试卷参考答案

一、填空题：（每题 4 分，共 48 分）

1、$a_t=0$，$a_n=g$，$v=v_0\cos\theta$，$\rho=v^2/a_n=v_0^2\cos^2\theta/g$

2、$\vec{a}=\dfrac{\vec{F}}{m}=4t\,\vec{i}$，$\vec{v}-\vec{v}_0=\int_0^t\vec{a}\mathrm{d}t=\int_0^t(4t\,\vec{i})\mathrm{d}t=2t^2\,\vec{i}$，$\vec{v}=2t^2\,\vec{i}+2\,\vec{j}$，

$\vec{r}-\vec{r}_0=\int_0^t\vec{v}\mathrm{d}t=\int_0^t(2t^2\,\vec{i}+2\,\vec{j})\mathrm{d}t=\dfrac{2}{3}t^3\vec{i}+2t\,\vec{j}$　得：$\vec{r}=\dfrac{2}{3}t^3\vec{i}+2t\,\vec{j}$ (SI)

3、$\dfrac{1}{2}k(x_0-x)^2=\dfrac{1}{2}m_1v_1^2+\dfrac{1}{2}m_2v_2^2$，$m_1v_1=m_2v_2$，$v_1=\sqrt{\dfrac{km_2(x-x_0)^2}{m_1(m_1+m_2)}}$

4、$(J+2mr_1^2)\omega_1=(J+2mr_2^2)\omega_2$，$\omega_2=\dfrac{J+2mr_1^2}{J+2mr_2^2}\omega_1=8\ \mathrm{rad}\cdot\mathrm{s}^{-1}$

5、$E=mc^2=5m_0c^2$，$E_k=mc^2-m_0c^2=4m_0c^2$

6、$l=l_0\sqrt{1-v^2/c^2}$，$v=c\sqrt{1-(\dfrac{l}{l_0})^2}$，$m=\dfrac{m_0}{\sqrt{1-v^2/c^2}}=m_0\dfrac{l_0}{l}$，$p=mv=m_0\dfrac{l_0}{l}c\sqrt{1-(\dfrac{l}{l_0})^2}$

7、$f=f_1+f_2=-k_1x-k_2x=-(k_1+k_2)x$，$\omega=\sqrt{\dfrac{k}{m}}=\sqrt{\dfrac{k_1+k_2}{m}}$，$A=x_0$，$\varphi=0$，$x=x_0\cos[\sqrt{\dfrac{k_1+k_2}{m}}\,t]$

8、$\Delta\phi=\varphi_2-\varphi_1-\dfrac{\omega}{u}(r_2-r_1)=\pi-\dfrac{2\pi}{0.2}(0.5-0.4)=0$

9、$\nu_1=\dfrac{u+v_R}{u-v_s}\nu_s=\dfrac{330}{330-2}\times400\approx402.4\ \mathrm{Hz}$，$\nu_2=\dfrac{330}{330+2}\times400\approx397.6\ \mathrm{Hz}$，$\Delta\nu=4.8\mathrm{Hz}$

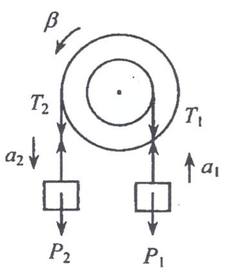
10、$p_1=2p_0$，$T_1=2T_0$，$p_2V_2=p_2 2V_1=p_1V_1$，$p_2=\dfrac{p_1}{2}=p_0$，$T_2=T_1=2T_0$，$\bar{\lambda}_2=\dfrac{kT_2}{\sqrt{2}\pi d^2p_2}=\dfrac{k2T_0}{\sqrt{2}\pi d^2p_0}=2\bar{\lambda}_0$　$P_2V_2=\cancel{\nu}RT_2.$

11、初态和末态的 $T_1=T_2$，用等温过程连接两状态，$\Delta S=\nu R\ln\dfrac{V_2}{V_1}=R\ln 2$ 或 5.76 J/K

12、带正电的完整圆环和带负电的缺口组成，完整圆环 $E_{O1}=0$，缺口看成点电荷，带电量为

$Q_d=-\dfrac{q}{(2\pi R-d)}d$，$E_O=\dfrac{|Q|}{4\pi\varepsilon_0R^2}=\dfrac{qd}{4\pi\varepsilon_0R^2(2\pi R-d)}\approx\dfrac{qd}{8\pi^2\varepsilon_0R^3}$，从 O 点指向缺口中心点

二、计算题：（共 6 题，共 52 分）



1 解：　$mg-T_2=ma_2$，　$T_1-mg=ma_1$，　$T_2(2r)-T_1r=9mr^2\beta/2$

$2r\beta=a_2$，　　$r\beta=a_1$

得：　$\beta=\dfrac{2g}{19r}$

2 解：　$mv_0\dfrac{l}{2}=m\dfrac{v_0}{2}\dfrac{l}{2}+\dfrac{1}{3}Ml^2\omega$，　$\dfrac{1}{2}\dfrac{1}{3}Ml^2\omega^2=Mg\dfrac{l}{2}$

$v_0=\dfrac{4M}{3m}\sqrt{3gl}$

3 解：(1)　$y=0.1\cos(4\pi t-\dfrac{2}{10}\pi x)=0.1\cos 4\pi(t-\dfrac{1}{20}x)$　(SI)

(2)　$y_1=0.1\cos 4\pi(T/4-\lambda/80)=0.1\cos 4\pi(1/8-\dfrac{1}{8})=0.1\text{m}$

(3)　$v=\dfrac{\partial y}{\partial t}=-0.4\pi\sin 4\pi(t-x/20)$.

$v_2=-0.4\pi\sin(\pi-\dfrac{1}{2}\pi)=-1.26\ \text{m/s}$

4 解：（1）$\int_0^{v_0}f(v)\mathrm{d}v=\int_0^{v_0}kv^3\mathrm{d}v=\dfrac{1}{4}kv_0^4=1$　得：$k=\dfrac{4}{v_0^4}$

（2）$\bar{v}=\int_0^{v_0}vf(v)\mathrm{d}v=\int_0^{v_0}kv^4\mathrm{d}v=\dfrac{1}{5}kv_0^5=\dfrac{4}{5}v_0$

$\overline{v^2}=\int_0^{v_0}v^2f(v)\mathrm{d}v=\int_0^{v_0}kv^5\mathrm{d}v=\dfrac{1}{6}kv_0^6=\dfrac{2}{3}v_0^2$　　得：　$\sqrt{\overline{v^2}}=\sqrt{\dfrac{2}{3}}v_0$，

（3）　$\dfrac{\Delta N}{N}=\int_0^{v_1}f(v)\mathrm{d}v=\int_0^{v_1}kv^3\mathrm{d}v=\dfrac{1}{4}kv_1^4=(\dfrac{v_1}{v_0})^4=\dfrac{1}{16}$　得：$v_1=\dfrac{1}{2}v_0$

5 解：(1)　$W_1=(p_1+p_2)(V_2-V_1)/2$　，$C_V=\dfrac{5}{2}R$

$\Delta E_1=C_V(T_2-T_1)=\dfrac{5}{2}R(T_2-T_1)=\dfrac{5}{2}(p_2V_2-p_1V_1)$

$Q_1=\Delta E_1+W_1=\dfrac{5}{2}(p_2V_2-p_1V_1)+\dfrac{1}{2}(p_1+p_2)(V_2-V_1)=2.02\times10^3\ \text{J}$.

(2)　$W_2=\int_{V_2}^{V_3}p\,\mathrm{d}V=p_2\sqrt{V_2}\int_{V_2}^{V_3}\mathrm{d}V/\sqrt{V}=2(p_3V_3-p_2V_2)$

又根据 $pV^{1/2}=C$ 得：$V_3=V_2(p_2/p_3)^2=32\times10^{-3}\text{m}^3$　∴ $W_2=4.85\times10^3$ J

$W=W_1+W_2=5.10\times10^3$ J

$\Delta E=C_V(T_3-T_1)=\dfrac{5}{2}R(T_3-T_1)=\dfrac{5}{2}(p_3V_3-p_1V_1)=7.83\times10^3$ J

$Q=\triangle E+W=1.29\times10^4$ J

6 解：$r<R$　　$q'=\int_0^r(ar-br^2)\cdot2\pi rl\mathrm{d}r=2\pi l(\dfrac{ar^3}{3}-\dfrac{br^4}{4})$

由高斯定理：$E_1=\dfrac{q'}{2\pi\varepsilon_0 rl}=\dfrac{1}{\varepsilon_0}(\dfrac{ar^2}{3}-\dfrac{br^3}{4})$

$r>R$　　$q'=\int_0^R(ar-br^2)\cdot2\pi rl\mathrm{d}r=2\pi l(\dfrac{aR^3}{3}-\dfrac{bR^4}{4})$

同理　　$E_2=\dfrac{q}{2\pi\varepsilon_0 rl}=\dfrac{1}{\varepsilon_0 r}(\dfrac{aR^3}{3}-\dfrac{bR^4}{4})$