Determining Young’s modulus by measuring guitar string frequency

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Christopher Chiaverina, Column Editor
4111 Connecticut Trail, Crystal Lake, IL 60012; fizzforfun@aol.com

Determining Young's modulus by measuring guitar string frequency

Robert D. Polak, Adam R. V. Davenport, Andrew Fischer, and Jared Rafferty, Loyola University Chicago, Chicago, IL

Values for physical constants are commonly given as abstractions without building strong intuition, and are too often utilized solely in the pursuit of more easily conceptualized properties. The goal of this experiment is to remove the obscurity behind Young's modulus by exploring the phenomena associated with it—namely, the frequency of a plucked guitar string in relation to a change in length. By tightening a guitar string around the tuning peg, the string is stretched by small increments, creating a tension force F_T on the string.

$$F_T = \frac{YA\Delta L}{L_0}. \tag{1}$$

Here, Y is Young's modulus, A is the cross-sectional area of the string, L_0 is the initial length, and ΔL is the change in string length. In this work, we assume that the tension force is constant throughout the string and is not affected by being wrapped around the tuning peg. When the guitar string is plucked, propagated waves travel at a speed v, expressed by

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}, \tag{2}$$

where f and λ are the frequency and wavelength of the wave, respectively, and μ is the mass per unit length of the string. Defining the mass per unit length divided by the cross-sectional area to be the density ρ, and combining Eqs. (1) and (2), we find

$$f^2 = \frac{Y\Delta L}{\rho\lambda^2 L_0}. \tag{3}$$

When plotting the frequency squared against the change in length and performing a linear fit, Young's modulus can be determined from

$$Y = m\lambda^2 \rho L_0, \tag{4}$$

where m is the slope of the linear fit.

A standard guitar is already designed to stretch a string and produce an audio frequency, making it an ideal experimental apparatus. We begin by installing a plain, unwound steel string[1] onto the guitar.[2] We determine the initial length L_0 of the string by measuring it from the bridge to the tuning peg, and then adding the length of the string wound around the peg. The string should initially be wound tight enough to produce a high steady tone without vibrating against the neck of the guitar but low enough that the tuning pegs can be tightened by several full turns without snapping the guitar string. This initial length is calculated by multiplying the circumference of the peg by the number of times the string is wound around it. To most accurately measure the initial length, we suggest that the string should be inserted into the hole in the peg before winding it, and the hole in the peg turned to its original orientation before the experiment begins, as shown in Fig. 1.

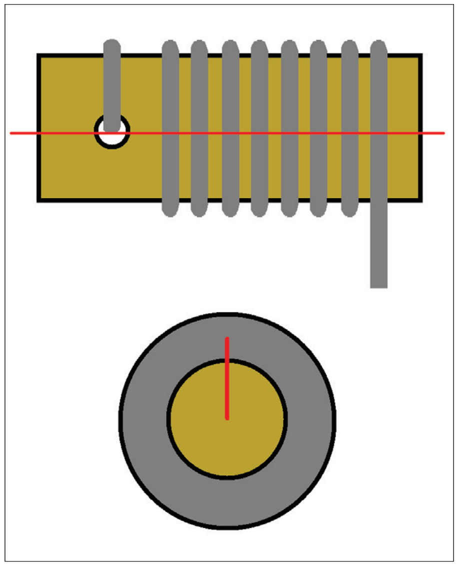

Fig. 1. The top image demonstrates the correct way to wind the guitar string around the peg in order to end up with a whole number of initial loops. The red line in the top image indicates that the hole is aligned with the point where the string is tangent to the peg. The bottom image displays the radius of the combined peg/string system that should be used in calculating their circumference. The images are not to scale.

We now wish to measure the frequency of the guitar string against ΔL, the change in string length. It is necessary before beginning the experiment to ascertain how much the length of the string is going to change for every turn of the tuning knob. On our guitar, the gearing is such that it takes 15 full turns of the knob to complete one full peg rotation. Each quarter turn of the tuning knob resulted in a ΔL of 1/60 of the combined peg and string circumferences as shown in Fig. 1. Our peg diameter was 5.8×10^{-3} m, which meant each quarter turn of the tuning peg resulted in the string being stretched by 3.2×10^{-4} m. The last initial condition to measure is the starting frequency, which we took using an iPhone 6S and the free iOS application iAnalyzer Lite.[3] In our experiment, we started with an initial frequency of 129.4 Hz for a 0.254-mm diameter string. However, different string thicknesses will create different frequencies. We tightened the string with the tuning knob

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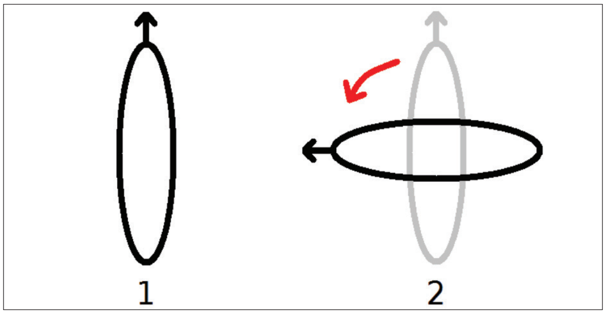

Fig. 2. A quarter turn of the peg from initial position 1 results in the 90° change in orientation demonstrated in position 2.

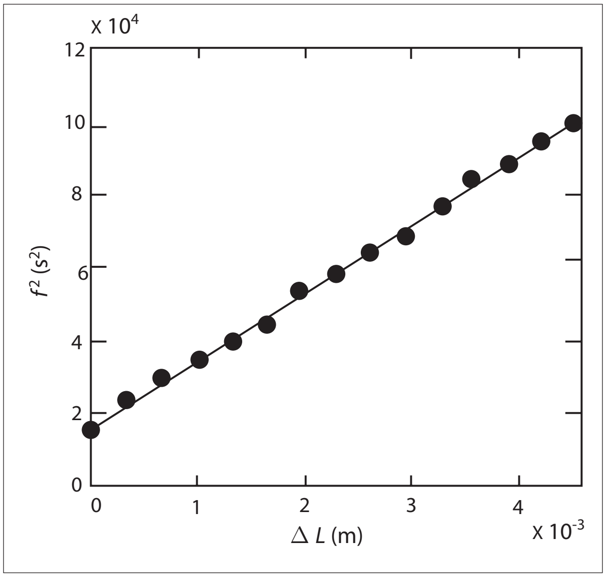

Fig. 3. Frequency squared is plotted against the change in length of the string as it is tightened. The slope of the best-fit line in conjunction with Eq. (4) is used to find Young's modulus. Error bars fall within the bounds of the data points.

in increments of quarter turns, as shown in Fig. 2. In order to maintain consistency in the data, it was necessary to continue increasing the frequency rather than adjusting it up and down. We measured the frequency after each quarter turn and plotted frequency squared against ΔL, as shown in Fig. 3.

After performing the best linear fit on the plot in Fig. 3, we can then find Young's modulus using Eq. (4). There is a noticeable fluctuation every fourth data point, which may be the result of imperfection in the guitar. This may be due to the machine gears not being round or centered, or the machine guitar teeth not being evenly spaced. Since the string was partially tightened when we began taking data, the y-intercept of the best fit will not equal zero, but it does not affect the slope of the line. The wavelength λ is taken to be twice the distance from the saddle to the nut of the guitar. In the case of our guitar, this length was 0.63 m. The initial length L_0 is determined using the procedure given above and for the data taken corresponding to Fig. 3. This length, in our experiments, was 0.77 m. Finally, we used the accepted value of ρ (7860 kg/m^3).[4] Though stretching the string can cause changes in the density of the string's material, we estimated this effect to be small (less than 0.4%). For the string measured in Fig. 3, we determined an experimental value for Young's modulus to be 198.1 GPa. Repeated measurements produced results within 10% of the accepted experimental value for the material, which is 210 GPa.[5]

The proprietary nature of guitar string production prevented us from being able to attribute a theoretical value for both density and Young's modulus to commercial guitar strings, which is why we used high-carbon steel strings to produce the results above. However, we also took frequencies using unwound steel acoustic guitar strings.[6] Experiments indicate that the high-carbon wires have the same value for Young's modulus as the guitar strings, and therefore can be used interchangeably. While other experiments have been devised to measure these physical constants,[7] this experiment is designed with versatility in mind, as it should be feasible to use materials instructors have on hand or can procure cheaply. Other stringed instruments can also be used, provided that a steady tone can be produced by an unwound string of a known material density that does not deviate greatly under tension, and that the ΔL can be measured accurately.

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4. H. Berns and W. Theisen, *Ferrous Materials: Steel and Cast Iron* (Springer, Berlin, 2008), p. 7.
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6. We used 0.228-mm diameter D'Addario Plain Steel Acoustic Singles.
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