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# REPARTIȚIA DIRICHLET GENERALIZATĂ

M. CRAIU și V. CRAIU

Fie  $x_1, x_2, \dots, x_{k+1}$  variabile aleatoare independente care au repartițiile gamma generalizate

$$f(x_i, a_i, \alpha_i, \beta_i) = \frac{\alpha_i}{(a_i)^{\beta_i/\alpha_i} \Gamma(\beta_i/\alpha_i)} x_i^{\beta_i-1} \exp\left(-\frac{x_i^{\alpha_i}}{a_i^{\alpha_i}}\right).$$

$$x_i > 0, \alpha_i > 0, \beta_i > 0$$

Fie

$$y_i = \frac{x_i}{x_1 + \dots + x_{k+1}}, \quad i = 1, \dots, k$$

Ne propunem să determinăm repartiția vectorului  $(y_1, \dots, y_k)$ , pe care o vom numi *repartiția Dirichlet generalizată*.

Repartiția lui  $(x_1, \dots, x_{k+1})$  este

$$\prod_{i=1}^{k+1} f(x_i) = \prod_{i=1}^{k+1} \frac{\alpha_i}{(a_i)^{\beta_i/\alpha_i} \Gamma(\beta_i/\alpha_i)} x_i^{\beta_i-1} \exp\left(-\frac{x_i^{\alpha_i}}{a_i^{\alpha_i}}\right).$$

Se face schimbarea de variabile

$$x_1 + \dots + x_{k+1} = z$$

$$x_1 = y_1 z$$

$$x_2 = y_2 z$$

$$\vdots$$

$$x_k = y_k z, \quad y_i \geq 0, \quad y_1 + y_2 + \dots + y_k \leq 1.$$

Cu aceasta repartiția lui  $(y_1, \dots, y_k)$  devine

$$h(y_1, \dots, y_k) = A \int_0^\infty \prod_{i=1}^k y_i^{\beta_i-1} z^{\sum_{i=1}^{k+1} \beta_i-1} (1-y_1-\dots-y_k)^{\beta_{k+1}-1} \times \\ \times \exp\left(-\left(\frac{y_1 z}{a_1}\right)^{\alpha_1} - \left(\frac{y_2 z}{a_2}\right)^{\alpha_2} - \dots - \left(\frac{y_k z}{a_k}\right)^{\alpha_k} - \left(\frac{z(1-y_1-\dots-y_k)}{a_{k+1}}\right)^{\alpha_{k+1}}\right) dz$$

unde

$$A = \prod_1^{k+1} \frac{\alpha_i}{a_i^{\beta_i/\alpha_i} \Gamma(\beta_i/\alpha_i)}$$

sau,

$$\begin{aligned} h(y_1, \dots, y_k) &= \prod_{i=1}^{k+1} \frac{\alpha_i}{a_i^{\beta_i/\alpha_i} \Gamma(\beta_i/\alpha_i)} \cdot \prod_{i=1}^k y_i^{\beta_i-1} (1-y_1-\dots-y_k)^{\beta_{k+1}-1} \times \\ &\times \int_0^\infty z^{\sum_1^{k+1} \beta_i - 1} \exp - \left( \left( \frac{y_1 z}{a_1} \right)^{\alpha_1} + \left( \frac{y_2 z}{a_2} \right)^{\alpha_2} + \dots + \left( \frac{y_k z}{a_k} \right)^{\alpha_k} + \right. \\ &\quad \left. + \left( \frac{z(1-y_1-\dots-y_k)}{a_{k+1}} \right)^{\alpha_{k+1}} \right) dz. \end{aligned}$$

În cazul particular în care

$$\alpha_1 = \alpha_2 = \dots = \alpha_{k+1} = 1$$

$$a_1 = a_2 = \dots = a_{k+1} = 1$$

se obține

$$\begin{aligned} h(y_1, \dots, y_k) &= \frac{1}{\Gamma(\beta_1) \dots \Gamma(\beta_{k+1})} \prod_{i=1}^k y_i^{\beta_i-1} (1-y_1-\dots-y_k)^{\beta_{k+1}-1} \times \\ &\times \int_0^\infty z^{\sum_1^{k+1} \beta_i - 1} e^{-z} dz \end{aligned}$$

sau

$$h(y_1, \dots, y_k) = \frac{\Gamma(\beta_1 + \dots + \beta_{k+1})}{\Gamma(\beta_1) \dots \Gamma(\beta_{k+1})} \prod_{i=1}^k y_i^{\beta_i-1} (1-y_1-\dots-y_k)^{\beta_{k+1}-1}$$

adică repartiția simplă a lui Dirichlet.

În cazul în care

$$\alpha_1 = \alpha_2 = \dots = \alpha_{k+1} = \alpha, \quad (1) \text{ devine}$$

$$\begin{aligned} (1') h(y_1, \dots, y_k) &= \frac{\alpha^{k+1}}{\prod_{i=1}^k \alpha^{\beta_i/\alpha} \Gamma\left(\frac{\beta_i}{\alpha}\right)} \prod_{i=1}^k y_i^{\beta_i-1} (1-y_1-\dots-y_k)^{\beta_{k+1}-1} \times \\ &\times \int_0^\infty z^{\sum_1^{k+1} \beta_i - 1} \exp - z^\alpha \left[ \left( \frac{y_1}{a_1} \right)^\alpha + \left( \frac{y_2}{a_2} \right)^\alpha + \dots + \left( \frac{1-y_1-\dots-y_k}{a_{k+1}} \right)^\alpha \right] dz. \end{aligned}$$

Cu schimbarea de variabilă

$$z = u^{\frac{1}{\alpha}} \left[ \left( \frac{y_1}{a_1} \right)^\alpha + \dots + \left( \frac{y_k}{a_k} \right)^\alpha + \left( \frac{1-y_1-\dots-y_k}{a_{k+1}} \right)^\alpha \right]^{-\frac{1}{\alpha}}$$

(1') devine

$$\begin{aligned}
 h(y_1, \dots, y_k) &= \frac{\alpha^k}{\prod_{i=1}^{k+1} a_i^{\beta_i/\alpha} \Gamma\left(\frac{\beta_i}{\alpha}\right)} \prod_{i=1}^k y_i^{\beta_i-1} (1-y_1-\dots-y_k)^{\beta_{k+1}-1} \times \\
 &\times \left[ \left(\frac{y_1}{a_1}\right)^\alpha + \dots + \left(\frac{y_k}{a_k}\right)^\alpha + \left(\frac{1-y_1-\dots-y_k}{a_{k+1}}\right)^\alpha \right]^{-\frac{\sum_{i=1}^{k+1} \beta_i}{\alpha}} \times \\
 &\times \int_0^\infty u^{\frac{\sum_{i=1}^{k+1} \beta_i}{\alpha} - 1} e^{-u} du, \quad \alpha > 0 \\
 &= \frac{\alpha^k \Gamma\left(\frac{\beta_1 + \beta_2 + \dots + \beta_{k+1}}{\alpha}\right)}{\prod_{i=1}^{k+1} a_i^{\beta_i/\alpha} \Gamma(\beta_i/\alpha)} \prod_{i=1}^k y_i^{\beta_i-1} (1-y_1-\dots-y_k)^{\beta_{k+1}-1} \times \\
 &\times \left[ \left(\frac{y_1}{a_1}\right)^\alpha + \dots + \left(\frac{y_k}{a_k}\right)^\alpha + \left(\frac{1-y_1-\dots-y_k}{a_{k+1}}\right)^\alpha \right]^{-\frac{\sum_{i=1}^{k+1} \beta_i}{\alpha}}
 \end{aligned}$$

*Observație:* În cazul  $k = 1$ , (1) devine repartiția  $\beta$  generalizată studiată în [2].

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#### ОБОБЩЕННОЕ РАСПРЕДЕЛЕНИЕ ДИРИХЛЕ

#### КРАТКОЕ СОДЕРЖАНИЕ

В работе определяется и изучается распределение Дирихле в интегральной форме

#### THE GENERALIZED DIRICHLET REPARTITION

#### ABSTRACT

In this paper one define and one utilise the Dirichlet repartition in an integral form.