试卷参考答案

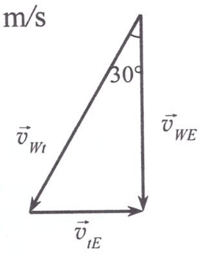
一、填空题：（每题 4 分，2 个空格的题每个空格给 2 分，共 48 分）

1. $\mathrm{d}v = a\mathrm{d}t = \frac{kx}{m}\mathrm{d}t = \frac{kx}{vm}\mathrm{d}x$，$\int_0^v v\mathrm{d}v = \int_0^{x_0}\frac{kx}{m}\mathrm{d}x$，$v = \sqrt{\frac{k}{m}}x_0$，$I = mv - 0 = \sqrt{mk}\,x_0$

2. $v = \sqrt{4^2 + (3t)^2}$ (SI)，$a_t = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{9t}{\sqrt{4^2+(3t)^2}} = \frac{9}{5}\,\mathrm{m/s^2}$，$\bar{a} = 3\,\vec{j}\ \mathrm{m/s^2}$，

$a_n = \sqrt{a^2 - a_t^2} = \frac{12}{5}\,\mathrm{m/s^2}$

3. W 雨滴，t 列车，E 地面，$v_{WE} = v_{tE}\,\mathrm{ctg}30° = 10\sqrt{3}$ m/s，$v_{Wt} = \frac{v_{tE}}{\sin 30°} = 20$ m/s



4. $2mv\frac{L}{2} = [\frac{1}{12}mL^2 + 2m(\frac{L}{2})^2]\omega$，$\omega = \frac{12v}{7L}$

5. $a = l_0\sqrt{1 - v^2/c^2}$，$v = c\sqrt{1 - (\frac{a}{l_0})^2}$

6. $E = \frac{m_0}{\sqrt{1 - v^2/c^2}}c^2 = 5.81\times10^{-13}$ J，$E_{kr} = \frac{m_0}{\sqrt{1-v^2/c^2}}c^2 - m_0c^2 = 4.99\times10^{-13}$ J

$E_{kc} = \frac{1}{2}m_0v^2 = \frac{1}{2}m_0(0.99c)^2 = 4.01\times10^{-14}$ J，$\frac{E_{kc}}{E_{kr}} = 8.04\times10^{-2}$

7. $E_p = \frac{1}{2}kx^2 = \frac{1}{2}k(\frac{x_0}{2})^2 = \frac{E}{4}$，$E_k = E - E_p = \frac{3}{4}E$；$k = \frac{mg}{\Delta l}$；$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\Delta l}{g}}$

8. $\Delta\varphi = -2\pi\frac{2L}{\lambda} = -\frac{4\pi L}{\lambda}$；$y_2 = A\cos(\omega t + \frac{2\pi x}{\lambda} - \frac{4\pi L}{\lambda})$

9. $C_V = \frac{i+2}{2}R = 29.1$，$i = 5$；$E_r = \frac{i-3}{2}kT = 3.77\times10^{-21}$ J

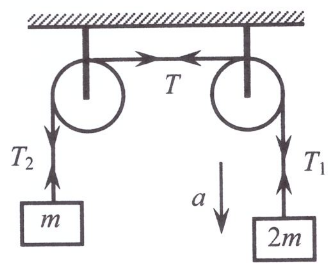
10. $\bar{v}_{12} = \frac{\int_{v_1}^{v_2} v\mathrm{d}N}{\int_{v_1}^{v_2}\mathrm{d}N} = \frac{\int_{v_1}^{v_2} Nvf(v)\mathrm{d}v}{\int_{v_1}^{v_2} Nf(v)\mathrm{d}v} = \frac{\int_{v_1}^{v_2} vf(v)\mathrm{d}v}{\int_{v_1}^{v_2} f(v)\mathrm{d}v}$

11. $\sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{\mu}}$，$\sqrt{\overline{v'^2}} = \sqrt{\frac{3kT'}{\mu'}} = \sqrt{\frac{3k2T}{\mu/2}} = 2\sqrt{\overline{v^2}}$；　2 倍

12. $E = \frac{\lambda_1}{2\pi\varepsilon_0 r}$；$\mathrm{d}F = E\mathrm{d}q = \frac{\lambda_1}{2\pi\varepsilon_0 r}\lambda_2\mathrm{d}r$，$F = \int_l^{2l}\frac{\lambda_1}{2\pi\varepsilon_0 r}\lambda_2\mathrm{d}r = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\ln 2$

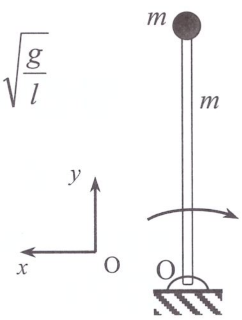
二、计算题：（6 题，共 52 分）

1. 解：$2mg - T_1 = 2ma$，$T_2 - mg = ma$，$T_1r - Tr = \frac{1}{2}mr^2\beta$



$Tr - T_2r = \frac{1}{2}mr^2\beta$，$a = r\beta$

解得：$T = \frac{11}{8}mg$



2．解：（1）$\frac{1}{2}J\omega^2 = mg\frac{l}{2} + mgl$，$J = \frac{1}{3}ml^2 + ml^2 = \frac{4}{3}ml^2$，$\omega = \frac{3}{2}\sqrt{\frac{g}{l}}$

（2）$2mgr_c = J\beta$，$r_c = \frac{ml + ml/2}{2m} = \frac{3}{4}l$，$\beta = \frac{9g}{8l}$

（3）$N_x = 2ma_{cn} = 2m\omega^2 r_C = \frac{27}{8}mg$，

$N_y - 2mg = -2ma_{Ct} = -2m\beta\, r_C = -\frac{27}{16}mg$，$N_y = \frac{5}{16}mg$

3．解：(1) $x = \lambda/4$ 处，$y_1 = A\cos(2\pi\nu t - \frac{1}{2}\pi)$，$y_2 = 2A\cos(2\pi\nu t + \frac{1}{2}\pi)$；∵ y_1，y_2 反相 ∴

$A_{\text{add}} = 2A - A = A$；合振动初相和 y_2 一样为$\pi/2$，合振动方程：$y = A\cos(2\pi\nu t + \frac{\pi}{2})$

(2) $x = \lambda/4$ 处质点的速度：$v = \mathrm{d}y/\mathrm{d}t = -2\pi\nu A\sin(2\pi\nu t + \frac{1}{2}\pi) = 2\pi\nu A\cos(2\pi\nu t + \pi)$

4．解：（1）$\omega = \frac{2\pi}{T} = \pi(\text{rad/s})$，$\lambda = 4\text{ m}$，$A = 10\,(\text{cm})$，$y_0 = 10\cos(\pi t + \varphi)$

$\cos(\frac{\pi}{3} + \varphi) = -\frac{1}{2}$，$\frac{\pi}{3} + \varphi = \frac{2\pi}{3}$，$\varphi = \frac{\pi}{3}$，$y_0 = 10\cos(\pi t + \frac{\pi}{3})\,(\text{cm})$

（2）$u = \frac{\lambda}{T} = 2\text{ m/s}$，$y = 10\cos[\pi(t - \frac{x}{2}) + \frac{\pi}{3}]\,(\text{cm})$

（3）$y_C = 10\cos(\pi \times \frac{1}{3} + \varphi_C) = 0$，$\frac{\pi}{3} + \varphi_C = -\frac{\pi}{2}$，$\varphi_C = -\frac{5\pi}{6}$

$\varphi - \varphi_C = \frac{7\pi}{6} = \frac{2\pi}{\lambda}x_C$，$x_C = \frac{7}{3}\,(\text{m})$

5．解：　$i = 5$　　　$\gamma = 7/5$　　　$T_c = T_a(V_a/V_c)^{\gamma-1} = (1/3)^{0.4}T_a$

（1）　$Q_{ab} = \nu RT_a \ln(V_b/V_a) = \nu RT_a \ln 3 = p_a V_a \ln 3$

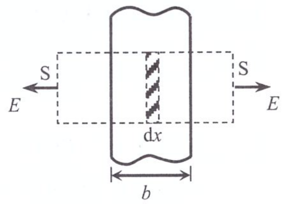
$Q_{bc} = \nu C_V(T_c - T_b) = \frac{5}{2}\nu R(T_c - T_a) = \frac{5}{2}(p_c V_b - p_a V_a) = -\frac{5}{2}(1 - 3^{-0.4})p_a V_a$

$Q_{ca} = 0$

（2）$\eta = 1 - \frac{|Q_{bc}|}{Q_{ab}} = 0.19$

（3）　$\Delta S_{bc} = \nu C_V \ln\frac{T_c}{T_b} = \frac{5}{2}\nu R\ln(\frac{1}{3})^{0.4} = -\nu R\ln 3 = -\frac{p_a V_a}{T_a}\ln 3 = -1.1\frac{p_a V_a}{T_a}$

6．解：(1) 按高斯定理：板外两侧



$2ES = \frac{1}{\varepsilon_0}\int_0^b \rho S\mathrm{d}x = \frac{kS}{\varepsilon_0}\int_0^b x^2\mathrm{d}x = \frac{kSb^3}{3\varepsilon_0}$，得到 $E = \frac{kb^3}{6\varepsilon_0}$

(2) 板内 $0 \le x \le b$ 处，由高斯定理有

$(E + E')S = \frac{kS}{\varepsilon_0}\int_0^x x^2\mathrm{d}x = \frac{kSx^3}{3\varepsilon_0}$，得到 $E' = \frac{k}{3\varepsilon_0}(x^3 - \frac{b^3}{2})$