# Master Thesis Presentation:

# The Application of a Hybrid Garch-type Model with Quantile Regression Neural Network on the Estimation of Value-at-Risk of Corporate Stocks

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# Introduction

- **Metric: Risk Measures**
- Models: Traditional vs Machine Learning
- Application: Tail Risk-managed Assets

$$\begin{aligned}\mathrm{VaR}^{\alpha}(X) :=\ & F_{-X}^{-1}(1-\alpha) \equiv -\inf\{x \in \mathbb{R} : F_X^{-1}(x) \geq \alpha\} \\ & \equiv -F_X^{-1}(\alpha), \forall\, X \in \mathcal{G}.\end{aligned}$$

Elicitable

$$\mathrm{ES}^{\alpha}(X) := \frac{1}{\alpha}\int_0^{\alpha} \mathrm{VaR}^{u}(X)\,du,\ \forall\, X \in \mathcal{G}.$$

Not elicitable

$$\mathrm{RVaR}^{\alpha,\beta}(X) = \begin{cases} \dfrac{1}{\beta-\alpha}\displaystyle\int_{\alpha}^{\beta} \mathrm{VaR}^{u}(X)\,du, & \text{if } \alpha < \beta; \\ -F_X^{-1}(\alpha), & \text{if } \alpha = \beta. \end{cases}$$

Joint elicitable

# Introduction

- Metric: Risk Measures
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Stock returns: heavy-tails, gain/loss asymmetry, volatility clustering, leverage effects

$$x_t = \mu + \sum_{i=1}^{m} a_i x_{t-i} + \sum_{j=1}^{n} b_j \epsilon_{t-j} + \epsilon_t \qquad \varepsilon_t = \sigma_t z_t$$

EWMA:

$$\sigma_t^2 = (1-\lambda)\epsilon_{t-1}^2 + \lambda \sigma_{t-1}^2$$

EGARCH:

$$\ln\left(\sigma_t^2\right) = \omega + \sum_{i=1}^{p} \alpha\left(|z_{t-i}| - \mathbb{E}\left[|z_{t-i}|\right]\right) + \gamma z_{t-i} + \sum_{j=1}^{q} \beta \ln\left(\sigma_{t-j}^2\right)$$

APARCH:

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta$$

# Introduction

- Metric: Risk Measures
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ML Models: GARCH-ANN, LSTM-ANN, BLSTM-ANN,...

Not quantile regression

Quantile Regression Neural Network

No Autoregressive Structure

Conditional Autoregressive Value at Risk by Regression Quantiles

No NN structure

**Quantile + Autoregressive + Neural Network?**

QRANN (refeeds model, computationally inefficient)
Deep Quantile NN (takes single GARCH volatility input)

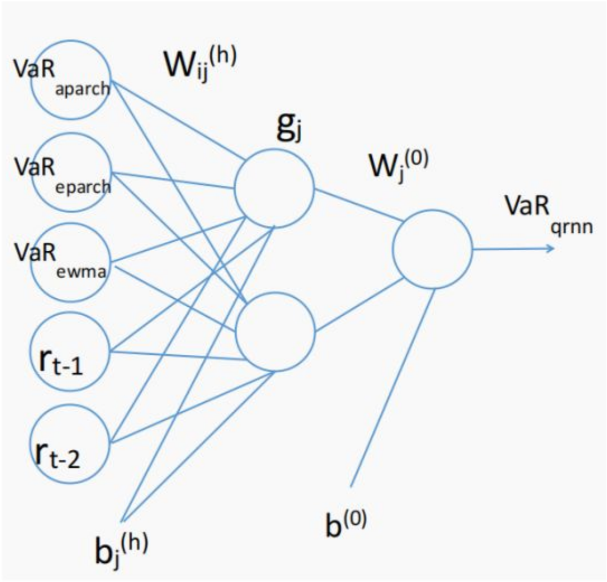
# Introduction

- Metric: Risk Measures
- **Models: Traditional vs Machine Learning**
- Application: Tail Risk-managed Assets

Proposed a hybrid model

- Forecasted VaRs from GARCH-type models as inputs
- Used QRNN structure
- Easy to implement

$$f\left(\mathbf{x_t}, \mathbf{v}, \mathbf{w}\right) = g_2\left(\sum_{j=0}^{m} v_j g_1\left(\sum_{i=0}^{n} w_{ji} x_{it}\right)\right)$$



# Introduction

- Metric: Risk Measures
- **Models: Traditional vs Machine Learning**
- Application: Tail Risk-managed Assets

$$\rho_\tau(u) = \begin{cases} \tau u & if u \geq 0 \\ (\tau - 1)u & if u \leq 0 \end{cases}$$

- NN: gradient-based nonlinear optimization algorithm
- Pinball losses not differentiable everywhere

$$h(u) = \begin{cases} \frac{u^2}{2\varepsilon} & if 0 \leq |u| \leq \varepsilon \\ |u| - \frac{\varepsilon}{2} & if |u| > \varepsilon \end{cases}$$

$$\rho_\tau^{(a)}(u) = \begin{cases} \tau h(u) & if u \geq 0 \\ (\tau - 1)h(u) & if u < 0 \end{cases}$$

$$E_\tau^{(a)} = \frac{1}{N}\sum_{t=1}^{N} \rho_\tau^{(a)}\left(y(t) - \hat{y}_\tau(t)\right) + \lambda \frac{1}{IJ}\sum_{i=1}^{I}\sum_{j=1}^{J}\left(w_{ij}^{(h)}\right)^2$$

# Introduction

- Metric: Risk Measures
- Models: Traditional vs Machine Learning
- **Application: Tail Risk-managed Assets**

- Adjusts the portfolio’s exposure based on the predicted risk or volatility of the market
    - Higher risk –X→ higher returns
- Sharpe Ratio: returns per risk

$$S_a = \frac{E[R_a - R_b]}{\sigma_a}$$

# Methodology

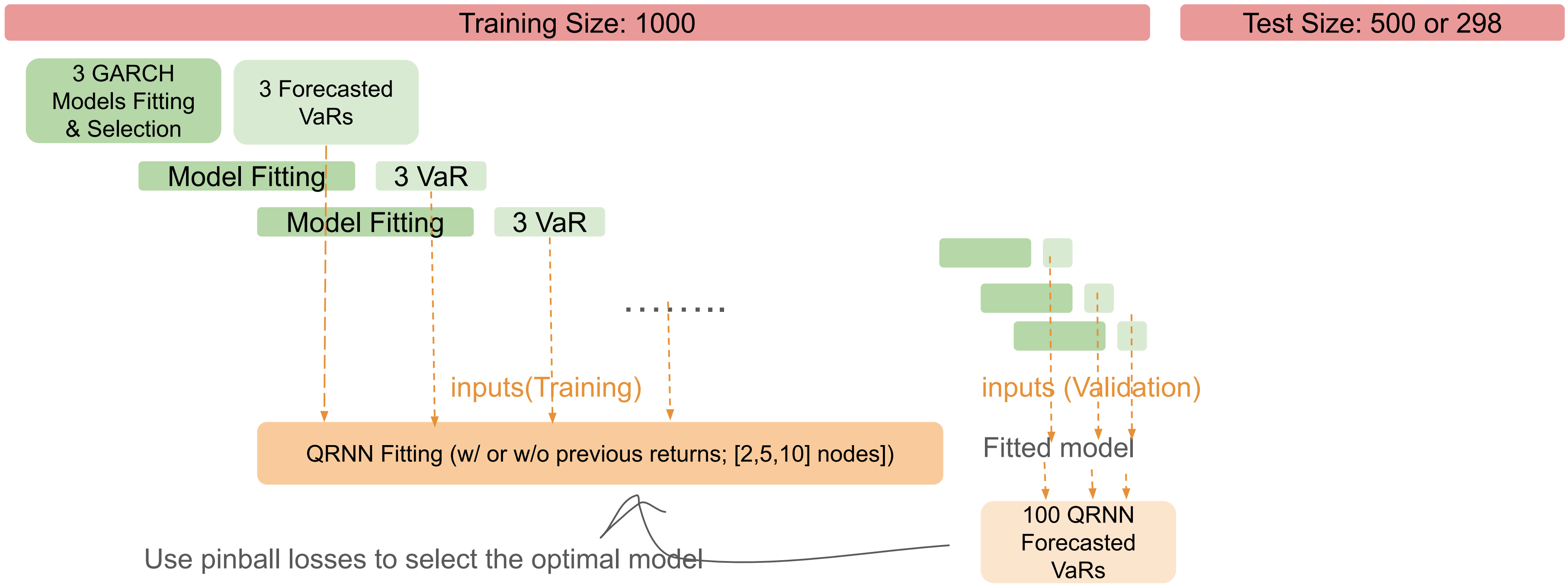
Models | Evaluations | Multiple Testing Correction | Tail-risk-management

- APARCH
  - Fit the first 1000 data by setting the GARCH orders p and q as 1 or 2, and setting the conditional distribution as normal, skewed normal, or student t
  - Chose the optimal configuration by BIC
  - Used a rolling window of 1000, step size of 1, to forecast the 1-day ahead VaR with levels [0.01,0.05,0.1]
- CAViaR
  - Used the first 300 data to calculate the initial value of the empirical VaR
  - Used a rolling window of 1000, step size of 1, to forecast the 1-day ahead VaR with levels [0.01,0.05,0.1]

# Methodology

**Models** | **Evaluations** | **Multiple Testing Correction** | **Tail-risk-management**

- Hybrid GARCH-QRNN



# Methodology

Models | **Evaluations** | Multiple Testing Correction | Tail-risk-management

one-day ahead predictions of VaR

- Violation-based Test

$$LR_{uc} = -2ln(\frac{(1-p)^{n-S_n}p^{S_n}}{(1-\frac{S_n}{n})^{n-S_n}(\frac{S_n}{n})^{S_n}}) \sim \chi_1^2$$

- Independence-based Test

$$\Pi = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix} \qquad H_{0,cc} : \pi_{01} = \pi_{11} = p$$

where $\pi_{ij}$ is the probability of an $i$ on day $t-1$ being followed by a $j$ on day $t$.

# Methodology

Models | Evaluations | Multiple Testing Correction | Tail-risk-management

- Storey's approach of controlling FDR

$$p - FalseDiscoveryRate(pFDR) = \mathbb{E}(\frac{V}{R}|R > 0)$$

1. Order original $p$-values $p_i, i = 1, \ldots, m$, from the smallest to the largest such that $p_{(1)} \leq p_{(2)} \leq p_{(m)}$;
2. Find $k$ as the largest $i$ for which $P_{(i)} \leq \frac{i}{m\hat{\pi}_0}\alpha$ where $\pi_0 = \frac{m_0}{m}$;
3. Reject all null hypothesis $H_i, i = 1, 2, \ldots, k$.

# Methodology

Models | Evaluations | Multiple Testing Correction | **Tail-risk-management**

- Rebalance positions of assets at the beginning of each month

$$f_t^{VAR} = \frac{c}{V\hat{A}R_t} f_t$$

$$\sum \left(f_t^{\mathrm{VAR}} - \overline{f_t^{\mathrm{VAR}}}\right)^2 = \sum \left(f_t - \overline{f_t}\right)^2, c = \sqrt{\sum \left(f_t - \overline{f_t}^2 / \sum \left(\frac{1}{\widehat{\mathrm{VAR}}_t^{\alpha}} f_t - \overline{\frac{1}{\widehat{\mathrm{VAR}}_t^{\alpha}} f_t}\right)\right.},$$

$$f_t^{VAR} = \alpha + \beta f_t + \epsilon_t$$

# Data Processing

① Simulation

$$\begin{cases} y_t = c + \sum_{i=1}^{m} \phi_i y_{t-i} + \varepsilon_t - \sum_{j=1}^{n} \varphi_j \varepsilon_{t-j}, \quad \varepsilon_t = \eta_t \sigma_t \\ \sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_{t-1}| - \gamma_i \varepsilon_{t-i}\right)^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta \end{cases}$$

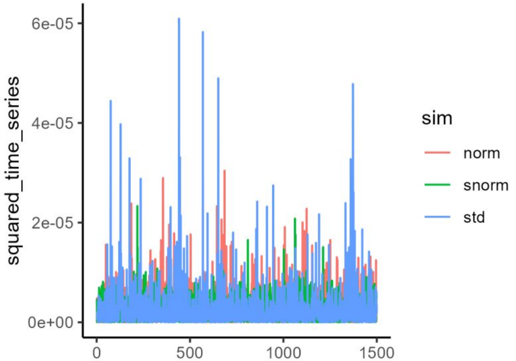
where parameters: $c = 0.001, \phi_1 = 0.05, \omega = 0, \alpha_1 = 0.05, \beta_1 = 0.8, \gamma_1 = 0, \delta = 1.8.$

Conditional distributions: normal, skewed normal, student t

Total: 1500 data

Training: 1000

Testing : 500



# Data Processing

② Empirical Data (AAPL, SMTC, ALAR)

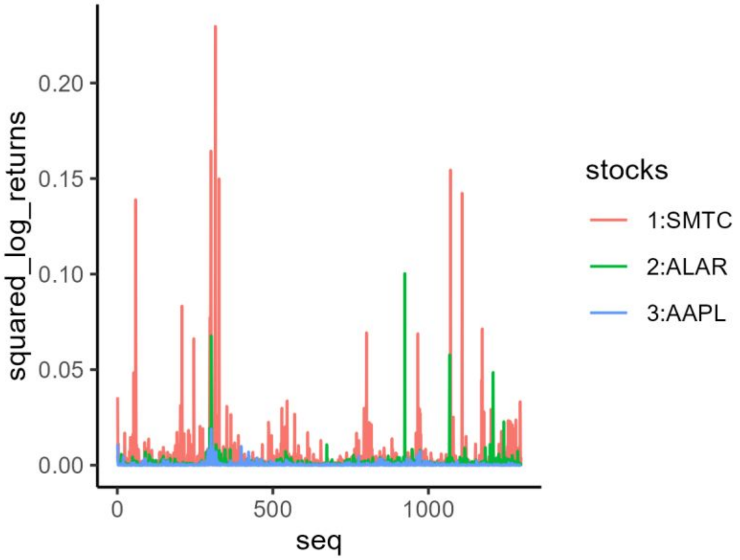
- Jan. 1, 2019 to Mar.1, 2024
- Log returns

$$ln(r_t) = ln(P_t) - ln(P_{t-1})$$

Total: 1298 data

Training: 1000

Testing : 298



# Results & Discussions

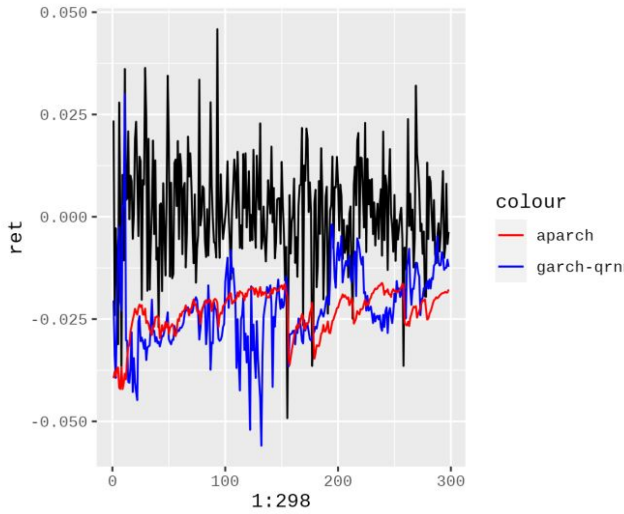
- p,q, conditional distribution
- QRNN parameters:

Table 1: Selected model structure of QRNN for simulated data

| | 0.01 | | 0.05 | | 0.1 | |
|---|---|---|---|---|---|---|
| | prev | nodes | prev | nodes | prev | nodes |
| norm | ✓ | 2 | ✓ | 2 | ✓ | 2 |
| snorm | × | 5 | × | 2 | × | 2 |
| std | ✓ | 2 | × | 5 | × | 5 |

Table 2: Selected model structure of QRNN for empirical data

| | 0.01 | | 0.05 | | 0.1 | |
|---|---|---|---|---|---|---|
| | prev | nodes | prev | nodes | prev | nodes |
| AAPL | ✓ | 2 | × | 5 | × | 2 |
| SMTC | ✓ | 5 | ✓ | 5 | ✓ | 2 |
| ALAR | × | 2 | ✓ | 2 | × | 2 |



# Results & Discussions

Table 4: Unadjusted p-values

| | | sim_norm | | sim_snorm | | sim_std | |
|---|---|---|---|---|---|---|---|
| | Models | uc_p | cc_p | uc_p | cc_p | uc_p | cc_p |
| 0.01 | APARCH | 1 | 0.951 | 0.641 | 0.869 | 0.641 | 0.869 |
| | CAViaR1 | 0.397 | **0.006** | 0.641 | 0.869 | 0.663 | 0.845 |
| | CAViaR2 | 0.397 | **0.006** | 1 | 0.951 | 0.397 | 0.632 |
| | CAViaR3 | 0.663 | 0.845 | 0.641 | 0.869 | 0.391 | 0.627 |
| | CAViaR4 | 0.663 | 0.845 | 0.397 | 0.632 | 0.048 | 0.115 |
| | Hybrid-QRNN | 0.663 | 0.845 | **0** | **0** | 0.663 | 0.845 |
| 0.05 | APARCH | 0.319 | 0.396 | 0.399 | 0.695 | 0.423 | 0.121 |
| | CAViaR1 | 0.836 | **0.012** | 0.288 | 0.28 | 0.423 | 0.703 |
| | CAViaR2 | 1 | 0.094 | 0.678 | 0.633 | 0.546 | 0.73 |
| | CAViaR3 | 0.678 | **0.008** | 0.199 | 0.417 | 0.838 | 0.842 |
| | CAViaR4 | 0.319 | 0.054 | 0.53 | 0.513 | 0.319 | 0.602 |
| | Hybrid-QRNN | 0.836 | 0.275 | 0.836 | 0.967 | 0.678 | 0.633 |
| 0.1 | APARCH | 0.244 | 0.487 | 0.286 | 0.442 | 0.881 | 0.617 |
| | CAViaR1 | 1 | 0.351 | 0.089 | 0.125 | 0.556 | 0.838 |
| | CAViaR2 | 0.882 | 0.665 | 0.167 | 0.254 | 0.462 | 0.675 |
| | CAViaR3 | 1 | 0.608 | 0.063 | 0.142 | 0.767 | 0.922 |
| | CAViaR4 | 0.244 | 0.307 | 0.764 | 0.489 | 0.244 | 0.204 |
| | Hybrid-QRNN | 0.306 | 0.188 | **0.019** | **0.019** | 0.449 | 0.25 |

Table 5: Unadjusted p-values

| | | AAPL | | SMTC | | ALAR | |
|---|---|---|---|---|---|---|---|
| | Models | uc_p | cc_p | uc_p | cc_p | uc_p | cc_p |
| 0.01 | APARCH | 0.991 | 0.970 | 0.284 | 0.517 | **0.014** | 0.050 |
| | CAViaR1 | 0.991 | 0.970 | 0.573 | 0.807 | 0.181 | 0.407 |
| | CAViaR2 | 0.991 | 0.970 | 0.284 | 0.517 | 0.181 | 0.407 |
| | CAViaR3 | 0.544 | 0.821 | 0.284 | 0.517 | 0.544 | 0.821 |
| | CAViaR4 | 0.544 | 0.821 | 0.122 | 0.077 | **0.016** | **0.043** |
| | Hybrid-QRNN | 0.544 | 0.821 | **0.005** | **0.000** | 0.991 | 0.970 |
| 0.05 | APARCH | **0.045** | 0.058 | 0.979 | 0.961 | **0.045** | 0.108 |
| | CAViaR1 | 0.167 | 0.272 | 0.809 | 0.893 | **0.020** | 0.056 |
| | CAViaR2 | 0.167 | 0.272 | 0.606 | 0.755 | **0.020** | 0.056 |
| | CAViaR3 | 0.278 | 0.364 | 0.809 | 0.893 | **0.007** | **0.025** |
| | CAViaR4 | **0.020** | **0.022** | 0.809 | 0.350 | 0.077 | 0.120 |
| | Hybrid-QRNN | 0.585 | 0.169 | 0.585 | 0.169 | 0.167 | 0.240 |
| 0.1 | APARCH | **0.026** | 0.066 | 0.583 | 0.515 | **0.004** | **0.008** |
| | CAViaR1 | **0.026** | **0.028** | 0.116 | 0.247 | **0.045** | 0.128 |
| | CAViaR2 | **0.026** | 0.066 | 0.116 | 0.277 | **0.008** | **0.028** |
| | CAViaR3 | **0.045** | 0.055 | 0.454 | 0.740 | **0.002** | **0.003** |
| | CAViaR4 | **0.000** | **0.000** | 0.877 | 0.170 | 0.088 | 0.153 |
| | Hybrid-QRNN | **0.015** | **0.036** | 0.173 | **0.007** | **0.008** | **0.018** |

# Results & Discussions

Table 8:
Adjusted p-values of Kupiec Test and Christoffersen Test by Storey Benjami
Hochberg approach on models fitted on simulation data

| | | sim_norm | | sim_snorm | | sim_std | |
|---|---|---|---|---|---|---|---|
| | Models | uc_p* | cc_p* | uc_p* | cc_p* | uc_p* | cc_p* |
| 0.01 | APARCH | 0.870 | 0.870 | 0.787 | 0.850 | 0.787 | 0.850 |
| | CAViaR1 | 0.670 | 0.088 | 0.787 | 0.850 | 0.787 | 0.845 |
| | CAViaR2 | 0.670 | 0.088 | 0.870 | 0.870 | 0.670 | 0.787 |
| | CAViaR3 | 0.787 | 0.845 | 0.787 | 0.850 | 0.670 | 0.787 |
| | CAViaR4 | 0.787 | 0.845 | 0.670 | 0.787 | 0.220 | 0.370 |
| | Hybrid-QRNN | 0.787 | 0.845 | **0.000** | **0.000** | 0.787 | 0.845 |
| 0.05 | APARCH | 0.588 | 0.670 | 0.670 | 0.802 | 0.680 | 0.370 |
| | CAViaR1 | 0.845 | 0.125 | 0.558 | 0.558 | 0.680 | 0.806 |
| | CAViaR2 | 0.870 | 0.321 | 0.787 | 0.787 | 0.766 | 0.832 |
| | CAViaR3 | 0.787 | 0.088 | 0.480 | 0.680 | 0.845 | 0.845 |
| | CAViaR4 | 0.588 | 0.229 | 0.766 | 0.759 | 0.588 | 0.787 |
| | Hybrid-QRNN | 0.845 | 0.558 | 0.845 | 0.870 | 0.787 | 0.787 |
| 0.1 | APARCH | 0.553 | 0.747 | 0.558 | 0.704 | 0.850 | 0.787 |
| | CAViaR1 | 0.870 | 0.635 | 0.310 | 0.373 | 0.774 | 0.845 |
| | CAViaR2 | 0.850 | 0.787 | 0.438 | 0.555 | 0.718 | 0.787 |
| | CAViaR3 | 0.870 | 0.787 | 0.247 | 0.411 | 0.845 | 0.870 |
| | CAViaR4 | 0.553 | 0.583 | 0.845 | 0.747 | 0.553 | 0.485 |
| | Hybrid-QRNN | 0.583 | 0.459 | 0.139 | 0.139 | 0.709 | 0.553 |

Note: The bold ones are below the significance level $\alpha = 0.05$. The CAViaR models are 1
SAV, 2 - AS, 3 - GARCH, 4 - ADAPTIVE.

Table 9:
Adjusted p-values of Kupiec Test and Christoffersen Test by Storey Benjamin-Hochberg approach on models fitted on empirical data

| | | AAPL | | SMTC | | ALAR | |
|---|---|---|---|---|---|---|---|
| | Models | uc_p* | cc_p* | uc_p* | cc_p* | uc_p* | cc_p* |
| 0.01 | APARCH | 0.870 | 0.870 | 0.558 | 0.759 | 0.139 | 0.224 |
| | CAViaR1 | 0.870 | 0.870 | 0.787 | 0.845 | 0.448 | 0.671 |
| | CAViaR2 | 0.870 | 0.870 | 0.558 | 0.759 | 0.448 | 0.671 |
| | CAViaR3 | 0.766 | 0.845 | 0.558 | 0.759 | 0.766 | 0.845 |
| | CAViaR4 | 0.766 | 0.845 | 0.370 | 0.278 | 0.139 | 0.212 |
| | Hybrid-QRNN | 0.766 | 0.845 | 0.088 | **0.000** | 0.870 | 0.870 |
| 0.05 | APARCH | 0.212 | 0.232 | 0.870 | 0.870 | 0.212 | 0.363 |
| | CAViaR1 | 0.438 | 0.558 | 0.845 | 0.852 | 0.139 | 0.229 |
| | CAViaR2 | 0.438 | 0.558 | 0.787 | 0.845 | 0.139 | 0.229 |
| | CAViaR3 | 0.558 | 0.652 | 0.845 | 0.852 | 0.088 | 0.153 |
| | CAViaR4 | 0.139 | 0.148 | 0.845 | 0.635 | 0.278 | 0.370 |
| | Hybrid-QRNN | 0.787 | 0.438 | 0.787 | 0.438 | 0.438 | 0.553 |
| 0.1 | APARCH | 0.153 | 0.248 | 0.787 | 0.759 | 0.088 | 0.088 |
| | CAViaR1 | 0.153 | 0.155 | 0.370 | 0.553 | 0.212 | 0.376 |
| | CAViaR2 | 0.153 | 0.248 | 0.370 | 0.558 | 0.088 | 0.155 |
| | CAViaR3 | 0.212 | 0.229 | 0.711 | 0.838 | 0.063 | 0.081 |
| | CAViaR4 | **0.000** | **0.000** | 0.850 | 0.438 | 0.310 | 0.436 |
| | Hybrid-QRNN | 0.139 | 0.193 | 0.440 | 0.088 | 0.088 | 0.139 |

Note: The bold ones are below the significance level $\alpha = 0.05$. The CAViaR models are 1 - SAV, 2 - AS, 3 - GARCH, 4 - ADAPTIVE.

# Results & Discussions

Table 3:The pinball loss of predictions on the test data

| | | norm | snorm | std | AAPL | SMTC | ALAR |
|---|---|---|---|---|---|---|---|
| 0.01 | APARCH | 0.018 | 0.018 | 0.022 | 0.144 | 0.540 | 0.523 |
| | CAViaR1 | 0.019 | 0.029 | 0.025 | 0.150 | 0.555 | 0.502 |
| | CAViaR2 | 0.021 | 0.031 | 0.022 | 0.146 | 0.565 | 0.516 |
| | CAViaR3 | 0.019 | 0.029 | 0.026 | 0.148 | 0.585 | 0.502 |
| | CAViaR4 | 0.019 | 0.032 | 0.027 | 0.141 | 0.612 | 0.583 |
| | Hybrid-QRNN | 0.022 | 0.043 | 0.026 | 0.143 | 1.586 | 0.495 |
| 0.05 | APARCH | 0.069 | 0.069 | 0.076 | 0.437 | 1.793 | 1.731 |
| | CAViaR1 | 0.070 | 0.086 | 0.079 | 0.454 | 1.297 | 1.679 |
| | CAViaR2 | 0.071 | 0.087 | 0.08 | 0.446 | 1.316 | 1.757 |
| | CAViaR3 | 0.069 | 0.086 | 0.077 | 0.461 | 1.315 | 1.654 |
| | CAViaR4 | 0.069 | 0.088 | 0.079 | 0.501 | 1.328 | 1.610 |
| | Hybrid-QRNN | 0.072 | 0.087 | 0.102 | 0.494 | 2.057 | 1.722 |
| 0.1 | APARCH | 0.112 | 0.113 | 0.119 | 0.693 | 2.856 | 2.670 |
| | CAViaR1 | 0.121 | 0.137 | 0.122 | 0.706 | 1.859 | 2.594 |
| | CAViaR2 | 0.122 | 0.136 | 0.123 | 0.696 | 1.886 | 2.641 |
| | CAViaR3 | 0.119 | 0.137 | 0.123 | 0.710 | 1.875 | 2.621 |
| | CAViaR4 | 0.12 | 0.137 | 0.123 | 0.789 | 1.929 | 2.597 |
| | Hybrid-QRNN | 0.124 | 0.141 | 0.15 | 0.720 | 2.126 | 2.607 |

Note: The CAViaR models are 1 - SAV, 2 - AS, 3 - GARCH, 4 - ADAPTIVE.

# Results & Discussions

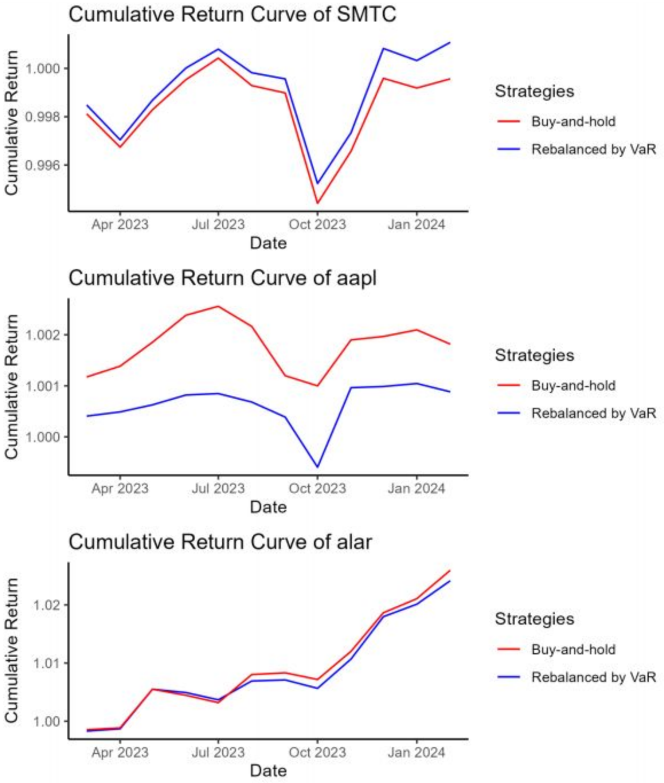



Table 7: The Estimated Intercept and Corresponding Test Statistics

| | SMTC | AAPL | ALAR |
|---|---|---|---|
| alpha | 0.013 | -0.003 | -0.011 |
| sd | 0.006 | 0.013 | 0.022 |
| p-value | **0.064** | 0.827 | 0.626 |

Note: The bold ones are below the significance level $\alpha = 0.1$.

# Conclusion

- Our model outperforms the other ones on 0.05 quantile when we used both violation and independence tests as criteria. For other quantiles, we do not see superior methods in terms of the tests;
- Our Hybrid-QRNN model performs poorly on smooth time series → the risk of overfitting;
- Using the predicted VaR, Tail-risk-management strategies shows that it increases the excessive returns compared to the buy-and-hold strategy for the stocks with higher volatilities

# Limitations & Further Studies

- Experimental Procedure
- Evaluation Methods
- Portfolio Risk

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