Multi-Antenna Transmission Using ESPAR with Peak Power Constraints

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Abstract—Electronically steerable parasitic array radiator (ESPAR) technology provides multi-antenna transmission with a single radio frequency (RF) unit. In order to achieve stable transmission using an ESPAR antenna (EA), two approaches have been proposed in literature. One is to increase the self-resistance of an EA, the other is to transmit signals closely approximating the actual signals that keep the EA stable. In both approaches, no constraint on the transmission power of an EA was considered. This is not the case in actual systems, as the practical power amplifier normally has limited peak power. Taking into account the limited power availability, an optimization problem is formulated with the objective to minimize the MSE between the currents corresponding to the ideal and the approximate transmission signals. The non-convex problem is solved analytically by coordination transformation and a novel algorithm is proposed. It is shown that the system employing the proposed transmission scheme gives similar performance to that of a standard multiple antenna system, especially at low SNRs. In addition, it is shown that increasing the self-resistance of an EA to achieve stability is highly power inefficient.

Index Terms—Reconfigurable antenna, ESPAR, MIMO transmission, single RF chain, optimization.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) transmission has been proposed in wireless communication applications due to its great benefits of spatial multiplexing and diversity gain. More antennas the transmitter/receiver is equipped with, the better performance is in terms of data rate and link reliability. However, from the hardware perspective, cost and size are two main factors that could affect the wide application. Each additional antenna element requires an additional radio frequency (RF) chain, resulting in increased cost and complexity. Therefore, Electronically steerable parasitic array radiator (ESPAR) has been proposed to reduce the cost and physical size of multiple antenna devices [4] and provide multi-antenna functionality with a single RF chain. For an ESPAR antenna (EA), the mutual coupling is exploited to control the currents at all of the parasitics. The overall radiation pattern is shaped by controlling the sole feeding at the active element and the impedance of the parasitics [5].

The model based on the currents at the ports of the transmit antenna was introduced in [1] [3]. The authors in [1] provided the design conditions that an EA was required to satisfy in order to support stable transmission and a new EA was designed to satisfy the condition. In [2], a different approach was proposed and approximate signals close to the ideal signals were proposed to support arbitrary signals transmission using an EA while maintaining stable operation. However, in both these works, [1] and [2], no constraint on the transmission power of an EA was taken into consideration. This is not the case in actual systems as the practical power amplifier normally supports a limited peak power [3]. Such peak power considerations have led to the new research in this paper.

In this work, we highlight the importance of enforcing a peak power constraint for the power amplifier. In an EA, all the antenna elements are fed centrally by a single power amplifier. This makes it more probable that an EAs power amplifier might reach maximum power during transmission. Considering the impact of limited power on EA transmission, we propose a new practical transmission scheme that enables an EA to provide stable multi-antenna functionality with instantaneous total power requirements. Specifically, we address a new algorithm to obtain EA configuration of approximate signals for transmission, which are close to the ideal signals, and satisfy the practical power requirement. It is formulated as a non-convex optimisation problem and solved analytically using coordination transformation. Closed-form expressions for optimal approximate transmission signals are derived from this problem. Our results show that a system employing EA transmitter and using our proposed algorithm gives similar performance as the system with a standard multiple antenna transmitter. Moreover, in [1], stable EA transmission was achieved by increasing the self-resistance of the active element. Our results show that if the EA has a large self-resistance, as in [1], the performance is significantly degraded.

II. SYSTEM MODEL FOR ESPAR MIMO

A. ESPAR Transmitter

Fig. 1 shows a block diagram of an EA consisting of a single active element with a RF unit and $M-1$ parasitic elements without any RF units. v_0 and z_s denotes the voltage feeding and the corresponding impedance at the active element, respectively.

In traditional multiple antenna transmitters with multiple RF chains, the currents are driven by the RF voltage supply of each antenna element through fixed impedances [6]. Whereas,
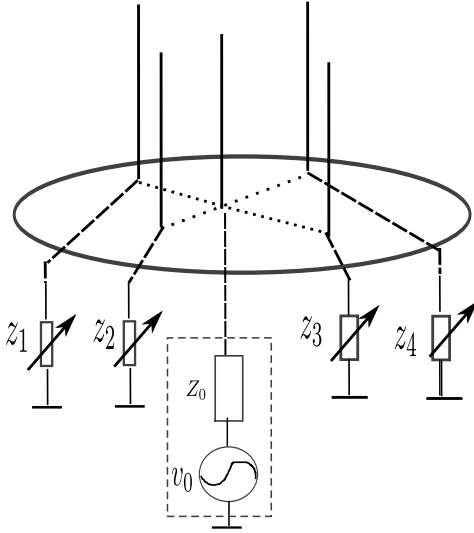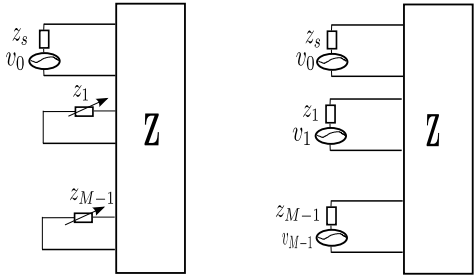
Fig. 1: Model of a circular EA

Fig. 2: Circuit comparison of a) the EA transmitter and b) a standard multiple antenna transmitter

the currents at the elements of EA, denoted by i_k where $k = \{0, 1, \ldots, M-1\}$, are varied by varying the input voltage v_0 at the active element and the tunable loads $z_1, z_2, \ldots, z_{M-1}$ at the parasitic elements. When feeding the active element, the currents are induced on the parasitics due to the mutual coupling between antenna geometry.

As depicted in Figure 2, $\mathbf{i} = [i_0, i_1, \ldots, i_{M-1}]^T$ is the vector of currents at the antenna elements, where T denotes the transpose operation. The matrix $\mathbf{Z} \in \mathbb{C}^{M \times M}$ denotes the MCM and it depends on the antenna geometry. $\mathbf{Z}_L = \mathrm{diag}(z_s, z_1, z_2, \ldots, z_{M-1})$ is the source impedance matrix where z_s is the source resistance [7] and $z_1, z_2, \ldots, z_{M-1}$ are the loads at the parasitic elements.

For the standard multiple antenna system, the vector of source voltages $\mathbf{v}$ is $[v_0, v_1, \ldots, v_{M-1}]^T$. The currents are driven by the RF voltage supply of each antenna element through fixed impedances $z_s, z_1, z_2, \ldots, z_{M-1}$ [6]. In the case of the EA system, $\mathbf{v}$ is $[v_0, 0, \ldots, 0]^T$. The currents at the elements are varied by varying the input voltage v_0 at the active element and the loads $z_1, z_2, \ldots, z_{M-1}$ at the parasitic elements. When feeding the active element, the currents are induced on the parasitic elements due to the mutual coupling

between antenna elements. The MCM $\mathbf{Z}$ is given as

$$\mathbf{Z} = \begin{bmatrix} Z_{00} & Z_{01} & \ldots & Z_{0(M-1)} \\ Z_{10} & Z_{11} & \ldots & Z_{1(M-1)} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{(M-1)0} & Z_{(M-1)2} & \ldots & Z_{(M-1)(M-1)} \end{bmatrix}. \quad (1)$$

According to Ohm's law, the port current vectors for both circuits can be expressed as

$$\mathbf{i} = (\mathbf{Z} + \mathbf{Z}_L)^{-1} \mathbf{v}. \quad (2)$$

If the current vector $\mathbf{i}$, which is the function of $\mathbf{Z}$ and $\mathbf{Z}_L$ in (2), flows through the antenna elements, then the input impedance, Z_{in}, is given by [1]

$$Z_{in} = Z_{00} + \frac{\sum_{m=1}^{M-1} Z_{0m} i_m}{i_0}. \quad (3)$$

where Z_{ij} is the mutual coupling impedance between the i-th and j-th elements in an EA. Note that Z_{in} is a function of the MC values Z_{ij} and currents. The currents i_k depend on the voltage feeding v_0 and tunable loads z_k.

Based on circuit theory, the current flowing through the antenna element of an EA can be mathematically expressed as

$$\mathbf{i} = (\mathbf{Z} + \mathbf{Z}_L)^{-1} \mathbf{v}. \quad (4)$$

where, $\mathbf{i} = [i_0, i_1, \ldots, i_{M-1}]^T$ is a vector of currents at the antenna elements, $\mathbf{Z}_L = \mathrm{diag}(z_s, z_1, z_2, \ldots, z_{M-1})$ is the impedance matrix composed of z_s at the active element and the variable loads at its diagonal entries, and $V = [v_0, 0, 0, \ldots, 0]$ is the vector of voltages at the antenna elements.

B. System Description

Consider a point-to-point link consisting of a transmitter having an array of M antennas and a receiver having n_r antennas, the corresponding signal model is

$$\mathbf{y} = \mathbf{H}\mathbf{i} + \mathbf{n}, \quad (5)$$

where $\mathbf{y}$ is the received signal at the receiver, $\mathbf{H} \in \mathbb{C}^{n_r \times M}$ is the channel matrix, $\mathbf{i}$ is the vector of currents flowing through the transmit antennas, and $\mathbf{n} \in \mathbb{C}^{n_r \times 1}$ denotes the noise vector. The noise is assumed to be additive white Gaussian noise (AWGN) with zero mean and unit variance. In order to radiate the signal, the currents at the antenna element need to be varied based on the transmission signals [8], [9].

C. Power Consideration Using an EA

Using the equivalent circuit model of an EA, the power delivered to an EA can be mathematically expressed as

$$P_S = i_0^2 \Re\{Z_{in}\} = \left| \frac{v_0}{z_s + Z_{in}} \right|^2 \Re\{Z_{in}\}. \quad (6)$$

where Z_{in} denotes the input impedance of an EA, and $\Re\{Z_{in}\}$ and $\Im\{Z_{in}\}$ denote its resistive and reactive components, respectively. In order to guarantee stable transmission, the input power to the antenna element should be positive, which

implies the $\Re\{Z_{in}\}$ should be positive. If $\Re\{Z_{in}\}$ is not positive, it means that the EA is reflecting power back and exhibiting oscillatory/unstable behavior [10]. From [2] [11], the power supplied to EA from the active element is given as

$$P_S = \mathbf{w}^T \mathbf{A} \mathbf{w}, \tag{7}$$

where $\mathbf{w} = [w_1, w_2, w_3, w_4, \ldots, w_{2M-1}, w_{2M}]^T$ and its elements w_{2m+1} and w_{2m+2} denote the real part and the imaginary part of i_m, respectively. A is given as

$$\mathbf{A} = \begin{bmatrix} R_0 & 0 & \frac{R_1}{2} & \cdots & -\frac{X_{M-1}}{2} \\ 0 & R_0 & \frac{X_1}{2} & \cdots & \frac{R_{M-1}}{2} \\ \frac{R_1}{2} & \frac{X_1}{2} & 0 & \cdots & 0 \\ -\frac{X_1}{2} & \frac{R_1}{2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{R_{M-1}}{2} & \frac{X_{M-1}}{2} & 0 & \cdots & 0 \\ -\frac{X_{M-1}}{2} & \frac{R_{M-1}}{2} & 0 & \cdots & 0 \end{bmatrix}. \tag{8}$$

where R_m and X_m denote the real part and the imaginary part of the mutual coupling from the active element to the m-th parasitic element, Z_{0m}.

Using (3) and after some mathematical manipulations, the voltage feeding and load values can be calculated from $\mathbf{w}$ as

$$v_0 = 2 \sum_{j=0}^{M-1} Z_{0j} (w_{2j} + j w_{2j+1}), \tag{9a}$$

$$z_m = -\frac{\sum_{j=0}^{M-1} Z_{mj} (w_{2j} + j w_{2j+1})}{(w_{2m} + j w_{2m+1})}. \tag{9b}$$

III. ALGORITHM FOR SIGNAL TRANSMISSION UNDER LIMITED POWER

A. Problem Formulation

For signal transmission, the currents at the antenna element need to be varied based on input symbols. This variation in current is achieved by varying the loads at the parasitic elements and the voltage feeding at the active element. In some cases, for a certain transmission signal, it is possible that the voltage and loadings take values which leads to a negative input resistance, causing an EA to exhibit unstable behaviour. For transmission of such signals, it was proposed to transmit signals closely approximating the ideal signal to keep the EA stable [2]. However, no limit on the power of an EA was assumed. This is not the case in real world systems which always have limited power. Thus, in this sequel, we ensure stability of an EA with limited power.

The problem to obtain the values of the voltage and the loadings can be formulated as an optimization problem to minimize the MSE between the currents corresponding to the ideal and approximate transmission signals, and is given as

$$\min_{v_0, \mathbf{Z}_L} \quad \left\| \hat{\mathbf{i}} - (\mathbf{Z} + \operatorname{diag}(z_s, z_1, z_2, \ldots, z_{M-1}))^{-1} [v_0, 0, \ldots, 0] \right\|_2 \tag{10a}$$

$$\text{st.} \quad P_S \geq P^{min}, \tag{10b}$$

$$P_S \leq P^{max}, \tag{10c}$$

where $\hat{\mathbf{i}}$ denotes the desired current vector corresponding to ideal signal required to be transmitted by the EA, P^{min} and P^{max} is the minimal input power and the saturation power of the power amplifier, respectively. The objective of this optimization problem is to find the voltage feeding v_0 and the loads $z_1, z_2, \ldots, z_{M-1}$ to minimize MSE between the ideal and approximate signal, the constraint (10b) is similar to the constraint in [2] and ensures that the input resistance is positive and that the EA does not exhibit unstable behaviour. The constraint (10c) is included to guarantee that the transmit power does not exceed the maximum power level supported by the transmitter. These constraints result in a non-convex optimization problem.

B. Power Consumption for an EA and Problem Reformulation

The optimization problem can be reformulated and represented in terms of real and imaginary part of current in the antenna elements as

$$\min_{\mathbf{w}} \quad \|\mathbf{w} - \hat{\mathbf{w}}\|^2, \tag{11a}$$

$$\text{st.} \quad \mathbf{w}^T \mathbf{A} \mathbf{w} > P_{min}, \tag{11b}$$

$$\mathbf{w}^T \mathbf{A} \mathbf{w} \leq P_{max}. \tag{11c}$$

where $\hat{\mathbf{w}} = [\hat{w}_1, \hat{w}_2, \hat{w}_3, \hat{w}_4, \ldots, \hat{w}_{2M-1}, \hat{w}_{2M}]^T$, where $\hat{w}_{2m+1}$ and $\hat{w}_{2m+2}$ denote the real part and the imaginary part of $\hat{i}_m$, respectively.

It can be noted that there is a quadratic objective function and two quadratic constraints in this problem. Moreover, the number of optimization variables is $2M$. In addition, $\mathbf{A}$ is an indefinite matrix, as shown in proposition below. Thus, the optimization problem (11) is non-convex.

Proposition 1 $\mathbf{A}$ *is an indefinite matrix and the eigenvalues of* $\mathbf{A}$ *are*

$$\begin{cases} \lambda_1 = \lambda_2 = R_0 - \sqrt{R_0^2 + \sum_{m=1}^{M-1} (R_m^2 + X_m^2)} < 0 \\ \lambda_{2M-1} = \lambda_{2M} = R_0 + \sqrt{R_0^2 + \sum_{m=1}^{M-1} (R_m^2 + X_m^2)} > 0 \\ \lambda_n = 0 \quad for \ n = 3, 4, \ldots, 2M - 2 \end{cases} \tag{12}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_{2M-1}, \lambda_{2M}$ *denotes the eigenvalues of the matrix* $\mathbf{A}$ *in ascending order.*

Proof: The proof is provided in [11]. ∎

Due to the non-convex nature of the constraint set, the strong duality employed in [2] and [12], cannot be applied to this problem. Instead we use coordinate transformation and geometric method to solve the optimization problem.

As $\mathbf{A}$ is a real symmetric matrix and it can be diagonalized as $\mathbf{A} = \mathbf{Q} \mathbf{\Lambda}_A \mathbf{Q}^T$, where $\mathbf{\Lambda}_A = [\lambda_1, \lambda_2, \ldots, \lambda_{2M-1}, \lambda_{2M}]$ is a real diagonal matrix with its elements are eigenvalues of $\mathbf{A}$, the columns of the real orthogonal matrix $\mathbf{Q}$ are corresponding eigenvectors. In order to simplify the optimization problem in (11), we introduce two vectors $\mathbf{e} = [e_1, e_2 \ldots, e_{2M-1}, e_{2M}]^T$ and $\mathbf{g} = [g_1, g_2, \ldots, g_{2M-1}, g_{2M}]^T$, where $\mathbf{e} = \mathbf{Q}^T \mathbf{w}$, $\mathbf{g} = \mathbf{Q}^T \hat{\mathbf{w}}$. The optimization problem can be simplified in the following proposition.

Proposition 2 *The optimization problem in* (11) *can be reformulated as*

$$\min_{\bar{\mathbf{e}}} \quad \|\bar{\mathbf{e}} - \bar{\mathbf{g}}\|^2, \tag{13a}$$

$$st. \quad \bar{\mathbf{e}}^T \operatorname{diag}\left(\lambda_1, \lambda_2, \lambda_{2M-1}, \lambda_{2M}\right) \bar{\mathbf{e}} > P_{min}, \tag{13b}$$

$$\bar{\mathbf{e}}^T \operatorname{diag}\left(\lambda_1, \lambda_2, \lambda_{2M-1}, \lambda_{2M}\right) \bar{\mathbf{e}} \leq P_{max}, \tag{13c}$$

where $\bar{\mathbf{e}} = [e_1, e_2, e_{2M-1}, e_{2M}]^T$ *and* $\bar{\mathbf{g}} = [g_1, g_2, g_{2M-1}, g_{2M}]^T$, *and the minimum is obtained when* $e_m = g_m$, for $m = 3, 4, \ldots, 2M - 2$.

Proof: The proof is provided in [11]. ∎

It can be noted that the optimization problem in (13) is more simplified and has 4 optimization variables instead of $2M$.

C. Solution of the Optimization Problem

The optimization problem in (13) can be further simplified using coordinate transformation. Representing the elements of $\bar{\mathbf{e}}$ and $\bar{\mathbf{g}}$ into polar coordinate system as $e_1 = r_a \cos\theta_a$, $e_2 = r_a \sin\theta_a$, $e_{2M-1} = r_b \cos\theta_b$, $e_{2M} = r_b \sin\theta_b$, $g_1 = r_c \cos\theta_c$, $g_2 = r_c \sin\theta_c$, $g_{2M-1} = r_d \cos\theta_d$, $g_{2M} = r_d \sin\theta_d$, respectively, where $r_a = \sqrt{e_1^2 + e_2^2}$, $\theta_a = \arctan\left(\frac{e_2}{e_1}\right)$, $r_b = \sqrt{s_{2M-1}^2 + e_{2M}^2}$, $\theta_b = \arctan\left(\frac{e_{2M}}{e_{2M-1}}\right)$, $r_c = \sqrt{g_1^2 + g_2^2}$, $\theta_c = \arctan\left(\frac{g_2}{g_1}\right)$, $r_d = \sqrt{g_{2M-1}^2 + g_{2M}^2}$, and $\theta_d = \arctan\left(\frac{g_{2M}}{g_{2M-1}}\right)$. Using this coordinate transformation, the problem can be further reformulated as shown in the following proposition.

Proposition 3 *By replacing Cartesian coordinate with polar system in* (13), *the objective function* $\|\bar{\mathbf{e}} - \bar{\mathbf{g}}\|^2$ *achieves its minimal value when when* $\theta_a = \theta_c$, $\theta_b = \theta_d$, *and* θ_a, θ_b, *and* e_m, *for* $m = 3, 4, \ldots, 2M - 2$.

Therefore, the problem in (13) *can be further simplified as*

$$\min_{r_a, r_b} \quad r_a^2 + r_b^2 - 2r_a r_b - 2r_c r_d + r_c^2 + r_d^2, \tag{14a}$$

$$st. \quad \lambda_1 r_a^2 + \lambda_{2M-1} r_b^2 \geq P_{min}, \tag{14b}$$

$$\lambda_1 r_a^2 + \lambda_{2M-1} r_b^2 \leq P_{max}. \tag{14c}$$

Proof: The proof is provided in [11]. ∎

It can be noted from Proposition 3 that the optimal value of θ_a and θ_b are obtained. Therefore, the number of optimization variables has been reduced to two and it is required to obtain values of r_a and r_b which minimizes (14a) under the constraints (14b) and (14c). The constraint set can be written as (15). Note that $\lambda_1 r_a^2 + \lambda_{2M-1} r_b^2 = P_{min}$ and $\lambda_1 r_a^2 + \lambda_{2M-1} r_b^2 = P_{max}$ are two hyperbolas with same asymptotes and different focus points. The constraint set is the area between the hyperbola $\lambda_1 r_a^2 + \lambda_{2M-1} r_b^2 = P_{min}$ and hyperbola $\lambda_1 r_a^2 + \lambda_{2M-1} r_b^2 = P_{max}$, which is not a convex set.

The optimization problem in (14) can be restated to find the optimal (r_a, r_b) to minimize the Euclidean distance between two points (r_a, r_b) and (r_c, r_d) when $(r_a, r_b) \in \mathbb{S}_1$. From the geometrical perspective, the problem is to find the distance from a point $\mathbf{r}_g = (r_c, r_d) \in \mathbb{R}^2$ to the set $\mathbb{S}_1$. Thus, it can be expressed as

$$\mathbf{dist}\left(\mathbf{r}_g, \mathbb{S}_1\right) = \inf\left\{\|\mathbf{r}_g - \mathbf{r}_e\| \mid \mathbf{r}_e \in \mathbb{S}_1\right\}. \tag{16}$$

This optimal point for this optimization problem is the projection of $\mathbf{r}_g$ on the hyperbola $\left\{r_e = (r_a, r_b) \mid \lambda_1 r_a^2 + \lambda_{2M-1} r_b^2 = P_{min}\right\}$ or $\left\{r_e = (r_a, r_b) \mid \lambda_1 r_a^2 + \lambda_{2M-1} r_b^2 = P_{max}\right\}$. Various methods are discussed in [13]. Considering the complexity and robustness, the approach which combines the bisection and Newton's method [13] is utilised to solve this problem. The bisection method is used to reduce the search area and then Newton's method is applied to finding the optimal point.

IV. NUMERICAL RESULTS

The SER performance of the system where the transmitter employs Alamouti code is shown in Fig 3. The modulation scheme is 16-QAM and the SER performance of an EA using our proposed algorithm is compared to the SER performance of a standard multiple antenna transmitter. The EA using our proposed algorithm is denoted by EA-P and the standard multiple antenna transmitter is denoted by SMA. For the simulation, we select three practical 2-element EAs with MC matries, $\mathbf{Z}^{(1)}$, $\mathbf{Z}^{(2)}$, and $\mathbf{Z}^{(3)}$, which has been considered in [1] and [14]. Without loss of generality $P_{min} = 0$ and $P_{max} = \{50, 100, 150\}$.

For an EA with $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{(2)}$, the SER performance of EA with our proposed algorithm matches the performance of the standard multiple antenna system at low P_T. As P_T increases, the power consumed by the antenna approaches P_{max} and an error floor occurs. The SER cannot be further reduced due to maximal power constraint P_{max}. The EA with $\mathbf{Z}^{(3)}$ was designed to overcome the stability problem for an EA transmitter in [1]. However, considering the maximal power requirement of EA, it consumes large power as the self-resistance at the active element is large. Due to this large self-resistance, in order to meet the power constraint the symbol transmission power is reduced which results in significantly degraded performance. This shows that achieving stability by increasing the self-resistance is highly power inefficient approach. Moreover, as P_{max} increases, the EA can transmit with more power and thus, the SER reduces.

When the transmitter has CSI, transmit diversity can be achieved by employing maximal ratio transmission (MRT) [15]. Assuming that symbol s is to be transmitted, the symbols are precoded and mapped to the antenna currents. Let $\mathbf{h} = [h_0, h_1, \ldots, h_{M-1}]^H \in \mathbb{C}^{M \times 1}$ and h_m denotes the channel from the $(m+1)$-th element of EA transmitter to the signal antenna receiver. h_m is a Rayleigh random variable with unit variance.

Considering MRT scheme, the SER performances are compared for EAs with different number of elements and spacings in Fig. 4. It can be observed that the SER performance of EAs with MCM $\mathbf{Z}_1$ and $\mathbf{Z}_2$ is similar to that of the standard multiple antenna transmitter especially at low SNRs. Similar to Fig. 3 as P_T increases, the power consumed by the antenna

$$\mathbb{S}_1 = \left\{ \mathbf{r}_e = (r_a, r_b) \mid \lambda_1 r_a^2 + \lambda_{2M-1} r_b^2 \geq P_{min}, \lambda_1 r_a^2 + \lambda_{2M-1} r_b^2 \leq P_{max} \right\}. \tag{15}$$
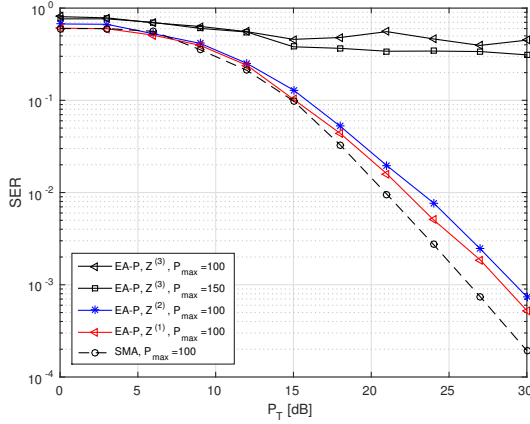
Fig. 3: SER performance comparison of the ESPAR transmitter and the standard multiple antenna transmitter employing Almouti scheme with 16-QAM modulation.
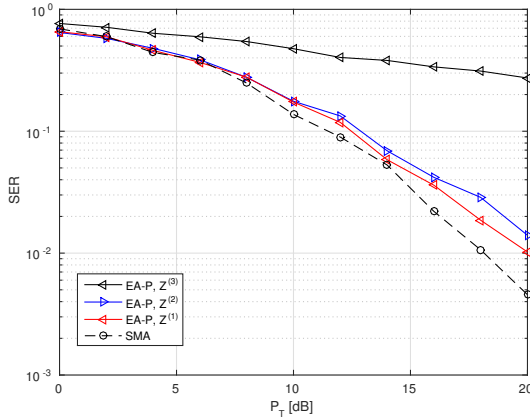
Fig. 4: SER performance comparison of the EA transmitter and the standard multiple antenna transmitter employing MRT scheme with 16-QAM modulation.

approaches P_{max} and an error floor occurs. However, again, with the maximal power constraint, the EA with $\mathbf{Z}^{(3)}$ is unable to achieve SER close to the SER of a standard multiple antenna system.

V. CONCLUSION

Considering limited power availability, we have proposed a new algorithm to achieve stable signal transmission using an EA. To transmit signals that might lead to power consumption beyond the limits of the transmitter and might lead to oscillatory/unstable behaviour of an EA transmitter, we have proposed a new method to formulate and solve this problem. Signals closely approximating the ideal signals are transmitted. Moreover, coordinate transformations has been

applied to find the optimal approximate signals. The SER performance of our proposed algorithm has been compared to that of a standard multiple antenna transmitter for Alamouti coded transmission and the maximum ratio transmission in the single-user scenario. Our results have shown that a system employing an EA transmitter and using our proposed algorithm gives performance similar to a system with a standard multiple antenna transmitter. In addition, it has been shown that improving the stability by increasing self-resistance [1] is inefficient and infeasible.

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