7 - Soluções - Produto Interno. Ortogonalidade.

7.1 $||\alpha \vec{u}||^2 = (\alpha \vec{u}) \cdot (\alpha \vec{u}) = \alpha^2 ||\vec{u}||^2$. Logo, $||\alpha \vec{u}|| = |\alpha| \, ||\vec{u}||$. Em particular,
$||\frac{\vec{u}}{||\vec{u}||}|| = |\frac{1}{||\vec{u}||}| \, ||\vec{u}|| = \frac{1}{||\vec{u}||} ||\vec{u}|| = 1$

7.2 Como $\vec{u}$ e $\vec{v}$ são vectores não nulos e linearmente dependentes, $\exists \alpha : \vec{u} = \alpha \vec{v} \wedge \alpha \neq 0$.
$\vec{u} \cdot \vec{v} = ||\vec{u}|| \, ||\vec{v}|| \cos(\sphericalangle(\vec{u}, \vec{v})) \Leftrightarrow (\alpha \vec{v}) \cdot \vec{v} = ||\alpha \vec{v}|| \, ||\vec{v}|| \cos(\sphericalangle(\vec{u}, \vec{v})) \Leftrightarrow$
$\Leftrightarrow \alpha ||\vec{v}||^2 = |\alpha| \, ||\vec{v}|| \, ||\vec{v}|| \cos(\sphericalangle(\vec{u}, \vec{v})) \Leftrightarrow \cos(\sphericalangle(\vec{u}, \vec{v})) = \frac{\alpha}{|\alpha|} \Leftrightarrow$
$\Leftrightarrow \cos(\sphericalangle(\vec{u}, \vec{v})) = \pm 1 \Leftrightarrow \sphericalangle(\vec{u}, \vec{v}) = 0 \vee \sphericalangle(\vec{u}, \vec{v}) = \pi$

7.3 $||\vec{u} + \vec{v}||^2 = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) = \vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{v} = \vec{u} \cdot \vec{u} + 0 + 0 + \vec{v} \cdot \vec{v} = ||\vec{u}||^2 + ||\vec{v}||^2$

7.4 $||\vec{u}_1|| = ||\vec{u}_2|| = \sqrt{2} \wedge \cos(\pi/3) = 1/2 \Rightarrow \vec{u}_1 \cdot \vec{u}_2 = \sqrt{2}\sqrt{2}(1/2) = 1$. Tem-se:
$(k\vec{u}_1 - \vec{u}_2) \cdot (\vec{u}_1 + 2\vec{u}_2) = 0 \Leftrightarrow k(\vec{u}_1 \cdot \vec{u}_1) + 2k(\vec{u}_1 \cdot \vec{u}_2) - \vec{u}_2 \cdot \vec{u}_1 - 2(\vec{u}_2 \cdot \vec{u}_2) = 0 \Leftrightarrow k||\vec{u}_1||^2 + (2k-1)\vec{u}_1 \cdot \vec{u}_2 - 2||\vec{u}_2||^2 = 0 \Leftrightarrow 2k + (2k-1) - 4 = 0 \Leftrightarrow k = 5/4$

7.5 $||\vec{u}|| = ||(-1, 0, 2, -2)|| = \sqrt{(-1)^2 + 2^2 + (-2)^2} = 3$ e
$||k\vec{u}|| = 15 \Leftrightarrow |k| \, ||\vec{u}|| = 15 \Leftrightarrow 3|k| = 15 \Leftrightarrow |k| = 5 \Leftrightarrow k = -5 \vee k = 5$

7.6 (a) $\vec{u} \cdot \vec{v} = 0 \Leftrightarrow (3, -1, -2) \cdot (k, 7, 1) = 0 \Leftrightarrow 3k - 7 - 2 = 0 \Leftrightarrow k = 3$
(b) $\vec{u} \cdot \vec{v} = 0 \Leftrightarrow (k, 1, k) \cdot (k, 6, -5) = 0 \Leftrightarrow k^2 + 6 - 5k = 0 \Leftrightarrow k^2 - 5k + 6 = 0 \Leftrightarrow k = 2 \vee k = 3$

7.7 (a) $\text{proj}_{\vec{u}} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}} \vec{u} = \frac{-2-1+2}{4+1+1}(2,1,1) = \frac{-1}{6}(2,1,1) = \left(-\frac{1}{3}, -\frac{1}{6}, -\frac{1}{6}\right)$ e
$\text{perp}_{\vec{u}} \vec{v} = \vec{v} - \text{proj}_{\vec{u}} \vec{v} = (-1,-1,2) - \left(-\frac{1}{3}, -\frac{1}{6}, -\frac{1}{6}\right) = \left(-\frac{2}{3}, -\frac{5}{6}, \frac{13}{6}\right)$;

(b) Sendo $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ a base canónica de $\mathbb{R}^3$, tem-se:
$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ 2 & 1 & 1 \\ -1 & -1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} \vec{e}_1 - \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} \vec{e}_2 + \begin{vmatrix} 2 & 1 \\ -1 & -1 \end{vmatrix} \vec{e}_3 = 3\vec{e}_1 - 5\vec{e}_2 - \vec{e}_3 = (3, -5, -1)$.

(c) $F = \langle \vec{u}, \vec{v} \rangle = \langle \vec{u}, \text{perp}_{\vec{u}} \vec{v} \rangle$, tendo em conta que $\text{perp}_{\vec{u}} \vec{v}$ é combinação linear de $\vec{u}$ e de $\vec{v}$ $\left[\text{perp}_{\vec{u}} \vec{v} = \vec{v} - \frac{\vec{u} \cdot \vec{v}}{\vec{u} \cdot \vec{u}} \vec{u}\right]$, logo, $F = \left\langle (2,1,1), \left(-\frac{2}{3}, -\frac{5}{6}, \frac{13}{6}\right) \right\rangle$ e estes 2 geradores são linearmente independentes e ortogonais. Formam, por isso, uma base ortogonal de F;

(d) $\dim(F) = 2 \Rightarrow \dim(F^\perp) = 1$ e $\vec{u} \times \vec{v} \in F^\perp$, donde, uma base ortonormada de $F^\perp$ é

$$\left\{ \frac{\vec{u} \times \vec{v}}{||\vec{u} \times \vec{v}||} \right\} = \left\{ \frac{(3, -5, -1)}{\sqrt{35}} \right\} = \left\{ \left(\frac{3}{\sqrt{35}}, -\frac{5}{\sqrt{35}}, -\frac{1}{\sqrt{35}} \right) \right\}$$

(e) Do que foi feito nas alíneas anteriores, e tendo em conta que vetores não nulos e ortogonais são linearmente independentes, uma base ortonormada de $\mathbb{R}^3$ contendo uma base de F é:

$$\left\{ \frac{u}{\|u\|}, \frac{\operatorname{perp}_u v}{\|\operatorname{perp}_u v\|}, \frac{u \times v}{\|u \times v\|} \right\} = \left\{ \frac{1}{\sqrt{6}}(2,1,1), \frac{1}{\sqrt{210}}(-4,-5,13), \frac{1}{\sqrt{35}}(3,-5,-1) \right\}$$

7.8 $\overrightarrow{u} = (2,2,-1)$ e $\overrightarrow{v} = (3,1,8)$

(a) $\overrightarrow{u} \cdot \overrightarrow{v} = (2,2,-1) \cdot (3,1,8) = 6 + 2 - 8 = 0$

(b) Tendo em conta que vetores não nulos e ortogonais são linearmente independentes, podemos calcular $\overrightarrow{w}$ de 2 formas:
(i) $\overrightarrow{w} = \overrightarrow{u} \times \overrightarrow{v}$
ou
(ii) $\overrightarrow{w} = (a,b,c)$ tal que $(a,b,c) \cdot (2,2,-1) = 0$ e $(a,b,c) \cdot (3,8,1) = 0$
Por exemplo, $\overrightarrow{w} = (-2,1,-2)$ é uma solução possível. Para determinar todas as soluções possíveis, basta resolver a condição $2a + 2b - c = 0 \wedge 3a + 8b + c = 0$, cujo conjunto-solução é $\{(-2b, b, -2b) : b \in \mathbb{R}\}$.

7.9 (a) $r = \langle (1,2,-1) \rangle$ e $\alpha \equiv x + 2y - z = 0 \Leftrightarrow (x,y,z) \cdot (1,2,-1) = 0$ logo $r^\perp = \alpha$.

(b) Uma base ortonormada de r é $B_r = \left\{ \frac{(1,2,-1)}{||((1,2,-1)||} \right\} = \left\{ \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \right) \right\}$.
Uma base de α é $B_\alpha = \{(-2,1,0),(1,0,1)\}$. Ortogonalizemos a base de B_α:
$\overrightarrow{u}'_1 = (-2,1,0)$
$\overrightarrow{u}'_2 = (1,0,1) - \frac{(1,0,1) \cdot (-2,1,0)}{(-2,1,0) \cdot (-2,1,0)}(-2,1,0) = (1,0,1) - \frac{-2}{5}(-2,1,0) = \left(\frac{1}{5}, \frac{2}{5}, 1 \right)$
Uma base ortonormada de F é $\left\{ \frac{\overrightarrow{u}'_1}{||\overrightarrow{u}'_1||}, \frac{\overrightarrow{u}'_2}{||\overrightarrow{u}'_2||} \right\} = \left\{ \left(\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0 \right), \left(\frac{1}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{5}{\sqrt{30}} \right) \right\}$
Logo, uma base ortonormada de $\mathbb{R}^3$ contituída por vetores de r e α é:

$$\left\{ \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \right), \left(\frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0 \right), \left(\frac{1}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{5}{\sqrt{30}} \right) \right\}$$

7.10 (a) $F = \langle (1,-2,1),(0,1,-1) \rangle$
Como os 2 geradores de F são linearmente independentes (não são colineares), formam uma base de F.
Por isso, $\dim(F^\perp) = 3 - \dim(F) = 1$ e
$F^\perp = \{(x,y,z) \in \mathbb{R}^3 : (x,y,z) \cdot (1,-2,1) = 0 \wedge (x,y,z) \cdot (0,1,-1) = 0\} =$
$\{(x,y,z) \in \mathbb{R}^3 : x - 2y + z = 0 \wedge y - z = 0\} = \{(z,z,z) : z \in \mathbb{R}\} = \langle (1,1,1) \rangle$.
Uma base ortonormada de $F^\perp$ é $\left\{ \frac{(1,1,1)}{||(1,1,1)||} \right\} = \left\{ \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \right\}$
Para obtermos uma base ortonormada de $\mathbb{R}^3$ formada só por vetores de F e de $F^\perp$, começamos por ortonormalizar a base de F dada:
$\overrightarrow{u}'_1 = (1,-2,1)$
$\overrightarrow{u}'_2 = (0,1,-1) - \frac{(1,-2,1) \cdot (0,1,-1)}{(1,-2,1) \cdot (1,-2,1)}(1,-2,1) = (0,1,-1) - \frac{-3}{6}(1,-2,1) = \left(\frac{1}{2}, 0, -\frac{1}{2} \right)$
Uma base ortonormada de F é $\left\{ \frac{\overrightarrow{u}'_1}{||\overrightarrow{u}'_1||}, \frac{\overrightarrow{u}'_2}{||\overrightarrow{u}'_2||} \right\} = \left\{ \frac{(1,-2,1)}{\sqrt{6}}, \frac{(1/2,0,-1/2)}{1/\sqrt{2}} \right\}$
Uma base ortonormada de $\mathbb{R}^3$ formada só por vetores de F e de $F^\perp$ é
$\left\{ \left(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right), \left(\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2} \right), \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \right\}$

(b) $F = \langle (1,-1,0) \rangle$
$\dim(F) = 1 \Rightarrow \dim(F^\perp) = 3 - 1 = 2$ e
$F^\perp = \{(x,y,z) \in \mathbb{R}^3 : (x,y,z)\cdot(1,-1,0) = 0\} = \{(x,y,z) \in \mathbb{R}^3 : x - y = 0\} =$
$\{(y,y,z) : y,z \in \mathbb{R}\} = \langle (1,1,0),(0,0,1) \rangle$
Os 2 geradores de $F^\perp$ são ortogonais e não nulos, donde, formam uma base ortogonal de $F^\perp$. Por isso, obtém-se uma base ortonormada para $F^\perp$ dividindo cada vetor pela sua norma:
$\left\{ \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right), (0,0,1) \right\}$
Uma base ortonormada de $\mathbb{R}^3$ formada só por vetores de F e de $F^\perp$ é
$\left\{ \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right), (0,0,1) \right\}$

(c) $F = \langle (1,0,1),(0,1,1),(2,2,4) \rangle$
$(2,2,4) = 2(1,0,1) + 2(0,1,1) \Rightarrow F = \langle (1,0,1),(0,1,1) \rangle$, donde, $\dim(F) = 2 \Rightarrow \dim(F^\perp) = 1$
$F^\perp = \{(x,y,z) \in \mathbb{R}^3 : (x,y,z)\cdot(1,0,1) = 0 \wedge (x,y,z)\cdot(0,1,1) = 0\} =$
$\{(x,y,z) \in \mathbb{R}^3 : x + z = 0 \wedge y + z = 0\} = \{(-z,-z,z) : z \in \mathbb{R}\} = \langle (-1,-1,1) \rangle$
Uma base ortonormada de $F^\perp$ é $\left\{ \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \right\}$
Para obtermos uma base ortonormada de $\mathbb{R}^3$ formada só por vetores de F e de $F^\perp$, começamos por ortonormalizar a base de F dada:
$\overrightarrow{u}'_1 = (1,0,1)$
$\overrightarrow{u}'_2 = (0,1,1) - \frac{(1,0,1)\cdot(0,1,1)}{(1,0,1)\cdot(1,0,1)}(1,0,1) = (0,1,1) - \frac{1}{2}(1,0,1) = \left(-\frac{1}{2}, 1, \frac{1}{2} \right)$
Uma base ortonormada de F é $\left\{ \frac{\overrightarrow{u}'_1}{||\overrightarrow{u}'_1||}, \frac{\overrightarrow{u}'_2}{||\overrightarrow{u}'_2||} \right\} = \left\{ \frac{(1,0,1)}{\sqrt{2}}, \frac{(-1/2,1,1/2)}{\sqrt{3/2}} \right\}$
Uma base ortonormada de $\mathbb{R}^3$ formada só por vetores de F e de $F^\perp$ é
$\left\{ \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right), \left(-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6} \right), \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \right\}$

(d) $F = \langle (0,1,-2),(0,-3,6) \rangle$
$(0,-3,6) = (-3)(0,1,-2) \Rightarrow F = \langle (0,1,-2) \rangle$, donde, $\dim(F) = 1 \Rightarrow \dim(F^\perp) = 2$
$F^\perp = \{(x,y,z) \in \mathbb{R}^3 : (x,y,z)\cdot(0,1,-2) = 0\} = \{(x,y,z) \in \mathbb{R}^3 : y - 2z = 0\} =$
$\{(x,2z,z) : x,z \in \mathbb{R}\} = \langle (1,0,0),(0,2,1) \rangle$
Os 2 geradores de $F^\perp$ são ortogonais e não nulos, donde, formam uma base ortogonal de $F^\perp$. Por isso, obtém-se uma base ortonormada para $F^\perp$ dividindo cada vetor pela sua norma:
$\left\{ (1,0,0), \left(0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right) \right\}$
Uma base ortonormada de $\mathbb{R}^3$ formada só por vetores de F e de $F^\perp$ é
$\left\{ \left(0, \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}} \right), (1,0,0), \left(0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right) \right\}$

7.11 $v_1 = (1,0,1), v_2 = (1,0,-2), v_3 = (0,3,4)$

$$u_1 = v_1 = (1,0,1)$$

$$u_2 = v_2 - \text{proj}_{u_1} v_2 = v_2 - \frac{v_2 \cdot u_1}{u_1 \cdot u_1} u_1 = v_2 + \frac{1}{2} u_1 = \frac{3}{2}(1,0,-1)$$

$$u_3 = v_3 - \text{proj}_{u_1} v_3 - \text{proj}_{u_2} v_3 = v_3 - \frac{v_3 \cdot u_1}{u_1 \cdot u_1} u_1 - \frac{v_3 \cdot u_2}{u_2 \cdot u_2} u_2 = v_3 - 2u_1 + \frac{4}{3} u_2 = (0,3,0)$$

Base ortonormada de $\mathbb{R}^3$:

$$\left\{\frac{u_1}{\|u_1\|}, \frac{u_2}{\|u_2\|}, \frac{u_3}{\|u_3\|}\right\} = \left\{\frac{1}{\sqrt{2}}(1,0,1), \frac{1}{\sqrt{2}}(1,0,-1), (0,1,0)\right\}$$

7.12 Sejam v_1, v_2, v_3 os 3 geradores de F. Temos $r\left(\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}\right) = 3$ logo os 3 geradores de F são linearmente independentes.

$\overrightarrow{u}_1 = v_1 = (-1,0,1,1)$

$\overrightarrow{u}_2 = (1,0,1,2) - \dfrac{(1,0,1,2)\cdot(-1,0,1,1)}{(-1,0,1,1)\cdot(-1,0,1,1)}(-1,0,1,1) = (1,0,1,2) - \dfrac{2}{3}(-1,0,1,1) = \left(\dfrac{5}{3}, 0, \dfrac{1}{3}, \dfrac{4}{3}\right) = \dfrac{1}{3}(5,0,1,4)$

Tomemos $\overrightarrow{u}''_2 = (5,0,1,4)$

$\overrightarrow{u}'_3 = (0,-2,0,1) - \dfrac{(0,-2,0,1)\cdot(-1,0,1,1)}{(-1,0,1,1)\cdot(-1,0,1,1)}(-1,0,1,1) - \dfrac{(0,-2,0,1)\cdot(5,0,1,4)}{(5,0,1,4)\cdot(5,0,1,4)}(5,0,1,4) =$
$(0,-2,0,1) - \dfrac{1}{3}(-1,0,1,1) - \dfrac{4}{42}(5,0,1,4) = (0,-2,0,1) - \dfrac{1}{3}(-1,0,1,1) - \dfrac{2}{21}(5,0,1,4) = \left(-\dfrac{1}{7}, -2, -\dfrac{3}{7}, \dfrac{2}{7}\right) =$
$-\dfrac{1}{7}(1,14,3,-2)$

Tomemos $\overrightarrow{u}''_3 = (1,14,3,-2)$

Uma base ortonormada de F nas condições pedidas é

$\left\{\dfrac{\overrightarrow{u}'_1}{||\overrightarrow{u}'_1||}, \dfrac{\overrightarrow{u}''_2}{||\overrightarrow{u}''_2||}, \dfrac{\overrightarrow{u}''_3}{||\overrightarrow{u}''_3||}\right\} = \left\{\left(-\dfrac{1}{\sqrt{3}}, 0, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right), \left(\dfrac{5}{\sqrt{42}}, 0, \dfrac{1}{\sqrt{42}}, \dfrac{4}{\sqrt{42}}\right), \left(\dfrac{1}{\sqrt{210}}, \dfrac{14}{\sqrt{210}}, \dfrac{3}{\sqrt{210}}, -\dfrac{2}{\sqrt{210}}\right)\right\}$

7.13 Sejam u_1, u_2 os geradores de F. Temos $u_1 \cdot u_2 = 0$ logo $\{u_1, u_2\}$ é uma base ortogonal de F.

$$\text{proj}_F\, v \in F \qquad \text{e} \qquad v - proj_F v \in F^\perp$$

$$\text{proj}_F\, v = proj_{u_1} v + \text{proj}_{u_2}\, v = \frac{1}{3}(7,5,6,-1)$$

$$v = \text{proj}_F\, v + \left(v - \text{proj}_F\, v\right) = \frac{1}{3}(7,5,6,-1) + \frac{1}{3}(-1,-2,3,1)$$

7.14 (a) $F = \{(x,y,z,w) \in \mathbb{R}^4 : x+y+z+w = 0\} = \{(x,y,z,w) \in \mathbb{R}^4 : (x,y,z,w)\cdot(1,1,1,1) = 0\}$
$\Rightarrow F^\perp = \langle(1,1,1,1)\rangle$

Uma base ortonormada de $F^\perp$ é: $\left\{\left(\dfrac{(1,1,1,1)}{||(1,1,1,1)||}\right)\right\} = \left\{\left(\dfrac{1}{\sqrt{4}}, \dfrac{1}{\sqrt{4}}, \dfrac{1}{\sqrt{4}}, \dfrac{1}{\sqrt{4}}\right)\right\}$

Uma base de F é: $B = \{(-1,1,0,0), (-1,0,1,0), (-1,0,0,1)\}$. Ortogonalizemos a base B:

$\overrightarrow{u}'_1 = (-1,1,0,0)$

$\overrightarrow{u}'_2 = (-1,0,1,0) - \dfrac{(-1,0,1,0)\cdot(-1,1,0,0)}{(-1,1,0,0)\cdot(-1,1,0,0)}(-1,1,0,0) = (-1,0,1,0) - \dfrac{1}{2}(-1,1,0,0) = (-\dfrac{1}{2}, -\dfrac{1}{2}, 1, 0) =$
$-\dfrac{1}{2}(1,1,-2,0)$

Tomemos $\overrightarrow{u}''_2 = (1,1,-2,0)$

$\overrightarrow{u}'_3 = (-1,0,0,1) - \dfrac{(-1,0,0,1)\cdot(-1,1,0,0)}{(-1,1,0,0)\cdot(-1,1,0,0)}(-1,1,0,0) - \dfrac{(-1,0,0,1)\cdot(1,1,-2,0)}{(1,1,-2,0)\cdot(1,1,-2,0)}(1,1,-2,0) =$
$(-1,0,0,1) - \dfrac{1}{2}(-1,1,0,0) - \dfrac{-1}{6}(1,1,-2,0) = \left(-\dfrac{1}{3}, -\dfrac{1}{3}, -\dfrac{1}{3}, 1\right) = -\dfrac{1}{3}(1,1,1,-3)$

Tomemos $\overrightarrow{u}''_3 = (1,1,1,-3)$

Uma base ortonormada de F é:
$\left\{\dfrac{\overrightarrow{u}'_1}{||\overrightarrow{u}'_1||}, \dfrac{\overrightarrow{u}''_2}{||\overrightarrow{u}''_2||}, \dfrac{\overrightarrow{u}''_3}{||\overrightarrow{u}''_3||}\right\} = \left\{\left(-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}, 0, 0\right), \left(\dfrac{1}{\sqrt{6}}, \dfrac{1}{\sqrt{6}}, -\dfrac{2}{\sqrt{6}}, 0\right), \left(\dfrac{1}{\sqrt{12}}, \dfrac{1}{\sqrt{12}}, \dfrac{1}{\sqrt{12}}, -\dfrac{3}{\sqrt{12}}\right)\right\}$

Uma base ortonormada de $\mathbb{R}^4$ formada só por vetores de F e de $F^\perp$ é

$$\left\{\left(\frac{1}{\sqrt{4}},\frac{1}{\sqrt{4}},\frac{1}{\sqrt{4}},\frac{1}{\sqrt{4}}\right),\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0,0\right),\left(\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},-\frac{2}{\sqrt{6}},0\right),\left(\frac{1}{\sqrt{12}},\frac{1}{\sqrt{12}},\frac{1}{\sqrt{12}},-\frac{3}{\sqrt{12}}\right)\right\}$$

(b) $F=\{(x,y,z,w)\in\mathbb{R}^4 : x=0\wedge z=0\}=\{(x,y,z,w)\in\mathbb{R}^4 : (x,y,z,w)\cdot(1,0,0,0)=0\wedge (x,y,z,w)\cdot(0,0,1,0)=0\}\Rightarrow F^\perp=\langle(1,0,0,0),(0,0,1,0)\rangle$
Uma base ortonormada de $F^\perp$ é: $\{(1,0,0,0),(0,0,1,0)\}$
Uma base ortonormada de F é: $\{0,1,0,0),(0,0,0,1)\}$
Uma base ortonormada de $\mathbb{R}^4$ formada só por vetores de F e de $F^\perp$ é:

$$\{(1,0,0,0),(0,0,1,0),(0,1,0,0),(0,0,0,1)\}$$