1) The Legendre polynomials $P_n(x)$ are defined by the Rodriguez formula:

$$P_n(x) = \frac{1}{2^n\, n!}\left(\frac{d}{d\,x}\right)^n [(x^2-1)^n] \tag{1}$$

2) Taking the Fourier transform (denoted by $\hat{P}_n(k)$) and using integration by parts $n$ times:

$$\begin{aligned}\hat{P}_n(k) =& \frac{1}{2^n\, n!}\int\left(\frac{d}{d\,x}\right)^n [(x^2-1)^n]\, e^{-ikx}\, d\,x & (2)\\ =& i^n\,(-1)^n\left(\frac{1}{2}\right)^n \left[\left(\frac{d}{d\,x}\right)^n (x^2-1)^n\right]_{x=1} - \left[\left(\frac{d}{d\,x}\right)^n (x^2-1)^n\right]_{x=-1} & (3)\end{aligned}$$

3) Using $(x^2-1)^n = (2\,i)^n\, j_n(x)$, where $j_n(x)$ are spherical Bessel functions:

$$\hat{P}_n(k) = i^n\,(-1)^n\left(\frac{k}{2}\right)^n [j_n(k) - (-1)^n\, j_n\,(-k)] \tag{4}$$

4) Expressing the spherical Bessel functions in terms of Lommel polynomials $s_{\mu,\nu}(z)$:

$$\begin{aligned}j_n(z) =& \left(\frac{z}{2}\right)^n s_{\nu,\mu}(z) \quad \text{where } \nu = n+\frac{1}{2}, \mu = -\frac{1}{2} & (5)\\ j_n\,(-z) =& \left(\frac{z}{2}\right)^n s_{\nu,\mu}\,(-z) \quad \text{where } \nu = n, \mu = -\frac{3}{2} & (6)\end{aligned}$$

5) Substituting this into the expression for $\hat{P}_n(k)$:

$$\begin{aligned}\hat{P}_n(k) =& i^n\,(-1)^n\left(\frac{k}{2}\right)^n \left[s_{-\frac{1}{2},n+\frac{1}{2}}(k) - (-1)^n\, s_{-\frac{3}{2},n}(k)\right] & (7)\\ =& i^n\,(-1)^n\left(\frac{k}{2}\right)^n \left[s_{-\frac{1}{2},n+\frac{1}{2}}(k) + (n+1)^{-1}\, s_{-\frac{3}{2},n}(k)\right] & (8)\end{aligned}$$

Therefore, the Fourier transform $\hat{P}_n(k)$ of the Legendre polynomial $P_n(x)$ is:

$$\hat{P}_n(k) = i^n\,(-1)^n\left(\frac{k}{2}\right)^n \left[s_{-\frac{1}{2},n+\frac{1}{2}}(k) + (n+1)^{-1}\, s_{-\frac{3}{2},n}(k)\right] \tag{9}$$

Where $s_{\mu,\nu}(z)$ are the Lommel polynomials defined in terms of the generalized hypergeometric function ${}_pF_q$ as:

$$s_{\mu,\nu}(z) = \left(\frac{z}{2}\right)^\nu {}_pF_q(\nu+1;\mu+\nu+1,\nu+1;-\left(\frac{z}{2}\right)^2) \tag{10}$$