The Odd Conjugacy Classes of the Symmetric Group: $A060632(n) = 2^{\mathrm{wt}(\lfloor \frac{n}{2} \rfloor)}$ in the OEIS

Definition

The sequence $a(n) = A060632(n)$ is defined as follows:

$$a(n) = 2^{\mathrm{wt}(\lfloor \frac{n}{2} \rfloor)}$$

Where:

$$\mathrm{wt}(m) \text{ is the number of 1s in the binary representation of } m$$
$$\lfloor x \rfloor \text{ is the floor function of } x$$

Sequence

The first few terms of the sequence are: 1, 1, 2, 2, 2, 2, 4, 4, 2, 2, 4, 4, 4, 4, 8, 8, 2, 2, 4, 4, 4, 4, 8, 8, 4, 4, 8, 8, 8, 8, 16, 16, 2, 2, 4, 4, 4, 4, 8, 8, 4, 4, 8, 8, 8, 8, 16, 16, 4, 4, 8, 8, 8, 8, 16, 16, 8, 8, 16, 16, 16, 16, 32

Offset

The sequence starts at offset 0, and the first 3 terms are omitted.

Comments

- Number of conjugacy classes in the symmetric group S_n that have an odd number of elements.
- Also, sequence A001316 doubled.
- Number of even numbers whose binary expansion is a child of the binary expansion of n. - Nadia Heninger and N. J. A. Sloane, Jun 06 2008
- First differences of A151566. Sequence gives the number of toothpicks added at the n-th generation of the leftist toothpick sequence A151566. - N. J. A. Sloane, Oct 20 2010

- The Fi1 and Fi1 triangle sums, see A180662 for their definitions, of Sierpiński's triangle A047999 equal this sequence. - Johannes W. Meijer, Jun 05 2011

- Also, the number of odd entries in the n-th row of the triangle of Stirling numbers of the first kind. - Istvan Mezo, Jul 21 2017

References

1. I. G. MacDonald: Symmetric functions and Hall polynomials. Oxford: Clarendon Press, 1979. Page 21.

Links

- Indranil Ghosh, Table of n, a(n) for n = 0..65536 (terms 0..1000 from Harry J. Smith)

- David Applegate, Omar E. Pol and N. J. A. Sloane, The Toothpick Sequence and Other Sequences from Cellular Automata, Congressus Numerantium, Vol. 206 (2010), 157-191. [There is a typo in Theorem 6: (13) should read u(n) $= 4 \cdot 3^{\mathrm{wt}(n-1)-1}$ for $n \geq 2$.]

- Christina Talar Bekaroğlu, Analyzing Dynamics of Larger than Life: Impacts of Rule Parameters on the Evolution of a Bug's Geometry, Master's thesis, Calif. State Univ. Northridge (2023). See p. 92.

- N. J. A. Sloane, Catalog of Toothpick and Cellular Automata Sequences in the OEIS

- Index entries for sequences related to toothpick sequences

Formulas

$$a(n) = \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{2\,k} \bmod 2^1$$

$$a(n) = \gcd(A\,056040(n), 2^n)$$

$$\text{Generating function: } (1+x) \cdot \prod_{k \geq 0} (1 + 2\,x^{2^{k+1}})$$

Example

$a(3) = 2$ because in S_3 there are two conjugacy classes with an odd number of elements: the trivial conjugacy class and the conjugacy class of transpositions consisting of 3 elements: (12),(13),(23).

Mathematica Code

```
a[n_] := 2^DigitCount[Floor[n/2], 2, 1];

Table[a[n], {n, 0, 94}] (* Jean-FranÃğois Alcover, Feb 25 2014 *)
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PARI Code

```
for (n=0, 1000, write("b060632.txt", n, " ", sum(k=0, floor(n/2),
binomial(n, 2*k) % 2)) )

\\ Harry J. Smith, Sep 14 2009
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Author

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Extensions

- More terms from James A. Sellers, Apr 16 2001
- Edited by N. J. A. Sloane, Jun 06 2008; Oct 11 2010
- $a(0) = 1$ added by N. J. A. Sloane, Sep 14 2009
- Formula corrected by Harry J. Smith, Sep 15 2009

Status

Approved