

The classical Nevanlinna–Pick interpolation problem, outlined in mathematical literature, deals with finding an analytic and bounded function within specific constraints. Here is a concise description based on the given reference material:

The Nevanlinna–Pick interpolation problem involves a set of n distinct points z_1, \dots, z_n within the open unit disc \mathbf{D} and associated complex numbers w_1, \dots, w_n . The objective is to determine an analytic function f from the Hardy space H^∞ (the space of all bounded analytic functions on \mathbf{D}), which satisfies:

- $f(z_j) = w_j \ \forall \ j = 1, \dots, n$
- The norm of f , $\|f\|_\infty = \sup \{|f(z)| : z \in \mathbf{D}\}$, is less than or equal to 1.

An alternative condition replaces the bounded norm requirement with a strict inequality (norm strictly less than one), referred to as the suboptimal problem. This problem extends to infinite collections of points.

The feasibility of solving the Nevanlinna–Pick interpolation problem hinges on the associated Pick matrix Λ :

$$\Lambda = \begin{pmatrix} \frac{1 - \bar{w}_1 w_1}{1 - \bar{z}_1 z_1} & \dots & \frac{1 - \bar{w}_1 w_n}{1 - \bar{z}_1 z_n} \\ \vdots & & \vdots \\ \frac{1 - \bar{w}_n w_1}{1 - \bar{z}_n z_1} & \dots & \frac{1 - \bar{w}_n w_n}{1 - \bar{z}_n z_n} \end{pmatrix} \quad (1)$$

This matrix must be positive semi-definite for a solution to exist. The solution is unique if Λ is also singular. For the suboptimal problem, the Pick matrix needs to be positive definite, and solutions involve a rational 2×2 matrix function $\Theta(z)$, where f can be represented in terms of $\Theta(z)$ and a free parameter g , a bounded analytic function with $\|g\|_\infty < 1$.

Moreover, matrix- and operator-valued extensions of this problem have been explored, significantly impacting fields such as engineering and control theory, where rational matrix functions and efficient algorithms for interpolants are crucial. The Nevanlinna–Pick problem has also adapted to non-stationary settings, involving lower-triangular operators and weighted shifts, extending its applicability and theoretical depth.

This rich area of interpolation has found extensive use in various mathematical and engineering applications, continuously evolving through contributions from many prominent mathematicians and engineers.