupling constant g vanishes, and hence the axillary vector field C_μ is not present in the definition of the action. It turns out that besides of symmetries (63) and (64), action (62) also has some other hidden symmetries which reduce the number of functional degrees of freedom of the system to three, like in the corresponding Abelian case when the axillary field C_μ is not present (see Refs. 10 and 18).

In Refs. 12 and 16 it is shown that a BRST invariant tree-level action can be constructed for the theory introduced in Refs. 4, 12 and 15. Therefore the theory is unitary in the physical sector. Note that the non-Abelian massive theory introduced in Refs. 4, 12 and 15 contains more field variables than the corresponding Abelian theory with $\mathrm{g} = 0$. As a result one can circumvent the no-go theorem (see Ref. 11) and prove that the non-Abelian theory is renormalizable (see Ref. 17).

Now we consider the Abelian counterpart W_0 of action (62) obtained by putting $\mathrm{g} = 0$ in (62),

$$W_0 = \int d^4x \,\mathrm{tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{6}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \frac{m}{2}\epsilon^{\mu\nu\rho\lambda}B_{\mu\nu}F_{\rho\lambda}\right), \tag{65}$$

where now $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and

$$H_{\mu\lambda\nu} = \partial_{[\mu}B_{\lambda\nu]} \,.$$

The action (65) is invariant under the Abelian gauge transformations

$$A_\mu \mapsto A_\mu + \partial_\mu\chi\,, \qquad B_{\mu\nu} \mapsto B_{\mu\nu}\,, \tag{66}$$

and under vector transformations

$$A_\mu \mapsto A_\mu\,, \qquad B_{\mu\nu} \mapsto B_{\mu\nu} + \partial_{[\lambda}\Lambda_{\nu]}\,, \tag{67}$$

where now χ and Λ_μ are arbitrary $\mathfrak{g}$-valued function and vector field, respectively.

The easiest way to prove that action (65) describes the massive vector field is as follows. Introducing a vector field $K^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda}\partial_\nu B_{\rho\lambda}$ and integrating by parts, one can rewrite W_0 in the form

$$W_0 = \int d^4x \,\mathrm{tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + K_\mu K^\mu - 2mK^\mu A_\mu\right), \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \tag{68}$$

This action gives the following equation of motion for K_μ:

$$K_\mu = mA_\mu \,.$$

Substituting K_μ given by the last formula into (68), we obtain the usual action for the massive $\mathfrak{g}$-valued vector field of mass m,

$$W_0 = \int d^4x \,\mathrm{tr}\left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - m^2 A_\mu A^\mu\right), \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,. \tag{69}$$

In Refs. 12 and 16 it is shown that the physical sector for action (65) describing the massive vector field can be defined with the help of the BRST cohomology corresponding to symmetry transformations (66) and (67). We show that the corresponding gauge fixed BRST-invariant action is equal to a gauge fixed Abelian version of action (5). This proves that in the Abelian case the theory introduced in Refs. 5 and 20 is equivalent to that defined in Refs. 4, 12 and 15.

First, following Refs. 12 and 16, we introduce the ghosts and gauge fixing conditions corresponding to transformations (66) and (67). We choose the gauge fixing terms for these transformations as in Refs. 12 and 16,

$$\mathcal{F}_1 = \partial^\mu A_\mu \,, \qquad \mathcal{F}_2^\mu = \partial_\nu B^{\mu\nu} \,. \tag{70}$$

The $\mathfrak{g}$-valued anticommuting ghosts, ω, $\bar{\omega}$, $\omega^* = \bar{\omega}$, corresponding to transformation (66) are introduced in the standard way. For transformation (67) we note that the r.h.s. of formula (67) only depends on the transversal part of the vector field Λ_μ. Therefore for transformation (67) only three pairs of $\mathfrak{g}$-valued anticommuting ghosts, ω_i, $\bar{\omega}_i$, $\omega_i^* = \bar{\omega}_i$, $i = 1, 2, 3$, corresponding to the three components of the transversal part of the vector field Λ_μ are required. This observation slightly simplifies the definition of the BRST cohomology comparing to the original definition given in Refs. 12 and 16. In order to define the corresponding BRST transformation, we introduce three arbitrary complex-valued orthonormal vectors $v^1(k)$, $v^2(k)$, $v^3(k)$ orthogonal to the position vector k in the Fourier dual to the Minkowski space. We shall also assume that these vectors satisfy the following conditions $v^i(k) = v^{i*}(-k)$. Let

$$\omega_\mu(x) = \frac{1}{(2\pi)^4}\int e^{ik\cdot(x-y)} v_\mu^i(k)\omega_i(y) d^4k\, d^4y \,, \quad \mu = 0,1,2,3 \,, \tag{71}$$

and

$$\bar{\omega}_\mu(x) = \frac{1}{(2\pi)^4}\int e^{ik\cdot(x-y)} v_\mu^i(k)\bar{\omega}_i(y) d^4k\, d^4y \,, \quad \mu = 0,1,2,3 \tag{72}$$

be the transversal vector combinations of the ghost fields, $\omega_\mu^* = \bar{\omega}_\mu$. The BRST transformation δ corresponding to gauge fixing conditions (70) has the form

$$\begin{aligned} \delta A_\mu &= \partial_\mu \omega \delta\lambda \,, & \delta B_{\mu\nu} &= \partial_{[\mu}\omega_{\nu]}\delta\lambda \,, \\ \delta\omega &= 0 \,, & \delta\bar{\omega} &= \partial^\mu A_\mu \delta\lambda \,, \\ \delta\omega_\mu &= 0 \,, & \delta\bar{\omega}_\mu &= \partial^\nu B_{\mu\nu}\delta\lambda \,, \end{aligned} \tag{73}$$

where $\delta\lambda$ is an anticommuting parameter independent of the space–time coordinates. Note that transformation (73) is in agreement with the transversality condition for the vector $\bar{\omega}_\mu$.

To obtain the BRST-invarant action, one has to add to action (65) a term $W_0^{\rm gf}$ containing gauge fixing conditions (70) and the corresponding Faddeev–Popov operator,

$$W_0^{\rm gf} = \int d^4x \, \mathrm{tr} \left(\partial^\mu A_\mu \partial^\nu A_\nu - \partial^\nu B_{\mu\nu} \partial_\lambda B^{\mu\lambda} - 2\bar{\omega}\Box\omega - 2\sum_{i=1}^{3} \bar{\omega}_i \Box \omega_i \right) . \tag{74}$$

In Refs. 12 and 16 it is proved the gauge fixed action $W_0' = W_0 + W_0^{\rm gf}$,

$$W_0' = \int d^4x \, \mathrm{tr} \left(\frac{1}{2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} B_{\mu\nu} \Box B^{\mu\nu} - \frac{m}{2} \epsilon^{\mu\nu\rho\lambda} B_{\mu\nu} F_{\rho\lambda} \right.$$
$$\left. + \, \partial^\mu A_\mu \partial^\nu A_\nu - 2\bar{\omega}\Box\omega - 2\sum_{i=1}^{3} \bar{\omega}_i \Box \omega_i \right) , \tag{75}$$

is invariant under BRST transformation (73). Therefore the physical sector for the model describing the massive vector field can be defined by quantizing gauge fixed action (75) and by taking the corresponding BRST cohomology.

Introducing the new variables $\Phi^{\mu\nu} = -\epsilon^{\mu\nu\lambda\rho} B_{\lambda\rho}$ and $\eta = \frac{\omega+\bar{\omega}}{\sqrt{2}}$, $\bar{\eta} = \frac{\omega-\bar{\omega}}{\sqrt{2}i}$, $\eta_i = \frac{\omega_i+\bar{\omega}_i}{\sqrt{2}}$, $\bar{\eta}_i = \frac{\omega_i-\bar{\omega}_i}{\sqrt{2}i}$, one can rewrite action (75) in the form

$$W_0' = \int \mathrm{tr} \left(\frac{1}{2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8} (\Box \Phi_{\mu\nu}) \Phi^{\mu\nu} + \frac{m}{2} \Phi_{\mu\nu} F^{\mu\nu} \right.$$
$$\left. - \, 2i \sum_{i=1}^{3} \bar{\eta}_i (\Box \eta_i) + \partial^\mu A_\mu \partial^\nu A_\nu - 2i \bar{\eta} (\Box \eta) \right) d^4x \, . \tag{76}$$

Action (76) coincides with the Abelian counterpart of action (5) with the additional gauge fixing and Faddeev–Popov terms for the Lorentz gauge $\partial^\mu A_\mu = 0$. Thus the theory with Lagrangian (26) is equivalent to the theory with action (65) in the physical sector. In that sector, both theories describe the massive $\mathfrak{g}$-valued vector field.

7. Conclusion

As we observed in this paper, one can suggest at least two mass generation mechanisms for gauge fields. They correspond to different quadratic terms for the $(2, 0)$-type tensor field. In the Abelian case, these mass generation mechanisms are equivalent. The problem of equivalence of the two theories in the non-Abelian case is still open. This question can be studied in the framework of BRST cohomology. If the two theories are equivalent, then there should exist a gauge fixing term and a set of ghosts corresponding to symmetry (64) for action (62) such that the corresponding BRST-invariant gauge fixed action defined as in Refs. 12 and 16 coincides with (5).

Actually, one can construct other non-Abelian gauge invariant actions which differ from (5) or (62) by terms quadratic in the $(2, 0)$-type tensor field. An interesting related question is which actions defined in this way generate unitary renormalizable theories describing a massive vector field only? At present it is only known that the theory suggested in Refs. 4, 12 and 15 has all these properties.

Another interesting problem is: can the gauge invariant actions with mass terms mentioned above be generated dynamically in a nonperturbative way when we consider initially massless models? If such a possibility was realized, one could obtain a dynamical gauge invariant generation mechanism of a mass parameter in a gauge theory.

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