Towards Standard Model without the Higgs boson

A. Sevostyanov
Department of Mathematical Sciences
University of Aberdeen
Aberdeen AB24 3UE, United Kingdom
e-mail: seva@maths.abdn.ac.uk
phone: +44-1224-272755, fax: +44-1224-272607

March 11, 2008

Abstract

We suggest a new formulation for the bosonic sector of the Standard Model. In the new formulation the Higgs field is not required.

PACS 12.15.-y

In paper [4] we introduced a new mass generation mechanism for gauge fields associated to compact simple Lie groups. Perturbatively the theory constructed in [4] describes massive vector fields, and no extra particles, like the Higgs boson, appear in the physical spectrum. It is natural to apply the results of [4] to the bosonic sector of the Standard Model.

Note that in [4] we only discussed the case of gauge fields associated to simple Lie groups, and the quantum field theory constructed in [4] describes fields of equal mass. On the other hand the underlying group related to the electroweak sector of the Standard Model is not simple, and, as a consequence, the vector mesons have different masses.

The main construction of this paper is based on a very simple observation: the underlying group $U(1) \times SU(2)$ of the electroweak sector of the Standard Model can be represented as a semideirect product $U(1) \rtimes SU(2)$ of $U(1)$ and $SU(2)$,

$$U(1) \times SU(2) \simeq U(1) \rtimes SU(2). \quad (1)$$

In order to establish isomorphism (1) one has to fix a homomorphism

$$U(1) \to SU(2),$$

i.e. a Cartan subalgebra in $SU(2)$, which gives rise to a representation $U(1) \to \mathrm{End}\ SU(2)$ via the adjoint action of $SU(2)$ on itself. This representation can be used to construct the semidirect product $U(1) \rtimes SU(2)$.

Let $\mathfrak{u}(1) \lhd \mathfrak{su}(2)$ be the Lie algebra of $U(1) \rtimes SU(2)$. Isomorphism (1) induces an isomorphism of Lie algebras,

$$\psi : \mathfrak{u}(1) \oplus \mathfrak{su}(2) \to \mathfrak{u}(1) \lhd \mathfrak{su}(2),\ (\xi, x) \mapsto (\mathrm{g}'\xi, \mathrm{g}x - \mathrm{g}'\xi h),\ \xi \in \mathfrak{u}(1) \simeq \mathbb{R},\ x \in \mathfrak{su}(2). \quad (2)$$

In formula (2) $h \in \mathfrak{su}(2)$ is a fixed element of the Cartan subalgebra in $\mathfrak{su}(2)$, and we assume throughout of this paper that the standard commutators in $\mathfrak{u}(1)$ and $\mathfrak{su}(2)$ are rescaled by

constants g$'$ and g, respectively. In our construction these constants will play the same role as in case of the usual formulation of the Standard Model, and isomorphism (2) will be used to obtain particles of different masses without gauge symmetry braking.
Now let B_μ and A_μ be the $\mathfrak{u}(1)$ and $\mathfrak{su}(2)$ -valued gauge fields (connections) on the Minkowski space. Denote by $\widetilde{\mathfrak{u}}(1)$ and $\widetilde{\mathfrak{su}}(2)$ the Lie algebras of the corresponding gauge groups $\widetilde{U}(1)$ and $\widetilde{SU}(2)$.
The basic ingredient of the Standard model is the direct sum (B_μ, A_μ) of connections B_μ and A_μ. (B_μ, A_μ) is a $\mathfrak{u}(1) \oplus \mathfrak{su}(2)$-valued gauge field on Minkowski space. Note that the isomorphism

$$\widehat{\psi} : \widetilde{\mathfrak{u}}(1) \oplus \widetilde{\mathfrak{su}}(2) \to \widetilde{\mathfrak{u}}(1) \lhd \widetilde{\mathfrak{su}}(2)$$

induced by (2) can not be applied directly to the connection (B_μ, A_μ) since the space of connections is not linear. But it can be applied to the curvature of (B_μ, A_μ). Indeed, the components of the curvature $(F_{\mu\nu}(B), F_{\mu\nu}(A))$ of (B_μ, A_μ),

$$F_{\mu\nu}(B) = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

$$F_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{g}[A_\mu, A_\nu],$$

are elements of the adjoint representation of the gauge group $\widetilde{U}(1) \times \widetilde{SU}(2)$, and hence

$$\widehat{\psi}(F_{\mu\nu}(B), F_{\mu\nu}(A)) = (\mathrm{g}' F_{\mu\nu}(B), \mathrm{g} F_{\mu\nu}(A) - \mathrm{g}' F_{\mu\nu}(B) h) \tag{3}$$

belongs to the adjoint representation of the gauge group $\widetilde{U}(1) \rtimes \widetilde{SU}(2) \simeq \widetilde{U}(1) \times \widetilde{SU}(2)$.
Note that from the definition of the semidirect product it follows that the $\mathfrak{su}(2)$ component in the decomposition $\mathfrak{u}(1) \lhd \mathfrak{su}(2)$ is invariant under the adjoint action of $U(1) \rtimes SU(2) \simeq U(1) \times SU(2)$. We denote by $\mathfrak{su}(2)_1$ this representation of $U(1) \times SU(2)$ in the space $\mathfrak{su}(2)$. By the definition of the representation $\mathfrak{su}(2)_1$ the component

$$F_{\mu\nu} = \mathrm{g} F_{\mu\nu}(A) - \mathrm{g}' F_{\mu\nu}(B) h \tag{4}$$

defined by the r.h.s. of (3) takes values in $\mathfrak{su}(2)_1$. Therefore $F_{\mu\nu}$ is invariant under the action of the gauge group $\widetilde{U}(1) \times \widetilde{SU}(2)$[1].
The last important ingredient in the new formulation of the Standard Model is a covariant d'Alambert operator $\Box_{(B,A)}$ associated to the connection (B_μ, A_μ),

$$\Box_{(B,A)} = D_\mu D^\mu,$$

where D_μ is the covariant derivative of the connection (B_μ, A_μ),

$$D_\mu = \partial_\mu - (\mathrm{g}' B_\mu, \mathrm{g} A_\mu).$$

The covariant d'Alambert operator can be applied to any tensor field defined on the Minkowski space and taking values in a representation space of the Lie group $U(1) \times SU(2)$, the $\mathfrak{u}(1) \oplus \mathfrak{su}(2)$-valued gauge field (B_μ, A_μ) acts on the tensor field according to that representation. Note that the operator $\Box_{(B,A)}$ is scalar, i.e. it does not change types of tensors.
In order to write down the Lagrangian of the Standard Model we have to fix a nondegenerate invariant under the adjoint action scalar product on $\mathfrak{su}(2)$. We denote this scalar product by tr (for instance, one can take the trace of the composition of the elements of $\mathfrak{su}(2)$ acting in the adjoint representation). Let t^a, $a = 1, 2, 3$ be a linear basis of $\mathfrak{su}(2)$ normalized in such a way that $\mathrm{tr}(t^a t^b) = -\frac{1}{2}\delta^{ab}$. We introduce the components A^a_μ of the gauge field A_μ by

$$A_\mu = A^a_\mu t^a.$$

[1] A term similar to (4) was also considered in [2]; however in [2] it was not observed that this term is gauge invariant.

We also use similar notation for the components of any $\mathfrak{su}(2)$-valued quantity.
We shall also put $h = t^3$ in formula (2).
The bosonic sector of the Standard Model can be described using the following gauge invariant Lagrangian

$$L = \int \Big[-\frac{1}{4}F_{\mu\nu}(B)F^{\mu\nu}(B) + \mathrm{tr}\big(\frac{1}{2}F_{\mu\nu}(A)F^{\mu\nu}(A) - \frac{1}{8}(\Box_{(B,A)}\Phi_{\mu\nu})\Phi^{\mu\nu} + \frac{m}{2}\Phi_{\mu\nu}F^{\mu\nu} - \\ -2i\sum_{i=1}^{3}\overline{\eta}_i(\Box_{(B,A)}\eta_i)\big)\Big]d^3x, \tag{5}$$

the gauge group of the theory being $\widetilde{U}(1)\times\widetilde{SU}(2)$. In the expression above $F_{\mu\nu}$ is defined by formula (4), $\Phi_{\mu\nu}$ is a skew-symmetric (2,0)-type tensor field with values in the representation $\mathfrak{su}(2)_1$ of the group $U(1)\times SU(2)$; $\eta_i, \overline{\eta}_i$, $i = 1, 2, 3$ are pairs of anticommuting scalar fields with values in $\mathfrak{su}(2)_1$; they satisfy the following reality conditions: $\eta_i^* = \eta_i$, $\overline{\eta}_i^* = \overline{\eta}_i$. We also use the standard convention about summations and lowering tensor indexes with the help of the standard metric $g_{\mu\nu}$ of the Minkowski space, $g_{00} = 1$, $g_{ii} = -1$ for $i = 1, 2, 3$, and $g_{ij} = 0$ for $i \neq j$.
The asymptotic states described by the quantized theory with Lagrangian (5) can be obtained from the abelian counterpart L_0 of Lagrangian (5),

$$L_0 = \int \Big[-\frac{1}{4}F_{\mu\nu}(B)F^{\mu\nu}(B) + \mathrm{tr}\big(\frac{1}{2}F_{\mu\nu}(A)F^{\mu\nu}(A) - \frac{1}{8}(\Box\Phi_{\mu\nu})\Phi^{\mu\nu} + \frac{m}{2}\Phi_{\mu\nu}F^{\mu\nu} - \\ -2i\sum_{i=1}^{3}\overline{\eta}_i(\Box\eta_i)\big)\Big]d^3x, \tag{6}$$

where now

$$F_{\mu\nu}(B) = \partial_\mu B_\nu - \partial_\nu B_\mu,$$
$$F_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

and

$$F_{\mu\nu} = \mathrm{g}F_{\mu\nu}(A) - \mathrm{g}'F_{\mu\nu}(B)t^3 = \partial_\mu(\mathrm{g}A_\nu - \mathrm{g}'B_\nu t^3) - \partial_\nu(\mathrm{g}A_\mu - \mathrm{g}'B_\mu t^3). \tag{7}$$

In view of the last formula it is convenient to introduce new fields A'_μ and Z_μ related to A^3_μ and B_μ by an orthogonal transformation,

$$A'_\mu = \sin\theta_W A^3_\mu + \cos\theta_W B_\mu, \tag{8}$$
$$Z_\mu = \cos\theta_W A^3_\mu - \sin\theta_W B_\mu, \tag{9}$$

where

$$\tan\theta_W = \frac{\mathrm{g}'}{\mathrm{g}},$$

and θ_W is the Weinberg angle. Now introducing the quantities $F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu$ and $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ one can rewrite Lagrangian (6) in the form

$$L_0 = \int \Big[-\frac{1}{4}(F'_{\mu\nu}F^{\mu\nu\prime} + \sum_{a=1,2}F^a_{\mu\nu}(A)F^{\mu\nu\, a}(A) + Z_{\mu\nu}Z^{\mu\nu}) + \frac{1}{16}\sum_{a=1,2,3}(\Box\Phi^a_{\mu\nu})\Phi^{\mu\nu\, a} - \\ -\frac{m\mathrm{g}}{4}\sum_{a=1,2}\Phi^a_{\mu\nu}F^{\mu\nu\, a}(A) - \frac{m\sqrt{\mathrm{g}^2+\mathrm{g}'^2}}{4}\Phi^3_{\mu\nu}Z^{\mu\nu} + i\sum_{i=1}^{3}\sum_{a=1,2,3}\overline{\eta}^a_i(\Box\eta^a_i)\Big]d^3x. \tag{10}$$

The first term in formula (10) obviously describes abelian gauge field A'_μ, and according to the results of [3], Sect. 4 the positive energy Poincaré invariant sector for the remaining terms

in (10) contains two massive vector fields of mass $m\mathrm{g}$ and one massive vector field of mass $m\sqrt{\mathrm{g}^2+\mathrm{g}'^2}$.
To identify these fields we introduce the following notation for the components $\Phi^a_{\mu\nu}$ of the field $\Phi_{\mu\nu}$:

$$G^a_k = \frac{1}{2}\varepsilon_{ijk}\Phi^a_{ij},\ \phi^a_k = \Phi^a_{0k},\ i,j,k,a=1,2,3,$$

and define the vector fields A^a, $a=1,2$, Z, A', G^a, ϕ^a, $a=1,2,3$ on $\mathbb{R}^3$, with spatial components A^a_i, Z_i, A'_i, G^a_i, ϕ^a_i, $i=1,2,3$, respectively.
We also introduce the longitudinal component $A^a_{\|}$ of A^a,

$$A^a_{\|}(\mathbf{x}) = \frac{i}{(2\pi)^3}\int e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\frac{k_j}{|\mathbf{k}|}A^a_j(\mathbf{y})d^3kd^3y, \tag{11}$$

and the transversal component $A^a_{\perp}$,

$$A^a_{\perp} = A^a - \mathrm{grad}\triangle^{-1}\partial_i A^a_i. \tag{12}$$

The transversal and longitudinal components of the vector fields Z, A', G^a, ϕ^a are defined by formulas similar to (11), (12). Here and below we write $\mathbf{x}=(x^1,x^2,x^3)$ for vectors in three-dimensional Euclidean space $\mathbb{R}^3$, and $\cdot$ stands for the usual scalar product in $\mathbb{R}^3$.
In the Coloumb gauge,

$$\sum_{i=1}^{3}\partial^i B_i = 0,\ \sum_{i=1}^{3}\partial^i A_i = 0,$$

the transversal components of the massive fields of masses $m_W = m\mathrm{g}$ and $m_Z = m\sqrt{\mathrm{g}^2+\mathrm{g}'^2}$ are the transversal components of A^a, $a=1,2$ and of Z, and the longitudinal components of the massive fields are the longitudinal components of the spatial vector fields ϕ^a, $a=1,2$ and of ϕ^3, respectively.
Therefore the transversal components of the spatial parts of the linear combinations

$$W^{\mp}_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \pm iA^2_\mu)$$

can be identified with those for the charged vector mesons of masses

$$m_W = m\mathrm{g},$$

and the transversal component of the spatial part of Z_μ with that of the neutral vector meson of mass

$$m_Z = m\sqrt{\mathrm{g}^2+\mathrm{g}'^2}.$$

The electromagnetic abelian gauge field A'_μ remains massless. The masses of particles listed above are the same as in case of the usual formulation of the Standard Model (see [1], Sect. 12-6).
The quantum field theory generated by Lagrangian (5) is unitary and satisfies the energy positivity condition. Actually both the unitarity and the energy positivity conditions hold in the physical space of states described in [3]. In [4] it is also proved that the theory generated by Lagrangian (5) is renormalizable.
The physical space of asymptotic states $\mathcal{H}^0_{\mathrm{phys}}$ is the bosonic Fock space for the operators $\mathbf{b}_i$, $\mathbf{b}^*_i$, $\mathbf{b}^a_i$, $\mathbf{b}^{a*}_i$, $i=1,2,3$, $a=1,2,3$, and $\mathbf{d}_i$, $\mathbf{d}^*_i$, $i=1,2$ obeying the standard commutation relations

$$[\mathbf{b}_i(\mathbf{k}),\mathbf{b}^*_j(\mathbf{k}')] = \delta_{ij}\delta(\mathbf{k}-\mathbf{k}'),\ [\mathbf{b}^a_i(\mathbf{k}),\mathbf{b}^{b^*}_j(\mathbf{k}')] = \delta_{ij}\delta^{ab}\delta(\mathbf{k}-\mathbf{k}'),\ [\mathbf{d}_i(\mathbf{k}),\mathbf{d}^*_j(\mathbf{k}')] = \delta_{ij}\delta(\mathbf{k}-\mathbf{k}'),$$

and all the operators with superscript $*$ being regarded as creation operators.
The normal symbol $\mathbf{S} = \mathbf{S}(b_i^{a*}, {b_i}^*, d_i^*; b_i^a, b_i, d_i)$ of the corresponding S–matrix in the Lorentz gauge can be expressed via Feynman path integral (see [3], Sect. 6),

$$\mathbf{S} = \int \mathcal{D}(A_\mu)\mathcal{D}(\Phi_{\mu\nu})\mathcal{D}(\eta)\mathcal{D}(\overline{\eta})\prod_{i=1}^{3}\mathcal{D}(\eta_i)\mathcal{D}(\overline{\eta}_i)\prod_x \delta(\partial^\mu A_\mu(x))\delta(\partial^\mu B_\mu(x)) \times \tag{13}$$
$$\times \exp\{i\int \Big(L + 2i\int \mathrm{tr}\ (\overline{\eta}\partial^\mu(\partial_\mu - \mathrm{g}A_\mu)\eta)d^3x\Big)dt\},$$

where $\overline{\eta}, \eta$ are the anticommuting scalar ghost fields (Faddeev-Popov ghosts) taking values in the adjoint representation of $\mathfrak{su}(2)$ and satisfying the following reality conditions $\eta^* = \eta$, $\overline{\eta}^* = \overline{\eta}$.
The components of the variables of integration in the Feynman path integral in formula (13) obey the following boundary conditions:
$A_\perp^a(\mathbf{x}, t)$, $a = 1, 2$ can be expressed as

$$A_\perp^a(\mathbf{x}, t) = A_{\perp 0}^a(\mathbf{x}, t) + \overline{A}_\perp^a(\mathbf{x}, t),$$

where

$$A_{\perp 0}^a(\mathbf{x}, t) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int \frac{d^3k}{\sqrt{2(\mathbf{k}^2 + m_W^2)^{\frac{1}{2}}}}\sum_{i=1,2}(b_i^a(\mathbf{k})e^i(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x} - i\sqrt{(\mathbf{k}^2 + m_W^2)}t} +$$
$$+ b_i^{a*}(\mathbf{k})e^i(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x} + i\sqrt{(\mathbf{k}^2 + m_W^2)}t}),$$

and $\overline{A}_\perp^a(\mathbf{x}, t)$ satisfies the radiation boundary condition for the operator $\Box + m_W^2$;
$\phi_\parallel^a(\mathbf{x}, t)$, $a = 1, 2$ can be expressed as

$$\phi_\parallel^a(\mathbf{x}, t) = \phi_{\parallel 0}^a(\mathbf{x}, t) + \overline{\phi}_\parallel^a(\mathbf{x}, t),$$

where

$$\phi_{\parallel 0}^a(\mathbf{x}, t) = \frac{2}{(2\pi)^{\frac{3}{2}}}\int \frac{d^3k}{\sqrt{2(\mathbf{k}^2 + m_W^2)^{\frac{1}{2}}}}(b_3^a(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x} - i\sqrt{(\mathbf{k}^2 + m_W^2)}t} +$$
$$+ b_3^{a*}(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x} + i\sqrt{(\mathbf{k}^2 + m_W^2)}t}),$$

and $\overline{\phi}_\parallel^a(\mathbf{x}, t)$ satisfies the radiation boundary condition for the operator $\Box + m_W^2$;
for $a = 1, 2$

$$G_\perp^a(\mathbf{x}, t) \underset{t \to -\infty}{\to} -\frac{2\ \mathrm{curl}}{m_W}A_\perp^a(\mathbf{x}, t)_0;$$

and

$$\phi_\perp^a(\mathbf{x}, t) \underset{t \to -\infty}{\to} -\frac{\partial}{\partial t}\frac{2}{m_W}A_\perp^a(\mathbf{x}, t)_0;$$

$Z_\perp(\mathbf{x}, t)$ can be expressed as

$$Z_\perp(\mathbf{x}, t) = Z_{\perp 0}(\mathbf{x}, t) + \overline{Z}_\perp(\mathbf{x}, t),$$

where

$$Z_{\perp 0}(\mathbf{x}, t) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int \frac{d^3k}{\sqrt{2(\mathbf{k}^2 + m_Z^2)^{\frac{1}{2}}}}\sum_{i=1,2}(b_i(\mathbf{k})e^i(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x} - i\sqrt{(\mathbf{k}^2 + m_Z^2)}t} +$$
$$+ {b_i}^*(\mathbf{k})e^i(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x} + i\sqrt{(\mathbf{k}^2 + m_Z^2)}t}),$$

and $\overline{Z}_\perp(\mathbf{x},t)$ satisfies the radiation boundary condition for the operator $\Box + m_Z^2$; $\phi_\parallel^3(\mathbf{x},t)$ can be expressed as

$$\phi_\parallel^3(\mathbf{x},t) = \phi_{\parallel 0}^3(\mathbf{x},t) + \overline{\phi}_\parallel^3(\mathbf{x},t),$$

where

$$\phi_{\parallel 0}^3(\mathbf{x},t) = \frac{2}{(2\pi)^{\frac{3}{2}}} \int \frac{d^3k}{\sqrt{2(\mathbf{k}^2+m_Z^2)^{\frac{1}{2}}}} (b_3(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x} - i\sqrt{(\mathbf{k}^2+m_Z^2)}t} + {b_3}^*(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x} + i\sqrt{(\mathbf{k}^2+m_Z^2)}t}),$$

and $\overline{\phi}_\parallel^3(\mathbf{x},t)$ satisfies the radiation boundary condition for the operator $\Box + m_Z^2$;

$$G_\perp^3(\mathbf{x},t) \underset{t\to-\infty}{\to} -\frac{2\ \mathrm{curl}}{m_Z} Z_\perp(\mathbf{x},t)_0;$$

$$\phi_\perp^3(\mathbf{x},t) \underset{t\to-\infty}{\to} -\frac{\partial}{\partial t}\frac{2}{m_Z} Z_\perp(\mathbf{x},t)_0;$$

$A'_\perp(\mathbf{x},t)$ can be expressed as

$$A'_\perp(\mathbf{x},t) = A'_{\perp 0}(\mathbf{x},t) + \overline{A}'_\perp(\mathbf{x},t),$$

where

$$A'_{\perp 0}(\mathbf{x},t) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \frac{d^3k}{\sqrt{2|\mathbf{k}|}} \sum_{i=1,2} (d_i(\mathbf{k}) e^i(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x} - i|\mathbf{k}|t} + \\ + {d_i}^*(\mathbf{k}) e^i(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x} + i|\mathbf{k}|t}),$$

and $\overline{A}'_\perp(\mathbf{x},t)$ satisfies the radiation boundary condition for the operator $\Box$; for $a = 1,2,3$

$$\begin{aligned}
\eta_i^a(\mathbf{x},t) &= \frac{1}{(2\pi)^{\frac{3}{2}}} \int \frac{d^3k}{\sqrt{2|\mathbf{k}|}} (c_i^a(\mathbf{k},t) e^{-i\mathbf{k}\cdot\mathbf{x}} + \overline{c}_i^{a*}(\mathbf{k},t) e^{i\mathbf{k}\cdot\mathbf{x}}), \\
& c_i^a(\mathbf{k},t) \underset{t\to-\infty}{\to} 0,\ \overline{c}_i^{a*}(\mathbf{k},t) \underset{t\to+\infty}{\to} 0, \\
& c_i^a(\mathbf{k},t) \underset{t\to+\infty}{\to} c_i^a(\mathbf{k})_{out} e^{i|\mathbf{k}|t},\ \overline{c}_i^{a*}(\mathbf{k},t) \underset{t\to-\infty}{\to} \overline{c}_i^{a*}(\mathbf{k})_{in} e^{-i|\mathbf{k}|t}; \\
\overline{\eta}_i^a(\mathbf{x},t) &= \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{d^3k}{\sqrt{2|\mathbf{k}|}} (\overline{c}_i^a(\mathbf{k},t) e^{-i\mathbf{k}\cdot\mathbf{x}} + c_i^{a*}(\mathbf{k},t) e^{i\mathbf{k}\cdot\mathbf{x}}), \\
& \overline{c}_i^a(\mathbf{k},t) \underset{t\to-\infty}{\to} 0,\ c_i^{a*}(\mathbf{k},t) \underset{t\to+\infty}{\to} 0, \\
& \overline{c}_i^a(\mathbf{k},t) \underset{t\to+\infty}{\to} \overline{c}_i^a(\mathbf{k})_{out} e^{i|\mathbf{k}|t},\ c_i^{a*}(\mathbf{k},t) \underset{t\to-\infty}{\to} c_i^{a*}(\mathbf{k})_{in} e^{-i|\mathbf{k}|t};
\end{aligned}$$

and

$$\begin{aligned}
G_\parallel^a(\mathbf{x},t) &= \frac{2}{(2\pi)^{\frac{3}{2}}} \int \frac{d^3k}{\sqrt{2|\mathbf{k}|}} (a^a(\mathbf{k},t) e^{-i\mathbf{k}\cdot\mathbf{x}} + a^{a*}(\mathbf{k},t) e^{i\mathbf{k}\cdot\mathbf{x}}), \\
& a^a(\mathbf{k},t) \underset{t\to-\infty}{\to} 0,\ a^{a*}(\mathbf{k},t) \underset{t\to+\infty}{\to} 0, \\
& a^a(\mathbf{k},t) \underset{t\to+\infty}{\to} a^a(\mathbf{k})_{out} e^{i|\mathbf{k}|t},\ a^{a*}(\mathbf{k},t) \underset{t\to-\infty}{\to} a^{a*}(\mathbf{k})_{in} e^{-i|\mathbf{k}|t};
\end{aligned}$$

for $a=1,2$ the longitudinal components $A^a_\parallel(\mathbf{x},t)$, and $A^a_0(\mathbf{x},t)$ can be expressed as

$$A^a_\parallel(\mathbf{x},t)=-\frac{m_W}{2}\Box_F^{-1}\frac{\partial}{\partial t}\phi^a_{\parallel 0}(\mathbf{x},t)+\overline{A}^a_\parallel(\mathbf{x},t),\ A^a_0(\mathbf{x},t)=-\frac{m_W}{2}\Box_F^{-1}\sqrt{-\triangle}\phi^a_{\parallel 0}(\mathbf{x},t)+\overline{A}^a_0(\mathbf{x},t),$$

where, $\Box_F^{-1}$ is the operator inverse to d'Alambert operator with Feynman (radiation) boundary conditions, and the variables $\overline{A}^a_\parallel(\mathbf{x},t)$, $\overline{A}^a_0(\mathbf{x},t)$ obey the radiation boundary conditions for the operator $\Box$;
$Z_\parallel(\mathbf{x},t)$ and $Z_0(\mathbf{x},t)$ can be expressed as

$$Z_\parallel(\mathbf{x},t)=-\frac{m_Z}{2}\Box_F^{-1}\frac{\partial}{\partial t}\phi^3_{\parallel 0}(\mathbf{x},t)+\overline{Z}_\parallel(\mathbf{x},t),\ Z_0(\mathbf{x},t)=-\frac{m_Z}{2}\Box_F^{-1}\sqrt{-\triangle}\phi^3_{\parallel 0}(\mathbf{x},t)+\overline{Z}_0(\mathbf{x},t),$$

where the variables $\overline{Z}_\parallel(\mathbf{x},t)$, $\overline{Z}_0(\mathbf{x},t)$ obey the radiation boundary conditions for the operator $\Box$;
$\overline{A}'_\parallel(\mathbf{x},t)$ and $\overline{A}'_0(\mathbf{x},t)$ obey the radiation boundary conditions for the operator $\Box$;
the ghosts η, $\overline{\eta}$ also obey the radiation boundary conditions for d'Alambert operator $\Box$.
In the formulas above the variables with subscripts *in* and *out* are arbitrary, and one has to integrate over them in (13). We also assume that the measure in the Feynman path integral is suitably normalized, and that the Feynman path integral over Φ in (13) is only taken with respect to the linearly independent components $\Phi_{\mu\nu}$, $\mu<\nu$ of the skew-symmetric tensor $\Phi_{\mu\nu}$.
From the results of [3], Sect. 5 and 6 it immediately follows that the S–matrix with normal symbol (13) is a unitary operator acting in the physical space of states $\mathcal{H}^0_{\rm phys}$.
In the formulas above $\mathbf{d}_i^*$ and $\mathbf{d}_i$ can be regarded as creation and annihilation operators for photons;

$$\frac{\mathbf{b}_i^{1*}-i\mathbf{b}_i^{2*}}{\sqrt{2}}\ \text{ and }\ \frac{\mathbf{b}_i^{1}+i\mathbf{b}_i^{2}}{\sqrt{2}}$$

as creation and annihilation operators for W^+ mesons;

$$\frac{\mathbf{b}_i^{1*}+i\mathbf{b}_i^{2*}}{\sqrt{2}}\ \text{ and }\ \frac{\mathbf{b}_i^{1}-i\mathbf{b}_i^{2}}{\sqrt{2}}$$

as creation and annihilation operators for W^- mesons;
$\mathbf{b}_i^*$ and $\mathbf{b}_i$ as creation and annihilation operators for Z mesons.

References

[1] Itzykson, C., Zuber, J.-B., Quantum field theory, McGraw-Hill (1980).

[2] Lahiri, A., Renormalizability of the Dynamical Two-Form, *Phys. Rev.* **D63** (2001), 105002.

[3] Sevostyanov, A.: A mass generation mechanism for gauge fields, preprint hep-th/0605050.

[4] Sevostyanov, A., A mass generation mechanism for gauge fields II. Renormalization, preprint hep-th/0605051.