SVD

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The purpose of the project was to utilize SVD for compression of dense matrices. Idea is to calculate eigenvalues of the matrix in parallel, then use MPI to pass each eigenvalue to a different process to calculate the corresponding eigenvector. During the calculation of the eigenvectors the process also utilized openMP for various matrix operations to improve speed.

The elements of a matrix A can be expressed as:

$$a_{ij} = \sum_{k=1}^{n} u_{ik} s_k v_{jk}$$

Where n is the number of eigenvalues of A.

If we lower n the size of the matrix can be decreased while maintaining a similar matrix. The composition of a matrix A can be expressed as:

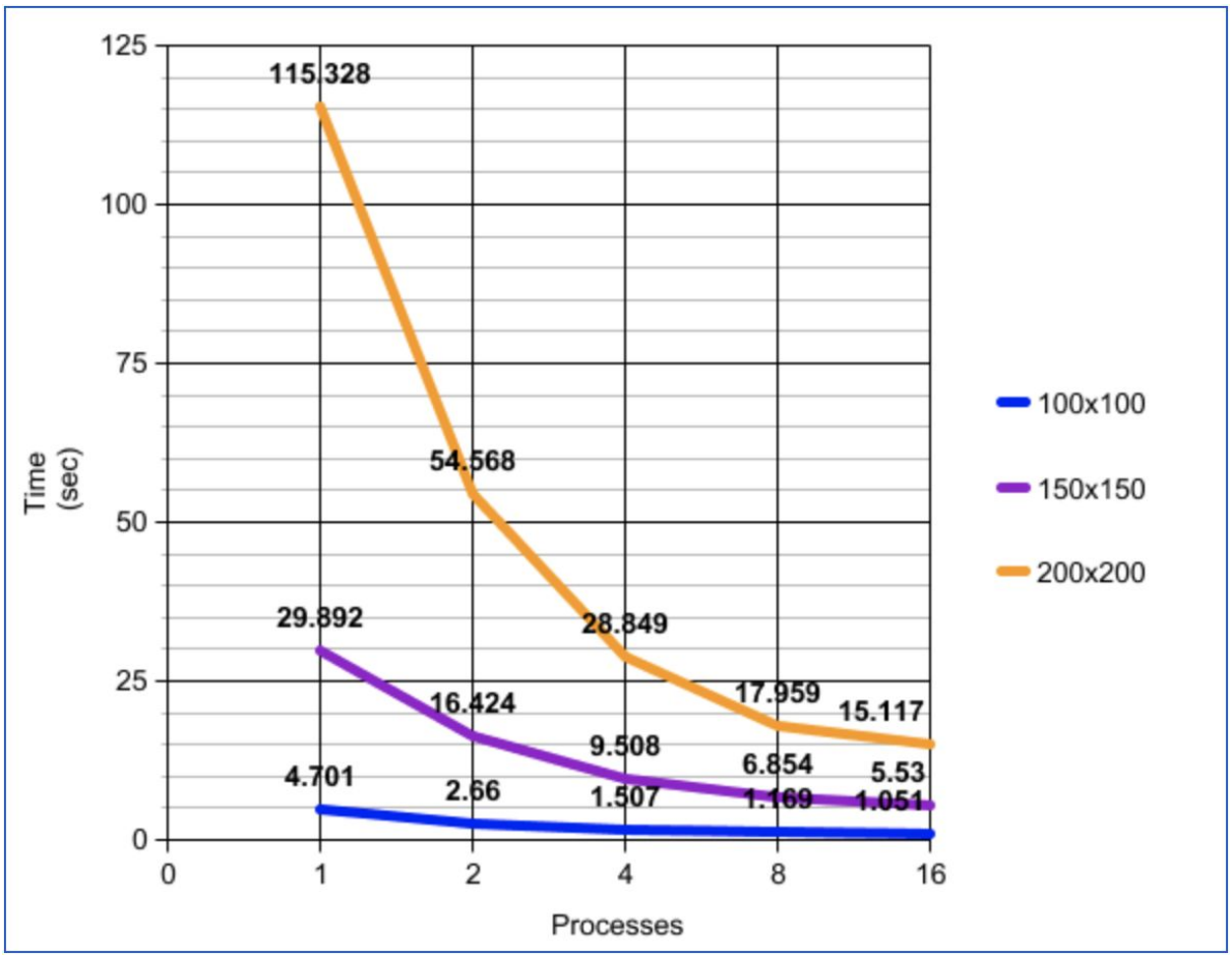
$$A = U \sum V^T$$

Where U is an mxn orthogonal matrix, Sigma is nxn diagonal matrix, and V is a nxn orthogonal matrix. U and V are computed from the eigenvectors of A multiplied by its transpose, and sigma is created by putting the square root of all the eigenvalues on the diagonal. If the condition number of a matrix is large, it will be noticeable based on the singular values and to produce a stable solution some of the singular values can be thrown out. SVD also has the advantage of working with non square matrices.

As far as parallelization goes, it was pretty simple to figure out how the parallelization was going to work, but difficult to implement. We thought of sending computed eigenvalues to different processes and then that process (rank greater than zero) will compute the corresponding eigenvector. Then the vector will then be sent to process zero and then the SVD matrix decomposition would be able to finish.

The hard part was row reducing for eigenvectors. The row reduction did not work if augmented with 0, but the way that we circumvented that was by augmenting with 1 and then normalizing the values.

Results:



Graph courtesy of Kidszone

Conclusions:

Increasing the number of nodes past 16 did not improve the time, as there was significant overhead. Sending eigenvalues to multiple nodes/processes increased the time it took for the calculation.

Sources:

http://web.mit.edu/be.400/www/SVD/Singular_Value_Decomposition.htm

https://rosettacode.org/wiki/Reduced_row_echelon_form#C

https://blog.statsbot.co/singular-value-decomposition-tutorial-52c695315254

https://www.mpi-inf.mpg.de/fileadmin/inf/d5/teaching/ss17_dmm/lectures/2017-05-08-lin_alg_and_svd.pdf

https://stackoverflow.com/questions/769/solving-a-linear-equation

https://dml.cz/bitstream/handle/10338.dmlcz/702748/PANM_15-2010-1_18.pdf

https://www.math.cuhk.edu.hk/~lmlui/CaoSVDintro.pdf

http://www.math.utah.edu/~goller/F15_M2270/BradyMathews_SVDImage.pdf