A Speed Convergence Least Squares Constant Modulus Algorithm For Smart Antenna Beamforming

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***Abstract* — Adaptive algorithms used in smart antenna systems have been thoroughly studied along the years. Well known algorithms like the LMS (Least Mean Squares) were improved and other more performant algorithms were created. An example is the VSSLMS (Variable Step Size LMS) algorithm, with Kwong and Aboulnasr variants. The LMS class of algorithms uses a reference signal in order to reach convergence. Other class of algorithms like the CMA (Constant Modulus Algorithm) does not use a reference signal and has a good performance for signals with a constant envelope. In this paper we propose a hybrid blind equalization algorithm based on the combination of the LMS and CMA algorithms plus a speed convergence module that provides a very fast convergence speed. We used this algorithm with a beamforming smart antenna network in order to point the main beam of the radiation pattern in certain directions.**

***Keywords* — Smart Antenna Systems; Least Mean Squares; Constant Modulus; Speed Convergence.**

I. Introduction

Smart antenna systems are used in areas like mobile communications, radar and even medicine. Such a system can be for example very efficient in detecting and locating underwater signal sources. This eliminates the use of active sonar. The technology is also very useful for anti-jamming applications. In our case a smart antenna system manages to create a beam pattern with the main lobe pointing in a specific direction with the help of an adaptive algorithm. At reception this algorithm processes the signals received by each element of the antenna array by multiply them with a set of complex weights. This way the main lobe of the radiation pattern is steered in a certain direction. We consider a linear antenna array made up of a number of equally spaced antennas. For a signal source in a certain direction in the far field, the wave traveling form the source to the antenna array is planar. This wave first reaches one of the elements and after that it reaches the next one with a slight delay in time and so on. Processing these delays is the job of the adaptive algorithm. These delays translate into phase difference. Each signal that arrives at the array is delayed with a certain phase and the adaptive processor multiplies each of these signals with a certain weight.

The radiation pattern will have maximum and minimum values in particular directions defined by the user. With the help of an antenna array with L elements we can create L-1 shapes for the radiation pattern with a theoretical maximum in the direction of the desired signal and nulls in the direction of the interfering signals.

The LMS algorithm is a non blind adaptive algorithm which uses a reference signal in order to reach convergence. This means it uses a training sequence to obtain an error. By minimizing this error we obtain the convergence. The error is minimized progressively. The algorithm is run for a number of iterations. With every iteration another set of weights is computed and the error gets smaller. On the other hand a blind equalization algorithm doesn't need a training sequence [10]. This is very useful in communication channels with limited bandwidth at our disposal where we do not want to occupy an important part of it with a training signal. The downside of the CMA algorithm is that it uses a lot of processing power. The downside of the LMS algorithm is that it needs a training sequence.

The idea in this paper is to take the CMA algorithm, use it to obtain a blueprint of a so called reference signal and use it for training the LMS algorithm. Good results were obtained. What we propose in the end though, is the use of a speed convergence module based on a deviation of Steffensen's method of speeding convergence. The Aitken's delta squared process which is a deviation of Steffensen method uses a function that has already been previously used with LMS as stated in [2]. We thought of combining it with the LSCMA (Least Squares Constant Modulus Algorithm) to see the results obtained for an algorithm that does not need a training sequence.

II. Adaptive Algorithms

A. The LMS Algorithm

The LMS algorithm is a well known algorithm. It works by minimizing the mean square error between the received signal and the reference signal. To be brief we will present only the main equations behind its functionality. Vectors will be used in our expressions [6]. The input signal is a vector consisting of all the input signals at each antenna element (L is the number of array elements):

$$x = [x_1\ x_2\ \dots\ x_L] \quad (1)$$

The weights vector has the following form:

$$w = [w_1\ w_2\ \dots\ w_L] \quad (2)$$

The output signal at instant k will be:

$$z(k) = \sum_{i=0}^{L-1} w_i(k) x_i(k-i) = w_{LMS}{}^H(k) x(k) \quad (3)$$

It is a product between the input signals vector and the transpose conjugate of the complex weights vector. The computed error is:

$$e(k) = d(k) - z(k) = d(k) - w_{LMS}{}^{H}(k)x(k) \qquad (4)$$

The signal d(k) above is the reference signal. In the end the weights update equation is:

$$w_{LMS}(k+1) = w_{LMS}(k) + \mu e(k)x(k) \qquad (5)$$

This is an iterative method. The weights obtained at iteration k+1 are computed with the help of the previous weights obtained at iteration k. As it can be seen from the set of equations above, x and w are vectors, y, e and d are scalars and μ is the step size. The step size influences the behavior of the algorithm. Choosing a small value for this parameter will take the algorithm a higher number of iterations to reach convergence but the precision will be in this case better. Choosing a higher value will speed up convergence paying a price in precisions.

B. The CMA Algorithm

This algorithm obtains a good performance if used on communication channels over which signals with modulations like PSK, BPSK, GMSK or QPSK [6] are sent. So the algorithm is best used with signals that have a constant envelope. On the other hand it requires a high amount of processing, and the convergence in case of the standard CMA algorithm is reached in a longer time than in the case of a non blind equalization algorithm. This algorithm does not introduce nulls in the radiation pattern. One of the ways to speed up convergence is by using the Normalized CMA. We consider x(k) the input signal. The output signal will be:

$$y(k) = w_{CMA}{}^{H}(k)x(k) \qquad (6)$$

The weights update equation is similar to the one used in LMS's case, except the error is computed from the actual received signal, not a training sequence. This equation has the following form:

$$w_{CMA}(k+1) = w_{CMA}(k) - \mu x(k)(|y(k)|^2 - 1)y^H(k) \qquad (7)$$

where H denotes the transpose conjugate.

For the NCMA algorithm the update equation is:

$$w_{NCMA}(k+1) = w_{NCMA}(k) - \frac{\mu}{x(k)x^*(k)}\Big(y(k) - y(k)|y(k)|yH(k) \qquad (8)$$

C. The SCLSCMA (Speed Convergence Least Squares Constant Modulus Algorithm)

As we stated earlier the proposed algorithm combines the CMA algorithm and the LMS algorithm together with a speed convergence module. This speed convergence module works on the principle of Aitken's method. This method proposed by Aitken is a root finding method. By defining a function and applying the method for finding the weights of the array, we will considerably increase the time needed to reach convergence [2].

The vector containing the weights computed by the LSCMA algorithm will be the input to the speed convergence module. First we will use the CMA algorithm in order to obtain a blueprint of a reference signal that we will use with the LMS algorithm.

The LSCMA algorithm computes the weights in the following manner:

$$y(k) = w_{CMA}{}^{H}(k)x(k) \qquad (9)$$

$$w_{CMA}(k+1) = w_{CMA}(k) - \mu x(k)(|y(k)|^2 - 1)y^H(k) \qquad (10)$$

$$z(k) = w_{LSCMA}{}^{H}(k)x(k) \qquad (11)$$

$$e(k) = d(k) - z(k) = d(k) - w_{LSCMA}{}^{H}(k)x(k) \qquad (12)$$

$$w_{LSCMA}(k+1) = w_{LSCMA}(k) + \mu e(k)x(k) \qquad (13)$$

The weights are updated by the following equation as part of Aitken's method [2]:

$$\zeta(w_{LSCMA}(k)) = M_k w_{LSCMA}(k) + N_k \qquad (14)$$

where

$$M_k = [I - 2\mu R_x]^{-1} \ , \ N_k = 2\mu R_x \qquad (15)$$

and R_x is an estimate of the array autocorrelation matrix computed recursively:

$$R_x = x(k)x^H(k) \qquad (16)$$

The value for the step size must be less than:

$$\mu = \frac{1}{Tr(R_x)} \qquad (17)$$

Tr(Rx) is the trace of the autocorrelation matrix, the sum of all the elements on the main diagonal of a matrix.

In the end, the weights values are speeded up to convergence by the following equation:

$$w_{sc} = \zeta(\zeta(w_{LSCMA}(k))) - \frac{\zeta(\zeta(w_{LSCMA}(k))) - \zeta(w_{LSCMA}(k)))^2}{\zeta\big(\zeta(w_{LSCMA}(k))\big) - 2\zeta(w_{LSCMA}(k))) + w_{LSCMA}(k)} \qquad (18)$$

The output signal for the whole system will be:

$$u(k) = w_{SC}{}^{H}(k)x(k) \qquad (19)$$

D. Block diagram and functionality of a SCLSCMA system

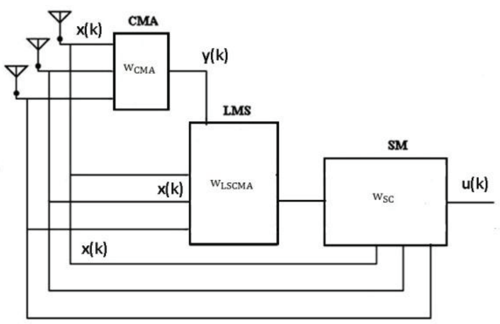

Figure 1. Block diagram of a SCLSCMA system

In figure 1 the whole adaptive circle can be seen. As mentioned before the reference signal for the LMS module is obtained from the CMA module. This signal is inputted together with the received user's signal into the LMS. This way we obtain the weights that we will use with the speed convergence module. As the whole hybrid algorithm is ran for a number of iterations we obtain a set of weights for each turn at the output of the LMS module. The idea is to get closer to the optimum weights in a faster way without the need of running the algorithm for a high number of iterations. This is done by the speed convergence module using Aitken's method of root finding.

In the next chapter we will simulate each of the adaptive algorithms described, independently and in the end we will simulate the hybrid algorithm proposed. After comparing the results obtained for each algorithm some advantages and disadvantages will be pointed for each case.

III. SIMULATIONS

The MATLAB environment is used in order to observe the behavior of the adaptive algorithms described in the above chapters.

The antenna array used in simulations is made up of 8 elements. The signals used will be randomly generated and the chosen frequency is 2.4GHz. This is the most widely used frequency in wireless communications. The final result will denote an increase in convergence speed. That is why the main point of interest in the simulations will be the number of iterations (loops) the algorithm is ran for, in order to reach adaptation. We mention that both the algorithms and the speed convergence module update their parameters in a same loop.

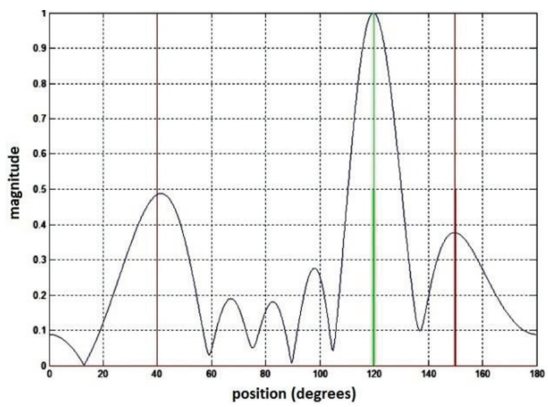

Figure 2. The radiation pattern in cartesian coordinates system for the LMS algorithm and a cycle of 10 iterations

In figure 2 the user's signal position is at 120 degrees. The interference signals are at 40 and 150 degrees. The step size has the value $\mu = 0.05$. For this value, the LMS algorithm reached convergence after approximately 90 iterations. After this number of iterations we can say that nulls are formed in the radiation pattern at angles corresponding to the interference positions.

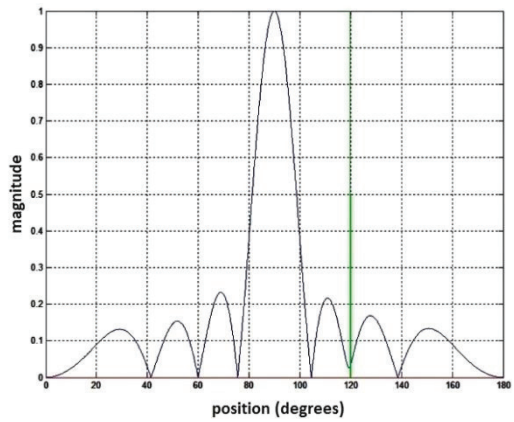

Figure 3. The radiation pattern in cartesian coordinates system for the CMA algorithm and 10 iterations

The radiation pattern shown in figure 3 is for the CMA algorithm that was run for a number of 10 iterations. We can see that after 10 iterations the convergence is far from being achieved. The step size has the value $\mu = 0.05$.

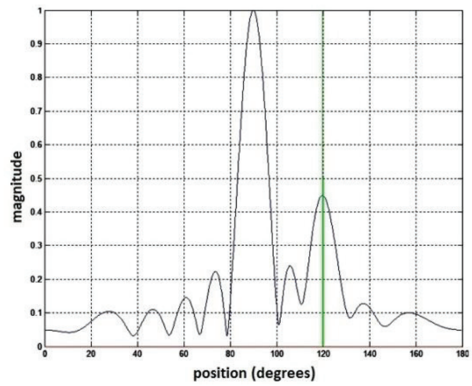

Figure 4. The radiation pattern in cartesian coordinates system for the NCMA algorithm and a cycle of 10 iterations

Figure 4 shows that in the case of the NCMA algorithm, the normalized version of the CMA algorithm, the user's signal direction is detected sooner, so the convergence is reached after a smaller number of iterations.

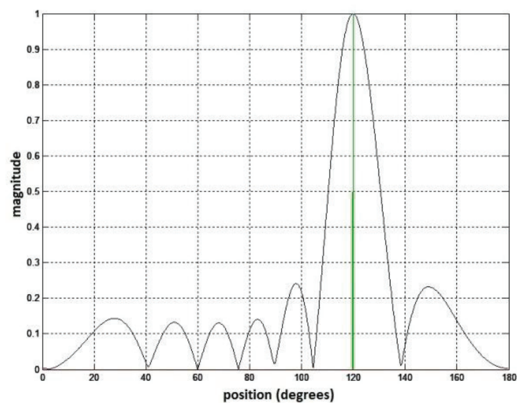

Figure 5. The radiation pattern in cartesian coordinates system for the CMA algorithm and a cycle of 80 iterations

The CMA algorithm reaches convergence after approximately 80 iterations for a value of the step size μ = 0.05. The NCMA algorithm converges for the same value of the step size, up to 4 times faster.

The NCMA algorithm is evidently faster compared to the CMA algorithm but it still needs to be run for at least 30 times in order to achieve convergence. Another solution would be the use of OCMA (Orthogonal Constant Modulus Algorithm) or the use of a block update for the CMA algorithm. In either case the algorithm can in no case outperform a combined algorithm as we will see below.

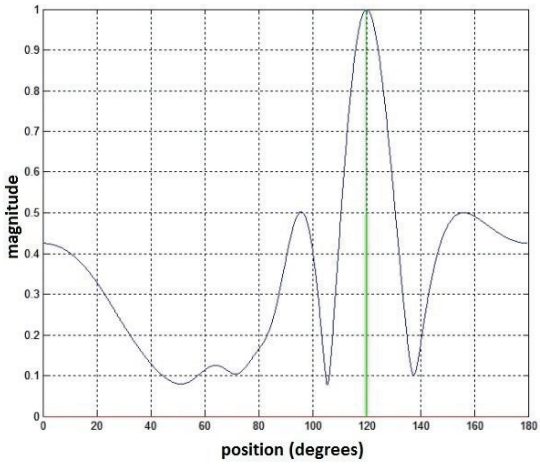

Figure 6. The radiation pattern in cartesian coordinates system for the LSCMA algorithm and a cycle of 10 iterations

It is obvious from figure 6 that the LSCMA algorithm performs better compared to the NCMA algorithm for the same number of iterations. The level of the side lobs is still high tough.

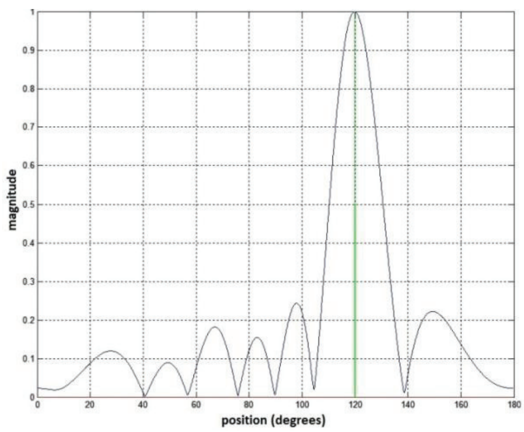

Figure 7. The radiation pattern in cartesian coordinates system for the SCLSCMA algorithm and a cycle of 10 iterations

In figure 7 the radiation pattern obtained in case of the proposed SCLSCMA algorithm shows the improvement in convergence speed. Although more complex, the convergence speed is better than in the case of any of the algorithms in the CMA class. For the CMA and the LMS algorithms we used a step size μ = 0.5. This high value for the step size is used to obtain faster, a set of values for the weights that are to be used by the speed convergence module. If a smaller value is used for the step size a better precision is obtained but the time in which the convergence takes place increases. The combined algorithms were run in the same loop for all iterations. We used a 20dB value for the SNR (Signal to Noise Ratio) in all simulations. The proposed algorithm obtained good performance for low values of the SNR also.

IV. Conclusions

As the results clearly state, the performance of the proposed hybrid algorithm is high. The speed with which the adaptation is obtained is higher compared to the other adaptive algorithms described. The algorithm is faster but of course comes with the other disadvantages of using a blind equalization algorithm. These kinds of algorithms are mainly used for signals with constant envelope such as PSK, BPSK, QPSK, MSK, and GMSK. A synchronization sequence must be used in order to discriminate between signals with the same modulation in case of using a blind equalization algorithm.

For further studies we propose the use of the speed convergence module in front of the LMS module and compare the results with the results obtained for the algorithm described in this paper.

Three step size parameters are used, one for each module. A study to determine the optimum values for each parameter and for different values of the SNR can be a subject of further investigation. These values for the step sizes must be chosen carefully as misadjustment can occur and the proposed algorithm might diverge instead of reaching adaptation.

References

[1] Abramowitz, M. şi Segun, "Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables", Dover, New York, 1972

[2] H.V Kumaraswamy, Aaquib Nawaz.S, Syed Younus, “A New Turbo LMS Beamforming for Mobile Communication”, dec. 2010

[3] Roy Y., Chouinard J.Y. şi Mahmoud S.A., "Selection diversity combining with multiple antennas for MM-wave indor wireless channels", IEEE J. Selected Areas Commn., 14, 674-682, 1996

[4] Schiff L., "Statistical supression of interference with diversity in a mobile-radio environement", IEEE Trans. Vehicular Technol., 21, 121-128, 1972

[5] Shah A. şi Haimovich A.M., "Performance analysis of maximum ratio combining and comparison with optimum combining for mobile radio communications with cochannel interference", IEEE Trans. Vehicular Technol.,49, 1454-1463,2000

[6] Godara, Lal Chand., "Smart Antennas", CRC Press Lcc, 2004

[7] Tomiuk, B.R., Beaulieu N.C. şi Abu-Dayya A.A., "General forms for maximal ratio diversity with weighting errors", IEEE Trans. Commn.,47, 488-492,1999

[8] Winters J.H., "Optimum combining in digital mobile radio with cochannel interference", IEEE J. Selected Areas Commn., 2, 528-539, 1984

[9] C.A. Balanis, "Antenna Theory: Analysis and design", ediția a 3-a, Wiley, 2005

[10] John Litva, Titus Kwok-Yeung Lo, "Digital Beamforming in wireless Communications",Artech, 1996

[11] M.T. Ma, "Theory and Application of Antenna Arrays", John Eiley & Sons, New York, 1974

[12] S. Applebaum, "Adaptive arrays", Technical Report SPL TR-66-001, Syracuse Univ, 1965

[13] W.F. Gabriel, "Adaptive arrays - an introduction", Proc. IEEE, vol.64, 239-272, 1976

[14] Jian Li şi Petre Stoica, "Robust Adaptive Beamforming", Wiley, 2006