Function Spectral

The Spectral Covariance Measure of $\frac{1}{\sqrt{1-t^2}}$

corresponding to the

$\int_0^x J_0(t)dt$ has a max at the first root of $J_0(x)$

The covariance function $r(t) = \pi J_0(2\pi t)$ can be expressed in terms of the spectral density $\dot{\rho}(s)$ as:

$$r(t) = \pi J_0(2\pi t) = \int_{-1}^{1} e^{2\pi its}\dot{\rho}(s)ds = \int_{-1}^{1} e^{2\pi its}d\rho(s) = \int_{-1}^{1} \frac{e^{2\pi its}}{\sqrt{1-s^2}}ds$$

where $\dot{\rho}(s) = \frac{1}{\sqrt{1-s^2}} \forall s \in [-1,1]$ and $d\rho(s) = \dot{\rho}(s)ds$. absolute continuous

The inverse Fourier transform of $\pi J_0(2\pi s)$ is the spectral ~~density:~~ ~~measure~~ density

$$\dot{\rho}(t) = \frac{1}{\sqrt{1-t^2}} = \int_{-\infty}^{\infty} r(s)e^{-2\pi its}ds = \int_{-\infty}^{\infty} \pi J_0(2\pi s)e^{-2\pi its}ds$$

whose integral is the spectral measure, also the inverse Fourier-Stieltjes integral

$$\rho(t) = \int_{-1}^{t} \dot{\rho}(s)ds = \int_{-1}^{t} \frac{1}{\sqrt{1-s^2}}ds = \arcsin(t) + \frac{\pi}{2}$$

The covariance function corresponding to the spectral measure $\frac{1}{\sqrt{1-t^2}}$

is

$\pi J_0(2\pi s)$, therefore

Since the Chebyshev Polynomials of the first kind orthogonalized (normalized (Fourier Transforms)) should be an eigenfunction expansion of $\pi J_0(2\pi s)$ are orthogonal under the inner product $\langle T_n | \frac{1}{\sqrt{1-x^2}} | T_m \rangle = \delta_{n,m}$