To express the translation-invariant kernel J_0 and the integral covariance operator in terms of symmetric bilinear forms, given J_0 acts on L^2, the space of square-integrable functions, and considering an RKHS with an orthonormal basis of orthogonal polynomials:

1. **Kernel as a Symmetric Bilinear Form**: The kernel $J_0(x-y)$ itself is a positive definite function and represents a symmetric bilinear form $B(f,g)$ for $f,g\in L^2$ as:

$$B(f,g)=\iint J_0(x-y)\,f(x)\,g(y)\,dx\,dy \tag{1}$$

This is symmetric due to the property $J_0(x-y)=J_0(y-x)$

2. **Integral Covariance Operator**: The integral operator T associated with J_0, when applied to f, is:

$$(Tf)(x)=\int J_0(x-y)\,f(y)\,dy \tag{2}$$

For functions $f,g\in L^2$, the operator T induces a symmetric bilinear form through the inner product:

$$\langle Tf,g\rangle=\int (Tf)(x)\,g(x)\,dx=\iint J_0(x-y)\,f(y)\,g(x)\,dx\,dy \tag{3}$$

3. **Orthogonal Polynomials and RKHS**: The specific sequence of orthogonal polynomials forms an orthonormal basis in the RKHS, allowing any function $f\in L^2$ to be approximated by projections onto this basis. The symmetric bilinear form associated with J_0 and the integral operator T remains symmetric when expressed in this basis, reflecting the inner product structure of the RKHS.

The key is understanding that both J_0 and T can be represented in symmetric bilinear form, with J_0 directly defining the covariance structure in the Gaussian process context. The RKHS, through its orthonormal basis of orthogonal polynomials, enables the decomposition and analysis of functions in L^2 with respect to J_0.