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COSMIC TIME DILATION

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ABSTRACT

The observed apparent time dilation of supernovae light curves claimed recently by Leibundgut et al. to establish directly the "expansion of the universe" is, rather, a general implication of fundamental physics. In particular, it applies to chronometric cosmology, in which it appears explicitly as a generalized Lorentz-Fitzgerald contraction. The dilation effect was earlier shown in the proper motion–to–redshift relation of superluminal sources, as analyzed in the frame of chronometric cosmology by Segal, providing an estimate of the cosmic distance scale.

Subject headings: cosmology: theory — distance scale — relativity — supernovae: general — supernovae: individual (SN 1995K)

1. INTRODUCTION

A recent paper of Leibundgut et al. (1996, hereafter L96) argues that the apparent time dilation observed in the light curves of a large-redshift supernova (SN) is uniquely indicative of the "expansion of the universe." A similar analysis is made by Goldhaber et al. (1997), who describe their finding as the "first clear observation of the cosmological time dilation for macroscopic objects."

In fact, however, time dilation complementary to energy loss has been a universal principle of fundamental physics since the origin of special relativity. Indeed, the principle was earlier used by Segal (1990) in explaining observations on extragalactic superluminal sources, within the frame of chronometric cosmology (CC). The proper motion–to–redshift relation of a sample of radio sources exhibits the corresponding relativistic dilation of the spatial distance scale, as further confirmed by an a posteriori statistical test. The analysis provides an objective estimate, free of cosmological or evolutionary parameters, of the cosmic distance scale, under the assumption of CC.

2. TIME DILATION IN CHRONOMETRIC COSMOLOGY

I recall the central features of CC and introduce notation.

1. CC assumes that empty (i.e., free, or reference) spacetime is the universal covering manifold M of the conformal compactification of Minkowski space, M_0.

The manifold M is the only four-dimensional conformal manifold other than M_0 that enjoys spatial and temporal isotropy and global causality and separability of time from space.

Every normalizable free solution of Maxwell's (or similar) equations in M_0 extends uniquely to a solution of the corresponding equation in M.

2. CC assumes also that physical spacetime, consisting of M together with its energetic contents, is representable on the cosmic scale by the Einstein universe E (Einstein 1917) as a Lorentzian manifold. M is conformally equivalent to E.

E consists of a time axis represented by the real line R^1 together with a three-dimensional spherical space S^3. Choosing henceforth natural (in fact, conformally invariant) units in which $R = c = 1$, $h = 2\pi$, where R denotes the radius of S^3, E has the metric $dt^2 - ds^2$, where t is the time and ds is the element of arc length on S^3 in radians.

3. Any conformal transformation on M will transform E into an equivalent form of the Einstein universe. The specific form involved here is interpreted as the inertial frame of the universe and is defined by minimization of the total energy of the contents of the universe, over all frames obtained by conformal transformation of any one version of the Einstein universe. It is assumed that discrete sources and cosmic backgrounds are generically substantially at rest in E (i.e., apart from small peculiar motions, etc.).

4. M_0 is conformally equivalent to an open submanifold of E. The imbedding of M_0 into E may be given in succinct form as follows, where the x_μ are the usual Minkowski coordinates at the origin, and t is the time and u_ν the space position in E ($\mu = 0, 1, 2, 3$; $\nu = 1, 2, 3, 4$; $u_1^2 + u_2^2 + u_3^2 + u_4^2 = 1$). Let $X = x_0 + x_1\sigma_1 + x_2\sigma_2 + x_3\sigma_3$, where the σ_μ are the usual Pauli matrices, and let $U = (1 + iX/2)(1 - X/2)^{-1}$. Then

$$t = (2i)^{-1} \log U; \quad u_1\sigma_1 + u_2\sigma_2 + u_3\sigma_3 + u_4 = e^{-it}U. \quad (1)$$

Any other conformal imbedding of M_0 into E is equivalent to this imbedding apart from possible conformal transformations in M_0 and E.

5. All conformal transformations on M_0, including those with singularities, extend to all of E and are then free of singularities. In particular, Lorentz transformations in M_0 extend uniquely and globally to conformal transformations in M or E. Temporal evolution, $t \to t + s$, in E is smooth infinitesimally on M_0 as imbedded in E, but its global action on M_0 is to carry it in part outside itself.

For more on CC, see Segal (1976a), Segal & Zhou (1995), and references therein.

3. SPECIAL RELATIVITY IN E

A localized observer in E is determined by his position and Lorentz frame. The frame is determined by the Jacobian matrix of the local Minkowskian coordinates x_μ of the observer, with respect to the global Einstein coordinates t and u. Any two localized observers in E are connected by a conformal transformation in E.

Special relativity was based on the principle that spatiotemporal coordinates are physically meaningful only as the rest frame coordinates of an observer. This implied, in particular, the Lorentz-Fitzgerald contraction, according to which the intrinsic time and length coordinates of an object in motion relative to an observer appear diminished and the energy momentum correspondingly increased. The same principle is naturally applicable in E, to all motions leaving invariant the Maxwell equations, which form the conformal group. Application to observation of a free photon emitted from a distant source involves the effect on localized observational coordinates of temporal evolution, $t \to t + s$, in E. To compute this effect, local Minkowskian coordinates may be chosen, both at the points of emission and observation, so that a common z-axis is along the line of sight. Cosmic uniformity is implemented by the use of the natural units defined above and is equivalent to the use of the same spectral frequency and other laboratory standards at all locations and in all frames in the universe.

In global terms, CC represents the redshift as the difference between the Einstein and Minkowski energies. These are, at the most rigorous level, the Hermitian operators in the Hilbert space of normalizable photon states corresponding to infinitesimal Einstein and Minkowski temporal displacement, i.e., $\partial/\partial t$ and $\partial/\partial x_0$, respectively. The Einstein energy of a free particle is conserved in CC, but the Minkowski energy is altered by temporal evolution in E. Computation in the photon Hilbert space of the redshift as thus defined (Segal 1976b) shows that $z = \tan^2(\rho/2)$ for localized photons of radio or higher energies, where ρ denotes the distance. In local terms, the same result may be deduced from the generalized Lorentz-Fitzgerald contraction in E by a variant of the argument used below to compute the time dilation effect.

4. TIME DILATION

To compute this effect in CC, let local Minkowskian coordinates be chosen, both at the points of emission and observation, so that a common z-axis is along the line of sight. When the zero points of both the Einstein time t and the local Minkowski time x_0 are chosen to coincide at a time of observation, the relation between the local Minkowski coordinates and the (global) Einstein coordinates given above takes the form

$$x_0 = 2 \sin t/(\cos t + \cos \rho); \quad x_1 = x_2 = 0;$$
$$x_3 = 2 \sin \rho/(\cos t + \cos \rho). \tag{2}$$

The zero point of Einstein time can be chosen arbitrarily; in particular, it is no essential loss of generality to assume that it coincides with the time of emission. If the distance to the point of observation is s, then the Einstein time at the point of observation will also have the value s, and the dilation factor will be $\partial x_0/\partial t_{|t=s}$. This is readily evaluated:

$$\partial/\partial t[2 \sin t/(1 + \cos t)] = 2/(1 + \cos t). \tag{3}$$

Thus, the dilation factor is $2/(1 + \cos t) = 1 + \tan^2(\rho/2)$ and so coincides with the factor $1 + z$ implied by the general principle of the complementarity of time and energy.

In particular, the apparent time dilation indicated by the observations on SN 1995K and others is just as predicted by CC. It is by no means consistent only with an "expanding universe," e.g., as represented by Friedman-Lemaitre cosmology (henceforth FLC).

L96 argue that time dilation would not take place in a so-called tired light theory and considers no other alternative theory of the redshift. Precisely what constitutes a "tired light theory" is not fully specified, however. The Einstein energy is well defined and conserved in CC, but free photons lose energy in terms of the respective Minkowski frames in which observations are made.

The emitted photon continues to live in the local Minkowskian frame of its source. (It should be noted that a *normalizable* photon is similar to a massive particle in having a well-defined frame of minimal energy.) However, time evolution in E alters this frame by the generalized Lorentz-Fitzgerald contraction. In consequence, the locally observed energy at a distant point of observation is less than the emitted energy (local Minkowskian *or* Einstein). For a photon emitted from a localized source, the Einstein and local Minkowskian energies (in the frame of the source) are the same, apart from a higher order effect that is negligible on a cosmic scale (e.g., Segal 1976b).

5. FURTHER ASPECTS

1. L96 note that "clear experimental proof of [the general expansion of the universe] has been lacking," but they add that "the main argument in favor or expansion ... is the cosmic background radiation." A priori, however, the observed isotropy of the cosmic background radiation appears inconsistent with the mechanism for its production in an expanding universe. A supplementary, ad hoc, mechanism is required to reconcile the expansion of the universe with the observed isotropy.

No such additional hypothesis is required in CC to imply the isotropy in conjunction with the classic mechanism of Planck (maximization of entropy subject to conservation of energy). It seems not to be generally realized that the Planck law for the cosmic background radiation is by no means uniquely implicative of a primeval global explosion, but follows from very general principles in a temporally homogeneous universe that enjoys global energy conservation (e.g., Segal 1983).

2. A possible means of testing cosmology and simultaneously exhibiting the dilation effect would be provided by observations of apparent diameter for a complete sample of high-redshift compact radio sources. A first step in this direction has been reported by Kellermann (1993). The *spatial* dilation corresponding to the cosmic *time* dilation should be observable in the $\theta - z$ relation. This, of course, is modulo evolution in the context of the expanding universe, which Kellermann argues should be slight for compact radio sources.

3. The expansion of the universe is, in any event, contraindicated by statistically efficient and equitable studies of the flux-redshift relation in complete samples in all four observed wave bands (e.g., Segal & Nicoll 1996a, 1996b; Segal, Nicoll, & Blackman 1994; Segal, Nicoll, & Wu 1994; Segal et al. 1993, and references therein).

4. I thank a referee for the information that observations on high-redshift Type Ia SNs have begun to provide samples that are suitable, in principle, to estimation of the basic parameters of FLC, modulo certain ancillary hypotheses, and the suggestion that these data be applied to CC. The cosmological parameters Ω and Λ have been estimated by Perlmutter et al. (1997) from a sample of seven SNs. The Hubble constant has been estimated by Kim et al. (1997) from the same sample. On the other hand, the reliability of these estimates depends on the intrinsic uniformity of the SNs involved. The results of

van den Bergh (1996) and Hoflich et al. (1996) indicate that this is generally more limited than in the sample treated by Perlmutter et al. (1997).

In any event, the extant SN samples do not appear to have application in the frame of CC analogous to their application in the frame of FLC. This is the case in part because CC involves no adjustable cosmological parameters comparable to Ω and Λ. Its only fundamental cosmological constant is R, which plays a role analogous to that of the Hubble constant. But R may be estimated from observation of the proper motions and redshifts of a reportedly statistically appropriate sample of superluminal quasars, independently of observations on Cepheids or other local sources, and without any corrections or ad hoc assumptions (Segal 1990). There is no apparent way to use the SN peak magnitude-redshift relation to estimate R in CC, nor is this relation suited to statistically rigorous cosmological testing, in view of the difficulty, if not impossibility, of observing a complete sample.

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