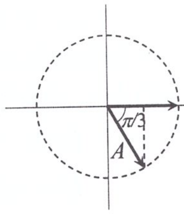
参考答案

一、填空题：（每题 4 分，共 48 分）



1、由旋转矢量，$\Delta\varphi = \pi/3$，$\Delta t = T\cdot(\pi/3)/2\pi = T/6$

2、$\frac{1}{2}kA^2 = E$，$k = 2\times10^2\ \mathrm{N/m}$，$v_m = \omega A$，$\nu = \omega/2\pi = v_m/(2\pi A) = 1.6\ \mathrm{Hz}$

3、$2\pi\sqrt{\frac{J}{mgL}}$

4、$I = P/S = P/(4\pi R^2) = 7.96\times10^{-2}\ \mathrm{W/m^2}$

5、$\nu_1 = \frac{u+v_R}{u-v_s}\nu_s = \frac{330+0}{330-(-20)}\times1500 = 1414.3\ \mathrm{Hz}$，$\nu_2 = \frac{330+0}{330-20}\times1500 = 1596.8\mathrm{Hz}$

6、$E = \frac{i}{2}kT = \frac{5}{2}kT$，$E_r = \nu\frac{i-3}{2}RT = RT$

7、$\sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{\mu}}$，$p = nkT$，$p_A : p_B : p_C = n_AT_A : n_BT_B : n_CT_C = n_A\overline{v_A^2} : n_B\overline{v_B^2} : n_C\overline{v_C^2} = 1:4:16$

8、$\frac{p_0^{\gamma-1}}{T_0^{\gamma}} = \frac{(2p_0)^{\gamma-1}}{T^{\gamma}}$，$\frac{T^{\gamma}}{T_0^{\gamma}} = \frac{(2p_0)^{\gamma-1}}{p_0^{\gamma-1}} = (2)^{\gamma-1}$，$\frac{T}{T_0} = 2^{\frac{\gamma-1}{\gamma}} = 2^{\frac{7/5-1}{7/5}} = 2^{\frac{2}{7}}$，

$$\frac{\bar{v}}{\bar{v}_0} = \sqrt{\frac{8kT}{\pi\mu}}\Big/\sqrt{\frac{8kT_0}{\pi\mu}} = \sqrt{\frac{T}{T_0}} = 2^{\frac{1}{7}}$$

9、不变，增加

10、$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0$，$\frac{-Q_2}{Q_1} = \frac{T_2}{T_1} = \frac{1}{n}$

11、$p = \frac{a^2}{V^2}$，$(-A) = \int_{V_1}^{V_2} p\mathrm{d}V = \int_{V_1}^{V_2}\frac{a^2}{V^2}\mathrm{d}V = a^2(\frac{1}{V_1} - \frac{1}{V_2})$，$pV = \frac{a^2}{V} = \nu RT$，$T = \frac{a^2}{\nu RV}$

$$T_2 - T_1 = \frac{a^2}{\nu R}(\frac{1}{V_2} - \frac{1}{V_1})$$

12、$\oiint_S \vec{E}\bullet\mathrm{d}\vec{S} = E_bS_b - E_aS_a = b\cdot2a\cdot a^2 - b\cdot a\cdot a^2 = a^3b = \frac{1}{\varepsilon_0}\sum q$，$\sum q = \varepsilon_0a^3b = 8.85\times10^{-12}\ \mathrm{C}$

二、计算题：（共 4 题，共 36 分）

1. 解：　$k = \frac{F}{x} = \frac{60}{0.3} = 200(\mathrm{N/m})$　　$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{4}} = 5\sqrt{2}(\mathrm{rad/s})$　　$T = \frac{2\pi}{\omega} = \frac{\sqrt{2}\pi}{5}(\mathrm{s})$

（1）$A = 0.1(\mathrm{m})$　　$\varphi = 0$　　$x = 0.1\cos5\sqrt{2}t\quad(\mathrm{m})$

（2）$a = -\omega^2x = -50\times(-0.05) = 2.5(\mathrm{m/s^2})$　　$F = mg - ma = 4(9.8-2.5) = 29.2(\mathrm{N})$

（3）$\frac{\Delta t}{T} = \frac{\pi/6}{2\pi}$　　$\Delta t = \frac{\sqrt{2}\pi}{60} = 0.074(\mathrm{s})$

2. 解：（1）$\omega = 4\pi\ (\mathrm{rad/s})$　$T = \frac{2\pi}{\omega} = \frac{1}{2}(\mathrm{s})$　$\lambda = uT = 10\ (\mathrm{m})$　$\Delta\varphi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{10}\cdot5 = \pi$

波动方程：$y = 3.0\cos[4\pi(t+\frac{x}{20})-\pi]$ (SI)

（2）反射波方程　$y' = 3.0\cos[4\pi(t-\frac{x}{20})-\pi\pm\pi] = 3.0\cos 4\pi(t-\frac{x}{20})$ (SI)

（3）驻波方程　$y_{合} = y + y' = 6.0\cos(\frac{\pi}{5}x-\frac{\pi}{2})\cos(4\pi t-\frac{\pi}{2})$ (SI)

波腹位置：　$0<x<5$　$\frac{\pi}{5}x-\frac{\pi}{2}=k\pi$　$k=0$　$x=2.5\,(\mathrm{m})$

3. 解：（1）$\int_0^{v_0} f(v)\mathrm{d}v = \int_0^{v_0} kv^3\mathrm{d}v = \frac{1}{4}kv_0^4 = 1$　$k=\frac{4}{v_0^4}$

（2）$\overline{v} = \int_0^{v_0} vf(v)\mathrm{d}v = \int_0^{v_0} kv^4\mathrm{d}v = \frac{1}{5}kv_0^5 = \frac{4}{5}v_0$

（3）$\frac{\Delta N}{N} = \int_0^{v_1} f(v)\mathrm{d}v = \int_0^{v_1} kv^3\mathrm{d}v = \frac{1}{4}kv_1^4 = (\frac{v_1}{v_0})^4 = \frac{1}{16}$，　$v_1 = \frac{1}{2}v_0$

4. 解：（1）bc 为等压过程：　$\frac{T_b}{T_c} = \frac{V_2}{V_1} = 2$　$T_c = \frac{T_b}{2} = \frac{T_a}{2}$

$Q_{ab} = RT_a\ln 2$，　$Q_{bc} = \frac{5}{2}R(T_c-T_b) = -\frac{5}{4}RT_a$，　$Q_{ca} = \frac{3}{2}R(T_a-T_c) = \frac{3}{4}RT_a$

$$\eta = 1-\frac{|Q_{bc}|}{Q_{ab}+Q_{ca}} = 1-\frac{5}{4\ln 2+3} = \frac{4\ln 2-2}{4\ln 2+3} = 13.38\,\%$$

（2）bc 为等压过程：$\mathrm{d}Q_{bc} = \frac{5}{2}R\mathrm{d}T$

$$\Delta S = \int\frac{\mathrm{d}Q}{T} = \frac{5}{2}R\ln\frac{T_c}{T_b} = -\frac{5}{2}R\ln 2 = -57.5\,(\mathrm{J/K})$$

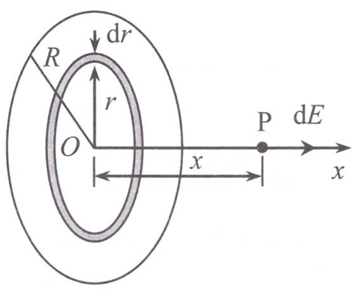
5. 解：设盘心 O 点处为原点，x 轴沿轴线方向，如图所示．
在任意半径 r 处取一宽为 $\mathrm{d}r$ 的圆环，其电荷为

$\mathrm{d}q = \sigma 2\pi r\mathrm{d}r$

$$\mathrm{d}E = \frac{\mathrm{d}qx}{4\pi\varepsilon_0\left(r^2+x^2\right)^{3/2}} = \frac{\sigma x}{2\varepsilon_0}\cdot\frac{r\mathrm{d}r}{\left(r^2+x^2\right)^{3/2}}$$

$$E = \int\mathrm{d}E = \frac{\sigma x}{2\varepsilon_0}\int_0^R\frac{r\mathrm{d}r}{\left(r^2+x^2\right)^{3/2}} = \frac{\sigma x}{2\varepsilon_0}\left[-\frac{1}{\sqrt{r^2+x^2}}\right]_0^R$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]$$



6. 解：取长度为 l 的同轴圆柱面为高斯面，

（1）$r<R$　$\oint_S\vec{E}\cdot\mathrm{d}\vec{S} = \frac{1}{\varepsilon_0}\sum q_i$　$E_1\cdot 2\pi rl = 0$　$E_1 = 0$

（2）$r>R$　$E_2\cdot 2\pi rl = \frac{q}{\varepsilon_0} = \frac{2\pi Rl\sigma}{\varepsilon_0}$　$E_2 = \frac{\sigma R}{\varepsilon_0 r}$