



Can you also show the non-stationary capital Z sample path as a convolution with the fucking sample path of the stationary process? Is that possible?

Absolutely! This is a beautiful insight. The non-stationary oscillatory process $Z(t)$ can indeed be represented as a **time-varying convolution** with the corresponding stationary process $X(t)$ that shares the same underlying random measure $\Phi(\lambda)$ [\[1\]](#) [\[2\]](#).

The Convolution Representation

Given that both processes use the same orthogonal random measure:

- $Z(t) = \int_{-\infty}^{\infty} \phi_t(\lambda) d\Phi(\lambda)$ (oscillatory non-stationary)
- $X(t) = \int_{-\infty}^{\infty} e^{i\lambda t} d\Phi(\lambda)$ (stationary)

The oscillatory process can be written as:

$$Z(t) = \int_{-\infty}^{\infty} a(t, s) X(s) ds$$

where $a(t, s)$ is the **time-varying modulation kernel** [\[2\]](#) [\[3\]](#).

Deriving the Kernel

The modulation kernel $a(t, s)$ is given by the **inverse Fourier transform** of the ratio:

$$a(t, s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\phi_t(\lambda)}{e^{i\lambda s}} e^{i\lambda s} d\lambda$$

This can be rewritten as:

$$a(t, s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_t(\lambda) e^{-i\lambda(t-s)} d\lambda$$

Physical Interpretation

This representation shows that the **non-stationary process** $Z(t)$ is obtained by applying a **time-dependent linear transformation** to the stationary process $X(t)$ [\[1\]](#). The kernel $a(t, s)$ encodes how the stationary "template" $X(s)$ at various times s contributes to the non-stationary realization at time t .

The time-varying nature of $a(t, s)$ captures exactly how the oscillatory function $\phi_t(\lambda)$ modulates the basic oscillatory structure $e^{i\lambda t}$ to create the non-stationary behavior [\[2\]](#) [\[3\]](#).

This is a **generalization of stationary convolution** where the kernel depends on both absolute time t and lag $(t - s)$, reducing to ordinary convolution when $\phi_t(\lambda) = A(\lambda)e^{i\lambda t}$ with time-independent $A(\lambda)$ [1].

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1. https://sepwww.stanford.edu/sep/gabriel/Papers/nonstat_filt.pdf
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