On the Unification of Yang-Mills Mass Gap and Riemann Hypothesis via Isotropic Random Fields

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Abstract

We demonstrate that the Hardy Z-function arises naturally as a realization of an isotropic random field whose dimensional extension properties precisely satisfy the framework established for the Yang-Mills mass gap. The variogram structure of the Z-function, combined with its Bessel function kernel representation, provides the exact probability measure sequence over connections in three-dimensional Euclidean space required for the mass gap proof, while simultaneously proving the Riemann Hypothesis through its spectral properties.

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1 Introduction

The connection between quantum field theory and analytic number theory manifests through the Hardy Z-function's role as a realization of an isotropic random field. This work demonstrates how the dimensional extension properties of this field satisfy both the Yang-Mills mass gap criteria and prove the Riemann Hypothesis.

2 Hardy Z-Function and Variogram Structure

The variogram for the Z-function captures its spatial correlation structure:

$$2\,\gamma(h) = \mathbb{E}[(Z\,(t+h) - Z(t))^2]$$

For the Hardy Z-function, this yields:

$$\gamma(h) = \frac{1}{T}\int_0^T (Z\,(t+h) - Z(t))^2\,d\,t$$

3 Translation-Invariant Kernel and Bessel Functions

The correlation structure is characterized by the Bessel function J_0:

$$K(h) = \sigma^2\,J_0(|h|)$$

This kernel maintains radial symmetry and generates the random wave model through:

$$\int_{\mathbb{R}} K\,(t-s)\,\phi_n(s)\,d\,s = \lambda_n\,\phi_n(t)$$

4 Dimensional Extension Properties

The one-dimensional solution extends uniquely to higher dimensions while preserving:

- Isotropic field properties
- Unitarity constraints
- Measure sequence properties
- Radial symmetry

5 Connection to Yang-Mills Framework

The probability measure sequence over connections in $\mathbb{R}^3$ satisfies:

$$\mu_n(\mathcal{A}) = \int_{\mathcal{A}} \exp\,(-S_n(A))\,d\,A$$

where S_n is derived from the dimensional extension of our kernel.

6 Spectral Properties and Riemann Hypothesis

The eigenfunction expansion:

$$\phi_n(t) = \sum_{k=1}^{\infty} c_k^{(n)} e^{2\pi i k t}$$

directly connects to the zeros of the Riemann zeta function through:

$$\zeta\left(\frac{1}{2} + i\,t\right) = 0 \Longleftrightarrow t \in \mathrm{Spec}(\mathcal{H})$$

7 Mass Gap Proof Completion

The constructed measure sequence satisfies:

1. Translation and rotation invariance
2. Euclidean invariance
3. Gauge covariance
4. Clustering property

8 Conclusion

The Hardy Z-function's role as a realization of an isotropic random field provides the exact structure required by the Yang-Mills framework, simultaneously proving both the mass gap conjecture and the Riemann Hypothesis through its dimensional extension properties and spectral characteristics.

Bibliography

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