Proof for Evaluating a General Complex Rational Function

Introduction

Apologies for the confusion. Let's establish a proof for evaluating a general complex rational function $f(z)$, defined as:

$$f(z) = \frac{P(z)}{Q(z)}$$

where:

$$P(z) = r(z) + i \cdot q(z)$$

$$Q(z) = s(z) + i \cdot t(z)$$

and $r(z), q(z), s(z), t(z)$ are rational functions of z.

Proof for Evaluating $f(z)$ Correctly

To evaluate $f(z)$, we must compute the quotient of the complex numbers $P(z)$ and $Q(z)$. The general formula for the division of two complex numbers $\frac{a+b\,i}{c+d\,i}$ is given by multiplying numerator and denominator by the conjugate of the denominator:

$$\frac{a+b\,i}{c+d\,i} = \frac{(a+b\,i)\,(c-d\,i)}{(c+d\,i)\,(c-d\,i)} = \frac{(a\,c+b\,d)+(b\,c-a\,d)\,i}{c^2+d^2}$$

Applying this formula to $P(z)$ and $Q(z)$:

1. **Multiply $P(z)$ by the conjugate of $Q(z)$**:

$$(r(z)+i\,q(z))\,(s(z)-i\,t(z)) = (r(z)\,s(z)+q(z)\,t(z)) + (q(z)\,s(z)-r(z)\,t(z))\,i$$

2. **Multiply $Q(z)$ by its own conjugate**:

$$(s(z)+i\,t(z))\,(s(z)-i\,t(z)) = s(z)^2+t(z)^2$$

3. **Form the quotient**:

$$f(z) = \frac{(r(z)\,s(z) + q(z)\,t(z)) + (q(z)\,s(z) - r(z)\,t(z))\,i}{s(z)^2 + t(z)^2}$$

Correctness of the Formula

The above formula ensures correct evaluation by adhering to the rules of complex number division. Each step explicitly handles the distribution and combination of real and imaginary parts, conforming to the algebraic principles of complex arithmetic. The computation maintains the structure and relationships of the components, ensuring that both the real and imaginary parts of the function are accurately represented in the final result.

Conclusion

This method provides a general, rigorous way to evaluate any complex rational function defined in the specified form, ensuring the accuracy of both real and imaginary parts. This approach correctly incorporates the algebraic handling of complex numbers with rational function components, covering all possible complex rational functions fitting the initial format.