Fast Beam Training in mmWave Multiuser MIMO Systems with Finite-Bit Phase Shifters

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***Abstract*—Hybrid analog/digital precoding has been widely utilized in millimeter wave multiple-input multiple-output (MIMO) systems to relieve the severe power consumption of gigasample/s mixed-signal devices. With conventional exhaustive beam searching, however, the beams formed by large antenna arrays can lead to a large time overhead at beam training stage. In this paper, we propose a novel multiuser downlink beam training algorithm to estimate the angle of arrival (AoA) and the angle of departure (AoD) between the base station(BS) and the mobile stations (MSs). The proposed algorithm realizes much reduction of beam training overhead and releases more time resources for users' data transmission.**

I. Introduction

As a result of rapidly increasing data rate demands, the sub-6 GHz spectrum is becoming very crowded. Millimeter wave (mmWave) communications operating in the super high frequency (SHF) and extremely high frequency (EHF) has been deemed to be a promising technology for future wireless communications due to the enormous available bandwidths [1][2]. mmWave can be used for multi-gigabit/s data transmission in both wireless backhaul and access networks as a part of 5G systems. The different effect of propagation environment on smaller wavelength leads to more sparsity of mmWave channels, which invites the application of beamforming technologies. Antenna arrays are utilized both in the base station (BS) and the mobile station (MS) to achieve considerable beamforming gains to overcome the severe environment loss.

On the other hand, the beamwidth for mmWave antenna arrays can be as low as 1° [3]. Accurately finding the optimum beam-pairs in the BS and the MSs requires a large time overhead. As a result, for single-user mmWave systems in [4], an efficient beam training algorithm is proposed while the analog precoder is based on unconstrained phase shifters. In [5], limited feedback hybrid precoding for multi-user mmWave systems is studied and could reach a high average rate but it requires accurate values of angle of arrival (AoA) and angle of departure (AoD) while no algorithm was presented to determine angles accurately. However, directly employing the hierarchical training method will incur a large quantity of time slots for beam training in multi-user systems. Some algorithms are proposed in multi-user scenario, but the time overhead is still unsatisfactory when the number of users increases [6].

In this paper, We propose a fast beam training algorithm in mmWave downlink multiuser multiple-input multiple-output (MIMO) systems with an extended orthogonal analog codebook inspired by [7], which is constituted by finite-bit analog phase shifters (APS) with constant amplitude. We focus on reducing the total training overhead when the BS communicates simultaneously with M MSs. The algorithm proves to be much faster in finding out the optimum beam pairs for all users compared with the exhaustive searching method (which is widely adopted in the literature [8]). For decreasing the spectral efficiency (SE) loss caused by quantization error, we further improve the accuracy of AoA/AoD estimation with very low additional overhead. The improved algorithm is able to realize approximately the same spectral efficiency compared with the works in [5], where the analog precoders are constructed by accurate AoA/AoD values.

II. System Model

Consider the BS (equipped with $N_{\rm BS}$ antennas and $N_{\rm RF,BS}$ radio frequency (RF) chains) communicating simultaneously with M MSs in Fig.1. Each MS is equipped with $N_{\rm MS}$ antennas and only one RF chain [5]. We assume that number ($N_{\rm s}$) of data streams at the BS is the same as the number of the MSs, i.e., $N_{\rm s} = M$. The structure in Fig.1 illustrates that the BS uses anolog/digital hybrid precoding while each MS only employs analog precoding. The RF precoder consists of multiple digitally controlled analog phase shifters,

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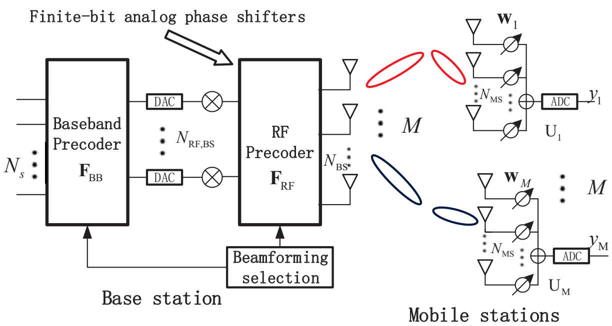

Fig. 1. Block diagram of analog/digital hybrid beamforming in BS and analog only beamforming in MS. The BS (MS) has $N_{\rm BS}$ ($N_{\rm MS}$) antennas with $N_{\rm RF,BS}$ (only one) RF chains. The analog precoder in the BS (MS) is composed of finite-bit phase shifters.

which can be written as $\mathbf{F}_{\rm RF} = [\mathbf{f}_1, \mathbf{f}_2, ..., \mathbf{f}_{N_{\rm RF,BS}}]$ and $[\mathbf{F}_{\rm RF}]_{m,n} = \frac{1}{\sqrt{N_{\rm BS}}} e^{j\phi_{m,n}}$, where $\phi_{m,n}$ represents the quantized phase value [5]. After digital precoding $\mathbf{F}_{\rm BB}$ [9] and analog combining $\mathbf{w}_u$, the combined signal of the uth user can be expressed as

$$y_u = \mathbf{w}_u^{\rm H}\mathbf{H}_u\mathbf{F}_{\rm RF}\mathbf{F}_{\rm BB}\mathbf{s} + \mathbf{w}_u^{\rm H}\mathbf{n_u} \tag{1}$$

where $\mathbf{H}_u$ is an $N_{\rm MS} \times N_{\rm BS}$ channel matrix, representing the narrowband block-fading channel coefficients between the BS and the uth MS. $\mathbf{n}_u \sim \mathcal{N}(0, \sigma^2\mathbf{I})$, is a standard white noise Gaussian vector. $\mathbf{s}$ is the $N_s \times 1$ vector of transmitted symbols, such that $\mathbb{E}[\mathbf{s}\mathbf{s}^{\rm H}] = (P_{\rm T}/N_{\rm S})\mathbf{I}_{N_{\rm s}}$, and $P_{\rm T}$ represents the average total transmit power. Like the entries of $\mathbf{F}_{\rm RF}$ in the BS, the element of $\mathbf{w}_u$ with the quantified phase value $\varphi_{m,u}$ can be written as $[\mathbf{w}_u]_{m,1} = \frac{1}{\sqrt{N_{\rm MS}}} e^{j\varphi_{m,u}}$.

According to [10] and [11], the channel matrix of the uth MS with L_u paths can be expressed as

$$\mathbf{H}_u = \sqrt{\frac{N_{\rm BS}N_{\rm MS}}{L_u}} \sum_{i=1}^{L_u} \alpha_{u,i}\mathbf{a}_{\rm MS}(\psi_{u,i})\mathbf{a}_{\rm BS}(\theta_{u,i})^{\rm H} \tag{2}$$

where $\alpha_{u,i}$ represents the complex path gain of the ith path for the uth MS. $\mathbf{a}_{\rm BS}(\theta_{u,i})$ is the response vector of the uniform linear array (ULA) employed in the BS, and can be written as

$$\mathbf{a}_{\rm BS}(\theta) = \frac{1}{\sqrt{N_{\rm BS}}}[1, e^{j\frac{2\pi}{\lambda}d\sin\theta}, ..., e^{j(N_{\rm BS}-1)\frac{2\pi}{\lambda}d\sin\theta}]^{\rm T} \tag{3}$$

where d is the antenna spacing and λ denotes the wavelength of operation. In this paper, we assume that $d = \frac{\lambda}{2}$. Similarly, the response vector of each MS has the same representation as (3).

Based on the uth channel matrix $\mathbf{H}_u$ with perfect channel knowledge, the uth user's effective channel is $\mathbf{h}_u^{\rm H} = \mathbf{w}_u^{\rm H}\mathbf{H}_u\mathbf{F}_{\rm RF}$. Adopting zero-force(ZF) precoding, the digital precoding matrix can be written as $\mathbf{F}_{\rm BB} = \mathbf{H}_{\rm eff}^{\rm H}(\mathbf{H}_{\rm eff}\mathbf{H}_{\rm eff}^{\rm H})^{-1}$ [5],[9], where $\mathbf{H}_{\rm eff}^{\rm H} = [\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_M]$. We can calculate the Signal-to-Interference-Noise-Ratio (SINR) for the uth MS and the information rate as

$$\mathrm{SINR}_u = \frac{\frac{P_{\rm T}}{M}|\mathbf{w}_u^{\rm H}\mathbf{H}_u\mathbf{F}_{\rm RF}[\mathbf{F}_{\rm BB}]_{:,u}|^2}{\sigma^2 + \sum_{i\neq u}^{M}\frac{P_{\rm T}}{M}|\mathbf{w}_u^{\rm H}\mathbf{H}_u\mathbf{F}_{\rm RF}[\mathbf{F}_{\rm BB}]_{:,i}|^2} \tag{4}$$

$$R = \sum_{i=1}^{M}\log_2(1 + \mathrm{SINR}_u) \tag{5}$$

In the training stage of mmWave multiuser communications, how to find the optimum beam pairs with relatively less time overhead between the BS and each MS is of great significance.

III. Multi-user Beam Training Scheme

In this section, the whole beam training period is divided to two sections, the hierarchical searching section and the partially exhaustive searching section, as shown in Fig.2.

A. Extended Hierarchical Codebook Design

The extended hierarchical codebook consists of two parts corresponding to the two stages of the whole beam training process. In the first part of the extended codebook, the searching period can be divided to S stages where $S = \max(S_{\rm BS}, S_{\rm MS})$ and $S_{\rm BS}$, $S_{\rm MS}$ respectively represent the numbers of hierarchical training stages in the BS and in the MS. In each beam training stage, the BS/MSs split each of the determined wide beams into K parts to perform beam training. Therefore, $S_{\rm BS}$ and $S_{\rm MS}$ can be calculated as $S_{BS} = \log_K N_{BS}$ and $S_{MS} = \log_K N_{MS}$. Here we assume N_{BS} and N_{MS} are the power of K (e.g. $K = 2$). Assume that the hierarchal codebook is denoted by $\mathcal{F}$ in the BS and $\mathcal{W}$ in the MS. The codewords used in the last stage of the hierarchical searching section are the response vectors with the angles of $\overline{\theta}_n = \arcsin\frac{2\pi n}{N_{\rm BS}}$ [7], where $n = 0, 1, ..., N_{\rm BS} - 1$. Namely, $\mathcal{F}_{S_{\rm BS}} = \mathbf{a}_{\rm BS}(\overline{\theta}_n)$.

Proposition 1: The beamforming vector of the sth stage, nth column in $\mathcal{F}$ can be written as

$$\mathcal{F}_s(:, n) = \frac{2^s}{N_{\rm BS}}\sum_{i=1}^{\frac{N_{\rm BS}}{2^s}}\mathcal{F}_{S_{\rm BS}}(:, n + (i-1)2^s) \tag{6}$$
$$s = 1, 2, ...S_{\rm BS}, n = 1, 2, ..., 2^s$$

Proof: The right side of (6) reflects a summation of several orthogonal beams in $\mathcal{F}_{S_{\rm BS}}$. The summed beam consequently points at multiple directions which together are identified as a "wide" beam. If the summed vector of each "wide" beam has constant amplitude, the right side of (6) would meet the requirement of an analog codebook and **Proposition 1** would be proved.

Adopting inverse method, we can deduct the mth element of $\mathcal{F}_s(:,n)$ as

$$\begin{aligned}\mathcal{F}_s(m,n) &= \frac{2^s}{N_{\mathrm{BS}}}\sum_{i=1}^{\frac{N_{\mathrm{BS}}}{2^s}} \mathcal{F}_{S_{\mathrm{BS}}}(m, n+(i-1)2^s) \\ &= \frac{2^s}{N_{\mathrm{BS}}}\frac{e^{j\frac{2\pi m}{N_{\mathrm{BS}}}(n-1)}(1-e^{j\frac{2\pi m}{N_{\mathrm{BS}}}2^s\frac{N_{\mathrm{BS}}}{2^s}})}{1-e^{j\frac{2\pi m}{N_{\mathrm{BS}}}2^s}} \\ &= \begin{cases} e^{j\frac{2\pi m}{N_{\mathrm{BS}}}(n-1)} & \text{if } n=\frac{hN_{\mathrm{BS}}}{2^m} \\ 0 & \text{others}\end{cases}\end{aligned} \tag{7}$$

where $h = 1, 2, ..., 2^s$ is a positive integer. The equation of (7) indicates that the property of the phase shifters behind the antennas, which have the antenna indices of $\frac{hN_{\mathrm{BS}}}{2^s}(h = 1, 2, ..., 2^s)$, meets the requirement of constant amplitude values. It means $\mathcal{F}(:, n, s)$ could be applied in a constrained analog beamformer.

In the second part of the extended codebook, the partially exhaustive searching method over the codewords of the $(S_{\mathrm{BS}}+1)$th stage is designed. Assume that the BS allocates the lth beam, which points the direction of θ_l, to the uth MS in the stage of S_{BS}. The directions of the adjacent beams, which have the indexes of $(l-1)$ and $(l+1)$, are represented as θ_{l-1} and θ_{l+1}. After accomplishing the hierarchical beam searching stage, the BS continues training the beams with the angles of θ_{l_1}, θ_{l_2} and θ_{l_3}, where $\theta_{l_1} = \frac{\theta_{l-1}+\theta_l}{2}, \theta_{l_2} = \theta_l, \theta_{l_3} = \frac{\theta_l+\theta_{l+1}}{2}$ which are also depicted in Fig.2. Also, we could divide the angle zone of $(\theta_{l-1}, \theta_{l+1})$ into more than 3 parts to perform the partially exhaustive beam training if a higher estimation accuracy is desired. The three codewords mapped by the determined training vector $\mathcal{F}_{S_{\mathrm{BS}}}(:, l)$ of the S_{BS}th stage can be represented as

$$\mathcal{F}_{S_{\mathrm{BS}}+1}(:, l_i) = \frac{[1, e^{j\pi\sin(\theta_{l_i})}, ..., e^{j\pi(N_{\mathrm{BS}}-1)\sin(\theta_{l_i})}]^{\mathrm{T}}}{\sqrt{N_{\mathrm{BS}}}} \qquad i = 1, 2, 3$$

Similarly, the design of $\mathcal{W}_s$, $s = 1, 2, ..., S_{\mathrm{MS}}+1$, follows the same process where N_{BS} is replaced by N_{MS}. For adapting the vectors to the finite-bit phase shifters, we quantify the phase values of $\mathcal{F}_{S_{\mathrm{BS}}+1}$ with 6 bit in the BS, and $\mathcal{W}_{S_{\mathrm{MS}}+1}$ with 5 bit in the MS.

B. Hierarchical Beam Searching Process

Consider M users communicating simultaneously with the BS with the structures of Fig.1. We present the hierarchical beam searching method in Algorithm 1 which operates as follows. In the second stage in Fig.2, the BS trains the beams with the indices of 1, 2, 3, and 4. The M MSs simultaneously receive the training symbols and detect the power distribution, and then feed back the optimal beams' indices (1, 2, 3, or 4) to the BS. Assume $M = 6$ and the received feedback indices in the BS are $[1, 3, 3, 1, 3, 1]$. Then $[1, 3]$ would be the effective beams while $[2, 4]$ would be noneffective beams. This means the 6 MSs are distributed at the zones of $[1, 3]$. The BS would trains the beams chronologically with the indexes under $[1, 3]$, namely $[1, 2, 5, 6]$, in the third stage.

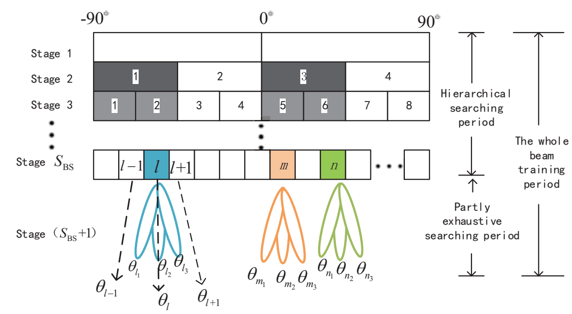

Fig. 2. The whole process of multi-user beam training in the BS. The training period is divided into two parts, hierarchical searching period and partially exhaustive searching period. The codewords in each stage have been numbered for simple analysis.

Due to the broadcast beam training rather than unicast beam training, the BS trains the beams without knowing any characteristics of users. Hence, we need to store the feedback information in trace matrix of $\mathbf{m}_{\mathrm{tr}}$ in the BS or $\mathbf{n}_{\mathrm{tr}}$ in the MS. In the steps of $2 \sim 10$ of Algorithm 1, we initialize the trace matrix and perform the hierarchical beam searching. Only when all users have accomplished training the whole effective beams in one stage, does the algorithm progress to the next stage. In the steps of $11 \sim 20$, the M MSs process the training signals from the BS and feed back the information of all the optimal beams. After receiving all users' feedback messages, the BS updates the trace matrix in the steps of 21 and 22. When $S = \max(S_{\mathrm{BS}}, S_{\mathrm{MS}})$ searching stages are finished, the optimal beam pairs of the hierarchical searching section between the BS and the MSs are determined also. In Algorithm 1, for overcoming the deficiency of transmitting power, caused by overly wide beams in the initial training stage, we can set the number of splitting parts in the first stage as $K^1 = 4$. Namely, we started the hierarchical beam training in the second stage in Fig.2.

In Stage s, the received signal of uth MS in the stage of s can be written as

$$\mathbf{Y}_{u,s} = \sqrt{\frac{P_{\mathrm{T}}}{M}}\mathbf{W}_{u,s}^H\mathbf{H}_u\mathbf{F}_s\mathbf{I}_{c\times c} + \mathbf{W}_{u,s}^{\mathrm{H}}\mathbf{N}_u \tag{8}$$

where $\mathbf{F}_s$ consists of c beamforming vectors representing c different directions of the sth beam training stage, c is the numbers of effective beams. P_{T} is the total transmitting power and $\mathbf{N}_u$ is $\mathbf{N}_{\mathrm{BS}} \times c$ Gaussian white noise matrix. Each MS detects the elements of $\mathbf{Y}_{u,s}$ and feed back the information of the strongest one to the BS.

The total numbers of time slots for fully exhaustive search is $N_{\mathrm{BS}}N_{\mathrm{MS}}$ while in [6] is approximately $2M(\log_2 N_{\mathrm{BS}} + \log_2 N_{\mathrm{MS}} - 2) + 4$, In our paper ,

Algorithm 1 Multi-user Hierarchical Beam training

Input: $N_{\mathrm{BS}}, N_{\mathrm{MS}}, K, K^1$

$\mathbf{f}(a,b)$ or $\mathbf{w}(a,b)$: the bth codeword in ath layer.
Initialization: $S_{\mathrm{BS}} = \log_K N_{\mathrm{BS}}, S_{\mathrm{MS}} = \log_K N_{\mathrm{MS}}$
$\mathbf{m}_{\mathrm{tr}} = \mathrm{zeros}(S_{\mathrm{BS}}, M)$, $\mathbf{n}_{\mathrm{tr}} = \mathrm{zeros}(1, M)$
for $p = 1 : K^1$ **do**
  for $q = 1 : K^1$ **do**
    $y_u(p,q) = \mathbf{w}^{\mathrm{H}}_{(1,q)} \mathbf{H}_u \mathbf{f}_{(1,p)} + \mathbf{w}^{\mathrm{H}}_{(1,q)} \mathbf{n}$
  end for
end for
$[\hat{\mathbf{m}}(1,u), \hat{\mathbf{n}}(u)] = \mathrm{argmax}_{p,q} |y_u(p,q)|$
$\mathbf{m}_{\mathrm{tr}}(1,u) = \hat{\mathbf{m}}(1,u) - 1$; $\mathbf{n}_{\mathrm{tr}}(1,u) = \hat{\mathbf{n}}(1,u) - 1$;
Hierarchical training:
for $s = \log_2 K^1 + 1 : S_{\mathrm{BS}}$ **do**
  $\mathbf{m}_{\mathrm{tr}} \to$ **beam_index** (Abbreviate as $\mathbf{b}_{\mathrm{in}}$)
  for $u = 1 : M$ **do**
    for $b_1 = 1 : \mathrm{length}(\mathbf{b}_{\mathrm{in}})$ **do**
      for $b_2 = 1 : (K-1)(1 - \mathrm{floor}(s/(S_{\mathrm{MS}} + 1))) + 1$ **do**
        $\mathbf{y}_u(b_2, b_1 * K - K + 1 : b_1 * K) = \sqrt{P}\mathbf{w}^H_u(s, u * \mathbf{n}_{tr}[u] + b_2)\mathbf{H}_u * \mathbf{f}(s, K * \mathbf{b}_{\mathrm{in}}(b_1) + 1 : K * \mathbf{b}_{\mathrm{in}}(b_1) + K) + \mathbf{w}^{\mathrm{H}}_u(s, u * \mathbf{n}_{tr}[u] + b_2) * \mathbf{n}$
      end for
    end for
  end for
  $[\hat{\mathbf{m}}(s,u), \hat{\mathbf{n}}(u)] = \mathrm{argmax}_{p,q} |y_u(p,q)|$
  Update $\mathbf{m}_{\mathrm{tr}}$ and $\mathbf{n}_{\mathrm{tr}}$
end for

the maximum time overhead of Algorithm 1 can be computed as

$$T_{\mathrm{overhead}} = MK^2[\log_K N - \mathrm{floor}(\log_k M) - 1] + \frac{K^2[1 - K^{\mathrm{floor}(\log_K M)+1}]}{1-K} \quad (9)$$

For convenient comparison, we assume $N = N_{\mathrm{BS}} = N_{\mathrm{MS}} = 128$. We note that (9) reflects the maximum time overhead with arbitrary users distribution. Assuming that the AoA of uth MS is φ_u and $\varphi_u \sim \mathbf{u}(-\frac{\pi}{2}, \frac{\pi}{2})$, we present the comparison of time overhead between Algorithm 1 and the beam searching method in [6] in TABLE 1. Also, the time overhead of directly employing hierarchical beam training method to the users one by one is presented in the table. The results show that Algorithm 1 offers almost 60% reduction in time overhead while the method in [6] has approximately the same overhead compared with the directly hierarchical training in the case of $M = 20$. This means Algorithm 1 is much faster in finding the optimal beams in mmWave multiuser MIMO systems.

C. Partially Exhaustive Beam Searching Process

The beam searching method of Algorithm 1 almost achieves the AoA/AoD estimation while the errors of the angle estimation partially depend on the number of the deployed antennas, and how the MSs are dispersive in spatial domain. The partially exhaustive beam searching process is to reduce the estimation errors and operates as follows. As shown in Fig.2, each of the allocated beams in the stage of S_{BS} further map out into 3 (or more if a higher accuracy is desired) sub beams to train respectively with another 3 sub beams formed in the MS considering that the MSs perform the same exhaustive search method. Hence, the process will cost 9 time slots in all. The mapping method can be seen in Fig.2, namely, θ_l maps θ_{l_1}, θ_{l_2}, and θ_{l_3}. As the sub beams further divide the estimated angle zone into 3 parts, the errors of AoA/AoD estimation would be reduced as well.

TABLE I
THE COMPARISON OF THREE BEAM TRAINING SCHEMES IN TRAINING OVERHEAD, SEQUENTIALLY BEAM TRAINING, THE METHOD IN [6], AND ALGORITHM 1 IN OUR PAPER.

Number of MSs (M)	T_{overhead}			
	Sequentially training	Works in [6]	Algorithm 1	
			$K^1 = 2$	$K^1 = 4$
$M = 4$	112	100	81	85
$M = 12$	336	292	170	173
$M = 20$	560	484	228	232

Considering the inter-user interference, the combined signal of the uth MS can be written as

$$y_{u,m,n} = \sqrt{\frac{P_{\mathrm{T}}}{M}}(\mathbf{w}^{\mathrm{H}}_{u,m}\mathbf{H}_u\mathbf{f}_{u,n} + \sum_{i \neq u} \mathbf{w}^{\mathrm{H}}_{u,m}\mathbf{H}_u\mathbf{f}_{i,n}) + \mathbf{w}^{\mathrm{H}}_{u,m}\mathbf{n}$$
$$m,n = 1,2,3. \quad u = 1,2,...,M.$$

where m,n denotes the index of the sub beams, $\mathbf{f}_{u,n} \in \mathcal{F}_{S_{\mathrm{BS}}+1}$, $\mathbf{w}_{u,m} \in \mathcal{W}_{S_{\mathrm{MS}}+1}$, and P_{T} is the total transmitting power.

IV. NUMERICAL RESULTS

In Fig.3, we present the average rate of success selection of Algorithm 1. A "success" means that the AoA and AoD of the selected beam pair are respectively closest to the most dominant path between the BS and the uth MS among the orthogonal beams of $\mathcal{F}_{S_{\mathrm{BS}}}$ and $\mathcal{W}_{S_{\mathrm{MS}}}$. Note that the total transmitting power is P_{T}, $N_{\mathrm{BS}} = N_{\mathrm{MS}} = 128, K = 2$, and $M = 6$. From Fig.3, only training multi-user beam pairs with Algorithm 1 would bring unsatisfactory success rate such as 60% with 5dB when $K^1 = 2$. The reason is that each user receives $K \times c$ transmitted beams, where c is the number of effective beams, in each stage for each of its receiving beams, and this would results in more inter-beam interference. Setting $K^1 = 4$, which means enhancement the initial training power, and employing improved angle estimation are both to mitigate the influence of inter-beam interference. It is found that the two solutions have equivalent effect while setting $K^1 = 4$ performs slightly better when SNR is low since enhancing transmitted signal power is easier to improve the performance at low SNR.

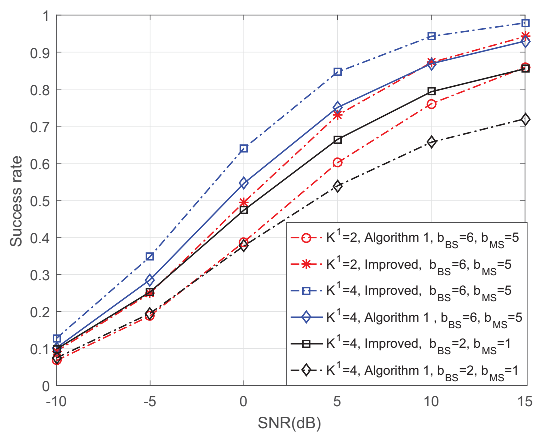

Fig. 3. The success rate versus SNR for Algorithm 1 and improved angle estimation with different number of initially divided parts ($K^1 = 2, 4$). The number of antennas employed in the BS (MS) is $N_{N_{\rm BS}} = 128$ ($N_{\rm MS} = 128$).

Form Fig.3, the success rate of Algorithm 1 evidently decrease when the number of bits reduces 4 bits. At low SNR, $K^1 = 4$ can compensate the success rate loss caused by few-bit phase quantization. Setting $K^1 = 4$ and adopting improved angle estimation with few-bit phase shifters still have a relatively high success rate such as 85% with 15dB.

In Fig.4, we compare the spectral efficiency (SE) with [5]. Assume that the BS communicates with 4 MSs simultaneously and is equipped with a 64 ULA while each MS is equipped with a 32 ULA. In [5], the BS performs hybrid beamforming with the accurate information of AoA/AoD. Due to the existence of spatial quantization error, the spectral efficiency of Algorithm 1 is 16% lower than [5]. The case of $L = 3$ would result in approximately $25 \sim 30\%$ SE loss at medium and high SNRs. Since our paper is to determine the most dominant path between the BS and each MS and further eliminates inter-user interference with in the dominant path by hybrid precoding, the suboptimal paths would inevitably reveal non-ignorable interference signal and further deteriorate the power distribution of receiving matrix. On the other hand, we also offer the SE changes with different-bit phase shifters, such as $b_{\rm BS} = 6$ bits, $b_{\rm MS} = 5$ bits and $b_{\rm BS} = 2$ bits, $b_{\rm MS} = 1$ bit considering different number of antennas in the BS and MSs. We find the effect of employing improved angle estimation based on Algorithm 1 with ignorable time overhead is sufficient to compensate the few-bit SE loss.

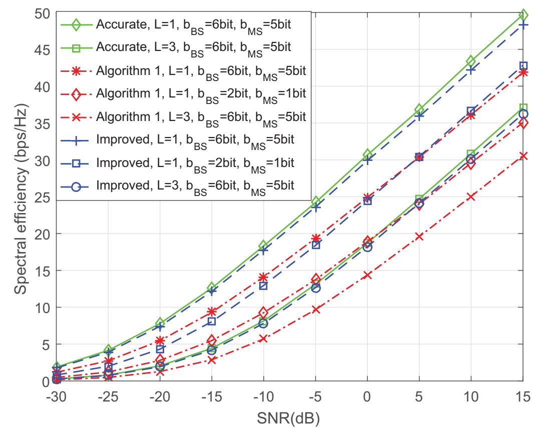

Fig. 4. The spectral efficiency versus SNR for finite-bit phase shifters with accurate AoA/AoD in [5], finite-bit and few-bit phase shifters with quantified AoA/AoD (Algorithm 1), and improved angle estimation.

V. CONCLUSION

In this paper, we proposed a fast beam training algorithm to find out the optimum beam pairs between the BS and M MSs in mmWave multi-user MIMO systems. To accomplish this, we extended an analog codebook with finite-bit phase values to perform beam training with joint method of hierarchical searching and partially exhaustive searching. The improved spectral efficiency can approximately reach the performance of [5] which however needs accurate AoA/AoD information.

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