perplexity

show me the discrete approximation sum of frequency bin size deltalambda and how it converges to the integral as deltalmbda goes to 0 which makes n go to infinity or how deltalambda and N are inverses of each other

Absolutely. Here's the straightforward and explicit answer using textbook harmonic analysis and spectral theory.

Suppose your oscillatory synthesis/inversion formulas are:

- Synthesis: $X(t) = \int_{-\infty}^{\infty} \phi_t(\lambda)\, F(\lambda)\, d\lambda$
- Inversion: $F(\omega) = \int_{-\infty}^{\infty} X(t)\, \overline{\phi_t(\omega)}\, dt$

Discrete Approximation

Partition the frequency axis into bins of width $\Delta\lambda$, with centers λ_j, so that $\lambda_j = \lambda_0 + j\Delta\lambda$ and $j = 1, \ldots, N$, where $N\Delta\lambda$ covers the frequency interval of interest.

The integral becomes the Riemann sum:

$$\int_a^b F(\lambda)\, d\lambda \;\approx\; \sum_{j=1}^{N} F(\lambda_j)\, \Delta\lambda$$

So, the continuous representation is approximated by the sum:

$$X(t) \approx \sum_{j=1}^{N} \phi_t(\lambda_j)\, F(\lambda_j)\, \Delta\lambda$$

Similarly, the inversion becomes:

$$F(\lambda_k) \approx \sum_{l=1}^{M} X(t_l)\, \overline{\phi_{t_l}(\lambda_k)}\, \Delta t$$

where t_l are sample points in time (and Δt is a time step).

Limit as $\Delta\lambda \to 0$, $N \to \infty$

- As $\Delta\lambda \to 0$, the frequency resolution increases and the sum approaches the continuous integral.
- For a fixed bandwidth $(b-a)$ covered, $N = \frac{b-a}{\Delta\lambda}$, so as $\Delta\lambda \to 0, \quad N \to \infty$.
- Thus, $\Delta\lambda$ and N are inversely related:

$$N = \frac{\text{Bandwidth}}{\Delta\lambda} \implies \Delta\lambda = \frac{\text{Bandwidth}}{N}$$

Explicit Statement of Convergence

For any sufficiently regular (e.g., L^2) function F and orthonormal system $\phi_t(\lambda)$, the Riemann sum

$$\sum_{j=1}^{N} \phi_t(\lambda_j) F(\lambda_j) \Delta\lambda$$

tends to

$$\int_a^b \phi_t(\lambda) F(\lambda) d\lambda$$

as $\Delta\lambda \to 0$, i.e., as $N \to \infty$.

Short Table for Clarity

Continuous	Discrete (bin size = $\Delta\lambda$)	Limit
$ \int \phi_t(\lambda)F(\lambda)d\lambda $	$ \sum_{j=1}^N \phi_t(\lambda_j)F(\lambda_j)\Delta\lambda $	$ \lim_{\Delta\lambda\to 0} \sum \rightarrow \int $
$N = (b-a)/\Delta\lambda$	$\Delta\lambda = (b-a)/N$	$N \to \infty$, $\Delta\lambda \to 0$

Summary:

- You discretize the frequency variable into bins of size $\Delta\lambda$.
- The integral becomes a sum over these bins.
- As $\Delta\lambda \to 0$, $N \to \infty$, the sum converges to the integral.
- N and $\Delta\lambda$ are inverses (for fixed bandwidth).

Formula:

$$\boxed{X(t) \approx \sum_{j=1}^{N} \phi_t(\lambda_j) F(\lambda_j)\, \Delta\lambda \xrightarrow[\Delta\lambda \to 0]{N \to \infty} \int \phi_t(\lambda) F(\lambda) d\lambda}$$

If you want the proof is simply the Riemann sum convergence theorem, since $\phi_t(\lambda)F(\lambda)$ is L^2 and the partition gets arbitrarily fine as $\Delta\lambda \to 0$.