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Quantum Field Theory

Quantum systems with a finite number of degrees of freedom have a complete and rigorous mathematical description. No such mathematical framework has yet been found for general systems with an infinite number of degrees of freedom. Since an infinite number of degrees of freedom are necessary to investigate systems involving interactions between particles and fields and interactions in which the creation or annihilation of particles are present, a satisfactory theory for such systems would have immense practical applications. In this chapter we present some of the recent progress which is bringing us closer to the solution of this problem. Much of the material of the previous two chapters will find application here and stochastic methods will prove to be extremely useful.

7.1. Canonical Commutation Relations

Let us begin with a mathematical description of a single spinless, nonrelativistic particle with one degree of freedom. Such a system is traditionally described by the Hilbert space $L^2(\mathbb{R})$. The position operator Q and the momentum operator P play fundamental roles. The position operator Q is the self-adjoint operator with domain

$$D(Q)=\{\phi\in L^2(\mathbb{R}):x\phi(x)\in L^2(\mathbb{R})\}$$

and defined by $(Q\phi)(x)=x\phi(x)$. The domain of P is the set of all absolutely continuous functions ϕ on $\mathbb{R}$ such that $\phi'\in L^2(\mathbb{R})$ and P is the self-adjoint operator $P=-id/dx$ (we assume $\hbar=1$). Notice that the