# Green Design for Smart Antenna System using Iterative Beamforming Algorithms

Rashi Mehrotra

Department of Electrical Engineering Indian Institute of Technology, Delhi New Delhi, India-110016 Email: rashi.mehrotra@ee.iitd.ac.in Ranjan Bose
Department of Electrical Engineering
Indian Institute of Technology, Delhi
New Delhi, India-110016
Email: rbose@ee.iitd.ac.in

Abstract—In wireless sensor networks operating over short inter-node distances, both computation power and radio power influence the battery life. In such a scenario, to evaluate the utility of Smart Antennas (SA) from a power perspective, one has to consider the power consumed in the beamforming (BF) unit (computation power) and the power consumed in the radio unit (radio power). Both computation power and radio power in turn depend on the number of iterations of the BF algorithms. In this paper, two iterative adaptive BF algorithms, Least Mean Square (LMS) algorithm and Recursive Least Square (RLS) algorithm are considered. Computation power measurements have been carried out for a StrongARM SA-1100 processor platform. A closed form expression for optimal number of iterations has been derived for a given bit error rate (BER) that minimizes the total power consumption. It is found that optimal number of iterations increases linearly with path loss exponent and decreases logarithmic with BER. We have analyzed the effect of different BERs and path loss exponents on the optimal number of iterations. Simulation results suggest that RLS algorithm becomes more effective compared to the LMS algorithm in terms of number of iterations at higher path loss exponents. This study yields a new, power optimal stopping criterion, thereby providing a green design for SA systems.

Index Terms—Computation power, beamforming gain, number of iterations

# I. INTRODUCTION

With the increasing energy cost in modern communication systems, 'Green communication design' has recently become a well known and appealing concept. The concept primarily aims at improving energy or power efficiency and reducing power consumption [1]. The energy efficiency [2], which is widely defined as the system throughput per unit energy, has become one of the critical performance metrics for future communication systems.

Recently researchers have studied energy efficient green communication to improve the mobile battery consumption in Multiple-input-multiple-output (MIMO) system [3]. In [3], energy-efficient design of uplink (UL) Multiuser-MIMO in a single cell environment is described. It accounts for both circuit and transmission power when designing power allocation schemes and emphasizes energy efficiency over peak rates or throughput. In these cases and in [4], [5], circuit and transmission powers are considered, but not the computation power. In a power limited devices such as broadband cellular networks and wireless sensor networks (WSN), energy consumption is

a critical issue. In [6], a methodology was proposed for design space exploration of WSN based on the trade-off between the radio energy and the computation energy. The work was targeted to find an energy-efficient sensor node configuration using error correcting codes (ECC). In this paper, we explore Smart Antennas (SA) in WSN, to improve the performance based on power consumption and increase in battery life. SA directs transmission or reception beams sharply and accurately. They are used to identify the direction of arrival of the signal and calculate the beamforming (BF) vectors in order to track and locate the antenna beam on the target through advanced signal processing techniques [7]. Energy efficient BF designs have been developed widely in [8], [9]. In these cases only the circuit and transmission powers are considered. The computation power is not included. In this paper, we consider radio power, circuit power and the computation power.

In SA, two major units contribute power consumption in the system. In radio unit, power is consumed due to signal transmission and due to electronics circuit. In computation unit, power is spent in the processing and computing the BF algorithms. Recently researchers have used well known Least Mean Square (LMS) algorithm and Recursive Least Square (RLS) algorithm [10], [11] for BF. BF algorithms are computed iteratively to update the weights. These weights converges after a certain number of iterations. Power consumed in computation varies with the number of iterations. In this paper, we have considered the computation power along with the transmit and circuit power of SA system. As power consumption varies significantly in the computation and the radio unit, at short inter-node distance, signal transmit power required is not very high and is comparable to the computation power. In SA, antenna gain is one of the key performance parameters. SA gain is obtained from the array response vector. As the array response vector or the array factor depend on the weight vectors of the BF algorithms, SA gain also depend on the weight vectors, which in turn depends on the number of iterations.

In this paper, we have proposed computation power model and SA gain model as a function of number of iterations. We have estimated computation power using Sim-Panalyzer power simulator based on ARM processor [12]. The measurement data is used to obtain an analytical expression for the computation power by curve fitting. These expressions are then used to obtain the total power consumption in SA. An expression for the optimal number of iterations are derived by minimizing the total power consumption for a given bit-error-rate (BER) constraint and a given transmitter-receiver separation. Our main contributions are as follows. We have obtained different values of optimal number of iterations for LMS and RLS algorithms. LMS algorithm consumes less power as compared to RLS algorithm at their respective optimal number of iterations. We have also analyzed the effect of different BERs and path loss exponents on the optimal number of iterations. RLS algorithm becomes more effective compared to the LMS algorithm in terms of number of iterations at higher path loss exponents. By fixing the number of iterations, we can provide a stopping criteria for the BF algorithms at the optimal iteration in order to achieve minimum power consumption for the desired BER. Through our study, we propose a green communication design for SA system that reduces power consumption and increases the battery life of WSNs. The paper is organized as follows. Section II describes the BF gain and power units of SA. A model for computation power and SA gain with respect to the number of iterations is proposed in section III. Section IV gives the simulation results. Finally, section V summarizes the conclusion of this paper.

#### II. BEAMFORMING GAIN AND POWER UNITS OF SA

We first consider antenna gain and power consumption in different units of SA. The antenna gain is found from the weights of the algorithms which perform beamforming, hence this is called beamforming (BF) gain.

# A. Beamforming Gain

It is well known in adaptive SA system, the signal are weighted and combined to achieve high SA gain. The SA or the BF gain is obtained by the array factor, which depends on the weight vectors. AF is found by summing the weighted outputs of each antenna element and is given by

$$AF = w^T . a(\theta) \tag{1}$$

where  $w^T$  is  $[w_M \ w_{M-1} \ ... \ w_1 \ ... \ w_{M-1}], \ a(\theta)$  is the steering array vector for uniform linear array and M is the total number of array elements.

Directivity is a measure of the antenna's ability to direct energy in a specific direction and is given in terms of array factor as

$$D = \frac{4\pi U_{max}}{P_{rad}} \tag{2}$$

where  $U_{max}$  is the radiation intensity defined as

$$U(\theta) = (max(|(AF)|))^2 \tag{3}$$

and  $P_{rad}$  is the total radiated power given by

$$P_{rad} = 2\pi \int_{0}^{\pi} U(\theta) \sin(\theta) d\theta \tag{4}$$

Hence, BF gain is [7]

$$G = \eta D \tag{:}$$

where  $\eta$  is the total antenna efficiency including the effects of losses and mismatches. We have considered  $\eta = 1$  in our case without loss of generality.

# B. Radio Power

Power in the radio unit is contributed by two components, signal transmit power  $P_t$  and the circuit power  $P_{ckt}$ .

1) Signal Power: The power required for signal transmission can be expressed as [13]

$$P_t = \left(\frac{4\pi}{\lambda}\right)^2 d^\alpha \frac{P_r}{G_r G_t} \tag{6}$$

where d is the distance between transmitter and receiver;  $\lambda$  is the wavelength of the transmitted signal;  $P_r$  is the received power;  $G_t$  and  $G_r$  are the gain of the transmitter and receiver antennas; and  $\alpha$  is the path loss exponent. In this paper, gain of the transmitter,  $G_t$ , is the BF gain,  $G_{BF}$ . The signal-tonoise ratio  $(\gamma)$  is defined as  $\gamma = P_r/\frac{N_0}{2}WNF$  where  $N_0$  is the noise spectral density for AWGN channel, NF is the receiver noise figure and W is the bandwidth of the signal. The modulation scheme considered is MQAM.

A bound on the probability of bit error (BER) for MQAM is given by [13]

$$P_e \le 4\left(1 - \frac{1}{\sqrt{2^b}}\right)Q\left(\sqrt{\frac{3}{2^b - 1}\gamma}\right) \tag{7}$$

where b is the bits per symbol. Hence by approximating the bound as an equality [14], the received power is obtained as

$$P_r = 4\left(1 - \frac{1}{\sqrt{2^b}}\right) \frac{2(2^b - 1)}{3} \frac{N_0 W N F}{P_e} \tag{8}$$

$$P_r = \frac{x(b)N}{P_c} \tag{9}$$

where  $x(b)=4\left(1-\frac{1}{\sqrt{2^b}}\right)\frac{2(2^b-1)}{3}$  and  $N=N_0WNF$ . Substituting (8) in (6), we obtain signal power for a fixed BER.

2) Circuit Power: The main components of the transmitter circuit are digital-to-analog converter (DAC), low-pass filter (LPF), mixer, frequency synthesizer (FS), power amplifier (PA) and band-pass filter (BPF). The receiver circuit components are mainly BPF, low-noise amplifier (LNA), mixer, FS, intermediate-frequency amplifier (IFA), LPF and analog-to-digital converter (ADC). Transceiver circuit power is the power consumed by power amplifier  $P_{PA}$  and other circuit components.

$$P_{ckt.tot} = P_{PA} + P_{ckt} (10)$$

where.

$$P_{ckt} = P_{DAC} + 2P_{LPF} + 2P_{FS} + 2P_{BPF} + P_{LNA} + P_{IFA} + P_{ADC}$$
 (11)

where  $P_{PA}$ ,  $P_{DAC}$ ,  $P_{LPF}$ ,  $P_{FS}$ ,  $P_{BPF}$ ,  $P_{LNA}$ ,  $P_{IFA}$ , and  $P_{ADC}$  are the power consumed in the respective components. The power consumption in PA is dependent on signal transmit

power as  $P_{PA}=\alpha P_t$  , where constant  $\alpha$  is related to drain efficiency  $\eta$  of RF PA.

Hence the radio power is given by

$$P_{radio} = P_t + P_{ckt \ tot} \tag{12}$$

# C. Computation Power

To estimate the computation power, we have used Sim-Panalyzer [15]. Sim-Panalyzer is a cycle-accurate power simulator for ARM instruction set architecture. Specifically, it simulates the StrongARM SA-1100 processor.

The prime use of Sim-Panalyzer is to estimate the power consumed by the application run on a given architecture system, (if simulator is a system simulator). The power model of the simulator consists of several components which models distinct parts of the processor, which includes, cache power models, datapath and execution unit power models, clock tree power models, and I/O power models. Sim-Panalyzer computes the power consumption of the BF algorithms, based on counting the number of transitions in these parts of the processor. The simulator loads the algorithm in C, compiles it for the target architecture and estimates the overall power consumed in the processor. Sim-Panalyzer is configured with a StrongArm SA-1100 processor operating at 200 MHz, 8-kB data cache, 16-kB instruction cache, and 1-MB RAM.

# III. PROPOSED BEAMFORMING GAIN AND COMPUTATION POWER MODELS

In this section we have described BF gain and the computation power model as a function of number of iterations.

# A. Beamforming Gain Model

BF gain is obtained in section II-A. The measured data is plotted as shown in Fig. 1. Using the approximation based on the curve-fitting method of least square, BF gain obtained in terms of number of iterations as

$$G(k) = A(1 - e^{-Bk}) (13)$$

where A and B are the constant unknown parameters. Hence (13), in general, can be used to model the gain as a function of iterations with fixed values of A and B. The graph stabilizes at

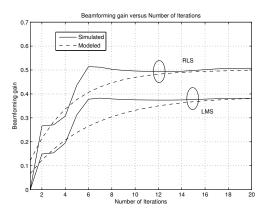


Fig. 1. Beamforming gain of SA with number of iterations for a) LMS algorithm, b) RLS algorithms.

a gain of 0.39 for LMS algorithm and 0.5 for RLS algorithm. This gives the value of A equal to 0.39 for LMS algorithm and 0.5 for RLS algorithm. Next we determine the best values of B so that G predict the function value that will correspond to k. The approximate value of B for RLS algorithm is 0.28 and for LMS algorithm B is 0.19. A and B are determined by using analytical software packages, e.g., Matlab and Maple.

As shown in Fig. 1, SA gain increases with the number of iterations. As the iterations increases, the gain stabilizes asymptotically. How fast it reaches the asymptotic value is modeled by B. Hence these two unknown parameters are directly linked to two important characteristics: the rate of convergence and the asymptotic value of the gain.

As mentioned earlier, the radio power depends on the BF gain  $(G_{BF})$  as

$$P_{radio} = \left(\frac{4\pi}{\lambda}\right)^2 d^{\alpha} \frac{P_r}{G_r G_{BF}} + P_{ckt}$$
 (14)

Hence using (13), the total radio power is given by

$$P_{radio}(k) = \left(\frac{4\pi}{\lambda}\right)^2 d^{\alpha} \frac{\frac{x(b)N}{P_e}}{(G_r)(A(1 - e^{-Bk}))} + P_{ckt} \quad (15)$$

# B. Computation power as a function of iterations

As given in section II-C, Sim-Panalyzer executes the algorithm written in C and after compiling it for the ARM processor, estimates the total power expended in computation. The program is iterated a number of times in the power simulator. The total computation power is plotted as a function of number of iterations in Fig. 2. The figure shows the simulated and modeled computation power of SA as a function of number of iterations. The expressions for LMS and RLS algorithms are obtained by curve-fitting method on the simulated computation power of SA and can be represented by

$$P_{comp,lms} = 0.024k + 0.019 \tag{16}$$

and for the RLS algorithm by

$$P_{comp.rls} = 0.063k + 1 \tag{17}$$

Hence the expression for the computation power in general is modeled as

$$P_{comp} = Ek + f (18)$$

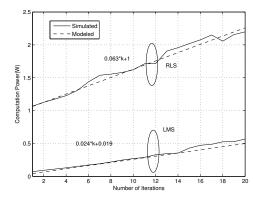


Fig. 2. Computation power with number of iterations for a) LMS algorithm b) RLS algorithm.

E and f are obtained similarly as A and B in (13). In general, (18) can be used to model the computation power as a function of number of iterations. The total power consumption is the sum of the power expended in the radio unit, circuit unit and the computation unit. Since the circuit power does not depend on the number of iterations, we do not consider its effect.

$$P_{Tot} = P_{radio} + P_{comp} \tag{19}$$

Thus on substituting from (15) and (18), we can express total power consumed as

$$P_{Tot}(k) = (\frac{4\pi}{\lambda})^2 d^{\alpha} \frac{\left(\frac{x(b)N}{P_e}\right)}{(G_r)(A(1 - e^{-Bk}))} + Ek + f \quad (20)$$

### C. Optimal number of Iterations

As seen from (20), the total power consumption of the SA is the function of number of iterations. The total power consumption problem for SA system with BER constraint is

$$\min_{k} P_{Tot}$$
s.t.  $P_e \le P_{e_{max}}$  (21)

where  $P_e$  is the fixed BER and  $P_{emax}$  is the maximum BER value. For a given BER, since  $\frac{\partial^2 P_{Tot}}{\partial^2 k} \geq 0$ ,  $P_{Tot}$  is the convex function of k. Taking the first derivative of  $P_{Tot}$  with respect to k and equating it to zero, we obtain  $k_{opt}$ .

$$E - \left(\frac{4\pi}{\lambda}\right)^2 d^{\alpha} \left(\frac{x(b)N}{P_e}\right) X = 0 \tag{22}$$

where X is given by

$$X = \frac{Be^{-Bk}}{AG_r(1 - e^{-Bk})^2}$$
 (23)

where  $G_r$  is the fixed receiver's gain. On solving (22), we obtain the roots of k as.

$$k_{opt} = \lceil \frac{1}{b} \ln \left( \frac{2}{c_1 \pm \sqrt{c_1^2 - 4}} \right) \rceil \tag{24}$$

where  $c_1$  is defined as

$$c_1 = \frac{2AG_rE + \frac{c_2d^{\alpha}}{P_e}}{AG_rE} \tag{25}$$

and  $c_2$  is given by

$$c_2 = \left(\frac{4\pi}{\lambda}\right)^2 x(b) NB \tag{26}$$

As  $c_1$  is a function of  $P_e$ , hence  $k_{opt}$  depends on  $P_e$ . Hence the optimum value of k is given by

$$k_{opt} = \lceil \frac{1}{B} \ln \left( \frac{2}{c_1 - \sqrt{c_1^2 - 4}} \right) \rceil \tag{27}$$

where  $\lceil x \rceil$  represents the smallest integer not less than x. As the iteration should be an integer value only, we have chosen  $\lceil . \rceil$ , instead of  $\lfloor . \rfloor$ , which will meet the system BER requirement. (27) can be approximated as

$$k_{opt} \approx \lceil \frac{1}{B} \ln \left( c_1 \right) \rceil$$
 (28)

TABLE I PARAMETERS FOR SIMULATIONS

$P_{FS}$	13.7 mW	$G_r$	1
$P_{LNA}$	0.55 mW	$\alpha$	3, 3.5, 4
$P_{BPF}$	6.12 mW	$N_0$	$4.1 \times 10^{-21}$
$P_{IFA}$	0.2 mW	NF	10 dB
$P_{LPF}$	0.29 mW	B	1 MHz
$P_{ADC}$	4.1 mW	$P_e$	$10^{-4}$
$P_{DAC}$	55 mW	$f_c$	2.4 GHz

On substituting  $c_1$ ,  $k_{opt}$  is given by

$$k_{opt} \approx \lceil \frac{1}{B} \ln \left( 2 + \frac{\frac{c_2 d^{\alpha}}{P_e}}{A G_r E} \right) \rceil$$
 (29)

$$\approx \lceil \frac{1}{B} \ln \left( \frac{\frac{c_2 d^{\alpha}}{P_e}}{A G_r E} \right) \rceil$$
 (30)

After simplification, we obtain

$$k_{opt} \approx \left\lceil \frac{1}{B} \ln \left( \frac{c_2}{AG_r E} \right) + \left( \frac{\alpha}{B} \right) \ln(d) - \frac{1}{B} \ln(P_e) \right\rceil$$
 (31)

Hence  $k_{opt}$  increases linearly with path loss exponent  $\alpha$  and decreases logarithmically with BER. This is required to provide a larger gain in order to guarantee the desired BER.

#### IV. SIMULATION RESULTS

For simulations we have considered a SA consisting of  $4 \times 4$  grid of antenna elements. Sim-Panalyzer power model and the circuit component power [16] listed in Table I are both based on 0.18- $\mu$ m technology.

As mentioned earlier, the BF gain of SA with number of iterations is plotted in Fig. 1. The figure also shows that the gain increases slowly for the LMS algorithm and rapidly for the RLS algorithm, that is fewer number of iterations will provide higher gain using RLS algorithm as compared to LMS algorithm. Fig. 2 shows that the computation power increases almost linearly with the number of iterations.

Fig. 3 shows the total power consumption for LMS and RLS algorithms with number of iterations for a given BER of

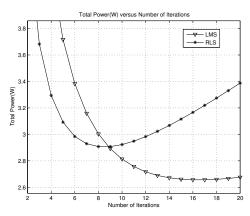


Fig. 3. Total power consumption of SA with number of iterations for a) LMS algorithm, b) RLS algorithm

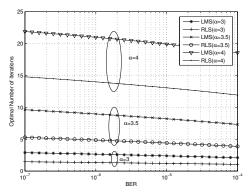


Fig. 4. Variation of Optimal number of iterations with different BERs for a) LMS algorithm, b) RLS algorithm

 $10^{-4}$ . It is observed that the LMS algorithm outperforms the RLS algorithm in terms of  $P_{tot}$  as the number of iterations increases. For the  $T_x-R_x$  separation of 200 m and a given BER of  $10^{-4}$ , the optimal number of iterations of the RLS algorithm is observed to be 8 by simulations and  $k_{opt}=13$  found analytically. For LMS algorithm at 200 m separation, the optimal number of iterations is observed to be 16 by simulations and  $k_{opt}=17$  found analytically. Thus the analytical expression over-estimates the optimal number of iterations.

From Fig. 3, we observe a crossover ( $k_{cross}$ ) at 9th iteration between the curves for LMS and RLS algorithms. Beyond this LMS becomes better in terms of power as compared to RLS. So, a 'green design' based strategy would make the user to switch from one algorithm to another at  $k_{cross}$ . The figure also shows that LMS achieves significant power saving (about 10%) compared to RLS at their respective optimal iteration. At  $k_{opt}$ , algorithm should be terminated in order to minimize power consumption for a desired BER. Thus, our technique provides a stopping criterion for iterative BF algorithms from the perspective of total power consumption and tells us the iteration at which one algorithm becomes less expensive in terms of power than the other.

Next, we consider the effect of the different BERs and the path loss exponent on the optimal number of iterations. Fig. 4 compares the variation of optimal number of iterations for LMS and RLS algorithms at different BERs and path loss exponents. As shown in figure, as the path loss exponent increases, number of iterations increases for a given BER. This can be explained as follows. As we increase the path loss exponent, the received signal becomes weak. In order to meet the BER constraint, a higher gain is required leading to a higher value of  $k_{opt}$ . Fig. 4 also shows that as the BER decreases logarithmically, optimal number of iteration increases. This is required to provide a larger gain in order to guarantee the desired BER. With an increase in the path loss exponent, RLS algorithm requires fewer number of iterations than the LMS algorithm.

It is also interesting to note that the gap between the number of iterations required for RLS and LMS algorithm increases for larger path loss exponents. This implies that the RLS algorithm becomes more effective in terms of the number of iterations at higher path loss exponent.

#### V. Conclusion

In this paper, we have proposed a 'Green design' for SA system from the perspective of total power consumption. A closed form expression for the optimal number of iterations has been derived by minimizing the total power consumed for a given BER constraint. It is found that, in order to guarantee the desired BER, a higher gain is required, leading to the higher value of optimal number of iterations for both the algorithms. We have shown that optimal number of iterations increases linearly with path loss exponent and decreases logarithmically with BER. We have also found that RLS algorithm becomes more effective in terms of number of iterations at higher path loss exponent for a given BER. It is also interesting to note that the gap between the number of iterations required for RLS and LMS algorithm increases for larger path loss exponents.1 This study yields a new, power optimal stopping criterion, thereby providing a green design for SA systems.

#### REFERENCES

- W. Vereecken, W. Van Heddeghem, D. Colle, M. Pickavet, and P. Demeester, "Overall ict footprint and green communication technologies," in 4th International Symposium on Communications, Control and Signal Processing (ISCCSP), March 2010, pp. 1–6.
- [2] D. Feng, C. Jiang, G. Lim, L. J. C. Jr., G. Feng, and G. Y. Li, "A survey of energy-efficient wireless communications." *IEEE Communications* Surveys and Tutorials, vol. 15, no. 1, pp. 167–178, 2013.
- [3] G. Miao, "Energy-efficient uplink multi-user mimo," *IEEE Transactions on Wireless Communications*, vol. 12, no. 5, pp. 2302–2313, 2013.
- [4] A. Wang, S. Cho, C. Sodini, and A. Chandrakasan, "Energy efficient modulation and mac for asymmetric rf microsensor systems," in *Low Power Electronics and Design, International Symposium on*, 2001., 2001, pp. 106–111.
- [5] S. Cui, A. Goldsmith, and A. Bahai, "Energy-constrained modulation optimization," Wireless Communications, IEEE Transactions on, vol. 4, no. 5, pp. 2349–2360, Sept 2005.
- [6] S. Chouhan, R. Bose, and M. Balakrishnan, "A framework for energy-consumption-based design space exploration for wireless sensor nodes," Computer-Aided Design of Integrated Circuits and Systems, IEEE Transactions on, vol. 28, no. 7, pp. 1017–1024, 2009.
- [7] F. Gross, Smart Antenna for Wireless Communications. McGraw-Hill, 2005.
- [8] C. Jiang and L. Cimini, "Energy-efficient multiuser mimo beamforming," in *Information Sciences and Systems (CISS)*, 2011 45th Annual Conference on, March 2011, pp. 1–5.
- [9] J. Feng, C.-W. Chang, S. Sayilir, Y.-H. Lu, B. Jung, D. Peroulis, and Y. Hu, "Energy-efficient transmission for beamforming in wireless sensor networks," in Sensor Mesh and Ad Hoc Communications and Networks (SECON), 2010 7th Annual IEEE Communications Society Conference on, June 2010, pp. 1–9.
- [10] S. S. Haykin, Adaptive filter theory. Pearson Education India, 2008.
- [11] S. F. Shaukat, R. Farooq, H. Saeed, and Z. Saleem, "Sequential studies of beamforming algorithms for smart antenna systems," World Applied Sciences Journal, vol. 6, no. 6, pp. 754–758, 2009.
- [12] T. Austin, E. Larson, and D. Ernst, "Simplescalar: an infrastructure for computer system modeling," *Computer*, vol. 35, no. 2, pp. 59–67, 2002.
- [13] J. G. Proakis, Digital Communications. McGraw-Hill, 1995.
- [14] Z. Wang and G. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *Communications, IEEE Transactions on*, vol. 51, no. 8, pp. 1389–1398, Aug 2003.
- [15] Sim-Panalyzer. [Online]. Available: http://www.eecs.umich.edu/~panalyzer.
- [16] H. Bergveld, K. van Kaam, D. Leenaerts, K. Philips, A. Vaassen, and G. Wetkzer, "A low-power highly digitized receiver for 2.4-GHz-band GFSK applications," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 53, no. 2, pp. 453–461, Feb. 2005.