The Complex Dynamics Of The Hyperbolic Tangent of The Logarithm of One Minus t Squared

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1 The S Function

Let the rational meromorphic quartic with a double-root at the origin and simple roots at $\pm\sqrt{2}$ be defined by

$$\begin{aligned} S(t) &= \tanh(\ln(1-t^2)) \\ &= \frac{(1-t^2)^2-1}{(1+t^2)^2+1} \end{aligned} \tag{1}$$

There are 4 inverse branches of $S(t)$ given by

$$S^{-1}(y) = \{t: S(t) = y\} = \pm\sqrt{\frac{y-1\pm\sqrt{1-y^2}}{y-1}} \tag{2}$$

The transfer operator of $S(t)$ is defined by·

$$\begin{aligned} [\mathcal{T}_S f](y) &= \sum_{\{t:S(t)=y\}} \frac{f(t)}{|\dot{S}(t)|} \\ &= \frac{f\left(\sqrt{\frac{y-1+\sqrt{1-y^2}}{y-1}}\right)}{\left|\dot{S}\left(\sqrt{\frac{y-1+\sqrt{1-y^2}}{y-1}}\right)\right|} + \frac{f\left(-\sqrt{\frac{y-1-\sqrt{1-y^2}}{y-1}}\right)}{\left|\dot{S}\left(-\sqrt{\frac{y-1-\sqrt{1-y^2}}{y-1}}\right)\right|} + \frac{f\left(\sqrt{\frac{y-1-\sqrt{1-y^2}}{y-1}}\right)}{\left|\dot{S}\left(\sqrt{\frac{y-1-\sqrt{1-y^2}}{y-1}}\right)\right|} + \frac{f\left(-\sqrt{\frac{y-1+\sqrt{1-y^2}}{y-1}}\right)}{\left|\dot{S}\left(-\sqrt{\frac{y-1+\sqrt{1-y^2}}{y-1}}\right)\right|} \end{aligned} \tag{3}$$