Estimating Evolutionary Power Spectra of Nonstationary Processes

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1 The Random response of Linear Single Degree-of-Freedom systems

Consider the random response of a linear single-degree-of-freedom (SDOF) system given by the equation:

$$m\,\ddot{x} + c\,\dot{x} + k\,x = f(t)$$

where m represents the mass, c the damping coefficient, k the stiffness of the system, and $f(t)$ is a broadband nonstationary stochastic process. Further, assume that $f(t)$ admits an integral representation of the form:

$$f(t) = \int_0^t K\,(t-s)\;d\,B(s)$$

where $K(t)$ is a slowly varying deterministic function, and $B(t)$ is an orthogonal process, such that:

$$E\,[B(t)\,B(s)] = \min\,(t, s)$$

Then, it can be proved that the EPS of $f(t)$ can be effectively estimated by considering the energy output of the system, which is quantified as:

$$E(t) = \frac{1}{2}\,m\,\dot{x}^2(t) + \frac{1}{2}\,k\,x^2(t)$$

The proposed methodology for estimating the evolutionary power spectrum (EPS) begins with the acquisition of energy data $E(t)$ from the SDOF system response to the nonstationary input $f(t)$. The first statistical moment of $E(t)$ is then evaluated, typically involving a complex oscillatory pattern, necessitating a smoothing procedure.

To refine the estimates, a Savitzky-Golay (S-G) filter is employed, which uses a polynomial model to smooth the fluctuating energy data, thereby facilitating a more accurate approximation of the EPS. This approach helps in reducing the noise and enhancing the stability of the estimated EPS, particularly when limited data records are available. Further refinements include monitoring the mean and the derivative of the energy function through a polynomial model, which provides a better understanding of the dynamics and variations of the system's energy over time.

The effectiveness and accuracy of this novel EPS estimation method are demonstrated through numerical simulations involving synthetic nonstationary processes, as well as real-world earthquake accelerograms, providing a comprehensive evaluation of its performance compared to existing methods like wavelet transform-based EPS estimation. This detailed analysis not only confirms the utility of the SDOF system as a spectral estimator but also highlights the importance of advanced smoothing techniques in dealing with real-world data.

2 Links To Hamiltonian Mechanics

Yes, you are correct in recognizing that the function $E(t)$ mentioned in the context of the SDOF system's response to a stochastic process resembles the Hamiltonian of a harmonic oscillator. In classical mechanics, the Hamiltonian H of a harmonic oscillator is expressed as:

$$H = \frac{1}{2\,m}\,p^2 + \frac{1}{2}\,k\,x^2$$

where $p = m\,\dot{x}$ is the momentum, m is the mass, k is the stiffness, and x is the displacement.

For the SDOF system under discussion, $E(t)$ is given by:

$$E(t) = \frac{1}{2}\,m\,\dot{x}^2(t) + \frac{1}{2}\,k\,x^2(t)$$

Here, $\frac{1}{2}\,m\,\dot{x}^2(t)$ represents the kinetic energy and $\frac{1}{2}\,k\,x^2(t)$ represents the potential energy of the system. The sum of these two forms the total mechanical energy of the system, which is analogous to the Hamiltonian in the case of conservative systems where the total energy is conserved.

In the case of the SDOF system being excited by a stochastic process, $E(t)$ also acts as an indicator of the energy content in the system at any given time, which is central to the methodology for estimating the evolutionary power spectrum (EPS) of the nonstationary process exciting the system. This similarity underscores the foundational principles of mechanics and energy conservation that apply across both deterministic and stochastic systems.

Absolutely, you're right. The function $E(t)$ for the SDOF system is indeed the Hamiltonian of a harmonic oscillator when considering only conservative forces (no non-conservative damping or external forcing). The formula represents the total mechanical energy, combining both kinetic and potential energies, which is exactly what the Hamiltonian describes in classical mechanics for conservative systems.

In the case of the SDOF system analysis, it maintains this form and function, making it not just a resemblance but essentially the same as the Hamiltonian for a harmonic oscillator. This relationship directly links the concepts used in mechanical vibrations and stochastic process analysis to fundamental physics principles found in classical mechanics.

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