Expression Parser

Complex 2 Real

# The Hyperbolic Tangent of The Logarithm of One Plus $t$ Squared

BY STEPHE CROWLEY

## 1 The S Function

Let $T(t)$, which is rational meromorphic quartic function with a double-root at the origin $t=0$ and simple roots at $t=\pm\sqrt{2}$, be defined by

$$\begin{aligned} S(t) &= \tanh(\ln(1+t^2)) \\ &= \frac{(1+t^2)^2-1}{(1+t^2)^2+1} \end{aligned} \tag{1}$$

There are 4 inverse branches of $S(t)$ given by

$$S^{-1}(y) = \{t : S(t) = y\} = \pm\sqrt{\frac{1+y}{\sqrt{1-y^2}} \pm 1} \tag{2}$$

### 1.1 The Real and Imaginary parts of $S(t)$

Semi Isotropic Space?

Given the function $S(t)$ in Formula (1) where $t$ is a complex number $t=x+i\,y$. This function can be represented as $S(t)=a(t)+i\cdot b(t)$, where $a(t)$ and $b(t)$ are the real and imaginary parts of $S(t)$ respectively.

The real part of the function $a(t)$ is derived as:

$$a(t) = \frac{x^4-6\,x^2\,y^2+y^4+2\,x^2-2\,y^2+2}{x^4+2\,x^2\,y^2+y^4+2\,x^2+2\,y^2+2}$$

form respective surfaces of revolution and see if it is a

seperable

And the imaginary part of the function $b(t)$ is:

$$b(t) = \frac{4\,x^3\,y-4\,x\,y^3}{x^4+2\,x^2\,y^2+y^4+2\,x^2+2\,y^2+2}$$

Both $a(t)$ and $b(t)$ are real-valued for any complex number $t$. The operations on $x$ and $y$ (addition, subtraction, multiplication, division, and exponentiation) yield real numbers, so both $a(t)$ and $b(t)$ are real-valued functions. The more general form of this way of splitting a function into simpler pieces is known as partial fraction decomposition.

Partial fraction decomposition

invert a(t) and b(t)