Proof that $\sqrt{\frac{8\,n+2}{\pi}}\,\sqrt{\frac{4\,n+1}{2\,\pi}}=\frac{4\,n+1}{\pi}$

Proof. We will prove that:

$$\sqrt{\frac{8\,n+2}{\pi}}\,\sqrt{\frac{4\,n+1}{2\,\pi}}=\frac{4\,n+1}{\pi} \tag{1}$$

1. Start with the left side of the equation:

$$=\sqrt{\frac{8\,n+2}{\pi}}\,\sqrt{\frac{4\,n+1}{2\,\pi}} \tag{2}$$

2. Multiply the terms under the square roots:

$$=\sqrt{\frac{(8\,n+2)\,(4\,n+1)}{\pi\,2\,\pi}}=\sqrt{\frac{(8\,n+2)\,(4\,n+1)}{2\,\pi^2}} \tag{3}$$

3. Expand the numerator:

$$=\sqrt{\frac{32\,n^2+8\,n+8\,n+2}{2\,\pi^2}} \tag{4}$$

4. Simplify:

$$=\sqrt{\frac{32\,n^2+16\,n+2}{2\,\pi^2}} \tag{5}$$

5. Divide both numerator and denominator by 2:

$$=\sqrt{\frac{16\,n^2+8\,n+1}{\pi^2}} \tag{6}$$

6. Factor the numerator:

$$=\sqrt{\frac{(4\,n+1)^2}{\pi^2}} \tag{7}$$

   where we see by expanding that

$$\begin{aligned}(4n+1)\,(4n+1) &= 4n\,4n+4n+4n+1\\ &= 16\,nn+4n+4n+1\\ &= 16n^2+4n+4n+1\\ &= 16n^2+8n+1\end{aligned} \tag{8}$$

7. Now, the square root of a square is the identity so finally :

$$= \frac{4\,n+1}{\pi} \tag{9}$$

This is exactly the right side of the equation whose proof was sought; therefore, it has been shown that:

$$\sqrt{\frac{8\,n+2}{\pi}}\,\sqrt{\frac{4\,n+1}{2\,\pi}} = \frac{4\,n+1}{\pi} \tag{10}$$

The proof is complete. $\square$