fourier series & GLM

March 19, 2016

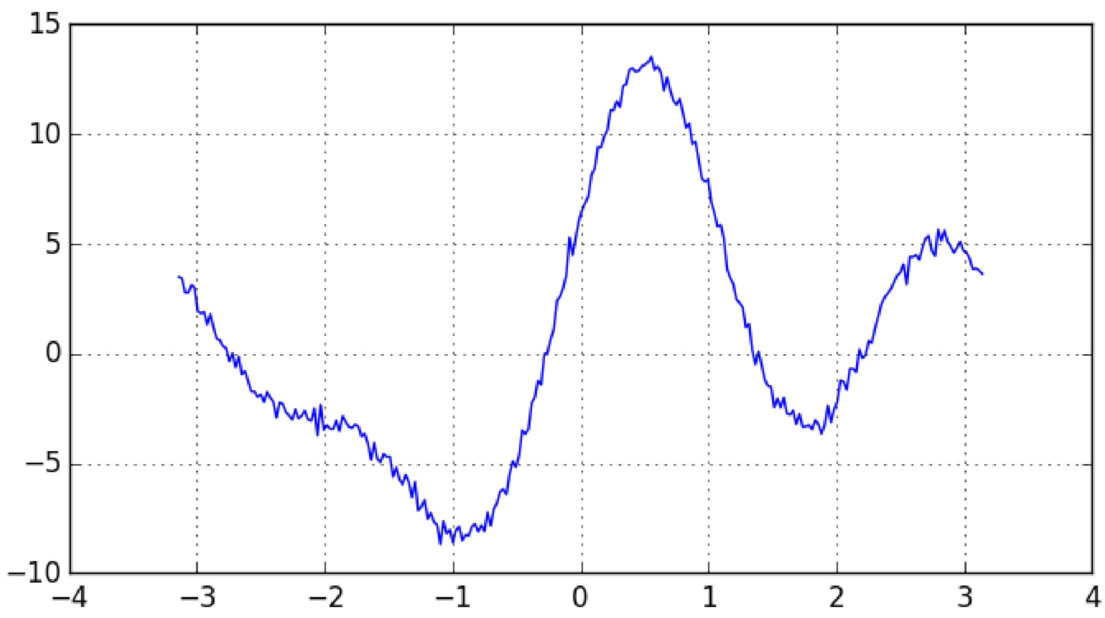
decomposition of periodic function given samples in $[-\pi, \pi)$

```
In [1]: using PyPlot
```

```
In [135]: x = linspace(-π, π, 256);
          coeff_0 = 5 * rand(3);
          coeff_1 = 5 * rand(3);
          offset = 2 * rand();
          noise = 0.3 * randn(length(x))

          cs = map(pack -> pack[2] * cos(pack[1] * x), enumerate(coeff_0))
          ss = map(pack -> pack[2] * sin(pack[1] * x), enumerate(coeff_1))
          y = reduce((x, xs) -> x + xs, cs) + reduce((x, xs) -> x + xs, ss) + offset + noise;
```

```
In [136]: figure(figsize=(7, 4))
          plot(x, y);
          grid();
          tight_layout();
```



solution #0

suppose we know $f(x) = c + \sum_{k=1}^{3} a_k \cdot cos(k \cdot x) + \sum_{k=1}^{3} b_k \cdot sin(k \cdot x)$
question becomes how to get c, a_k, b_k from samples
GLM can be used

In [137]:
```
# a naive GLM implementation
A = [ones(length(x)) cos(x) cos(2*x) cos(3*x) sin(x) sin(2*x) sin(3*x)];
result = pinv(A) * y;

@printf("for (offset, a_1, a_2, a_3, b_1, b_2, b_3)\n")
@printf("GLM gets      (%.3f, %.3f, %.3f, %.3f, %.3f, %.3f, %.3f)\n", result...)
@printf("actual params (%.3f, %.3f, %.3f, %.3f, %.3f, %.3f, %.3f)\n", offset,
        coeff_0..., coeff_1...)
```

```
for (offset, a_1, a_2, a_3, b_1, b_2, b_3)
GLM gets      (0.704, 1.310, 4.242, 0.016, 4.654, 3.652, 3.632)
actual params (0.720, 1.269, 4.228, 0.010, 4.635, 3.657, 3.625)
```

solution #1 fourier series, projection to complete orthogonal basis
$f(x) = \frac{c}{2} + \sum_{k=1}^{\text{inf}} a_k \cdot cos(k \cdot x) + \sum_{k=1}^{\text{inf}} b_k \cdot sin(k \cdot x)$
because
$\int_{-\pi}^{\pi} sin(mx)sin(nx)dx = \int_{-\pi}^{\pi} \frac{cos((m-n)x) - cos((m+n)x)}{2} = \pi\delta_{mn}$
$\int_{-\pi}^{\pi} cos(mx)cos(nx)dx = \int_{-\pi}^{\pi} \frac{cos((m+n)x) + cos((m-n)x)}{2} = \pi\delta_{mn}$
$\int_{-\pi}^{\pi} sin(mx)cos(nx)dx = \int_{-\pi}^{\pi} \frac{sin((m+n)x) + sin((m-n)x)}{2} = 0$
$\int_{-\pi}^{\pi} sin(x)dx = 0$
$\int_{-\pi}^{\pi} cos(x)dx = 0$
we get
$c = \frac{\int_{-\pi}^{\pi} f(x)dx}{\pi}$
$a_k = \frac{\int_{-\pi}^{\pi} f(x)cos(k \cdot x)dx}{\pi}$
$b_k = \frac{\int_{-\pi}^{\pi} f(x)sin(k \cdot x)dx}{\pi}$

In [138]:
```
# naive numerical integration
function integrate(f)
    return sum(f(x[2:length(x)]) .* y[2:length(x)] .* diff(x))
end

funcs = [t->1, t->cos(t), t->cos(2t), t->cos(3t), t->sin(t), t->sin(2t), t->sin(3t)];
result = map(f -> integrate(f) / π, funcs)

@printf("for (c, a_1, a_2, a_3, b_1, b_2, b_3)\n")
@printf("fourier gets  (%.3f, %.3f, %.3f, %.3f, %.3f, %.3f, %.3f)\n", result...)
@printf("actual params (%.3f, %.3f, %.3f, %.3f, %.3f, %.3f, %.3f)\n", 2 * offset,
        coeff_0..., coeff_1...)
```

```
for (c, a_1, a_2, a_3, b_1, b_2, b_3)
fourier gets  (1.409, 1.309, 4.242, 0.015, 4.654, 3.652, 3.632)
actual params (1.440, 1.269, 4.228, 0.010, 4.635, 3.657, 3.625)
```

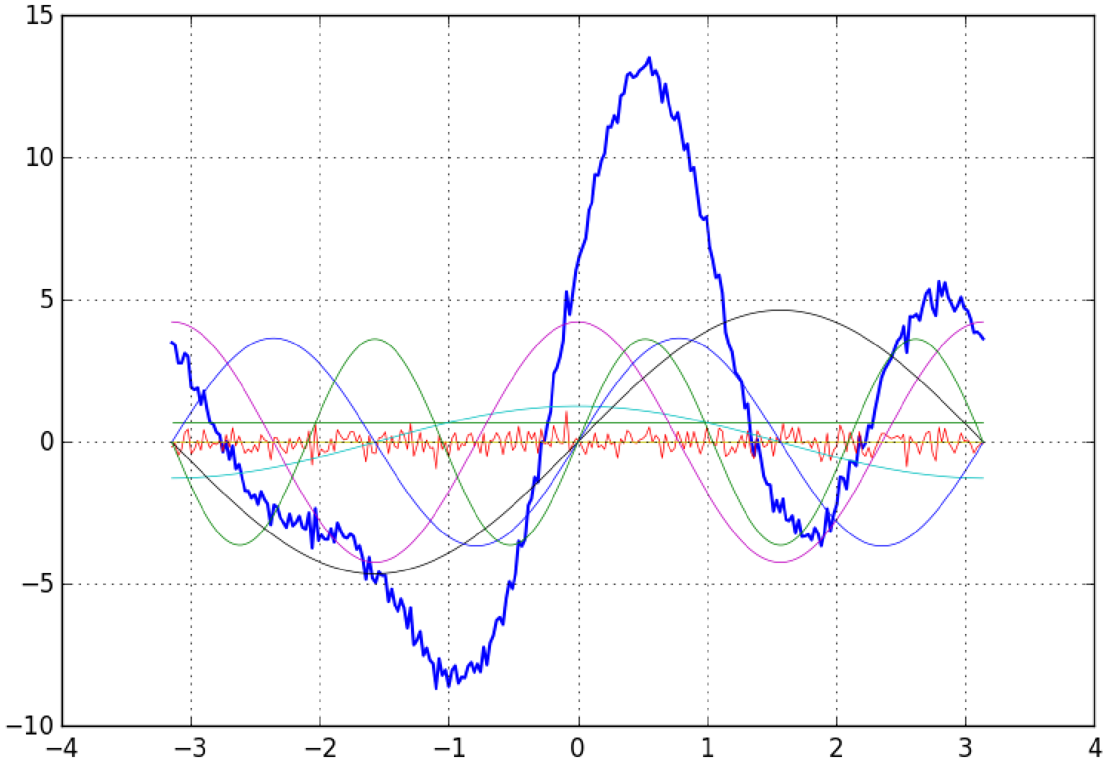
plot superposition

In [141]:
```
figure(figsize=(9, 6))
grid("on")
hold("on")

plot(x, y, linewidth=1.5)
plot(x, offset * ones(length(x)), "-", linewidth=0.5)
plot(x, noise, linewidth=0.5)
```

```
for ys in cs
    plot(x, ys, "-", linewidth=0.5)
end

for ys in ss
    plot(x, ys, "-", linewidth=0.5)
end
```



TODO: use FFT

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In [ ]:
```