Let $\theta(t)$ be the Riemann-Siegel vartheta function then define:

$$K(t,s) = J_0(t-s) \cdot e^{\frac{1}{\pi}(\theta(t)-\theta(s))^2}$$

The first derivative with respect to t is:

$$K_t(t,s) = \frac{\partial}{\partial t}\left[J_0(t-s) \cdot e^{\frac{1}{\pi}(\theta(t)-\theta(s))^2}\right]$$

Using the product rule, we get:

$$K_t(t,s) = J_0'(t-s) \cdot e^{\frac{1}{\pi}(\theta(t)-\theta(s))^2} + J_0(t-s) \cdot \frac{\partial}{\partial t}\left[e^{\frac{1}{\pi}(\theta(t)-\theta(s))^2}\right]$$

Applying the chain rule to the second term:

$$K_t(t,s) = J_0'(t-s) \cdot e^{\frac{1}{\pi}(\theta(t)-\theta(s))^2} + J_0(t-s) \cdot e^{\frac{1}{\pi}(\theta(t)-\theta(s))^2} \cdot \frac{2}{\pi}(\theta(t)-\theta(s)) \cdot \theta'(t)$$

So,

$$K_t(t,s) = e^{\frac{1}{\pi}(\theta(t)-\theta(s))^2}\left[J_0'(t-s) + J_0(t-s) \cdot \frac{2}{\pi}(\theta(t)-\theta(s)) \cdot \theta'(t)\right]$$

Evaluating the second derivative at $t=s$:

$$K_{tt}(t,t) = \frac{\partial}{\partial t}\left[J_0'(0) \cdot e^0 + J_0(0) \cdot e^0 \cdot \frac{2}{\pi}(\theta(t)-\theta(t)) \cdot \theta'(t)\right]$$

Given $J_0(0)=1$ and $J_0'(0)=0$:

$$K_{tt}(t,t) = \frac{\partial}{\partial t}\left[0 + 1 \cdot \frac{2}{\pi}(\theta(t)-\theta(t)) \cdot \theta'(t)\right] = \frac{2}{\pi}(\theta'(t))^2$$

The Kac-Rice formula for the expected number of zeros is:

$$E[N(a,b)] = \int_a^b \sqrt{\frac{-K_{tt}(t,t)}{2\pi K(t,t)}}\, dt$$

Given $K(t,t)=1$:

$$E[N(a,b)] = \int_a^b \sqrt{\frac{\frac{2}{\pi}(\theta'(t))^2}{2\pi}}\, dt$$

Simplifying:

$$E[N(a,b)] = \int_a^b \frac{\sqrt{\frac{2}{\pi}}|\theta'(t)|}{\sqrt{2\pi}}\, dt = \int_a^b \frac{\sqrt{2/\pi}|\theta'(t)|}{\sqrt{2\pi}}\, dt$$

Further simplification:

$$E[N(a,b)] = \int_a^b \frac{|\theta'(t)|}{\pi}\, dt$$

$$E[N(a,b)] = \frac{1}{\pi}(\theta(b)-\theta(a))$$