1 Approximation of $\mathbb{E}X_j$

We have that

$$m_j(\alpha,\lambda) = \frac{(\alpha_j/\lambda_j)}{\theta}\sum_i \lambda_i m_i(\alpha^{(j)},\lambda)$$

and

$$\sum_i m_i(\alpha^{(j)},\lambda) = 1$$

so letting $\lambda_m = min\{\lambda_i\}$ and $\lambda_M = max\{\lambda_i\}$, we can write

$$\lambda_m \sum m_i(\alpha^{(j)},\lambda) \le \sum \lambda_i m_i(\alpha^{(j)},\lambda) \le \lambda_M \sum m_i(\alpha^{(j)},\lambda)$$

$$\lambda_m \le \sum \lambda_i m_i(\alpha^{(j)},\lambda) \le \lambda_M$$

Also,

$$\frac{\alpha_j/\lambda_j}{\theta/\lambda_m} \le \frac{\alpha_j/\lambda_j}{\sum \alpha_i/\lambda_i} \le \frac{\alpha_j/\lambda_j}{\theta/\lambda_M}$$

Therefore, letting $F(\alpha,\lambda) = \frac{\alpha_j/\lambda_j}{\sum \alpha_i/\lambda_i}$, both $F(\alpha,\lambda)$ and $m_j(\alpha,\lambda)$ are in the interval $[\frac{\alpha_j/\lambda_j}{\theta/\lambda_m}, \frac{\alpha_j/\lambda_j}{\theta/\lambda_M}]$. This results in the bound

$$|m_j(\alpha,\lambda) - F(\alpha,\lambda)| \le \frac{\alpha_j/\lambda_j}{\theta/\lambda_M} - \frac{\alpha_j/\lambda_j}{\theta/\lambda_m} = \frac{\alpha_j(\lambda_M - \lambda_m)}{\theta\lambda_j}$$