add $\lambda[]\sigma(\cdot)$ method to LinearOperator

Let's consider a linear operator T acting on a Banach space (a complete normed vector space). The spectrum of T, denoted as $\sigma(T)$, is the set of all scalars λ for which the operator $(T - \lambda I)$ is not invertible, where I is the identity operator.

The spectrum of an operator is made up of three different types of points:

Discrete

1. **Eigenvalues:** These are the values of λ for which the operator $(T - \lambda I)$ has a non-zero vector that it sends to the zero vector (i.e., there is some non-zero vector x such that $(T - \lambda I)x = 0$). The corresponding x are called eigenvectors. Eigenvalues reveal important aspects of the operator's behavior, such as its stability and how it transforms the space.

Continuous

2. **Continuous Spectrum:** These are values of λ for which the operator $(T - \lambda I)$ is injective (one-to-one), has dense range, but is not surjective (onto). This means there are vectors that are arbitrarily close to being sent to zero, but no non-zero vector is actually sent to zero.

Singular

3. **Residual Spectrum:** These are values of λ for which the operator $(T - \lambda I)$ is not injective (not one-to-one), or it is injective but does not have dense range.

In quantum mechanics, the spectrum of an operator has a direct physical interpretation: it represents the set of possible outcomes for a measurement of the corresponding physical quantity. For example, the spectrum of the Hamiltonian operator, which represents the total energy of a system, gives all the possible energies that can be measured. Each eigenvalue in the spectrum corresponds to a possible result of the measurement, and the associated eigenvector represents the state the system will be in after the measurement.

In quantum mechanics, the operators we deal with typically represent observable physical quantities, like momentum, position, or energy. Each possible measurement outcome of these quantities corresponds to an eigenvalue of the operator, and the state of the system after the measurement corresponds to the associated eigenvector.

what does this imply

When an operator is not invertible, it means there's some "loss of information" or ambiguity involved. In the context of quantum mechanics, the points where an operator fails to be invertible correspond to its spectrum - the possible values one can measure for the physical quantity associated with that operator.

In other words, the non-invertible points are exactly those that could be "observed" or "measured" when applying the operator to the system. So, the spectrum of an operator (the set of points where it's non-invertible) has direct physical meaning in quantum mechanics. It tells us the set of possible outcomes we could get when we measure the physical quantity represented by the operator.

ie the eigenvalues (transformed via S(t) or T(t)) really are possible universes?

Would this imply the entire universe is a single "measurement" made by what or whom?

measurements aren't what they think they are