Definition 1

The inner product for two functions $f(t)$ and $g(t)$ in a Hilbert space, such as $L^2(\mathbb{R})$, is defined as:

$$\langle f, g\rangle = \int_{-\infty}^{\infty} f(t)\overline{g(t)}\, dt \tag{1}$$

where $\overline{g(t)}$ denotes the complex conjugate of $g(t)$.

Definition 2

Bessel's Inequality can be formulated using the inner product as follows. Given a set of orthonormal functions $\{e_n\}$ in a Hilbert space and any function f in that space, Bessel's Inequality states:

$$\sum_{n=1}^{N} |\langle f, e_n\rangle|^2 \leq \|f\|^2 \tag{2}$$

where $\|f\|^2 = \langle f, f\rangle$ represents the norm of f, calculated as the inner product of f with itsel

Theorem 3

Parseval's Theorem, when expressed in terms of the inner product for a complete orthonormal system, states that for any function f in $L^2(\mathbb{R})$, the sum of the squares of the inner product of f with the orthonormal basis functions equals the norm of f:

$$\sum_{n=-\infty}^{\infty} |\langle f, e_n\rangle|^2 = \|f\|^2 \tag{3}$$