# Peaks in the CMB—sensitively testing the gaussian hypothesis

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# Peaks in the CMB - Sensitively Testing the Gaussian Hypothesis

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**Abstract.** Presently the most popular early Universe theories are based on inflation or topological defects. Inflationary theories have been developed more, providing predictions for cosmic microwave background (CMB) fluctuations, dependent on 10 or more cosmological parameters. Almost all of the inflationary models, differ from the defect models in that they predict a nearly scale-invariant spectrum of density perturbations which produce a microwave background temperature map which is very close to being a random gaussian field. Therefore a test of gaussian fluctuations in the CMB is a possible discriminant between the two models. We have used the form of the probability distribution function of the peaks of a gaussian random field distributed over sphere, and computed exactly the correlation function for points of the CMB sky separated by arbitrary angles, thus generalising the small-angle calculations of Heavens and Sheth (1999). The technique we have developed provides a sensitive and exact test of the gaussian hypothesis. This is a vital first step before the new high quality CMB observations can be used to fit the various unknown cosmological parameters.

## INTRODUCTION

The cosmic microwave background radiation (CMB) presents an ideal opportunity to test early Universe models. The physics is free from the effects of non-linear gravitational evolution which make interpretation of the present-day matter distribution more complicated. A test can be made between the generic properties of two classes of structure-formation models, based on inflation and cosmic defects respectively. Most inflationary models predict that the microwave background temperature map will be very close to a random gaussian field, whereas generically defect models predict a non-gaussian temperature map.

Current evidence from balloon experiments ( [2], [3], [1]) favours inflation models, since the power spectrum is acceptable for certain combinations of cosmological parameters. Indeed, the major scientific goal of these and future experiments such as the *Microwave Anisotropy Probe* (MAP) and *Planck Surveyor*, is to use the power spectrum for cosmological parameter estimation. The methods are, however, based on the assumption that the temperature map is created by inflation or some similar process, not by defects, and that the map is not seriously contaminated by

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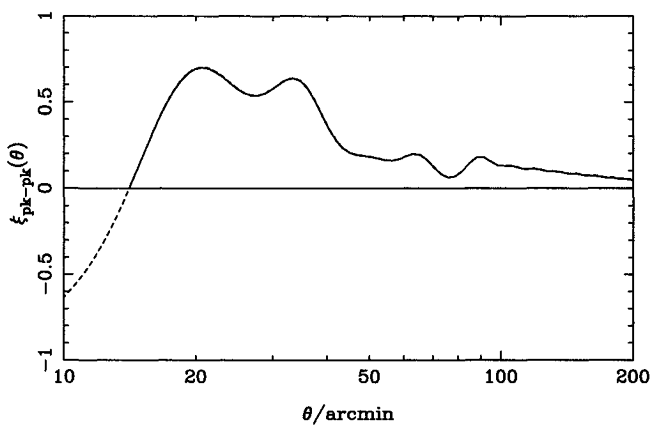


**FIGURE 1.** (Solid line) The correlation function for peaks above a $+1\sigma$ threshold, in a mixed dark matter model with CDM, vacuum and baryon density parameters $\Omega_{CDM} = 0.8$, $\Omega_\nu = 0.15$ and $\Omega_B = 0.05$. Hubble constant is $H_0 = 60$ km s$^{-1}$ Mpc$^{-1}$. For comparison, the flat sky results of Heavens & Sheth (1999) are shown dotted. The results coincide to an accuracy of better than 0.004.

foregrounds. The statistics of peaks can be a valuable tool; in lending support to the inflation hypothesis, and constraining possible contaminants.

## THE CORRELATION FUNCTION OF PEAKS

The two-point correlation function of local maxima above a threshold of a multiple, $\nu$, of the r.m.s in 2D gaussian random fields on the surface of a sphere is

$$1 + \xi(\theta|\nu) = \frac{1}{4\theta_*^4[n_{pk}(>\nu)]^2} \int_{\nu_1=\nu}^{\infty} \int_{\nu_2=\nu}^{\infty} \int_{X_1=0}^{\infty} \int_{X_2=0}^{\infty} \int_{Y_1=-X_1}^{X_1} \int_{Y_2=-X_2}^{X_2}$$

$$\int_{Z_1=-\sqrt{X_1^2-Y_1^2}}^{\sqrt{X_1^2-Y_1^2}} \int_{Z_2=-\sqrt{X_2^2-Y_2^2}}^{\sqrt{X_2^2-Y_2^2}} d\nu_1 d\nu_2 dX_1 dX_2 dY_1 dY_2 dZ_1 dZ_2 \qquad (1)$$

$$\times \left(X_1^2 - Y_1^2 - Z_1^2\right)\left(X_2^2 - Y_2^2 - Z_2^2\right) p(\nu_1, X_1, Y_1, Z_1, \eta^{(1)}_{\phi,\theta} = 0, \nu_2, X_2, Y_2, Z_2, \eta^{(2)}_{\phi,\theta} = 0)$$

The derivation and details are presented in Heavens & Gupta (2000) [4]. It is a multivariate gaussian distribution involving the correlations of properties of points separated by angle $\theta$. Two of the eight integrals can be done analytically, leaving a 6D integration which can be evaluated rapidly. Fig.1 shows the peak correlation function for a mixed dark matter model.

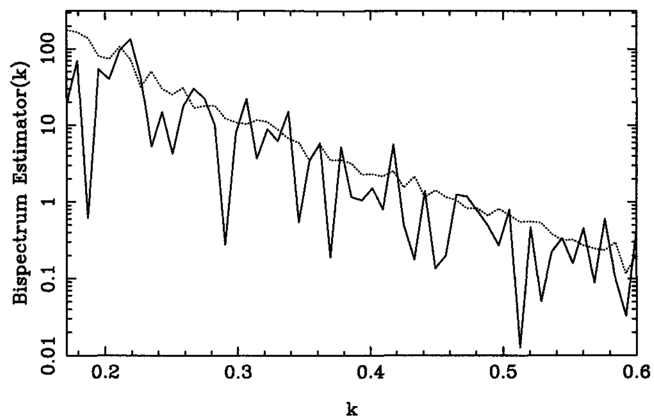


**FIGURE 2.** The equilateral bispectrum as estimated from map with string foreground and gaussian temperature on last-scattering surface (solid), and cosmic r.m.s. (dotted).

## The Correlation Function vs. Bispectrum

Although the visual appearance of a temperature map may be evidently non-gaussian, it is not necessarily easy to find statistics which will unambiguously distinguish them from gaussian fields.

Here we focus on one particular non-gaussian field, produced by a network of cosmic strings. We analysed two realisations of the temperature map expected from cosmic strings, kindly given to us by François Bouchet, using the bispectrum and the peak-peak correlation function as distinguishing statistics.

The bispectrum (Fig.2) shows no significant excess over cosmic variance.

Fig. 3 shows the correlation function of peaks above $1\sigma$ (where $\sigma^2$ is the map variance) for the same map. The map is smoothed with a gaussian beam of FWHM $5.5'$ to model the Planck beam. The peak correlation function of the string map is significantly different from that of a gaussian map with the same power spectrum, the clustering of peaks below 15 arcminutes appearing to show the obvious signature of strings.

## Conclusions

We have computed the predictions for the correlation function of local maxima (and minima) for a gaussian field [4], generalising the work of Heavens and Sheth (1999) [5] in dropping the small-angle approximation: the results of this paper can be used for all valid separations on the sky. Our test is quite straightforward: given a power spectrum, the statistical properties of peaks of a gaussian field are fully determined - there are no free parameters. If the peak correlation function is not consistent with the predictions, then either the CMB temperature map is not

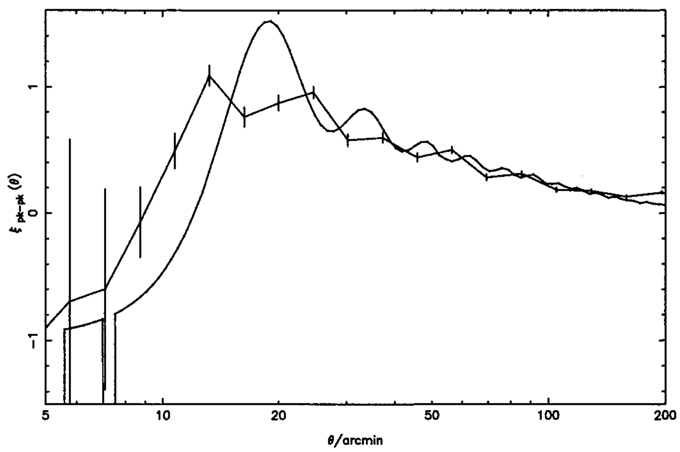


**FIGURE 3.** The correlation function of peaks above $1\sigma$ calculated from the map. Errors are Poisson, and hence underestimates. Superimposed is the correlation function from a gaussian map with the same power spectrum. Note the excess of string peaks around 10-15 arcminutes.

gaussian, or it is significantly contaminated by foregrounds, or both. In either of these cases, the derived cosmological parameters from the power spectrum will be suspect.

There are, of course, many ways to test the gaussian hypothesis, and it is tempting to ask which is the best. Unfortunately the question is badly posed, as methods will fare differently depending on the exact properties of the non-gaussian field considered. It appears from our calculations that cosmic variance can make the bispectrum difficult to use on a small patch of sky whereas the peak-peak correlation function can still be successful.

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