

# Comments on “A Note on the Predictability of Band-Limited Processes”

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## Abstract

A simpler proof is provided for the problem presented by Papoulis regarding past uniform sampling of random signals, extending the result to deterministic signals and nonuniform sampling.

## 1 Introduction

The problem of past uniform sampling of random signals, as presented by Papoulis[papoulis1985], is shown to have been previously solved by others using different methods. This note provides a simpler proof that covers both random and deterministic signals with uniform or nonuniform sampling.

## 2 Proof of Uniqueness for Band-Limited Signals

**Theorem 1.** *If the past samples have a rate slightly greater than the Nyquist rate, then no band-limited signal (having a bandwidth equal to or smaller than half the Nyquist rate) can be found to have zero crossings at the past instances.*

**Proof.** The average zero crossings per interval of a band-limited signal is equal to the Nyquist rate. Therefore, the past samples represent uniquely the band-limited signal [requicha1980]. This argument is valid for deterministic and random signals with uniform or nonuniform samples.  $\square$

## 3 Discussion

The proof presented here is implied in [requicha1980]. Previous solutions for this problem include:

- Wainstein and Zubakov [wainstein1962] at three times the Nyquist rate,
- Brown [brown1972] at two times the Nyquist rate,
- Beutler [beutler1966] for deterministic and random signals with general nonuniform samples,

- Fjallbrandt [fjallbrandt1975] for past uniform samples.

## 4 Conclusion

The proof by Papoulis, while different, can be simplified and generalized to cover both deterministic and random signals with uniform or nonuniform sampling, as shown above.

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