A Study on MVDR Beamforming Applied to an ESPAR Antenna

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***Abstract*—The adaptive beamforming algorithm-minimum variance distortionless response (MVDR) has been studied based on the electronically steerable parasitic array radiator (ESPAR) antenna. The ESPAR antenna uses a single radio-frequency (RF) front end, and its beamforming is achieved by adjusting reactance loads of parasitic elements coupled to the central active element. In the proposed beamforming method, the MVDR beamformer optimizes weights applied to outputs of beams. The optimization problem is formulated as a second-order-cone programming (SOCP) problem including a Euclidean distance metric to approximate the optimal equivalent weight vector to a feasible solution. Then the ESPAR beampattern design strategy iterates between the SOCP problem and a simple projection of reactance loads. The simulations show that the proposed MVDR beamforming method based on an ESPAR antenna gives a beam steering at the desired direction and placing nulls at the interfering directions, and it converges fast. However, when the desired source is close to the interferer, the output signal-to-interference-plus-noise ratio (SINR) degrades and where we use the interference-plus-noise sample covariance matrix to improve the beamforming performance.**

***Index Terms*—Adaptive beamforming, ESPAR, MVDR, SOCP.**

I. Introduction

THE adaptive array is a key technique for communications and radar applications to adaptively steer beam to a direction of interest, while placing nulls at interference directions. The well-known MVDR beamformer [1] is designed to linearly combine outputs of sensors in order to minimize the power from interference while maintaining desired gain at the interest direction, where the MVDR is developed on the conventional multi-sensor arrays connecting with multiple RF chains. Recently, a smart antenna, named ESPAR [2], has gained a great deal of attention, as it uses a single RF chain and mutual coupling of parasitic elements to the sole active element resulting in smaller element spacing. Thus, it provides a practical solution to radio equipment with constrains on size, complexity, power and cost. The ESPAR antenna has been studied to perform spectrum sensing for cognitive radio systems [3], and estimate direction of arrival (DoA) by the modified MUSIC [4] and ESPRIT algorithms [5]. The ESPAR antenna also has been proposed with interference mitigation applications using beampattern switching where MVDR beam designs will bring additional benefits [6].

ESPAR beamforming is achieved by tuning the reactance loads of parasitic elements. However, mapping of these reactance loads to a desired beampattern has a nonlinear relationship; thus, traditional adaptive beamforming methods cannot be directly applied to the ESPAR antenna due to lack of a closed-form solution to this problem. And ESPAR beampatterns are commonly designed using iterative methods. In [7], [8], stochastic based beampattern design methods are presented, where the beampattern is designed by maximization of the cross correlation coefficient between the received signal (or the achievable beampattern) and reference signal (or desired beampattern). In stochastic algorithms, proper choice of some key parameters (e.g. the update step size and perturbation size) requires a large amount of experiments and/or meta-optimization to find, and they may need large iterations to converge. In [3], authors present a beampattern design method iterating between a convex optimization problem and a projection operation of reactance loads. The method based on convex optimization is a better way to find a solution within a small number of iterations; however, the method assumes knowledge of DoAs of both desired and interfering signals. The work on adaptive beamforming for ESPAR antennas has not been studied extensively yet, and to our knowledge, this letter is the first work on MVDR beamforming for ESPAR antennas.

In this work, we study the MVDR beamforming applied to the ESPAR antenna. In the single-RF port antenna array, designed beamformer weights are applied to outputs over different beams formed sequentially. The MVDR beamforming algorithm is formulated as a SOCP problem which uses a Euclidean distance metric to remain close to the set of feasible solutions. The beamforming strategy iterates between the SOCP problem and a simple projection of reactance loads of parasitic elements. Moreover, the use of interference-plus-noise covariance matrix helps to improve the beamforming performance especially when the desired source is close to the interferer.

The rest of the letter is organized as follows. The ESPAR antenna and signal model are described in Section II, and the MVDR algorithm based on ESPAR antennas is presented in Section III. In Section IV, simulations of performance of the proposed beamforming method are given. The letter is concluded in Section V.

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II. Signal Model of ESPAR Antenna

A. ESPAR Antenna Structure

Consider an $(M+1)$-element ESPAR which is comprised of an active element and M parasitic elements placed on a circle around the active one with a radius of d. The active element is connected to the single RF chain and fed to a low noise amplifier with a loading impedance Z_s. Parasitic elements are short-circuited and loaded by variable reactors which control their reactance loads. The ESPAR beamforming is achieved by adjusting the reactance loads denoted by a vector $\hat{\boldsymbol{x}} = [x_1, x_2, \cdots, x_M]$. Here, we focus on the 2-dimensional propagation analysis through azimuth angle (θ). The ESPAR beampattern is given by [7]

$$B(\theta) = \boldsymbol{w}^T \boldsymbol{a}(\theta), \tag{1}$$

where $\boldsymbol{a}(\theta)$ is the steering vector, and $\boldsymbol{w} \in \mathbb{C}^{M+1}$ is the equivalent weight vector of the form

$$\boldsymbol{w} = (\boldsymbol{Z} + \boldsymbol{X})^{-1} \boldsymbol{u}, \tag{2}$$

where the matrix $\boldsymbol{Z} \in \mathbb{C}^{(M+1)\times(M+1)}$ represents mutual impedance between elements. $\boldsymbol{u} = [1, 0], \cdots, 0]^T$ is an $(M+1)$-dimensional selection vector, and $\boldsymbol{X}$ is a diagonal loading matrix given by

$$\boldsymbol{X} = \mathrm{diag}([Z_s \quad j\hat{\boldsymbol{x}}]). \tag{3}$$

B. Signal Model

Signals impinging on the ESPAR elements are combined due to strong mutual coupling and collected from the sole RF port. This is different to the conventional multi-sensor array where signals are observed at individual elements. Assume a line-of-sight (LOS) propagation environment, and there are P ($P < M$) signals impinging on the ESPAR antenna, where the P signals include the desired signal and interferences. In this work, the multipath components are not considered. The output of the sole RF port is the sum of combined signals expressed as

$$y(t) = \sum_{p=1}^{P} \boldsymbol{w}^T \boldsymbol{a}(\theta_p) s_p(t) + n(t) = \boldsymbol{w}^T \boldsymbol{A}\boldsymbol{s}(t) + n(t), \tag{4}$$

where $s_p(t)$ is the p-th signal from the DoA $\theta_p, p \in \{1, \cdots, P\}$, and the vector $\boldsymbol{s}(t) \in \mathbb{C}^P$ stores P incident signals. The matrix $\boldsymbol{A} \in \mathbb{C}^{(M+1)\times P}$ stores steering vectors corresponding to DoAs of incoming signals. $n(t) \sim \mathcal{CN}(0, \sigma_n^2)$ is the additive noise with zero-mean and variance σ_n^2.

It is noteworthy that, although the MVDR algorithm is effective in the LOS case, its performance is limited when multipath components are not ignorable. The beamformer may treat the multipath arrivals of the desired signal as interferences and try to null them. Therefore, the optimal combining method is more suitable for the multipath case [9].

III. MVDR Beamforming for ESPAR Antennas

The conventional MVDR beamforming algorithm cannot be directly applied to the ESPAR antenna. The reasons are: 1) only the output of the sole RF port connected to the active element can be observed, while signals impinging on parasitic elements are unknown; 2) the ESPAR beampattern is controlled by adjusting reactance loads of parasitic elements having a nonlinear relationship with the desired beampattern.

First, to obtain the spatial covariance matrix, an ESPAR array samples signal over different beampatterns using a set of reactance loads, that creates spatial diversity for the array. This method has been used in [4] to detect DoA by the modified MUSIC algorithm based on ESPAR antennas, and also in [3] to perform spectrum sensing for cognitive radios equipped with ESPAR antennas. In particular, N beampatterns are formed in N sequential time slots. Signal measurements obtained by N beams are represented as

$$\begin{aligned} y_1(t_1) &= \boldsymbol{w}_1^T \boldsymbol{A}\boldsymbol{s}(t_1) + n(t_1) \\ &\vdots \\ y_N(t_N) &= \boldsymbol{w}_N^T \boldsymbol{A}\boldsymbol{s}(t_N) + n(t_N), \end{aligned} \tag{5}$$

where $y_n(\cdot), n = 1, \cdots, N$ is the signal received over the n-th beam. Here, we assume the training signals are kept the same during the block of N time slots, i.e. $\boldsymbol{s}(t_1) = \cdots = \boldsymbol{s}(t_N)$. This condition can be achieved by periodically sending signals during the block of N time slots, which is the same as the method used in [4] for signal direction estimation. Rewrite (5) in vector version and omit time index for notation simplicity, then we have the N-dimensional received signal vector

$$\boldsymbol{y} = \underbrace{[\boldsymbol{w}_1, \cdots, \boldsymbol{w}_N]^T}_{\boldsymbol{W}^{\mathrm{T}}} \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n} = \boldsymbol{A}_{eq}\boldsymbol{s} + \boldsymbol{n}, \tag{6}$$

where $\boldsymbol{A}_{eq} = \boldsymbol{W}^T \boldsymbol{A}$ is the equivalent manifold matrix of the ESPAR in beamspace domain.

In order to obtain L measurement vectors $\boldsymbol{Y} = [\boldsymbol{y}(1), \cdots, \boldsymbol{y}(L)]$, L blocks of time are required, and thus a total of $L \times N$ sampling time slots are used. Perform the aforementioned periodical signal transmission during each of L blocks of time. Then we can obtain the estimate spatial covariance matrix by the ESPAR antenna as

$$\hat{\boldsymbol{R}} = \frac{1}{L} \sum_{l=1}^{L} \boldsymbol{y}(l) \boldsymbol{y}^H(l). \tag{7}$$

For an adaptive array with N elements, its beampattern is shaped by giving appropriate weights to outputs of N elements. Similarly, we consider applying appropriate weights to outputs of N beams of the ESPAR array. Thus, we can design the MVDR beamformer vector $\boldsymbol{v} \in \mathbb{C}^N$ by solving the following optimization problem

$$\min_{\boldsymbol{v}} \quad \boldsymbol{v}^H \hat{\boldsymbol{R}} v, \quad s.t. \quad \boldsymbol{v}^H \boldsymbol{a}_{eq}(\theta_d) = 1. \tag{8}$$

Suppose the solution to (8) is $\boldsymbol{v}_{opt}$, then the designed beampattern is given as $B_{opt}(\theta) = \boldsymbol{v}_{opt}^H \boldsymbol{a}_{eq}(\theta)$. Thus, we have the optimal equivalent weight vector $\boldsymbol{w}_{opt}^T = \boldsymbol{v}_{opt}^H \boldsymbol{W}^T$, which should be obtained by adjusting the reactance loads of parasitic elements $\hat{\boldsymbol{x}}$.

It is noted that there is no known closed form solution to problem (8) for reactance loads of parasitic elements. Thus we consider solving the ESPAR-based MVDR problem by iterative calculation between a convex optimization problem and a projection of reactance loads first introduced in [3]. We reformulate the optimization problem (8) as a SOCP problem. Let $\hat{\boldsymbol{R}} = \boldsymbol{U}^H \boldsymbol{U}$ be the Cholesky decomposition of $\hat{\boldsymbol{R}}$, and introduce a new scalar nonnegative variable μ_1, then the objective function in (8) is rewritten as a constraint $\boldsymbol{v}^H \hat{\boldsymbol{R}} v = \|\boldsymbol{U}\boldsymbol{v}\|^2 \leq \mu_1$. And the distortionless response constraint is rewritten as $\|\boldsymbol{v}^H \boldsymbol{a}_{eq}(\theta_d) - 1\|^2 \leq \mu_2$, where μ_2 is another introduced nonnegative variable. Then, we have the following SOCP problem

$$\begin{aligned} &\min_{\mu_1,\mu_2,\boldsymbol{v}} \quad \beta_1\mu_1 + \beta_2\mu_2, \\ &s.t. \quad \|\boldsymbol{U}\boldsymbol{v}\|^2 \leq \mu_1, \quad \|\boldsymbol{v}^H \boldsymbol{a}_{eq}(\theta_d) - 1\|^2 \leq \mu_2, \end{aligned} \tag{9}$$

where β_1 and β_2 are constant weights determining the importance of components in the Pareto-optimal solution.

Define $\mathcal{W}$ as the set of equivalent vectors $\boldsymbol{w}$ with the loadings $\boldsymbol{X}$ of the form given in (3). In general, the solution to (9) has $\boldsymbol{w}_{opt} \notin \mathcal{W}$ (i.e. the $\boldsymbol{X}$ related to $\boldsymbol{w}_{opt}$ does not satisfy the form given in (3)), so a projector $\mathbb{P}_{\mathcal{W}}(\cdot)$ is required to approximate $\boldsymbol{w}_{opt}$ to a point in $\mathcal{W}$. In order to avoid poor approximation to the desired beampattern, constraints are added to ensure $\boldsymbol{w}_{opt}$ being 'close' to $\mathcal{W}$ around a given feasible point.

Rearrange (2) and (3) as

$$\boldsymbol{Z}\boldsymbol{w} + \boldsymbol{X}\boldsymbol{w} = \boldsymbol{u}. \tag{10}$$

The first element of $\boldsymbol{X}$ is the loading impedance of the active element Z_s, and it is constrained to a constant value (e.g. the characteristic value 50Ω) by the constraint given in (11e). Given a proposed value $\boldsymbol{X}$ corresponding to $\boldsymbol{w} \in \mathcal{W}$, and let $\bar{\boldsymbol{Z}}_L$ be the matrix obtained by removing the first row of $\boldsymbol{Z} + \boldsymbol{X}$. Then $\boldsymbol{w}_{opt}$ can be kept close to the feasible point defined by $\boldsymbol{X}$ using a Euclidean distance metric, $\|\bar{\boldsymbol{Z}}_L(\boldsymbol{v}^H \boldsymbol{W}^T)^T\|^2 \leq \epsilon$, where ϵ is a small constant controlling how 'close' between the beampattern defined by a feasible $\boldsymbol{X}$ and the optimized beampattern by solving the SOCP problem. In the simulations given in the next section, we adopt $\epsilon = 0.1$, and it is observed that the difference of the projected beampattern to the optimized one is small enough.

In summary, the MVDR beamforming problem taking into account reactance loads design is reformulated as the following SOCP problem

$$\min_{\mu_1,\mu_2,\boldsymbol{v}} \quad \beta_1\mu_1 + \beta_2\mu_2, \tag{11a}$$

$$s.t. \quad \|\boldsymbol{U}\boldsymbol{v}\|^2 \leq \mu_1, \tag{11b}$$

$$\left\|\boldsymbol{v}^H \boldsymbol{a}_{eq}(\theta_d) - 1\right\|^2 \leq \mu_2, \tag{11c}$$

$$\left\|\bar{\boldsymbol{Z}}_L(\boldsymbol{v}^H \boldsymbol{W}^T)^T\right\|^2 \leq \epsilon, \tag{11d}$$

$$[\boldsymbol{Z}(\boldsymbol{v}^H \boldsymbol{W}^T)^T](1) + Z_s[(\boldsymbol{v}^H \boldsymbol{W}^T)^T](1) = 1 \tag{11e}$$

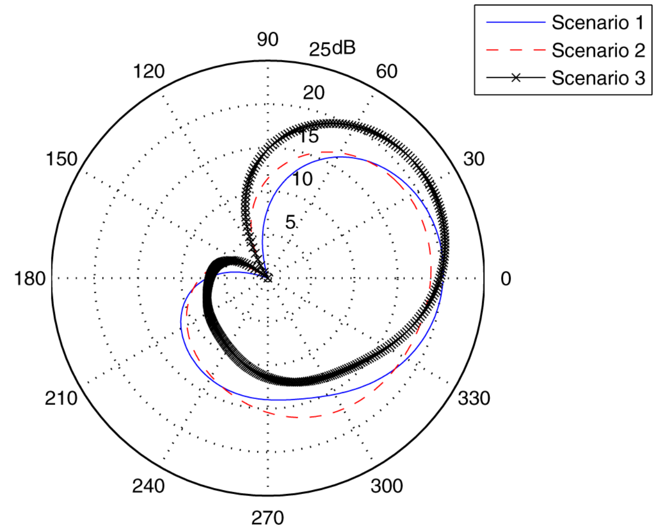

Fig. 1. ESPAR-based MVDR beamforming results.

The iterative beamforming strategy is described as following. First select an initial value $\boldsymbol{X}_0$. It is used as input to SOCP problem (11) to optimize $\boldsymbol{v}$. Then obtain $\boldsymbol{w}_{opt}$, and a corresponding $\boldsymbol{X}_1^*$ is calculated by (10), where the calculated $\boldsymbol{X}_1^*$ is unique with respect to a particular $\boldsymbol{w}_{opt}$ due to the ESPAR structure. The projected solution $\boldsymbol{X}_1$ is achieved by getting rid of real parts of $\boldsymbol{X}_1^*$ except the first element which is constrained to a constant due to (11e). The reactance load of each parasitic element is required to be kept in the range of $[-250\Omega, 250\Omega]$ which is achievable in practice. The above procedure is repeated with updated values of $\boldsymbol{X}$ until convergence.

In [10], it is demonstrated that the presence of the desired signal during the signal measurements under limited sampling numbers may lead to significant SINR degradation of the MVDR beamforming. This impact on our proposed method will be shown in the simulation results later. To overcome this problem, we consider replacing of the signal sample covariance matrix $\hat{\boldsymbol{R}}$ by the interference-plus-noise sample covariance matrix $\hat{\boldsymbol{R}}_{I+n}$. This can be simply achieved when the desired user is silent, but it requires cooperation among users. Another method is using the pilot-assisted algorithm like the one developed in [10] to subtract the desired signal from measured signals before compute the estimate spatial covariance matrix.

IV. Simulations

In this section, we evaluate performance of the proposed MVDR beamforming based on ESPAR antennas. The simulated ESPAR antenna has $M + 1 = 7$ thin electrical dipoles with length of $\lambda/2$ each (λ is the wavelength). The element spacing is set to $d = \lambda/4$. In this work we use a set of $N = 7$ reactance loading vectors to realize signal sampling over 7 beams. As aforementioned, in order to get $L = 100$ sampling vectors $\boldsymbol{Y}$ the ESPAR requires a total of $L \times N = 700$ sampling periods. Problem (11) was solved using YALMIP [11] as the modelling tool and the free optimization software SeDuMi [12].

Simulations of the ESPAR-based MVDR beamforming are implemented under three scenarios: 1) desired signal is from 0° and interference is at 120°; 2) desired signal is from 90° and interference is at 120°; 3) the desired and interfering directions

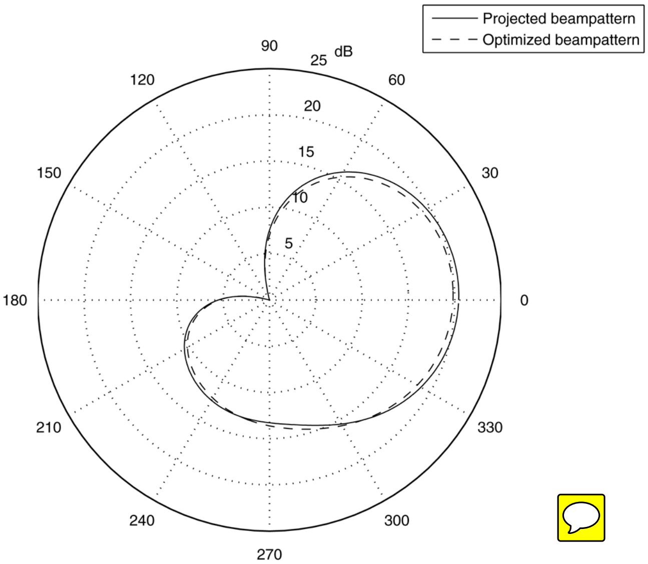

Fig. 2. Convergence performance of MVDR beamforming.

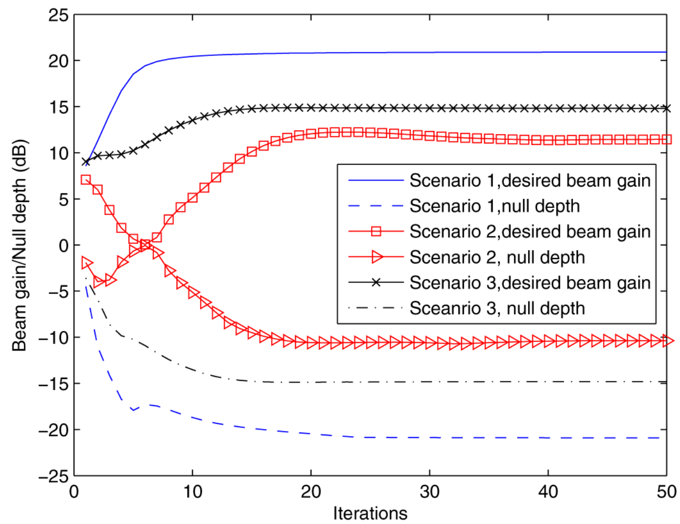

Fig. 3. Difference between the projected and optimized beampatterns with the Euclidean metric ($\epsilon = 0.1$).

are set as the scenario 2, and consider the measurements include only interference and noise, i.e., desired signal is free from the measurements similar to the formulation in minimizing output energy as proposed in [10]. Here, signal-to-noise ratio (SNR) is set to 10 dB and signal-to-interference ratio (SIR) is set to 0 dB. Fig. 1 shows that designed beampatterns place nulls at the interfering directions and steer to the desired direction; however the performance of scenario 2 is worse than that of scenario 1, as the interferer is closer to the desired source. The performance of scenario 3 is improved compared to that of scenario 2, because the desired signal is not present in signal measurements. Fig. 2 shows the convergence performance of the ESPAR-based MVDR beamforming. All the results are converged within 50 iterations. And it illustrates again that subtracting the desired signal from measurements leads to improvement of the beamforming performance when interference is close to the desired source. Fig. 3 gives the performance difference between the projected and optimized beampatterns for scenario 1, where with the Euclidean metric ($\epsilon = 0.1$) the difference between them is small. Similar results are achieved for scenario 2 and 3.

V. Conclusion

In this letter, an MVDR beamforming method is developed for the ESPAR antenna. The MVDR beamformer optimizes weights applied to outputs of different beams, where the MVDR optimization problem is formulated as a SOCP problem with an Euclidean metric to make sure that the optimized equivalent weight vector is close to a feasible solution. The reactance loadings are designed by iterating between the SOCP problem and a simple projector to achieve a desired beampattern. The simulation results show that with the proposed method the ESPAR beampattern can be designed steering to desired direction and placing null towards interference within a small number of iterations. Moreover, the use of interference-plus-noise covariance matrix improves the beamforming performance especially when the desired signal is close to the interference.

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