

## Upper Bound on the Covering Number for Self-Adjoint Compact Operators

For a self-adjoint compact operator  $T$  with eigenvalues  $\lambda_i$ , sorted in decreasing absolute value and converging to zero, the upper bound on the covering number  $N(\epsilon)$  of the unit ball under  $T$  with respect to  $\epsilon$ -balls in the operator norm is given by:

$$N(\epsilon) \leq \sum_{i: \lambda_i > \epsilon} \left( 1 + \left\lfloor \frac{\lambda_i}{\epsilon} \right\rfloor \right).$$

This formula uses the eigenvalues  $\lambda_i$  to determine how many  $\epsilon$ -balls are necessary to cover the range of the unit ball under  $T$  in the induced norm, based on the magnitude of each significant eigenvalue relative to  $\epsilon$ .