
Soluções - 5 -Aplicações Lineares

5.1 (a) f_1 não é linear, uma vez que $f_1(0,0) = (0,0,1) \neq (0,0,0)$

(b) f_2 é linear:

- $f_2((x,y)+(a,b)) = f_2(x+a,y+b) = ((x+a)-(y+b), 2(y+b), 2(x+a)+3(y+b)) = (x-y, 2y, 2x+3y)+(a-b, 2b, 2a+3b) = f_2(x,y)+f_2(a,b)$
- $f_2(\alpha(x,y)) = f_2(\alpha x, \alpha y) = (\alpha x - \alpha y, 2\alpha y, 2\alpha x + 3\alpha y) = \alpha(x-y, 2y, 2x+3y) = \alpha f_2(x,y)$

(c) f_3 não é linear. Por exemplo,

$$\left.\begin{array}{l} f_3((-1)(1,0)) = f_3(-1,0) = |-1+0| = 1 \\ (-1)f_3(1,0) = (-1)|1+0| = -1 \end{array}\right\} \Rightarrow f_3((-1)(1,0)) \neq (-1)f_3(1,0)$$

(d) f_4 não é linear: $f_4(0,0,0) = (1,0) \neq (0,0)$

(e) f_5 não é linear. Por exemplo,

$$\left.\begin{array}{l} f_5((1,1,0)+(1,0,0)) = f_5(2,1,0) = (2,2,2) \\ f_5(1,1,0)+f_5(1,0,0) = (1,2,1)+(0,0,1) = (1,2,2) \end{array}\right\} \Rightarrow f_5((1,1,0)+(1,0,0)) \neq f_5(1,1,0)+f_5(1,0,0)$$

(f) f_6 não é linear. Por exemplo,

$$\left.\begin{array}{l} f_6(2(1,0)) = f_6(2,0) = (4,0) \\ 2f_6(1,0) = 2(1,0) = (2,0) \end{array}\right\} \Rightarrow f_6(2(1,0)) \neq 2f_6(1,0)$$

(g) f_7 é linear:

- $f_7((x,y,z)+(a,b,c)) = f_7(x+a,y+b,z+c) = \begin{vmatrix} 1 & -1 & x+a \\ 3 & 0 & y+b \\ -1 & 2 & z+c \end{vmatrix} = \begin{vmatrix} 1 & -1 & x \\ 3 & 0 & y \\ -1 & 2 & z \end{vmatrix} + \begin{vmatrix} 1 & -1 & a \\ 3 & 0 & b \\ -1 & 2 & c \end{vmatrix} =$

  $= f_7(x,y,z)+f_7(a,b,c)$
- $f_7(\alpha(x,y,z)) = f_7(\alpha x, \alpha y, \alpha z) = \begin{vmatrix} 1 & -1 & \alpha x \\ 3 & 0 & \alpha y \\ -1 & 2 & \alpha z \end{vmatrix} = \alpha \begin{vmatrix} 1 & -1 & x \\ 3 & 0 & y \\ -1 & 2 & z \end{vmatrix} = \alpha f_7(x,y,z)$

(h) f_8 é linear:

- $f_8((x,y)+(a,b)) = f_8(x+a,y+b) = (2(x+a)-(y+b), (x+a)+6(y+b)) =$

$$= (2x-y, x+6y)+(2a-b, a+6b) = f_8(x,y)+f_8(a,b)$$

- $f_8(\alpha(a,b)) = f_8(\alpha a, \alpha b) = (2\alpha a - \alpha b, \alpha a + 6\alpha b) = \alpha(2a-b, a+6b) = \alpha f_8(a,b)$

(i) f_9 é linear:

- $f_9\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = f_9\left(\begin{bmatrix} x+a & y+b \\ z+c & w+d \end{bmatrix}\right) = \begin{bmatrix} (x+a)+(w+d) & (y+b)-(z+c) \\ 0 & 2(z+c)+(w+d) \end{bmatrix} =$
$$= \begin{bmatrix} x+w & y-z \\ 0 & 2z+w \end{bmatrix} + \begin{bmatrix} a+d & b-c \\ 0 & 2c+d \end{bmatrix} = f_9\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) + f_9\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$$
- $f_9\left(\alpha\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = f_9\left(\begin{bmatrix} \alpha x & \alpha y \\ \alpha z & \alpha w \end{bmatrix}\right) = \begin{bmatrix} \alpha x + \alpha w & \alpha y - \alpha z \\ 0 & 2\alpha z + \alpha w \end{bmatrix} = \alpha\begin{bmatrix} x+w & y-z \\ 0 & 2z+w \end{bmatrix} = \alpha f_9\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right)$

(j) f_{10} não é linear. Por exemplo,

$$\left.\begin{array}{l} f_{10}\left((-1)\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = f_{10}\left(\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ (-1)f_{10}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = (-1)\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \end{array}\right\} \Rightarrow f_{10}\left((-1)\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) \neq (-1)f_{10}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right)$$

(k) f_{11} é linear:

- $f_{11}\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = f_{11}\left(\begin{bmatrix} x+a & y+b \\ z+c & w+d \end{bmatrix}\right) = ((x+a)+(w+d), (y+b)+(z+c)) =$
$$= (x+w, y+z) + (a+d, b+c) = f_{11}\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) + f_9\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$$
- $f_{11}\left(\alpha\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = f_{11}\left(\begin{bmatrix} \alpha x & \alpha y \\ \alpha z & \alpha w \end{bmatrix}\right) = (\alpha x + \alpha w, \alpha y + \alpha z) = \alpha(x+w, y+z) = \alpha f_9\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right)$

(l) f_{12} é linear:

- $f_{12}(X+Y) = A(X+Y) = AX + AY = f_{12}(X) + f_{12}(Y)$
- $f_{12}(\alpha X) = A(\alpha X) = \alpha(AX) = \alpha f_{12}(X)$

5.2 $f(x,y,z,w) = (2x - 2y + 3z + w, -x - 2z + w)$ e $f(1,3,4,-2) = (6,-11)$

5.3 $T(x,y,z) = (2x + 2y + z, 3x - 2y, -2x + 6y - 11z)$ e $T(2,0,1) = (5,6,-15)$

5.4 $g(x,y) = \dfrac{1}{7}(3x - y, -9x - 4y, 5x + 10y)$ e $g(2,-3) = \left(\dfrac{9}{7}, -\dfrac{6}{7}, -\dfrac{20}{7}\right)$

5.5

- $f_2(x,y) = (0,0,0) \Leftrightarrow \begin{cases} x - y = 0 \\ 2y = 0 \\ 2x + 3y = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = 0 \\ 0 = 0 \end{cases}$

$$\mathrm{Nuc}(f_2) = \{(x,y) \in \mathbb{R}^2 : f_2(x,y) = (0,0,0)\} = \{(0,0)\}$$

$f_2(x,y) = (x-y, 2y, 2x+3y) = x((1,0,2) + y(-1,2,3)$

$$\mathrm{Im}(f_2) = \langle f_2(1,0), f_2(0,1)\rangle = \langle (1,0,2), (-1,2,3)\rangle = \{(x,y,z) \in \mathbf{R}^3 : -4x - 5y + 2z = 0\}$$

- $f_7(x,y,z) = 0 \Leftrightarrow \begin{vmatrix} 1 & -1 & x \\ 3 & 0 & y \\ -1 & 2 & z \end{vmatrix} = 0 \Leftrightarrow 6x - y + 3z = 0$

$$\mathrm{Nuc}(f_7) = \{(x,y,z) \in \mathbb{R}^3 : f_7(x,y,z) = 0\} = \{(x,y,z) \in \mathbb{R}^3 : 6x - y + 3z = 0\}$$

$$f_7(x,y,z) = \begin{vmatrix} 1 & -1 & x \\ 3 & 0 & y \\ -1 & 2 & z \end{vmatrix} = 6x - y + 3z$$

$$\mathrm{Im}(f_7) = \langle f_7(1,0,0), f_7(0,1,0), f_7(0,0,1) \rangle = \mathbb{R}$$

- $f_8(a,b) = (0,0) \Leftrightarrow \begin{cases} 2a - b = 0 \\ a + 6b = 0 \end{cases} \Leftrightarrow \begin{cases} -13b = 0 \\ a = -6b \end{cases} \Leftrightarrow \begin{cases} b = 0 \\ a = 0 \end{cases}$

$$\mathrm{Nuc}(f_8) = \{(a,b) \in \mathbb{R}^2 : f_8(a,b) = (0,0)\} = \{(0,0)\}$$

$$\mathrm{Im}(f_8) = \langle f_8(1,0), f_8(0,1) \rangle = \langle (2,1), (-1,6) \rangle = \mathbf{R}^2$$

- $f_9\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} x+w & y-z \\ 0 & 2z+w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Leftrightarrow \ldots \Leftrightarrow \begin{cases} x = 2z \\ y = z \\ w = -2z \end{cases}$

$$\mathrm{Nuc}(f_9) = \left\{ \begin{bmatrix} x & y \\ z & w \end{bmatrix} \in \mathbb{R}^{2\times 2} : f_9\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} 2z & z \\ z & -2z \end{bmatrix} : z \in \mathbb{R} \right\}$$

$$\begin{aligned} \mathrm{Im}(f_9) &= \left\langle f_9\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right), f_9\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right), f_9\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right), f_9\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) \right\rangle = \left\langle \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\rangle = \\ &= \left\langle \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\rangle = \left\{ \begin{bmatrix} x & y \\ z & w \end{bmatrix} \in \mathbf{R}^{2\times 2} \colon z = 0 \right\} \end{aligned}$$

- $f_{11}\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = (0,0) \Leftrightarrow (x+w, y+z) = (0,0) \begin{cases} x = -w \\ y = -z \end{cases}$

$$\mathrm{Nuc}(f_{11}) = \left\{ \begin{bmatrix} x & y \\ z & w \end{bmatrix} \in \mathbb{R}^{2\times 2} : f_{11}\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = (0,0) \right\} = \left\{ \begin{bmatrix} -w & -z \\ z & w \end{bmatrix} : z, w \in \mathbb{R} \right\}$$

$$\begin{aligned} \mathrm{Im}(f_{11}) &= \left\langle f_{11}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right), f_{11}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right), f_{11}\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right), f_{11}\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) \right\rangle = \langle (1,0), (0,1), (0,1), (1,0) \rangle = \\ &= \langle (1,0), (0,1) \rangle = \mathbb{R}^2 \end{aligned}$$

- $\mathrm{Nuc}(f_{12}) = \{X \in \mathbb{R}^{n\times 1} \colon f(X) = O\} = \{X \in \mathbb{R}^{n\times 1} \colon AX = O\}$ é o subespaço de $\mathbb{R}^{n\times 1}$ formado pelas soluções do sistema homogéneo $AX = O$, também designado núcleo de A.

$$f_{12}(X) = AX = \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \ldots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1 \begin{bmatrix} a_{11} \\ \vdots \\ a_{n1} \end{bmatrix} + \cdots + x_n \begin{bmatrix} a_{1n} \\ \vdots \\ a_{nn} \end{bmatrix}$$

$$\mathrm{Im}(f_{12}) = \left\langle \begin{bmatrix} a_{11} \\ \vdots \\ a_{n1} \end{bmatrix}, \ldots, \begin{bmatrix} a_{1n} \\ \vdots \\ a_{nn} \end{bmatrix} \right\rangle$$

é o subespaço de $\mathbb{R}^{n\times 1}$ gerado pelas colunas de A, também designado espaço-coluna de A.

5.6 (a) $(1,-4), (-3,12) \in \mathrm{Im}(f)$;

(b) $(5,10) \in \mathrm{Nuc}(f)$.

5.7 (a) $T(1,0,-1,2,3) = (5,4,10,3)$.

(b) $\dim \mathrm{Im}(T) = r(M_T) = 4$

$\{(1,1,2,0),(-1,0,-1,-1),(3,3,5,1),(-1,-1,-1,0)\}$ é uma base de $\mathrm{Im}(T)$

$\dim \mathrm{Nuc}(T) = 1$, $\{(0,1,0,0,0)\}$ é uma base de $\mathrm{Nuc}(T)$

(c) $\dim \mathrm{Im}(T) = 4 = \dim \mathbb{R}^4$ logo $\mathrm{Im}(T) = \mathbb{R}^4$.

$\mathrm{Nuc}(T) = \{(0,y,0,0,0) : y \in \mathbb{R}\}$.

(d) É sobrejetiva porque $\dim \mathrm{Im}(T) = 4 = \dim \mathbb{R}^4$.

Não é injetiva porque $\dim \mathrm{Nuc}(T) = 1 \neq 0$.

(e) $T(v) = (2,2,4,1) \Leftrightarrow v = (0,y,0,1,1), y \in \mathbb{R} \Leftrightarrow v \in (0,0,0,1,1) + \mathrm{Nuc}(T)$

5.8 $\dim \mathrm{Im}(g) = r(M_g) = 3$ logo $\mathrm{Im}(g) = \mathbb{R}^3$;

$\dim \mathrm{Nuc}(g) = 0$ logo $\mathrm{Nuc}(g) = \{(0,0,0)\}$.

Como $\mathrm{Im}(g) = \mathbb{R}^3$, g é um endomorfismo sobrejetivo. Como $\mathrm{Nuc}(g) = \{(0,0,0)\}$, g é um monomorfismo. Assim g é um automorfismo de $\mathbb{R}^3$.

5.9 $\{(1,0)\}$ é uma base de $\mathrm{Im}(h)$, logo, $\mathrm{Im}(h) = \{(x,0) : x \in \mathbb{R}\}$

$\mathrm{Nuc}(h) = \{(x,y) \in \mathbb{R}^2 : y = 0\} = \{(x,0) : x \in \mathbb{R}\} = \mathrm{Im}(h)$.

5.10 (a) Se $n > m$ então $r(M_f) \leq m < n = \dim \mathbb{R}^n$ logo $\dim \mathrm{Nuc}(f) > 0$ e f não é injetiva.

(b) Se $n < m$ então $r(M_f) \leq n < m$ logo $\dim \mathrm{Im}(f) < m = \dim \mathbb{R}^m$ e f não é sobrejetiva.

(c) Suponha-se que $n = m$.

f é injetiva $\Leftrightarrow \dim \mathrm{Nuc}(f) = 0 \Leftrightarrow r(M_f) = n \Leftrightarrow \dim \mathrm{Im}(f) = n = \dim \mathbb{R}^n \Leftrightarrow f$ é sobrejetiva.

5.11 (a) $(T_2 \circ T_1) : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ tal que $(T_2 \circ T_1)(x,y,z) = (x+y-z, -x+y, 0, 2x-z)$;

(b) $(T_3 \circ T_2) : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ tal que $(T_3 \circ T_2)(x,y) = (x+y, 2x+2y, 3x)$

(c) $(T_1 \circ T_3) : \mathbb{R}^4 \longrightarrow \mathbb{R}^2$ tal que $(T_1 \circ T_3)(x,y,z,w) = (-2x+2y-2z-w, x+z+2w)$

(d) $(T_1 \circ T_3 \circ T_2) : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ tal que $(T_1 \circ T_3 \circ T_2)(x,y) = (3y, x-2y)$

(e) $(T_3 \circ T_2 \circ T_1) : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ tal que $(T_3 \circ T_2 \circ T_1)(x,y,z) = (-x+y, -2x+2y, 3x+3y-3z)$

5.12 $p : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ tal que $p(x,y,z) = (x,y,0)$

(a) $p \circ p : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ tal que $(p \circ p)(x,y,z) = p(p(x,y,0)) = p(x,y,0) = (x,y,0) = p(x,y,z)$;

(b) $\mathrm{Im}(p) = \langle (1,0,0),(0,1,0) \rangle$ é o plano de equação geral $z = 0$ (plano xy);

(c) $\mathrm{Nuc}(p) = \langle (0,0,1) \rangle$ é a reta de equações reduzidas $\begin{cases} x = 0 \\ y = 0 \end{cases}$ (eixo dos zz).

5.13 (a) $M_g = M(g; b.c.(\mathbb{R}^3), b.c.(\mathbb{R}^3)) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 1 & 2 & -1 \end{bmatrix}$

$r(M_g) = \dim \mathrm{Im}(g) = 3 = \dim \mathbb{R}^3$, logo, g é sobrejetiva;

$\dim \mathrm{Nuc}(g) = \dim \mathbb{R}^3 - \dim \mathrm{Im}(g) = 3 - 3 = 0$, logo, g é injetiva;

(b) $r(M_g) = 3$, logo, M_g é invertível. Por isso, g é também invertível;

$g^{-1} : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ tal que $g^{-1}(x, y, z) = \dfrac{1}{9}(3x + 3y, x - 2y + 3z, 5x - y - 3z)$.

5.14 (a) Se $a \neq 1$ então:

- $\dim \mathrm{Im}(T) = r(M_T) = r(A) = 3$, donde, $\mathrm{Im}(T) = \mathbb{R}^3$ e qualquer base de $\mathbb{R}^3$ é base de $\mathrm{Im}(T)$;
- $\dim \mathrm{Nuc}(T) = 0$ e $\varnothing$ é base de $\mathrm{Nuc}(T)$.

Se $a = 1$ então:

- $\dim \mathrm{Im}(T) = r(M_T) = r(A) = 2$ e $\{(1, 2, 0), (0, 1, 1)\}$ é uma base de $\mathrm{Im}(T)$;
- $\dim \mathrm{Nuc}(T) = 1$ e $\{(-2, 2, 1)\}$ é uma base de $\mathrm{Nuc}(T)$.

(b) Seja $a = 1$.

i. $\dim \mathrm{Nuc}(T) = 1 \neq 0$, logo, T não é injetiva.
Por exemplo, os vetores $(-2, 2, 1)$ e $(0, 0, 0)$ pertencem ao Núcleo de T, donde,

$$T(-2, 2, 1) = T(0, 0, 0) = (0, 0, 0)$$

ii. $(1, 2, k) \in \mathrm{Im}(T) = \langle (1, 2, 0), (0, 1, 1) \rangle$ sse $k = 0$.

T não é sobrejetiva porque, por exemplo, $(1, 2, 1) \notin \mathrm{Im}(T)$.

5.15 (a) $f_A : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ tal que $f_A(x, y, z) = (x - y + 3z, 5x + 6y - 4z, 7x + 4y + 2z)$;

$f_B : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ tal que $f_B(x, y, z) = (2x - z, 4x - 2z, 0)$;

$f_C : \mathbb{R}^4 \longrightarrow \mathbb{R}^2$ tal que $f_C(x, y, z) = (4x + y + 5z + w, x + 2y + 3z)$.

(b) $\dim \mathrm{Im}(f_A) = r(A) = 2$, $\dim \mathrm{Nuc}(f_A) = 1$;

$\dim \mathrm{Im}(f_B) = r(B) = 1$, $\dim \mathrm{Nuc}(f_B) = 2$;

$\dim \mathrm{Im}(f_C) = r(C) = 2$, $\dim \mathrm{Nuc}(f_C) = 2$.

(c) f_A não é injetiva pois $\dim \mathrm{Nuc}(f_A) = 1 \neq 0$

f_A não é sobrejetiva pois $\dim \mathrm{Im}(f_A) = 2 \neq 3 = \dim \mathbb{R}^3$;

f_B não é injetiva pois $\dim \mathrm{Nuc}(f_A) = 2 \neq 0$;

f_B não é sobrejetiva pois $\dim \mathrm{Im}(f_B) = 1 \neq 3 = \dim \mathbb{R}^3$;

f_C não é injetiva pois $\dim \mathrm{Nuc}(f_C) = 2 \neq 0$;

f_C é sobrejetiva pois $\dim \mathrm{Im}(f_C) = 2 = \dim \mathbb{R}^2$.

(d) $\{(1, 5, 7), (-1, 6, 4)\}$ é uma base de $\mathrm{Im}(f_A)$; $\left\{\left(-\dfrac{14}{11}, \dfrac{19}{11}, 1\right)\right\}$ é uma base de $\mathrm{Nuc}(f_A)$;

$\{(2, 4, 0)\}$ é uma base de $\mathrm{Im}(f_B)$; $\{(1, 0, 2), (0, 1, 0)\}$ é uma base de $\mathrm{Nuc}(f_B)$;

$\{(4, 1)), (1, 2)\}$ é uma base de $\mathrm{Im}(f_C)$; $\left\{\left(-2, 1, 0, \dfrac{7}{2}\right), \left(-3, 0, 1, \dfrac{7}{2}\right)\right\}$ é uma base de $\mathrm{Nuc}(f_C)$;

5.16 $g\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x & y \\ z & w \end{bmatrix} - \begin{bmatrix} x & y \\ z & w \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2x & -2y \\ 3z & 3w \end{bmatrix} - \begin{bmatrix} -2x & 3y \\ -2z & 3w \end{bmatrix} = \begin{bmatrix} 0 & -5y \\ 5z & 0 \end{bmatrix}$

(a) $g\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 0 & -5y \\ 5z & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Leftrightarrow \begin{cases} y = 0 \\ z = 0 \end{cases}$

$$\mathrm{Nuc}(g) = \left\{ \begin{bmatrix} x & y \\ z & w \end{bmatrix} \in \mathbb{R}^{2\times 2} : g\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} x & 0 \\ 0 & w \end{bmatrix} : x, w \in \mathbb{R} \right\}$$

Uma base de $\mathrm{Nuc}(g)$ é $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.

(b) $g\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = \begin{bmatrix} 0 & -5y \\ 5z & 0 \end{bmatrix} = -5y\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 5z\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$$\mathrm{Im}(g) = \left\{ g\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) : y, z \in \mathbb{R} \right\} = \left\langle \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right\rangle$$

(c) $\dim \mathrm{Im}(g) = 2 \neq 4$, donde, g não é sobrejetiva, logo, não é bijetiva, por isso, não é invertível.

5.17 (a) $f(a,b,c) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Leftrightarrow \begin{bmatrix} a & b-c \\ 2b & a \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Leftrightarrow \begin{cases} a = 0 \\ b - c = 0 \\ 2b = 0 \\ a = 0 \end{cases} \Leftrightarrow \begin{cases} a = 0 \\ b = 0 \\ c = 0 \end{cases}$

$$\mathrm{Nuc}(f) = \left\{ (a,b,c) \in \mathbb{R}^3 : f(a,b,c) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\} = \{(0,0,0)\}$$

(b) $f(a,b,c) = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \Leftrightarrow \begin{cases} a = 2 \\ b - c = 2 \\ 2b = 2 \\ a = 2 \end{cases} \Leftrightarrow \begin{cases} a = 2 \\ c = -1 \\ b = 1 \end{cases}$

$$f(2,1,-1) = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \in \mathrm{Im}(f)$$

(c) Como $\dim \mathrm{Nuc}(f) = 0$, pelo Teorema da Dimensão, $\dim \mathrm{Im}(f) = 3 \neq 4$, logo, f não é sobrejetiva

(d) $f(a,b,c) = \begin{bmatrix} a & b-c \\ 2b & a \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$

$$\mathrm{Im}(f) = \{f(a,b,c)\colon a,b,c \in \mathbb{R}\} = \left\langle \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \right\rangle$$

5.18 $h\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = \begin{bmatrix} x & x+y \\ z+w & 2w \end{bmatrix}$

(a) $\mathcal{B}_c = \left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right)$

$h\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, h\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, h\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, h\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}$

$M_h = M(h; \mathcal{B}_c, \mathcal{B}_c) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

(b) $r(M_h) = 4$, donde, M_h é invertível e, por isso, h também o é.

$M(h^{-1}; \mathcal{B}_c, \mathcal{B}_c) = M_h^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1/2 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$

$h^{-1}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = 1\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - 1\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$

$h^{-1}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = 0\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$h^{-1}\left(\begin{bmatrix}0&1\\0&0\end{bmatrix}\right)=0\begin{bmatrix}1&0\\0&0\end{bmatrix}+0\begin{bmatrix}0&1\\0&0\end{bmatrix}+1\begin{bmatrix}0&0\\1&0\end{bmatrix}+1\begin{bmatrix}0&0\\0&1\end{bmatrix}=\begin{bmatrix}0&0\\1&0\end{bmatrix}$$
$$h^{-1}\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)=0\begin{bmatrix}1&0\\0&0\end{bmatrix}+0\begin{bmatrix}0&1\\0&0\end{bmatrix}-\tfrac{1}{2}\begin{bmatrix}0&0\\1&0\end{bmatrix}+\tfrac{1}{2}\begin{bmatrix}0&0\\0&1\end{bmatrix}=\begin{bmatrix}0&0\\-\frac{1}{2}&\frac{1}{2}\end{bmatrix}$$

$$\begin{aligned}h^{-1}\left(\begin{bmatrix}x&y\\z&w\end{bmatrix}\right)&=xh^{-1}\left(\begin{bmatrix}1&0\\0&0\end{bmatrix}\right)+yh^{-1}\left(\begin{bmatrix}0&1\\0&0\end{bmatrix}\right)+zh^{-1}\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)+wh^{-1}\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)\\&=\begin{bmatrix}x&-x+y\\z-\frac{1}{2}w&\frac{1}{2}w\end{bmatrix}\end{aligned}$$

5.19 (a) $f(2,0,0)=(2,0)\Rightarrow 2f(1,0,0)=(2,0)\Rightarrow f(1,0,0)=(1,0)$
$f(2,-1,1)=(0,0)\Rightarrow f(2,0,0)-f(0,1,0)+f(0,0,1)=(0,0)$
$\Rightarrow f(0,0,1)=-f(2,0,0)+f(0,1,0)\Rightarrow f(0,0,1)=-(2,0)+(2,1)=(0,1)$
$f(x,y,z)=xf(1,0,0)+yf(0,1,0)+zf(0,0,1)=x(1,0)+y(2,1)+z(0,1)=(x+2y,y+z)$

(b) $\begin{bmatrix}1&-1\\-1&2\\3&-1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}\Rightarrow x=y=0\Rightarrow \mathrm{Nuc}(g)=\{(0,0)\}$, donde, g é injetiva.
$\mathrm{Im}(g)=\langle g(1,0),g(0,1)\rangle=\langle(1,-1,3),(-1,2,-1)\rangle$. Como $\dim\mathrm{Im}(g)=2\neq 3$, g não é sobrejetiva

(c) $F=M(f;b.c.(\mathbb{R}^3),b.c.(\mathbb{R}^2))=\begin{bmatrix}1&2&0\\0&1&1\end{bmatrix}$ e

$FG=M(f\circ g;b.c.(\mathbb{R}^2),b.c.(\mathbb{R}^2))\Rightarrow M(f\circ g;b.c.(\mathbb{R}^2),b.c.(\mathbb{R}^2))=\begin{bmatrix}-1&3\\2&1\end{bmatrix}$, donde,

$(f\circ g)(x,y)=(-x+3y,2x+y)$. Como $\det(FG)=-7\neq 0$,
$r(FG)=2\Rightarrow \dim\mathrm{Im}(f\circ g)=2\Rightarrow\dim\mathrm{Nuc}(f\circ g)=0$.
Donde, $f\circ g$ é um automorfismo de $\mathbb{R}^2$

5.20 (a) Como $r(A)=3$, A é invertível, logo, h é um automorfismo de $\mathbb{R}^3$.

(b) $\begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}x+y+z\\x+y\\x\end{bmatrix}$, donde, $h(x,y,z)=(x+y+z,x+y,x)$.
$h(1,1,0)=(2,2,1)=2(1,1,0)+(-1)(0,1,0)+1(0,1,1)$
$h(0,1,0)=(1,1,0)=1(1,1,0)+0(0,1,0)+0(0,1,1)$
$h(0,1,1)=(2,1,0)=2(1,1,0)+(-1)(0,1,0)+0(0,1,1)$. Logo,
$M(h;\mathcal{B},\mathcal{B})=\begin{bmatrix}2&1&2\\-1&0&-1\\1&0&0\end{bmatrix}$.
Alternativamente,
$M(h;\mathcal{B},\mathcal{B})=[B]^{-1}A\,[B]=\begin{bmatrix}1&0&0\\-1&1&-1\\0&0&1\end{bmatrix}\begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}1&0&0\\1&1&1\\0&0&1\end{bmatrix}=\begin{bmatrix}2&1&2\\-1&0&-1\\1&0&0\end{bmatrix}$

(c) $A^{-1}=M(h^{-1};b.c.(\mathbb{R}^3),b.c.(\mathbb{R}^3))=\begin{bmatrix}0&0&1\\0&1&-1\\1&-1&0\end{bmatrix}$, $A^{-1}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}z\\y-z\\x-y\end{bmatrix}$, logo,

$h^{-1}(x,y,z)=(z,y-z,x-y)$

5.21 (a) $M(T;b.c.(\mathbb{R}^2),b.c.(\mathbb{R}^2))=M_T=\begin{bmatrix}1&1\\-2&4\end{bmatrix}$;

(b) $M(T;\mathcal{B},\mathcal{B})=[B]^{-1}M_T\,[B]=\begin{bmatrix}1&1\\1&2\end{bmatrix}^{-1}\begin{bmatrix}1&1\\-2&4\end{bmatrix}\begin{bmatrix}1&1\\1&2\end{bmatrix}=\begin{bmatrix}2&0\\0&3\end{bmatrix}$,
ou, alternativamente, tendo em conta que $T(x,y)=(x+y,-2x+4y)$,

$T(1,1) = (2,2) = 2(1,1) + 0(1,2)$,
$T(1,2) = (3,6) = 0(1,1) + 3(1,2)$, donde,
$M(T;\mathcal{B},\mathcal{B}) = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

5.22 (a) $M(g; b.c.(\mathbb{R}^3), b.c.(\mathbb{R}^2)) = M_g = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$;

(b) $M(g;\mathcal{B},\mathcal{B}') = \left[B'\right]^{-1} M_g \left[B\right] = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}$, ou, alternativamente, tendo em conta que $g(x,y,z) = (2x - y, 2y - z)$,
$g(1,1,1) = (1,1) = 0(0,1) + 1(1,1)$
$g(0,1,1) = (-1,1) = 2(0,1) + (-1)(1,1)$
$g(0,0,1) = (0,-1) = (-1)(0,1) + 0(1,1)$, donde,
$M(g;\mathcal{B},\mathcal{B}') = \begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}$

5.23

$$M_f = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

(a) $M_f \begin{bmatrix} 1 \\ -2 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \\ -2 \end{bmatrix}$, donde, $f\left(\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}\right) = \begin{bmatrix} 0 & 2 \\ 2 & -2 \end{bmatrix}$;

$M_f \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a+b+c+d \\ b+d \\ -a-c \\ b \end{bmatrix}$, logo, $f\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+b+c+d & b+d \\ -a-c & b \end{bmatrix}$.

(b) $M(f;\mathcal{B},\mathcal{B}) = \left[B\right]^{-1} M_f \left[B\right] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 2 & 2 \\ 1 & 0 & 1 & 0 \\ -1 & 1 & -1 & -1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$.

5.24 (a) $M(f;\mathcal{B},\mathcal{B}') = \left[B'\right]^{-1} M_f \left[B\right] = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -\frac{1}{2} & 1 \\ \frac{8}{3} & \frac{4}{3} \end{bmatrix}$.

(b) $\left[f(1,3)\right] = M_f \left[(1,3)\right] = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$;

$\left[f(-2,4)\right] = M_f \left[(-2,4)\right] = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix}$;

(c) $\left[f(1,3)\right]_{\mathcal{B}'} = M(f;\mathcal{B},\mathcal{B}')\left[(1,3)\right]_{\mathcal{B}} = \begin{bmatrix} 0 & 0 \\ -\frac{1}{2} & 1 \\ \frac{8}{3} & \frac{4}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{2} \\ \frac{8}{3} \end{bmatrix}$; **Nota:** $\left(0,-\frac{1}{2},\frac{8}{3}\right)_{\mathcal{B}'} = (7,-1,0)$ $\left[f(-2,4)\right]_{\mathcal{B}'} =$

$M(f;\mathcal{B},\mathcal{B}')\left[(-2,4)\right]_{\mathcal{B}} = \begin{bmatrix} 0 & 0 \\ -\frac{1}{2} & 1 \\ \frac{8}{3} & \frac{4}{3} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \frac{4}{3} \end{bmatrix}$; **Nota:** $\left(0,1,\frac{4}{3}\right)_{\mathcal{B}'} = (6,2,0)$

5.25 (a) $M(T;\mathcal{B},\mathcal{B}) = \left[B\right]^{-1} M_T \left[B\right] = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 3 & 0 \\ 0 & -4 & 1 \end{bmatrix}.$

(b) $\left[T(1,-1,0)\right] = M_T\left[(1,-1,0)\right] = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix};$

$\left[T(1,0,-1)\right] = M_T\left[(1,0,-1)\right] = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix};$

$\left[T(1,0,0)\right] = M_T\left[(1,0,0)\right] = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix};$

(c) $\left[T(1,-1,0)\right]_{\mathcal{B}} = M(T;\mathcal{B},\mathcal{B})\left[(1,-1,0)\right]_{\mathcal{B}} = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 3 & 0 \\ 0 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix};$

Nota: $(2,0,0)_{\mathcal{B}} = (2,-2,0)$

$\left[T(1,0,-1)\right]_{\mathcal{B}} = M(T;\mathcal{B},\mathcal{B})\left[(1,0,-1)\right]_{\mathcal{B}} = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 3 & 0 \\ 0 & -4 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix};$

Nota: $(2,3,-4)_{\mathcal{B}} = (1,-2,-3)$

$\left[T(1,0,0)\right]_{\mathcal{B}} = M(T;\mathcal{B},\mathcal{B})\left[(1,0,0)\right]_{\mathcal{B}} = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 3 & 0 \\ 0 & -4 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix};$

Nota: $(1,0,1)_{\mathcal{B}} = (2,-1,0)$

5.26 (a) $M_f = \left[B'\right] M(f;\mathcal{B},\mathcal{B}')\left[B\right]^{-1} = \begin{bmatrix} 2 & 1 \\ 5 & 3 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{5}{7} & \frac{3}{7} \\ 2 & 1 \\ \frac{4}{7} & -\frac{6}{7} \end{bmatrix}$

$f : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ tal que $f(x,y) = \left(\frac{5}{7}x + \frac{3}{7}y, 2x+y, \frac{4}{7}x - \frac{6}{7}y\right)$.

5.27 (a) $M_g = \left[B'\right] M(g;\mathcal{B},\mathcal{B}')\left[B\right]^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 3 \\ -3 & 3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -1/2 & 3/2 \\ -1/2 & 3 & 1/2 \\ -3/2 & -5/2 & -2 \end{bmatrix}$

$g : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ tal que $g(x,y,z) = \left(2x - \frac{1}{2}y + \frac{3}{2}z, -\frac{1}{2}x + 3y + \frac{1}{2}z, -\frac{3}{2}x - \frac{5}{2}y - 2z\right)$.