The Kubo-Martin-Schwinger (KMS) condition is a fundamental property that naturally emerges in quantum statistical mechanics. It can be derived from the principles of quantum mechanics and the concept of thermal equilibrium.

In quantum statistical mechanics, the KMS condition relates the correlation functions of a quantum system at different points in time. It specifies the behavior of these correlation functions under a time translation.

The KMS condition can be understood as a boundary-value condition because it imposes constraints on the behavior of correlation functions at the boundary between the forward and backward evolution of a system in time.

Here's a simplified explanation of how the KMS condition arises:

1. Consider a quantum system in thermal equilibrium, such as a system described by a canonical ensemble.
2. Thermal equilibrium implies that the system has reached a steady-state where observables no longer change with time.
3. In this steady-state, correlation functions between observables at different points in time become time-independent and only depend on the time difference between them.
4. The KMS condition arises when we examine the behavior of correlation functions for imaginary time (τ), which is obtained by analytically continuing the time variable (t) into the complex plane.
5. By requiring the correlation functions to be analytic functions of τ, except for a possible branch cut along the real axis, we arrive at the KMS condition.

Mathematically, the KMS condition can be expressed as follows: for a given observable A and temperature β (inverse of temperature), the correlation function satisfies

$$\langle A(t) B \rangle = \langle B A(t - i\beta) \rangle$$ $\langle e^{iH t} B \rangle$

where $\langle\rangle$ represents the thermal expectation value.

The KMS condition reflects the underlying symmetry of time translation invariance in thermal equilibrium. It provides a powerful constraint on the behavior of correlation functions in quantum statistical mechanics and allows for the derivation of various important results, such as fluctuation-dissipation theorems and the existence of Kubo formulas for transport coefficients.

In summary, the KMS condition emerges naturally in quantum statistical mechanics due to the constraints imposed by thermal equilibrium and the requirement of analyticity in the imaginary time plane. It acts as a boundary-value condition for correlation functions, specifying their behavior under time translations.

Chromo Colorado