"One-dimensional spectral problems are smoothly deformable like C∞-functions, while multi-dimensional problems are rigid like analytic functions (at least in Euclidean spaces)." This statement provided touches on a fundamental distinction in the behavior of spectral problems depending on the dimensionality of the space in which they are considered. This distinction is grounded in the mathematical properties of functions and the nature of differential equations governing the spectral problems.

One-Dimensional Spectral Problems

In one-dimensional spaces, spectral problems involve solving differential equations with boundary conditions on a line or interval. The solutions to these problems, which determine the spectrum (the set of eigenvalues) of the associated differential operator, are highly sensitive to smooth deformations of the operator or the boundary conditions. This sensitivity means that small, smooth changes in the problem's parameters can lead to smooth changes in the spectrum. This behavior is analogous to that of C^∞-functions, which are infinitely differentiable and can thus be smoothly modified.

Multi-Dimensional Spectral Problems

In contrast, for multi-dimensional spectral problems, which involve partial differential equations on domains in Euclidean spaces of dimension two or higher, the situation is markedly different. These problems exhibit a form of rigidity, meaning that the spectrum of the differential operator is more stable under small deformations. This stability is akin to the behavior of analytic functions, which are defined not just by their infinite differentiability but also by the condition that their Taylor series converge to the function in some neighborhood of every point in their domain. Analytic functions are rigid in the sense that their values over an entire domain are determined by their values (and the values of their derivatives) in an arbitrarily small neighborhood.

Mathematical Foundations

The difference in behavior between one-dimensional and multi-dimensional spectral problems is fundamentally linked to the mathematical structure of the differential equations involved. In one dimension, the solutions to differential equations can often be expressed in terms of smooth functions whose properties allow for a great deal of flexibility. In higher dimensions, however, the solutions are subject to more complex conditions that stem from the interplay between different directions in space. This complexity leads to a form of rigidity, as changes in one part of the domain can have far-reaching implications due to the interconnectedness of the space.

Conclusion

Thus, the assertion that one-dimensional spectral problems are smoothly deformable like C^∞-functions, while multi-dimensional problems are rigid like analytic functions, is an insightful observation on the intrinsic nature of these mathematical problems. It highlights the profound impact that the dimensionality of space has on the behavior of spectral problems and the solutions to the differential equations that define them.