9.4 Do you agree with the following argument? The ℓ_1-norm of a vector $x \in \mathbf{R}^m$ can be expressed as

$$\|x\|_1 = (1/2) \inf_{y \succ 0} \left(\sum_{i=1}^m x_i^2/y_i + \mathbf{1}^T y \right).$$

Therefore the ℓ_1-norm approximation problem

$$\text{minimize} \quad \|Ax - b\|_1$$

is equivalent to the minimization problem

$$\text{minimize} \quad f(x,y) = \textstyle\sum_{i=1}^m (a_i^T x - b_i)^2/y_i + \mathbf{1}^T y, \tag{9.62}$$

with $\mathbf{dom}\, f = \{(x,y) \in \mathbf{R}^n \times \mathbf{R}^m \mid y \succ 0\}$, where a_i^T is the ith row of A. Since f is twice differentiable and convex, we can solve the ℓ_1-norm approximation problem by applying Newton's method to (9.62).

5.7 Solve the problem

$$\text{minimize} \ \ f(\mathbf{x}) = (x_1^2 + x_2^2 - 1)^2 + (x_1 + x_2 - 1)^2$$

by applying Algorithm 5.1. Use $\varepsilon = 10^{-6}$ and try the following initial points: $[4\ 4]^T$, $[4\ \ -4]^T$, $[-4\ 4]^T$, $[-4\ \ -4]^T$. Examine the solution points obtained.

5.8 Solve the problem in Prob. 5.7 by applying Algorithm 5.2. Compare the computational efficiency of Algorithm 5.2 with that of Algorithm 5.1.

5.17 Solve Prob. 5.7 by applying the algorithm in Prob. 5.15. Examine the solution points obtained and compare the amount of computation required with that of Algorithm 5.1.

5.20 (*a*) Find the global minimizer of the objective function

$$f(\mathbf{x}) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 100(x_1 - x_4)^4$$

by using the fact that each term in the objective function is nonnegative.

(*b*) Solve the problem in part (*a*) using the steepest-descent method with $\varepsilon = 10^{-6}$ and try the initial points $[-2\ \ -1\ 1\ 2]^T$ and $[200\ \ -200\ 100\ \ -100]^T$.

(*c*) Solve the problem in part (*a*) using the modified Newton method in Prob. 5.15 with the same termination tolerance and initial points as in (*b*).

(*d*) Solve the problem in part (*a*) using the Gauss-Newton method with the same termination tolerance and initial points as in (*b*).

(*e*) Based on the results of (*b*)–(*d*), compare the computational efficiency and solution accuracy of the three methods.