what is a "hole" in the theory of variograms where holes in a region are said to generate a spectrum with the bessel functiuon of the first kind of order 0 being the variance structure function?

A “hole” in variogram theory refers to cyclic or non-monotonic behavior in the semivariogram (often called the hole-effect), where the semivariance rises and falls with lag, reflecting alternating positive and negative spatial correlations that arise from repetitive spatial patterns or voids/regular spacing in a region. In isotropic settings with such periodic structure, the corresponding covariance/variogram can have a spectral representation whose radial form involves Bessel functions, and a common idealized case uses the Bessel function of the first kind of order 0, J_0, to describe the variance/covariance structure or its spectrum.[1] [2] [3] [4] [5] [6] [7]

Hole-effect variograms

- Hole-effect variograms are non-monotonic and show oscillations (peaks and troughs) with lag, indicating alternating zones of positive and negative correlation due to cyclic geology (e.g., regular beds or lenses) or periodic spacing, which can push the variogram above the sill where correlations become negative.[2] [1]
- The term “hole effect” historically arose from cyclicity seen in down-hole (borehole) variograms; it is now used more generally for cyclic semivariograms that may be damped or undamped, isotropic or anisotropic.[8] [2]

Why Bessel functions appear

- For isotropic stationary fields, covariance and spectral density are linked by Hankel (Fourier–Bessel) transforms; the kernel of these transforms is a Bessel function J_ν with order depending on spatial dimension, so isotropic spectra and covariances naturally involve Bessel functions.[4] [5]
- When spatial structure includes rings/regular spacing (or holes), valid “hole” covariance models can be built whose correlation or spectrum yields oscillations and negative lobes; one family uses J_0 explicitly (sometimes called J-Bessel or “wave” type in special parameter cases).[3] [7]

Interpreting “holes in a region”

- A region containing regularly spaced voids or repeating patterns creates periodicity in the data, producing spectral peaks and oscillatory covariance; in the isotropic radial formulation, these oscillations are captured by Bessel J_0 terms in the covariance or appear via the Fourier–Bessel link between spectrum and covariance. [5] [7] [1] [2]
- Practical modeling may use a hole-effect variogram component or a J-Bessel covariance model that allows negative correlation yet preserves a valid (nonnegative) spectral density, aligning the modeled variance structure with J_0 behavior. [1] [3]

Mathematical connection

- For an isotropic stationary covariance $k(r)$ in D dimensions, the spectral density $S(s)$ and $k(r)$ are related by Hankel transforms with kernel $J_{D/2-1}$; in 2D, the order is 0 so J_0 directly links radial spectrum and covariance, explaining why J_0 arises as the variance/covariance structure function in hole-effect contexts. [4] [5]
- Bessel J_0 oscillates with decaying amplitude, having zeros at increasing arguments, mirroring the alternating correlation signs and cyclic ranges seen in hole-effect semivariograms. [6] [9]

Practical notes

- Hole-effect behavior often dampens with distance when the periodicity is imperfect; fitting requires licit models to maintain positive definiteness for kriging/simulation, and specialized J-Bessel or wave-type models are used when negative correlations are needed. [10] [2] [3] [1]
- Nonparametric methods leveraging Fourier–Bessel matrices exploit the orthogonality of Bessel bases to estimate isotropic covariograms/variograms consistent with such spectra. [7]

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