

# A Reactance Domain Fourth-order MUSIC Algorithm Using 13-element ESPAR Antenna

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**Abstract**—Electronically steerable parasitic array radiator (ESPAR) antenna is well known for its low-power consumption, small-size antenna. In this paper, a novel fourth-order MUSIC algorithm using a 13-element ESPAR antenna for direction of

h-order MUSIC algorithm is that it is insensitive to additive Gaussian noise regardless of whether it is white or colored so it can improve the precision of the DOA estimation. By comparing with the conventional algorithm, our proposed algorithm can obtain more than 45dB gain under SNR=10dB, which can obviously show an outstanding performance improvement. Finally, computer simulation results show that the novel algorithm can resolve up to one impinging signal with less than  $0.15^\circ$  under SNR=0dB and provide an estimation with less than  $0.6^\circ$  under SNR=5dB for two impinging signals coming from different angles.

**Keywords**— *MUSIC algorithm; DOA; fourth-order MUSIC algorithm; ESPAR antenna*

## I. INTRODUCTION

Direction of arrival (DOA) estimation technology plays an important role in enhancing the performance of adaptive arrays for mobile wireless communications [1]. A number of DOA estimation algorithms have been developed. For the most recent ones being MUSIC [2] and ESPRIT [3] algorithms, who both utilizing subspace-based on exploiting the eigen structure of the input covariance matrix. Among many DOA algorithms, MUSIC algorithm which is based on the correlation matrix decomposition of the received signals has a good estimation performance and alleviate the system complexity comparing to other algorithms.

As we all know, mobile terminals have rigorous limitation on hardware and algorithm complexity as well as power consumption. Unlike conventional antenna array systems which require one receiver chain per antenna branch, the ESPAR antennas can be controlled by means of its electronically controllable reactance, which only need a single-port output and requires less complex hardware and low power consumption. Thus due to these advantages, the

arrival (DOA) estimation is proposed based on processing the covariance matrix of the array output signals. Unlike the conventional MUSIC algorithm which is an effective method for DOA estimation, the main motivation of the four ESPAR antennas can be used for application to wireless communication system, especially to mobile user terminals.

Recently, a great deal of effort has been focused on taking the advantages of ESPAR antennas to improve the performance of localization system [4]. According to the ESPAR antennas configuration, the RF currents on the element, which depend on the values of the reactances, are not independent but mutually coupled with each other. The single output is a highly nonlinear of the reactances. And the antenna pattern is formed due to the reactance on the parasitic radiators [5]. All of these ESPAR antennas features make it complex to adapt to conventional algorithms.

In this paper, we combine an reactance domain MUSIC algorithm [6-8] based on fourth-order MUSIC algorithm with a 13-element ESAPR antenna to evaluate and estimate the DOA performance. The main motivation is that fourth-order comulant is insensitive to additive Gaussian noise regardless of whether it is white or colored. The covariance matrix of the output can be computed by the fourth-order cumulants. The cumulants have an more important suppression property for not only Gaussian noise but also non-Gaussian noise. By using the reactance domain fourth-order MUSIC algorithm, we create the correlation matrix for the single-port output ESPAR antenna. Finally we evaluate the DOA estimation performance for both one and two sources cases in a feasible SNR environment and the results show a good performance.

This paper is organized as follows. In Section II illustrate the system model of the 13-element ESPAR antennas. In Section III, the proposed reactance domain fourth-order MUSIC algorithm is briefly described. In Section IV, the performance of the reactance domain fourth-order MUSIC algorithm is evaluated. Finally, Section VI concludes the paper.

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## II. SYSTEM MODEL

In this section, we review the working principle of ESPAR antenna and present a signal model. The 13-element ESPAR antenna is composed of a single active monopole antenna element at the center of the circular plane while the other twelve parasitic elements are surrounding at the same spacing with 30 degree encircles the central element as shown in Fig. 1. The parasitic elements are loaded with variable reactors that can control the imaginary part of the parasitic element' input impedances. Therefore, the radiation patterns of the ESPAR antenna system can be controlled by adjust values of the reactors connected to the parasitic radiators and it can be formed twelve directions as Fig. 2. The current on the monopoles are induced by mutual coupling with the central active element. The RF current vector  $i$  is given as

$$i = V_s (Z + X)^{-1} u_o \quad (1)$$

where  $V_s$  represents the transmitted voltage source of the central active radiator.  $Z$  is the mutual impedance matrix given by  $(M+1)$  by  $(M+1)$  dimension whose element are assumed  $Z_{ij} = Z_{ji}$ ,  $\{i, j\} \in \{0, 1 \dots M\}$  due to the symmetry of the ESPAR antenna structure.

$X$  is called reactance matrix given as:

$$X = \text{diag}[Z_0, jx_1, jx_2 \dots jx_M] \quad (2)$$

where  $Z_0$  is the characteristic impedance of  $50\Omega$  and finally  $u_o$  is a select vector defined as:

$$u_o = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T \quad (3)$$

Assume there are a number of  $D$  transmitted signals  $u_d(t)$  with DOA  $\theta_d$  ( $d=1, 2, \dots, D$ ). Let  $s_m(t)$  ( $m=1, 2, \dots, M$ ) denote the signal impinging on the  $m$ th element of the antenna and define  $s(t)$  as the column vector with the  $m$ th component  $s_m(t)$ . Therefore, the column vector  $s(t)$  can be written as:

$$s(t) = \sum_{d=1}^D a(\theta_d) u_d(t) \quad (4)$$

where  $a(\theta_d)$  is the steering vector based on the ESPAR antenna structure and can be expressed as:

$$a(\theta_d) = [1, e^{jkd \cos(\theta_d - \phi_1)}, \dots, e^{jkd \cos(\theta_d - \phi_M)}]^T \quad (5)$$

where  $k = 2\pi / \lambda$  is the wavenumber and  $d$  is the ESPAR inter-element spacing and  $\phi_m = (2\pi / M) / (m-1)$  is the position corresponding to the  $m$ th antenna.

Due to the signal model, the output of the ESPAR antenna is given as

$$y(t) = \sum_{d=1}^D i^T a(\theta_d) u_d(t) + n(t) = i^T s(t) + n(t) \quad (6)$$

where  $n(t)$  is an additive white Gaussian noise and  $i$  is the RF current vector.

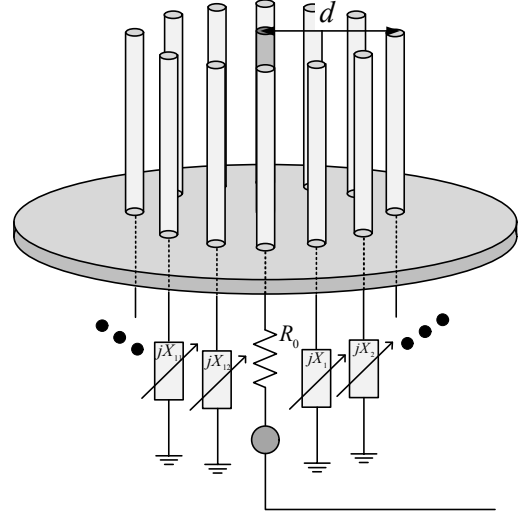


Fig. 1. 13-element ESPAR antenna structure.

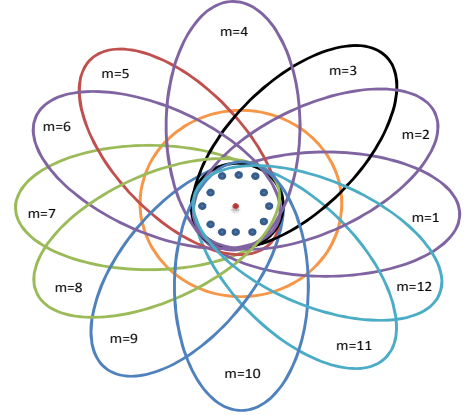


Fig. 2. Rotation of the directional pattern in the received side.

## III. REACTANCE DOMAIN FOURTH-ORDER MUSIC ALGORITHM

### IV. SIMULATION

In this section we describe a set of simulation to simulate the reactance domain fourth-order MUSIC algorithm using 13-element ESPAR antenna. The parameters of simulation are given in Table 1.

Table 1. Simulation Parameters

Parameters	Values
Element spacing	$0.25\lambda$
Reactance value( $\Omega$ )	$[-100 \ 100 \ 100 \ 100 \ 100 \ 100 \ 100 \ 100 \ 100 \ 100 \ 100 \ 100 \ 100]$
Parasitic elements	$M=12$
Snapshots Number	$N=512$
transmitted voltage $V_s$	$10V$

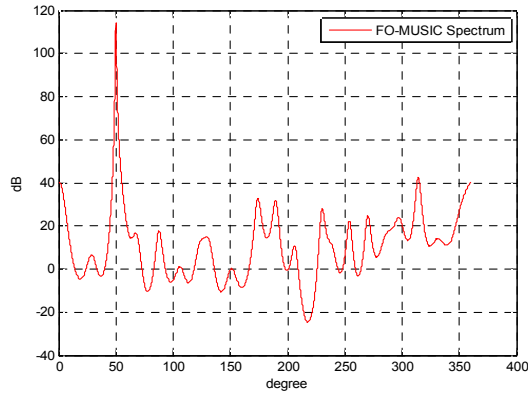


Fig. 2. DOA estimation performance under SNR=10dB with desired angle 50.

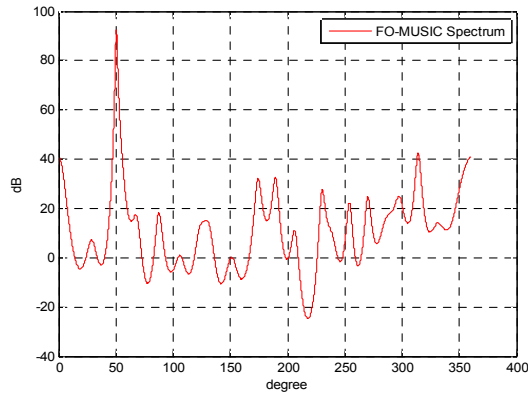


Fig. 3. DOA estimation performance under SNR=5dB with desired angle 50°.

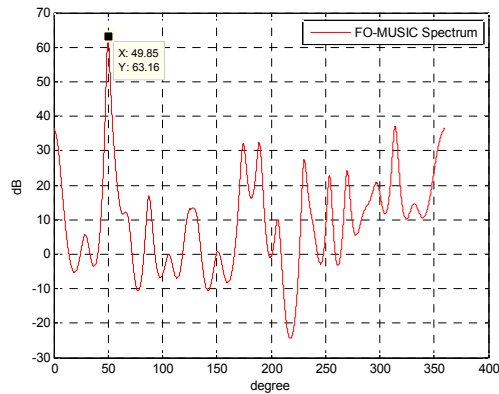


Fig. 4. DOA estimation performance under SNR=0dB with desired angle 50°.

We first evaluate the case of DOA estimation performance for one source given a SNR range from 0dB to 10dB. The desired angle for three situations is the same given as 50°. Fig. 2 shows the DOA estimation under SNR=10dB while Fig. 3 and Fig. 4 show the DOA estimation under SNR=5dB and SNR=0dB, respectively. As we can see, **in the relatively high SNR environment like SNR=10dB, our system can show an accurate DOA estimation performance with high resolution.**

Meanwhile, in the relatively low SNR environment given like SNR=0dB which is feasible in practical, we estimate the DOA with the error of less than 0.15° which show a realizable significance in application implantation.

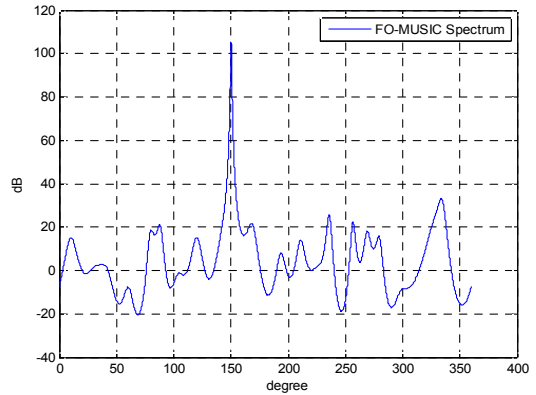


Fig. 5. DOA estimation performance under SNR=10dB with desired angle 150°.

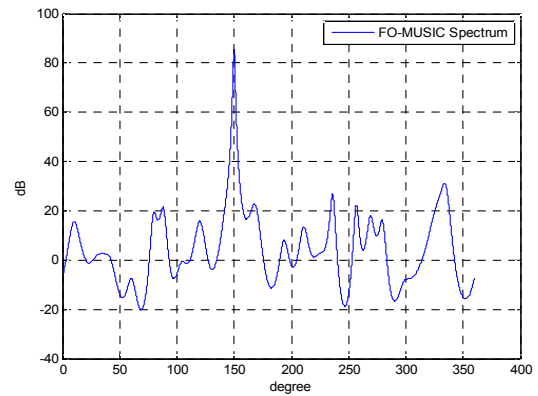


Fig. 6. DOA estimation performance under SNR=5dB with desired angle 150°.

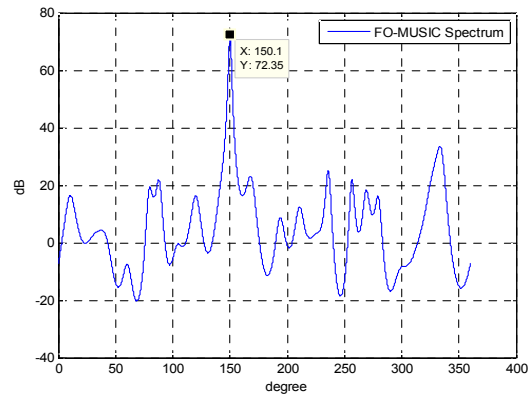


Fig. 7. DOA estimation performance under SNR=0dB with desired angle 150°.

We then evaluate another case of DOA estimation performance for one source given a SNR range from 0dB to 10dB. The desired angle for three situations is the same given as 150°. Fig. 5 shows the **DOA estimation under SNR=10dB**

while Fig. 6 and Fig. 7 show the DOA estimation under SNR=5dB and SNR=0dB, respectively. As we can see, in the relatively high SNR environment like SNR=10dB, our system can show an accurate DOA estimation performance with high resolution. Meanwhile, in the relatively low SNR environment given like SNR=0dB which is feasible in practical, we also estimate the DOA with the error of less than  $0.1^\circ$  which is the same to case one.

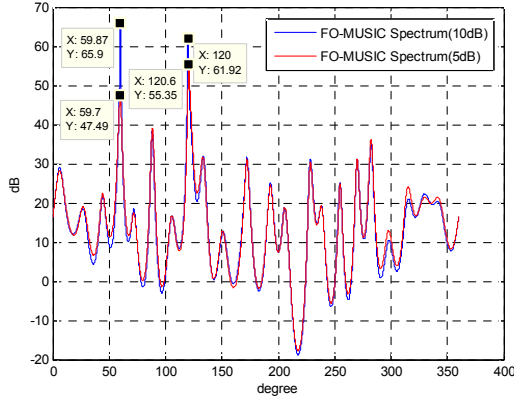


Fig. 8. DOA estimation performance with desired angles  $60^\circ$  and  $120^\circ$  for SNR=5dB and SNR=10dB.

In the case three, we evaluate the DOA estimation performance for two sources given a SNR range from 5dB to 10dB. The desired angles for the two situations are given as  $60^\circ$  and  $120^\circ$ , respectively. Fig. 8 shows the DOA estimation under SNR=10dB and SNR=5dB. As we can see, in the relatively high SNR environment like SNR=10dB, our system can show an accurate DOA estimation performance with high resolution. Meanwhile, in the relatively low SNR environment given like SNR=5dB which is feasible in practical, we estimate the DOA with the error of less than  $0.13^\circ$  and  $0.6^\circ$  for the situations of both SNR=10dB and SNR=5dB which show a realizable significance in application implantation.

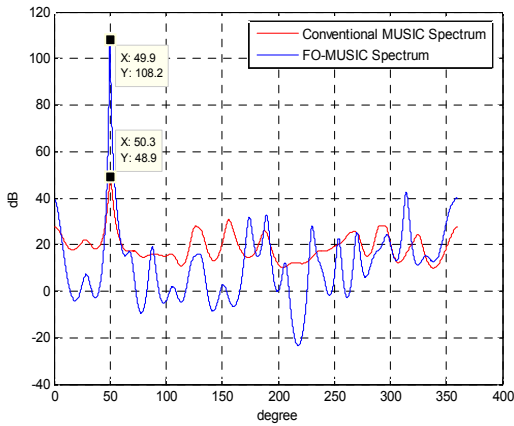


Fig. 9. Compare of DOA estimation of conventional MUSIC algorithm and proposed fourth-order MUSIC algorithm under SNR=10dB.

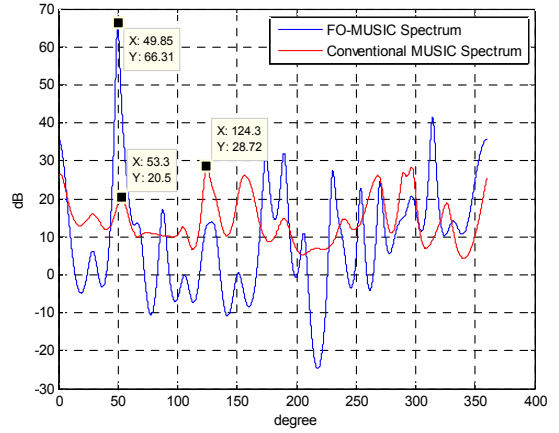


Fig. 10. Compare of DOA estimation of conventional MUSIC algorithm and proposed fourth-order MUSIC algorithm under SNR=0dB.

In the case four, we compare of the DOA estimation of our proposed novel fourth-order MUSIC algorithm and conventional MUSIC algorithm. Fig. 9 and Fig. 10 give the performance comparison under SNR=10dB and 0dB, respectively. The desired angles for the two situations are given as  $50^\circ$ . As we can see from Fig. 9, the proposed algorithms not only can estimate the DOA with the error of  $0.15^\circ$  while conventional algorithm is  $3.3^\circ$  but also have more than 45dB spectrum gain than conventional algorithm. Meanwhile, in the relatively low SNR environment given as SNR=0dB which is feasible in practical, conventional algorithm fail to estimate the desired angel while our proposed algorithm can still show an accurate DOA estimation performance with high resolution. From above simulation results, our proposed algorithm can obviously show outstanding performance improvement.

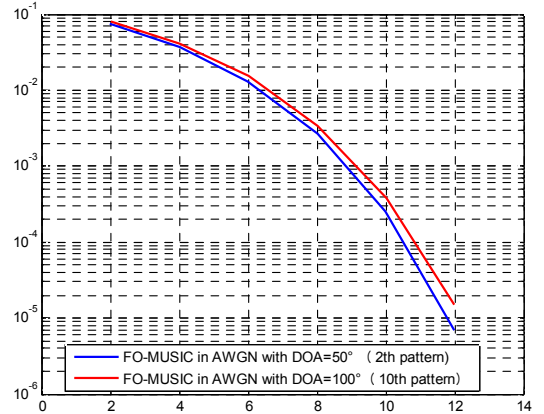


Fig. 11. BER performance of fourth-order MUSIC algorithm in AWGN with DOA =  $50^\circ$  and  $100^\circ$ .

Fig. 11 shows BER performance of fourth-order MUSIC algorithm in AWGN. Proposed DOAs are  $50^\circ$  and  $100^\circ$ . Through this algorithm, we confirm that BER performance is  $10^{-5}$  approximately at 12dB. We can find the best BER

performance through selecting the optimal pattern of all directional patterns.

## V. CONCLUSIONS

In this paper, we proposed a novel MUSIC algorithm based on 13-element ESPAR antenna by processing the covariance matrix of the array output signals using fourth-order cumulant. By using the fourth-order MUSIC algorithm, the DOA estimation performance is improved for the main motivation of the fourth-order MUSIC algorithm is that it is insensitive to additive Gaussian noise regardless of whether it is white or colored. By comparing with the conventional algorithm, our proposed algorithm can obtain more than 45dB gain under SNR=10dB, which can obviously show an outstanding performance improvement. Finally, simulation results show that the novel algorithm can resolve up to one impinging signal with less than  $0.15^\circ$  under SNR=0dB and provide an estimation with less than  $0.6^\circ$  under SNR=5dB for two impinging signals coming from different angles, which has an important significance in real application for implantation.

## REFERENCES

- [1] R. Schmidt, "Multiple emitter location and signal parameter estimation," *Antennas and Propagation*, IEEE Transactions on, vol. 34, no. 3, pp. 276–280, Mar. 1986.
- [2] A. Paulraj, R. Roy, and T. Kailath, "A subspace rotation approach to signal parameter estimation," *Proceedings of the IEEE*, vol. 74, no. 7, pp. 1044 – 1046, Jul. 1986.
- [3] T. Lavate, V. Kokate, and A. Sapkal, "Performance analysis of MUSIC and ESPRIT DoA estimation algorithms for adaptive array smart antenna in mobile communication," *International Journal of Computer Networks (IJCN)*, vol. 2, no. 3, pp. 152 – 172, Jul. 2010.
- [4] C. Sun and N. C. Karmakar, "A DOA estimation technique based on a single-port smart antenna for position location services," in *Proc. Asia-Pacific Microwave Conf.*, Kyoto, Japan, Nov. 2002, pp. 196–199.
- [5] R. Schlub, J. Lu, and T. Ohira, "Seven element ground skirt monopole ESPAR antenna design using a genetic algorithm and the finite element method," *IEEE Trans. Antennas Propagat*, vol. 51, pp. 3033–3039, Nov. 2003.
- [6] C. Plapous, J. Cheng, E. Taillefer, A. Hirata, and T. Ohira, "Reactance domain MUSIC algorithm for electronically steerable parasitic array radiator," *IEEE Trans. Antennas Propagat*, vol. 52, 2004.
- [7] A. Hirata, H. Yamada, and T. Ohira, "Reactance-domain MUSIC DOA estimation using calibrated equivalent weight matrix of ESPAR antenna," in *Proc. IEEE Antennas and Propagation Soc. Int. Symp.*, June 2003, pp. 252–255.
- [8] E. Taillefer, C. Plapous, J. Cheng, K. Iigusa, and T. Ohira, "Reactance-domain MUSIC for ESPAR antennas (experiment)," in *Proc. Wireless Communications Network Conf.*, WCNC2003, Mar. 2003, pp. 98–102.