Juan Maldacena's Lecture on Cosmological Inflation: Closed-Captioning Brought To You

by Steve (via perpleity-flabored collective unconscious)

Table of contents

1 Introduction

It's a pleasure to be here. I was asked to give some talks on inflation. The talks will be somewhat based on the paper I wrote on astro-ph that contains a review of the basics of inflation. When I mention experimental results, it will be based on the latest Planck paper. But the talk will not be too much about phenomenology, but mostly about the theoretical aspects of inflation and various theoretical ideas behind inflation.

2 The Copernican vs. Aristotelian Principles

When the ancients thought about cosmology, they invented what we could call the Aristotelian principle - the idea that we are at a special place in the universe. Perhaps the Earth is at the center of the universe. Later in the history of science, there was the so-called Copernican principle, which was the idea that we are not in a special place in the universe, and that the universe at large scales is uniform.

But if you think about it, in reality, both principles are correct because we are at a special place in the universe. The universe at large scales has a mean density of one atom per cubic meter. Our galaxy has a mean density of a million atoms per cubic meter. Here at Earth, we have 10^{29} to 10^{30} atoms per cubic meter. So we are definitely at a special place.

The question is: how do we explain both things? We have to explain both why the universe looks uniform (the Copernican principle) and why it's not exactly uniform - why there are regions with high density like the one we have here.

3 The Big Bang Theory and Initial Conditions

You've probably heard about the Big Bang Theory. This theory has a set of initial conditions that are exceedingly simple. We only have to specify what the ratio of ordinary matter to dark matter is, the ratio of photons to baryons, and so on. Importantly, we have to assume the universe is almost homogeneous, but not exactly homogeneous.

There are some primordial inhomogeneities which, in the context of Big Bang Theory, we have to postulate. We have to say the universe was almost homogeneous with some small fluctuations - some amplitude for these fluctuations and some spectrum of fluctuations.

Based on these initial conditions, the tiny fluctuations get amplified by the force of gravity. The initial fluctuations are very small - they're one part in 10^5, more or less. These are the fluctuations that we see in the cosmic microwave background. These fluctuations then get amplified and form galaxies, stars, planets, etc.

4 The Metric and Conformal Time

When we talk about the Big Bang Theory, we assume the universe is uniform. We typically think about the expanding universe with the metric:

$$d s^2 = -d t^2 + a(t)^2 \, d x^2 \tag{1}$$

Sometimes it's convenient to use another time coordinate called conformal time:

$$d s^2 = a(\eta)^2 \, (-d \eta^2 + d x^2) \tag{2}$$

There is a simple relationship between the two times: $dt = a\,d\eta$.

The coordinates x that appear here are called comoving coordinates. They're called comoving because if you have two observers with no velocity in the x-directions, those are geodesics going with the cosmological flow. The proper distance between these observers increases if the universe is expanding, but the comoving distance Δx is just the difference in coordinates.

5 Radiation-Dominated Universe

For concreteness, consider a radiation-dominated universe where the Hubble constant is defined as $H = \dot{a}/a$. The Friedmann equation is:

$$H^2 \propto \frac{1}{a^4} \tag{3}$$

Since $H = \dot{a}/a$, this implies $\dot{a} \propto \frac{1}{a}$, which gives us $a \propto t^{1/2}$.

We can calculate what happens with η: putting $a \propto t^{1/2}$, we get $\eta \propto t^{1/2}$, so $a \propto \eta$.

6 The Horizon Problem

The purpose of this calculation is to understand the causal structure. If we look at the Penrose diagram of spacetime, we can think of the spacetime as having some initial $\eta = 0$ and then extending to the future. The horizontal line is x, and $\eta = 0$ is where we have the curvature singularity - the initial singularity of the Big Bang.

If we are here in the future and look back, we see a certain region of the universe. We can receive signals from points within our past light cone, but not from regions outside it.

Now, let's imagine some time very close to the beginning of the Big Bang. If you are an observer there, when you look to the past, you see some region, but different points see separate regions that cannot send signals to each other since the beginning of the universe.

We can ask: how many of these causally disconnected regions do we have? If we extrapolate the Big Bang backwards to nucleosynthesis (temperature $\sim$1 MeV), the number of these regions is roughly the ratio $\eta_{\text{today}}/\eta_{\text{nucleosynthesis}}$. This gives something like e^{25}.

If we go all the way back to the Planck time, we get a number proportional to $T_{\text{Planck}}/T_{\text{CMB}}$, which gives something of order e^{70}.

So there are e^{25} to e^{70} little regions that are causally disconnected at the very beginning. This creates a mystery: why is the universe synchronized when different regions that were not in causal contact appear to have evolved in the same way?

7 The Inflationary Solution

There is a theory that explains this - a theory of what happened before the Big Bang (meaning before the hot Big Bang phase). The idea is that the Big Bang phase extends into the past up to a certain time, and before that time, we have a different evolution called the inflationary phase.

Consider a universe where the Hubble constant is actually constant: $\dot{a} = H\, a$, so $a \propto e^{H t}$. The conformal time becomes:

$$\eta \propto -\frac{1}{a\, H} \propto -e^{-H t} \tag{4}$$

The metric is:

$$d\, s^2 = \frac{1}{H^2\, \eta^2} (-d\, \eta^2 + d\, x^2) \tag{5}$$

This gives us a Penrose diagram where η goes from $-\infty$ to 0. An observer in this spacetime has a particle horizon - they can only see a limited region of spacetime.

The key insight is that we can join these two spacetimes: we cut off the de Sitter spacetime at some η_{cutoff} and join it to the radiation-dominated spacetime.

In this combined picture, regions that were causally disconnected in the radiation-dominated phase become causally connected when we extend back into the inflationary phase. Moreover, the proper distances that remain constant during inflation can become very small in the past, much smaller than the Planck length, and then grow to the size of the observable universe.

8 The Inflaton Field

To achieve this, we need two elements:

1. The spacetime should be close to de Sitter for some time (but not exactly de Sitter, or it would continue forever)

2. There should be some kind of clock telling the universe when to stop inflating

The clock will be a scalar field called the inflaton. The basic picture is that we have a scalar field with a vacuum expectation value that is roughly constant in space but time-dependent. There will be a potential for this scalar field.

During inflation, the potential has a region where it's almost constant but with a small slope. The scalar field rolls down this slope slowly, such that what dominates the energy of the universe is the potential energy - like a cosmological constant. Eventually, when the field gets to some region, it starts moving fast and oscillating, and the energy is transferred to Standard Model particles, reheating the universe.

9 The Action and Equations of Motion

Consider a model with gravity and a scalar field:

$$S = \int d^4 x \sqrt{-g} \left[\frac{M_{\mathrm{Pl}}^2}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right] \tag{6}$$

where $M_{\mathrm{Pl}}^2 = 1/(8\pi G_N)$ is the reduced Planck mass.

Examples of potentials that work include:

- Power-law potentials: $V = \kappa \phi^n$
- Exponential potentials: $V = V_0 (1 - e^{-\alpha\phi/M_{\mathrm{Pl}}})$

The equations of motion are:

$$H^2 = \frac{1}{2 M_{\mathrm{Pl}}^2} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) \tag{7}$$

$$\ddot{\phi} + 3 H \dot{\phi} + V'(\phi) = 0 \tag{8}$$

$$M_{\mathrm{Pl}}^2 \dot{H} = -\frac{1}{2} \dot{\phi}^2 \tag{9}$$

10 Slow-Roll Conditions

For inflation to occur, we need $\dot{H}$ to be small, meaning the fractional variation of H during one e-fold should be much less than 1:

$$\left| \frac{d H / d N}{H} \right| = \left| \frac{\dot{H}}{H^2} \right| \ll 1 \tag{10}$$

where $d N = H d t$ is the number of e-folds.

This gives us: $\frac{\dot{\phi}^2}{2 M_{\mathrm{Pl}}^2 H^2} \ll 1$.

We also assume that the acceleration term $\ddot{\phi}$ can be neglected compared to the friction term $3 H \dot{\phi}$. This leads to:

$$\dot{\phi} \approx -\frac{V'(\phi)}{3 H} \tag{11}$$

We can define slow-roll parameters:

$$\epsilon = \frac{M_{\mathrm{Pl}}^2}{2} \left(\frac{V'}{V} \right)^2 \tag{12}$$

$$\eta = M_{\mathrm{Pl}}^2 \frac{V''}{V} \tag{13}$$

For slow-roll inflation, we need $\epsilon \ll 1$ and $|\eta| \ll 1$.

11 Quantum Fluctuations

Classical inflation predicts a perfectly uniform universe, but quantum effects introduce small inhomogeneities. By analogy with Hawking radiation, we expect the de Sitter spacetime to have a temperature $T \sim H/(2\pi)$.

The typical fluctuations of the scalar field over a horizon-sized region will be of order H, so:

$$\langle (\delta\phi)^2 \rangle \sim H^2 \tag{14}$$

For gravitational waves:

$$\langle h^2 \rangle \sim \frac{H^2}{M_{\mathrm{Pl}}^2} \tag{15}$$

For scalar curvature fluctuations, we need to consider how fluctuations in ϕ translate to curvature fluctuations. The result is:

$$\langle (\delta\mathcal{R})^2 \rangle \sim \frac{H^4}{\dot{\phi}^2} = \frac{H^2}{2 M_{\mathrm{Pl}}^2 \epsilon} \tag{16}$$

The tensor-to-scalar ratio is:

$$r = \frac{\langle h^2 \rangle}{\langle (\delta\mathcal{R})^2 \rangle} = 16\,\epsilon \tag{17}$$

12 Conclusion

Inflation solves several problems of the standard Big Bang model:

- The horizon problem (synchronization of causally disconnected regions)
- The flatness problem (why the universe is spatially flat)
- The monopole problem (dilution of unwanted relics)

It also provides a mechanism for generating the primordial fluctuations that seed structure formation, with specific predictions for the spectrum and properties of these fluctuations that can be tested observationally.