Reactively Controlled Directive Arrays

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Abstract—The radiation characteristics of an N-port antenna system can be controlled by impedance loading the ports and feeding only one or several of the ports. Reactive loads can be used to resonate a real port current to give a radiation pattern of high directivity. The theory of resonance is extended to include complex port currents and impedance loads. The initial design of an array is obtained by resonating a desired port current vector, which is then improved by an optimum seeking univariate search method. The direction of maximum gain can be controlled by varying the load reactances. Several numerical examples are given for a circular array of seven dipole elements.

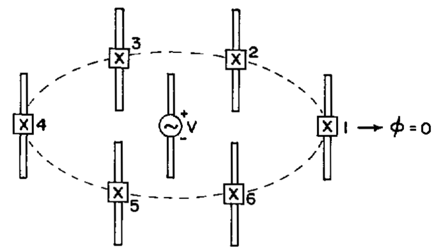

Fig. 1. A seven-element circular array of reactively loaded dipoles.

I. INTRODUCTION

A REACTIVELY loaded antenna is defined to be an N-port radiating system with reactive elements at the ports. The excitation of the antenna is taken to be a voltage source at one of the ports. For the theory the antenna structure can be viewed as completely arbitrary, while for the examples the antenna is considered to be a wire structure. A particular case of a reactively loaded array is shown in Fig. 1. It consists of six reactively loaded dipoles equispaced on a circle, plus a central dipole which is fed. Another special case of a reactively loaded array is a linear array of dipoles with all dipoles reactively loaded, and one or more dipoles excited by a source.

A reactively controlled antenna array is one for which the reactive loads are varied to control the antenna radiation characteristics. We consider in detail the reactive control of the array of Fig. 1 to obtain a directive beam which can be steered in various directions. Another case that has been studied is the reactive control of linear arrays [1], but we do not consider that case in detail here. A special case of the linear array is the reactively loaded Yagi–Uda array studied by Bojsen *et al.* [3], [4]. They used an optimum seeking method (Rosenbrock [5]) to find the maximum gain as the load reactances were varied. Their procedure finds a local optimum point, which may or may not be a global optimum.

A closely related problem is that of reactively loading an N-port scatterer to control its electromagnetic scattering properties [6], [7]. A procedure for optimizing the radar cross section by reactive loading was developed [8]. The theory is basically the same as that for optimizing the gain of a reactively loaded antenna system, as will be shown in this paper. When the ports of the radiating system (antenna or scatterer) are close together with respect to wavelength, exceptionally high values of gain or cross section can be obtained. This is basically the phenomenon of supergain, which has been studied extensively over the years. (For a bibliography or supergain phenomena, see [9].) Unfortunately, supergain antennas are extremely sensitive to variations in frequency, excitation, geometry, and other parameters. They also tend to have high power loss due to imperfect conductors and dielectrics [10]. For these reasons it is impractical to design antennas with more than moderate amounts of supergain.

Manuscript received August 24, 1976; revised March 5, 1977. This work was supported by the Office of Naval Research under Contract N00014-76-C-0225.

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II. FORMULATION OF THE PROBLEM

The analysis will be carried out in terms of N-port impedance parameters. Fig. 2 shows the Thevenin equivalent of an active feed system connected to an N-port antenna system. For the particular problem of Fig. 1, the feed network consists of lumped reactances across each port of the antenna system, and a voltage source at just one port. All coupling to the other ports occurs through the mutual impedances of the antenna system. The terminal equation for the feed and antenna system of Fig. 2 is

$$\vec{V}^{oc} = [Z_A + Z_L]\vec{I} \tag{1}$$

where $\vec{V}^{oc}$ and $\vec{I}$ are the column vectors of the Thevenin equivalent voltages and port currents, respectively. The matrices $[Z_A]$ and $[Z_L]$ are the open circuit impedance matrices of the antenna system and feed system, respectively. The electric field $\mathbf{E}$ radiated by the antenna is the superposition

$$\mathbf{E} = \sum_{n=1}^{N} I_n \mathbf{E}_n{}^{oc} \tag{2}$$

where $\mathbf{E}_n{}^{oc}$ is the field radiated when a unit current exists at port n, and all other ports are open circuited. In matrix form (2) can be written as

$$\mathbf{E} = \tilde{\mathbf{E}}^{oc}\vec{I} \tag{3}$$

where $\tilde{\mathbf{E}}^{oc}$ is the row vector of the $\mathbf{E}_n{}^{oc}$. Equation (1) can be solved for the port current, and the result substituted into (3) to obtain

$$\mathbf{E} = \tilde{\mathbf{E}}^{oc}[Z_A + Z_L]^{-1}\vec{V}^{oc}. \tag{4}$$

The current on the antenna system may also be of interest. It is linearly related to $\mathbf{E}$, and hence must also be of the form (4),

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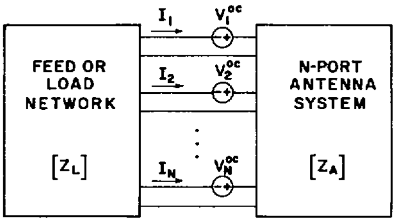

Fig. 2. General Thevenin equivalent of feed system with sources connected to N-port antenna system.

or

$$\mathbf{J} = \tilde{\mathbf{J}}^{oc}[Z_A + Z_L]^{-1}\vec{V}^{oc}. \tag{5}$$

Here $\tilde{\mathbf{J}}^{oc}$ is the row vector with elements $\mathbf{J}_n{}^{oc}$, the current on the antenna when a unit current exists at port n and all other ports are open circuited.

The above formulation is general, applying to any feed network connected to any antenna system. For the special case of Fig. 1, the feed matrix $[Z_L]$ is simply a diagonal matrix with diagonal elements jX_n. The problem can be equally well formulated in terms of admittance parameters, in which case equations dual to (1)-(5) apply. This admittance formulation is given in detail for loaded scatterers in [11]. The analysis of this reference applies equally well to antenna systems.

The network parameters needed in the theory can be either measured or calculated. When constructing an actual antenna system it would be desirable to measure the parameters, since computations are usually only approximate. For testing the theory it is convenient to use computed parameters. These can be computed to various approximations in a number of ways. Two possible methods are moment methods with triangular expansion functions [12], and the induced EMF method with assumed sinusoidal current [13]. The first method is the more accurate one, but the second method gives good answers when the wire dipoles are a half wavelength or less in length.

III. MODES, RESONANCE, AND GAIN OPTIMIZATION

The general appraoch taken is to determine the current required for maximum gain, and then to resonate that current. This is most conveniently done in terms of the characteristic modes for the N-port system. These are the same modes as used for N-port scatterers [11]. The general theory is given in [11], and only the important concepts will be summarized here.

The impedance matrix in (1) is first expressed in Hermitian parts as

$$[Z_A + Z_L] = [R] + j[X] \tag{6}$$

where $[R]$ is called the resistance matrix and $[X]$ the reactance matrix. If $[Z]$ is symmetric, then $[R]$ and $[X]$ are real and symmetric. The characteristic modes are defined by

$$[X]\vec{I}_n = \lambda_n[R]\vec{I}_n \tag{7}$$

where $\vec{I}_n$ are the port eigencurrents and λ_n are the eigenvalues. Equation (7) is a real symmetric eigenvalue equation, hence all eigenvalues λ_n are real and all eigencurrents $\vec{I}_n$ are real or equiphasal. The eigencurrents satisfy the following orthogonality relationships:

$$\tilde{I}_m{}^*[R]\vec{I}_n = \delta_{mn} \tag{8}$$

$$\tilde{I}_m{}^*[X]\vec{I}_n = \delta_{mn}\lambda_n \tag{9}$$

where δ_{mn} is the Kronecker delta (1 if $m = n$ and 0 if $m \neq n$). Equation (8) includes the normalization relationship $\tilde{I}^*[R]\vec{I}_n = 1$. (The tilde denotes transpose, and the asterisk denotes complex conjugate.)

Eigencurrents can be used for a modal representation of the solution in the usual way. For example, the modal solution for the port current is

$$\vec{I} = \sum_{n=1}^{N} \frac{\tilde{I}_n{}^*\vec{V}^{oc}}{1 + j\lambda_n}\vec{I}_n. \tag{10}$$

The derivation of (10) is given in [11]. The modal solution for the radiated field is

$$\mathbf{E} = \sum_{n=1}^{N} \frac{\tilde{I}_n{}^*\vec{V}^{oc}}{1 + j\lambda_n}\mathbf{E}(\vec{I}_n) \tag{11}$$

where $\mathbf{E}(\vec{I}_n)$ is the field produced by $\vec{I}_n$. The modal solution for the current distribution $\mathbf{J}$ on the antenna is similar in form to (11).

A mode is said to be in resonance when its eigenvalue λ_n is zero. Any real port current $\vec{I}$ can be resonated by choosing the load (or feed) network to be a set of reactances at the antenna system ports [11]. For example, if the reactances X_i in Fig. 1 are properly chosen, any desired real current $\vec{I}$ can be resonated. As shown in [11], to reactively resonate a real current $\vec{I}$ the feed network should be chosen to be a diagonal matrix with diagonal elements jX_i where

$$X_i = \frac{-1}{I_i}([X_A]\vec{I})_i. \tag{12}$$

Here $([X_A]\vec{I})_i$ denotes the ith component of the column vector $[X_A]\vec{I}$.

When one current is resonated and all other modes have relatively large eigenvalues, then the principal contribution to the modal solutions (10) and (11) comes from the resonated mode. This assumes that the mode is well excited, that is, $\tilde{I}^*\vec{V}^{oc}$ is not small. In this case the radiation field (11) becomes approximately

$$\mathbf{E} \approx \tilde{I}\vec{V}^{oc}\mathbf{E}(\vec{I}). \tag{13}$$

By choosing $\vec{I}$ to obtain some desirable characteristic, say a given pattern or high gain, one has a method for synthesizing a reactively loaded antenna system.

The directive gain of an N-port antenna system excited by a port current $\vec{I}$ is given by [14]:

$$G = \frac{k^2\eta}{4\pi} \frac{|\tilde{I}\vec{V}^0|^2}{\tilde{I}^*[R]\vec{I}} \tag{14}$$

where k is the wavenumber, η is the intrinsic impedance of space, and $\vec{V}^0$ is the open circuit port voltage of the antenna system when excited by a plane wave from the direction of gain evaluation. For the current $\vec{I}$ to be a characteristic mode it must be real. The maximization of (14) when $\vec{I}$ is restricted to be real is considered in [8] and [15]. The method developed in these references is used for the present work. Once the real current for maximum gain is found, it is resonated according to the preceding concepts.

IV. ALTERNATIVE DEFINITION OF RESONANCE

A resonant load as defined in the above section is that load which supplies the reactive power required by the antenna system for a given port current $\vec{I}$. Alternatively, a resonant load can be defined as that load for which the source voltage norm is minimized for a given antenna port current $\vec{I}$. This alternative definition is taken in this section.

The voltage source required to produce a given port current $\vec{I}$ in a loaded antenna system is

$$\vec{V} = [Z_A + Z_L]\vec{I}. \tag{15}$$

The square of the norm of the port voltage is

$$|\vec{V}|^2 = |[Z_A + Z_L]\vec{I}|^2. \tag{16}$$

It is desired to find the load $[Z_L]$ which minimizes $|\vec{V}|^2$. Consideration will be restricted to diagonal load networks for which

$$[Z_L] = [\operatorname{diag} Z_{Li}] \tag{17}$$

with elements

$$Z_{Li} = R_i + jX_i. \tag{18}$$

The minimization of (16) is with respect to the X_i and R_i, which are real. Furthermore, consideration will be restricted to passive loads, in which case the R_i are constrained to be nonnegative.

The minimum of $|\vec{V}|^2$ may be either at an interior stationary point, for which

$$\frac{\partial |V|^2}{\partial R_i} = \frac{\partial |V|^2}{\partial X_i} = 0 \tag{19}$$

for all i, or at a point on the boundary. From (16),

$$\frac{\partial |V|^2}{\partial R_i} = 2R_i|\vec{I}_i|^2 + 2\operatorname{Re}\{I_i^*([Z_A]\vec{I})_i\} \tag{20}$$

$$\frac{\partial |V|^2}{\partial X_i} = 2X_i|\vec{I}_i|^2 + 2\operatorname{Im}\{I_i^*([Z_A]\vec{I})_i\} \tag{21}$$

where $(\cdot)_i$ denotes the ith component of the enclosed vector. Since the partial derivatives (20) and (21) are linear in the variable of differentiation, and depend only on this variable, it follows that the minimum of $|\vec{V}|^2$ is attained by

$$R_i = \max\left(\frac{-1}{|I_i|^2}\operatorname{Re}\{I_i^*([Z_A]\vec{I})_i\}, 0\right) \tag{22}$$

$$X_i = -\frac{1}{|I_i|^2}\operatorname{Im}\{I_i^*([Z_A]\vec{I})_i\} \tag{23}$$

where max $(A, 0)$ denotes the larger of A or 0. If the load is restricted to being purely reactive, the minimum of $|\vec{V}|^2$ is attained under condition (23). Furthermore, if $\vec{I}$ is real, (23) reduces to (12). Hence, the new definition of resonance includes the previous one as a special case.

V. A UNIVARIATE SEARCH METHOD

The port current vector $\vec{I}$ is that vector having the port currents I_i as elements. It is related to the excitation vector $\vec{V}$ by (15). The directive gain of the array is given by (14). If the inverse of (15) is substituted into (14), one obtains

$$G = \frac{k^2\eta}{4\pi} \frac{|\tilde{V}^0[Z_A + Z_L]^{-1}\vec{V}|^2}{\tilde{I}^*[R]\vec{I}}. \tag{24}$$

We are now considering the loads to be pure reactances, hence $[Z_L]$ is the diagonal matrix

$$[Z_L] = \begin{bmatrix} jX_1 & 0 & 0 & \cdots & 0 \\ 0 & jX_2 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & jX_N \end{bmatrix} \tag{25}$$

It is desired to obtain the maximum G of (24) by varying the X_i of (25).

The maximization of (24) is, in general, difficult because G is a complicated function of the X_i. Hence, it is appropriate to use an optimum seeking procedure. Rather than using one of the standard methods, which are often slowly convergent and expensive to run, we have devised a univariate search procedure which makes use of the particular form (24). The method is a modification of one described in references [16] and [17]. The basic method is as follows.

The quantity G to be maximized is given by (24). Since the loads are purely reactive, $[Z_L] = j[X_L]$. The loads are to be varied one at a time, and it is convenient to separate out that load being varied. Hence, when the ith load is varied, we write

$$[Z_A + jX_L] = [Z^i] + j[X_L{}^i] \tag{26}$$

where $[Z^i]$ is $[Z_A + jX_L]$ with $X_i = 0$, and $[X_L{}^i]$ consists of only one nonzero element X_i. The inverse of (26) is given by [18]

$$[Z_A + jX_L]^{-1} = [Z^i]^{-1} - \frac{jX_i}{1 + jX_i(Z^i)_{ii}{}^{-1}} \cdot [Z^i]^{-1}[U^i][Z^i]^{-1} \tag{27}$$

where $(Z^i)_{ii}{}^{-1}$ is the iith element of $[Z^i]^{-1}$, and all elements of $[U^i]$ are zero except the iith which is unity. Substituting (27) into (24), and taking advantage of the symmetry of $[Z^i]^{-1}$, we have

$$G = \frac{k^2\eta}{4\pi\tilde{I}^*[R]\vec{I}} \left| \tilde{V}^0[Z^i]^{-1}\vec{V} - \frac{\tilde{V}^0[Z^i]^{-1}[U^i][Z^i]^{-1}\vec{V}}{jB_i + (Z_i)_{ii}{}^{-1}} \right|^2. \tag{28}$$

Here $B_i = -1/X_i$ is the parameter to be varied to optimize G. When B_i is positive, the load is capacitive, and when B_i is negative, the load is inductive.

We can gradually increase G of (28) by first maximizing it with respect to B_1, then with respect to B_2, next with respect to B_3, and so on. After G is maximized with respect to the last B_i, the process is repeated. This general iteration process is called univariate search [19]. The process eventually converges because G can only increase with each iteration and G has an upper bound. The process converges to a point at which a small change in any one of the B_i must decrease G. There is no assurance that this point is the absolute maximum of G or even a relative maximum of G in the space of the variables B_i. Furthermore, convergence may be slow. The details are given in [2].

VI. APPLICATION TO A CIRCULAR ARRAY

The above theory is here applied to the seven-element circular array of Fig. 1. The array consists of seven dipoles, six equispaced on a circle of radius a and the seventh located at the center. The six outer elements have reactances X_i across their input terminals, and the center element is fed by a voltage V. It is desired to determine those reactances X_i which give maximum gain in a given direction ϕ_0, where ϕ_0 is arbitrary. Once the X_i are found as a function of ϕ_0, the directive beam can be scanned by appropriately varying the reactances X_i.

Computations were made for dipoles of length $L = \lambda/2$, of diameter $D = L/100$, and the spacing S between the dipoles was variable. Initial computations were made with $S = \lambda/4$, chosen to be small enough to provide efficient electromagnetic coupling between elements, yet large enough to preclude extreme supergain behavior. The array parameters, such as impedance matrix and element patterns, were calculated using computer programs given in [1]. These programs used the sinusoidal current approximation, which is reasonably accurate for half wavelength long dipoles.

The univariate search program was used with several different starting points. The two used most often were a) the reactive loads which resonate the maximum gain real currents (Section III), and b) the reactive loads which resonate the maximum gain complex currents (Section IV). For the case $S = \lambda/4$, these two starting points yielded the same design via the univariate search. Fig. 3 shows the gain patterns obtained for the seven element reactively loaded array. The solid curves are the patterns for the final reactively loaded array, and the small triangles show the maximum gain pattern for the array when all dipoles are fed. The design was carried out for maximum gain at the angles a) $\phi_0 = 0$, b) $\phi_0 = 10^\circ$, c) $\phi_0 = 20^\circ$, d) $\phi_0 = 30^\circ$. Because of the symmetry of the array, gain patterns for other cases $\phi_0 = 10n^\circ$, $n = 4, 5, \cdots, 35$, are just rotations and reflections of the four cases shown. Note that the maximum gain when all dipoles are fed varies considerably as a function of angle, while the gain of the reactively loaded antenna varies only slightly. Table I lists the gains in the direction ϕ_0 for the three cases a) maximum gain, real current, b) maximum gain, complex current, and c) gain of reactively loaded antenna. Note that the reactively loaded array has a gain intermediate between the maximum gain, real current, and the maximum gain, complex current, as was expected. Note also that the maximum gain, real current, is always at least one-half the maximum gain, complex current, a property which has been proved theoretically [15].

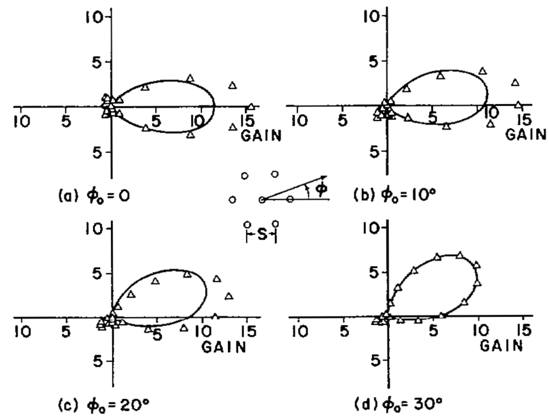

Fig. 3. Radiation gain patterns for seven-element circular array, $S = \lambda/4$. Solid curve is reactively loaded array pattern, triangles show maximum gain pattern.

TABLE I
GAIN IN DIRECTION ϕ_0 FOR SEVEN-ELEMENT CIRCULAR ARRAY OF FIG. 3, $S = \lambda/4$

ϕ_0	max. gain, real I	max. gain, complex I	gain of loaded array
0°	8.67	15.45	11.41
10°	7.40	14.42	11.23
20°	7.50	12.37	10.95
30°	7.73	11.35	10.99

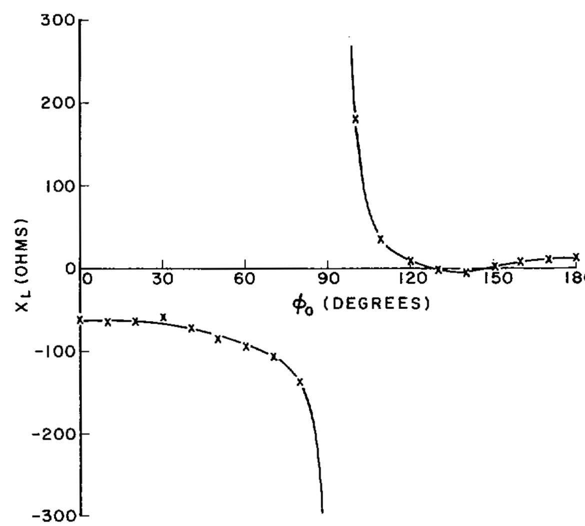

Fig. 4. Reactive load X_L versus beam angle ϕ_0 for dipole at $\phi = 0$.

The beam direction ϕ_0 can now be steered by varying the reactive loads on each element. Fig. 4 shows the load reactance X_L versus beam angle ϕ_0 for the element located at $\phi = 0$. The computed values are shown by crosses, and the solid line is a smoothed graph. Only the X_L for angles $0 \leqslant \phi_0 \leqslant 180^\circ$ are shown since X_L is an even function about $\phi_0 = 0$. Because of the circular symmetry of the array, X_L for the element located at $\phi = 60^\circ$ is given by Fig. 4 with 60° added to each abscissa value. Similarly, X_L for an element located at $\phi = n \times 60^\circ$ $n = 2, 3, 4, 5$, is given by Fig. 4 with $n \times 60^\circ$ added to each abscissa value.

Computations were also made for the seven-element circular array with other element spacings S (see Table II). In each case

TABLE II
REACTIVE LOADS OBTAINED FROM UNIVARIATE SEARCH FOR SEVEN-ELEMENT CIRCULAR ARRAY, $S = \lambda/4$

element position	Beam direction 0°	10°	20°	30°
0°	-63.6Ω	-64.3Ω	-64.7Ω	-57.4Ω
60°	-94.6	-85.2	-71.0	-57.4
120°	9.0	34.8	180.0	-398.1
180°	10.8	11.4	9.7	1.7
240°	9.0	-1.0	-4.1	1.7
300°	-94.6	-105.6	-134.1	-401.0

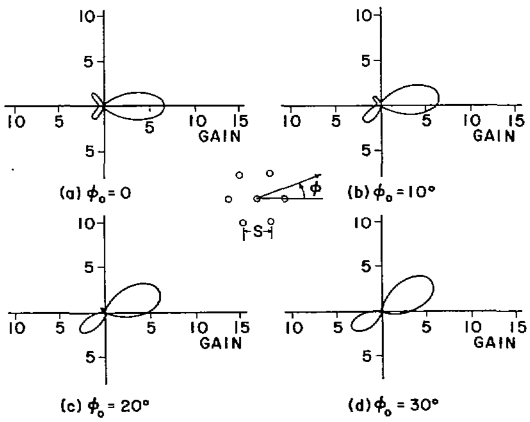

Fig. 5. Radiation gain patterns for seven-element reactively loaded circular array, $S = \lambda/2$.

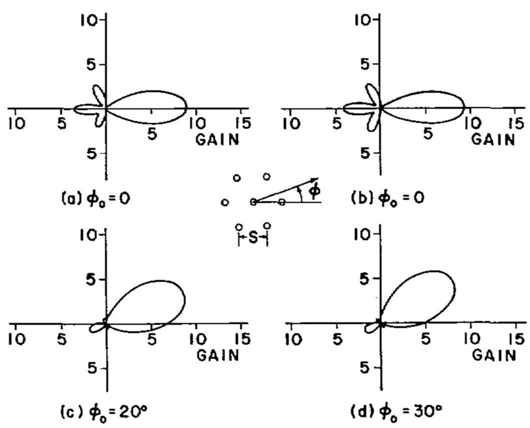

Fig. 6. Radiation gain patterns for seven-element reactively loaded circular array, $S = \lambda/8$.

the maximum gain current was first computed, and then the reactive loads for resonance found according Section IV. These loads were then used in the univariate search program to obtain the final design. Fig. 5 shows the final gain patterns for the case $S = \lambda/2$, and Fig. 6 for the case $S = \lambda/8$. For element separation $S = \lambda/2$, the pattern control by reactive loads was less than when $S = \lambda/4$ (Fig. 3), as was expected. For example, at the angles $\phi_0 = 20^\circ$ and 30°, the back lobe is higher than desirable. However, the patterns are still satisfactory for many uses. For element separation $S = \lambda/8$ more directive patterns were obtained than when $S = \lambda/4$, but these tend to be supergain excitations of the array. In order to obtain a realistic design when $S = \lambda/4$, we would have to exclude the high Q

TABLE III
GAIN IN DIRECTION ϕ_0 FOR SEVEN-ELEMENT CIRCULAR ARRAY OF FIG. 5, $S = \lambda/2$

ϕ_o	max. gain real I	max. gain, complex I	gain of loaded array
0°	6.84	11.23	6.76
10°	5.44	10.68	6.62
20°	6.99	9.60	6.45
30°	7.91	9.08	7.43

TABLE IV
GAIN IN DIRECTION ϕ_0 FOR SEVEN-ELEMENT CIRCULAR ARRAY OF FIG. 6, $S = \lambda/8$

ϕ_o	max. gain, real I	max. gain complex I	gain of loaded array	alternative design
0°	9.37	16.83	9.37	9.55
10°	8.00	15.52	8.15	9.48
20°	7.63	12.91	9.30	9.28
30°	7.69	11.61	9.39	9.35

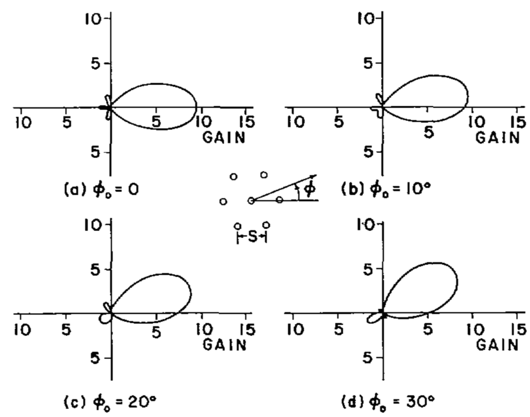

Fig. 7. Radiation gain patterns for alternative design of seven-element reactively loaded circular array, $S = \lambda/8$.

modes from our design procedure. Values of the gains achieved for the two cases, $S = \lambda/2$ and $S = \lambda/8$ are given in Tables III and IV, respectively. The corresponding table for the case $S = \lambda/4$ is Table I. The last column of Table IV is for an alternative starting point, to be discussed.

Different designs can be obtained by using different starting points in the univariate search. This is because there are many local optima in the gain function. Fig. 7 shows that the gain patterns obtained from one such alternative starting point. We shall not go into details here on how the starting loads were chosen for this case. Suffice it to say that it involved a preliminary attempt to exclude high Q modes from the synthesis procedure.

VII. DISCUSSION

The optimum seeking method of designing reactively loaded arrays has been found to give good results for arrays of moderate numbers of dipoles. The final array design depends to some extent on the initial array to which the search program is applied. Good starting points are those reactive loads which resonate either the real current or the complex current for maximum gain. Initial studies of larger arrays indicate that convergence of the search program is slower, and that there are more points of local optima, than for small arrays. Hence, the design of large arrays is more difficult than that of small arrays.

One improvement which could be made in the design procedure is the exclusion of high Q modes. This would result in faster convergence of the search procedure, fewer local optimum points, and a final design less sensitive to frequency and design variations. This improvement could be accomplished by using only low Q modal currents as a basis, instead of the port currents presently used.

The type of antenna array being considered in this paper has several advantages over the more conventional types of arrays. a) There are no transmission lines to the individual elements, the excitation of elements being accomplished by electromagnetic interaction. b) There is only one reactive element per input port, although this results in some loss of pattern control. c) There is only one feed point, which simplifies the problem of matching the antenna to the transmitter. d) Variable reactance loads can provide a means for beam steering, either mechanically or electronically. There are two principal disadvantages to the reactively loaded arrays being considered. a) The elements must be relatively closely spaced to provide sufficient electromagnetic interaction. b) There is incomplete control of the radiation characteristics of the array. Complete control requires two reactive elements per port, plus transmission lines [20]-[22].

Reactively controlled antenna arrays should prove useful for directive arrays of restricted spacial extent. For example, shipboard antennas at low and intermediate frequencies must make use of regions small compared to wavelength. By using several reactively loaded elements, without connecting transmission lines, one can obtain remotely controlled directive antenna systems. Another possible use, requiring an extension of the theory, might be to reduce the number of phase shifters required for a phased array. This might be accomplished by using phase shifters to feed some of the elements of an array, and reactive loads to control the excitation of other elements.

ACKNOWLEDGMENT

The author gratefully acknowledges the contributions of Joseph R. Mautz, Robert F. Wallenberg, and Alan R. Harvey to the research described in this paper.

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