For office use only	Team Control Number	For office use only
T1 ________________	**61011**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**A**	F4 ________________

2017

(MCM/ICM) Summary Sheet

Managing The Zambezi River

Summary

To remove the Kariba Dam and replace it with a series of smaller dams, it is necessary to confirm the location and number of small dams along the Zambezi River. The rainfall, topography and population distribution are the main factors to select the location and the number of the dams.

First, draw the elevation lines along the Zambezi River, and a multi-seed growth model are established to describe and evaluate the locations, so that we could choose the best place to build the dams. The regional division of rainfall and terrain are set as the smallest observation unit. By solving this model, a conclusion came out that 12 dams is the best solution.

Second, after obtaining the number of dams, we found that this dam system presents a trapezoidal structure, so a trapezoidal dam model was built to get the respective position and the reservoir capacity of each dam, to get the same or higher overall water management capacity (reservoir capacity, power generation capacity) with Kariba Dam.

Third, using the data, we established the safety & cost model. First, analyze the relationship between safety and the cost. Use SPSS to do curve fitting on the total cost and safety guarantee, and get the relation equation, which is expected to provide a reasonable balance between safety and cost. By doing the single factor sensitivity analysis with safety and cost, the conclusion is that the life sensitivity coefficient of dam is 1.070, the investment cost sensitivity coefficient is 1.746, which indicates that the cost is sensitive to safety.

In order to deal with the extreme situation such as flood and drought in time, we set up a $\mathrm{GM}(1,1)$ and dam joint scheduling model. Use the gray prediction model to forecast the future water flow and demand. The expected flow of the river is less than or equal to its safe flow. In the coordination process, the model first determines the future flood situation of each reservoir, and store the floods to reduce the downstream flood control burden.

By solving the trapezoidal dam model, the maximum expected daily discharge is $254\mathrm{M}m^3$, the minimum expected discharge should be the dead water level of the dam system. The most dangerous location of Zambezi River is near the downfall of the Victoria Falls, whose flood time period is from April to May. With the dam system near this place, this period could be spent safely.

Keywords: Continuous Multi-seed Growth Model; Trapezoidal Dam Model; Safety & Cost Model; GM (1,1) and Dam Joint Scheduling Model; Sensitivity Analysis.

Contents

Evaluation Report

Assessment of Repairing the Existing Kariba Dam

The drop water from 6 spillways of the Kariba Dam caused a deep hole in the downstream of the Kariba Dam, which caused eddy currents and reflux phenomenon, leading to the erosion of the bottom dam rock. The repair of large dam is extremely costly. Considering about the detailed situation of Kariba Dam, first, cut rocks downstream of the dam, and re-shape it to prevent water from splashing back into the bottom rocks. The amount of rock that should be reshaped is about 300,000m^3, and the expected cost of this program is about $100M. Then, renew the six spillways. The overall cost is about $300M, including $70M for institutional and project management, $100M for water tunnel remodeling, $130M for spillway renovation. This will improve the long-term safety and reliability of the Kariba Dam, which could continue to operate for 10 years. However, this project could only run during the dry seasons, and the expected time of construction is about 8 years. The potential cost would be pretty large to this region. This article only calculates the benefits from the electricity supplied by the dam, and the restoration of existing Kariba Dam will probably last for about 10 years, with an total yield potential gain of $ 24.065 million.[2]

Assessment of Reconstruction of Kariba Dam

The construction of a complete Kariba Dam is $480M. At the current engineering level, the cost of handling and construction and the usage of labor will be reduced. A portion of materials after removal of the dam could be used for the reconstruction of Kariba Dam, which will reduce the cost of materials and manpower. But the management cost will increase, and the demolition of the dam also requires a lot of cost, and the recovery of the terrain will also take a long time. So the potential cost may be greater than the estimated cost. The construction of New Kariba Dam is expected to run for about 60 years. Rebuilding the Kariba Dam will also yield potential gains of approximately $ 144.39 million for approximately 60 years.

Assessment of the Replacement of Kariba Dam

The third measure is removing the Kariba Dam. Affected by the regional distribution of the reservoir and the power transmission, It is reasonable to build a series of smaller dams along the Zambezi River, to replace the power generation and the reservoir capacity of Kariba Dam. That is, there will be a series of dams along the Zambezi River, which provide convenience to most of the residents in this area. Small dams are not costly for maintenance, and they could decrease the cost of cable, water pipes and other facilities.

To get the same or higher reservoir & power generation capacity than the Kariba Dam, By solving the model, the conclusion has come out that there are 12 dams in total, to achieve the same or higher overall water management capacity (reservoir capacity, power generation) than Kariba Dam. Their positions and scales are as follows:

A dam with capacity of 40MW and storage capacity of 18 billion m^3 will be built at about 50km from the upper reache of Victoria Falls;

A reservoir with a capacity of 12 billion dams was built at Batoka gorge, 94 km downstream from Victoria Falls;

In the upstream and downstream of Kariba Dam plan to build three dams seperately with the power capacity of 120MW and the storage capacity of 19 billion m^3;

Three dams will be located at 60km, 110 km and 140 km downstream of the Chaora Bassa Dam, The planning installed capacity is 1.8MW 440KW and 660 KW respectively;

The remaining one will be built in the most downstream with the capacity of 9 billion m^3.

According to the actual local terrain, the building materials could be estimated. And the cables, transportation, pipelines and other facilities could be built according to the distance of demanded places. So, the cost and profit could be estimated.

Removing the Kariba Dam, and replacing it with a series of smaller dams along the Zambezi River would yield $ 240.65 million, if they provide the same amount of power as Kariba Dam.

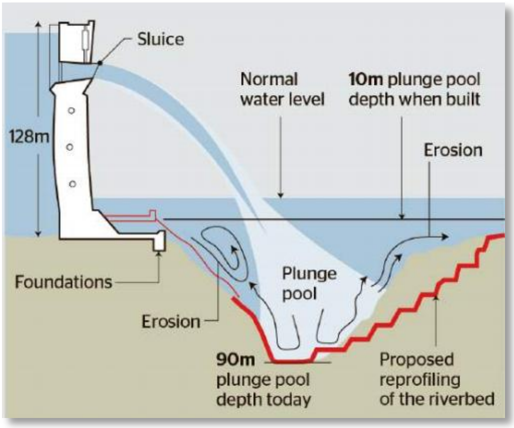
Sincerely,

MCM Team Members

1. Introduction

1.1 Background

The Zambezi River basin is at the tropical grassland climate zone. The river has a clear flood and dry season. The flood period is between November and the rainy season of April, and the dry period in the dry season. The precipitation in the Zambezi River Basin decreases from north to south (1560mm-650mm). The basin area of Zambezi River is 1.39Mkm^2, which has a total resource of water power about 130 billion kWh. It has 4 major power stations. The Kariba Dam, built between 1956 and 1959, has 58 years of history. The maximum dam height is 128m, and the total reservoir is 184 billion m^3. It is one of the world's largest water reservoirs. The existing plant capacity is 1.26MW, the annual generating capacity is up to 8 billion kWh. The construction of Kariba Dam has brought great convenience to the local population. It provides sufficient power to 40% of the local residents, and the Lake of Kariba supplies the surrounding residents in a large area.[1]



However, this dam has some severe problems. The Kariba Dam was built in 1959, but just 3 years later, the water drained from the spillway drilled a 30-meter-deep hole. By 1981, the hole had extended to 80 meters deep, resulting in the erosion of the rock near the basis of the dam. In 2015, it was detected to have serious design problems, and it needs immediate maintenance.

1.2 Our Work

The ZRA has three options to choose, the options are as follows:

1. Repair---- The Kariba Dam have to be repaired or it would have bigger problems several years later. If we repair the Kariba Dam, it is possible to extend its life for further.
2. Reconstruction---- Remove the old Kariba Dam and build a new one, which could possibly solve the original structural weakness when rebuilding, and it could work better after rebuilding.
3. Replacement---- Remove the Kariba Dam, and replace it with a series of smaller dams (10 to 20) along the Zambezi River, which is expected to have the same or higher water managing capabilities, handling ability in extreme weather conditions than the Kariba Dam.

Our work is helping ZRA to make assessments to these options and provide the best choice for solving the dam problem with sufficient evidence and reasonable analysis between cost and profit. Then, build mathematical model to locate and design the small dams in option 3, calculate the balance between the profit and safety.

2. Assumptions and Symbol Description

2.1 Symbol Description

Serial number	Symbol	Symbol description
1	a_i	Divided area of the dam
2	b_i	Rainfall, terrain
3	n_i	Number of dams by region
4	S	A collection of rainfall in the Zambezi River
5	P	Dams Arrangement along the Zambe River
6	V_{total}	Total reservoir capacity
7	F	Catchment area of a reservoir
8	S_0	Sediment transport modulus in watershed
9	β	Adjustment factor of storage capacity
10	E	Bed load is the percentage of annual sediment discharge
11	W	Average annual incoming water
12	Q_M	Peak discharge
13	C_p	Coefficients related to river basin physical geography and frequency
14	α	Empirical coefficient
15	C_h	Cost of safety assets
16	C_j	Safety Management Costs
17	C_k	Safety Loss Costs

2.2 **Assumptions**

1) Assume that only the precipitation, topography and population distribution will affect the location and number of dams.

2) Assume that the benefit is calculated considering only the power generation.

3) Assume that the effects of natural disasters are ignored.

4) Assume extreme cases refer to the occurrence of floods, droughts.

5) Assume that the total emissions are replaced by daily expected emissions.

3. Selection of Dam Locations

In order to solve the problem of how to select locations of dams on the upper, middle, lower of the Zambezi River, the first work is analyzing the rainfall, topography and population distribution along the Zambezi River.

3.1 Hydrological Data

Most of the Zambezi River flows through the South African Plateau at an altitude of 500 to 1500m, and the total water resources of the Zambezi River amounts to 130 billion kW • h. Zambezi River Basin is in the tropical grassland zone, and the river has a clear flood season and dry season. The flood season is in the rainy season from November to April, and the dry period is in the dry season. The rainfall in the Kafue River basin is reduced from 500 mm to 750 mm from north to south. As the beginning of each rainy season has different time, the flood period is not the same month. The upper peak usually occurs in 2 ~ 3 months, midstream peak is postponed to 4 ~ 5 months. The seasonal variation of runoff is large, with the maximum flow from March to April, and the minimum flow from October to November.

3.2 The Model Analysis of Dam Location Selection

The growth segmentation model could be selected to determine the number of dams in the middle and lower stream of the Zambezi River. In the growth segmentation model, the rainfall and terrain are taken as the smallest unit and takes two steps to solve the problem of dam division in the middle and lower reaches of the Zambezi River, and quantitatively analyze the three criteria of dam division: Whether the regional rainfall of the coastal dam is the same or whether to meet the continuity and compactness. First, the multiple seeds growth models are used to generate multiple seeds in a region along the Zambezi River, and seed growth rules are built to create the division pattern of the dams. Then, use the optimal segmentation method, the dam division boundary is further adjusted by the idea of stochastic climbing so that the dams with the same rainfall amount are built. According to whether the regional rainfall similarity and compactness and consistency to determine the quality of dam selection.

3.3 Establishment of Dam Location Selection Model

According to the conventional thought, we say that the Zambezi River is divided into upstream, middle and downstream, and the rainfall and terrain are classified as the smallest unit. The rainfall meets the requirements of dam water storage and flood control, and the terrain is similar in the same area. At the same time, rainfall is similar, the river terrain, economic conditions are the same, so this model will rainfall, terrain as the basic unit of dam selection.

3.3.1 Continuous Multi-seed Growth Model

Mark a_i as the areas divided by the dam, n_i as the number of dams in each region, b_i as the precipitation and the terrain, $\mathrm{S} = \{b_i\}_{1 \le i \ll 20}$ as the set of rainfall in the Zambezi River, $\mathrm{P} = \{a_i\}_{1 \ll i \ll 20}$ as the division of the Zambezi River. Define two adjacent regions of rainfall$\left(b_i, b_j\right)$, mark as $b_i \sim b_j$. If $a_i\left(b_j\right)$ contains the rainfall area b_j, the concept of adjacency can be extended to the selection of the region. The total amount of rainfall that can be adjoined to b_i could be expressed as:

$$C_b(b_i) = \left\{b_j \epsilon S \middle| b_j \sim b_i\right\} \tag{1}$$

$C_a(b_i) = a\big(a_b(b_i)\big)$ Represents the set of all the regions that are adjacent to the rainfall b_i. Since each rainfall has at least one adjacency, both $C_b(b_i)$ and $C_a(b_i)$ are non-null.

First, assign the Zambezi rainfall to a virtual area a_0. Then, an initial seed is then assigned to each of the n_i regions, each of which initially has only one rainfall. When $a_0 > 0$, the rainfall in the virtual area is gradually moved to each region, then all the moving set can be expressed as

$$\mathrm{P}(a_0, a_1, \cdots a_n) = \cup_{i=1}^{n} \cup_{b_j \epsilon f_i} M\big(b_j, a_i, a_j\big) \tag{2}$$

Where $M\big(b_j, a_i, a_j\big)$ shows that rainfall b_j moves from a_i to a_j. Each move can be valued according to whether it is worthwhile to accept (value is according to rainfall bias and consistency). Here, the top 3% of the scoring moves. Note that the motion preserves continuity, since $b_j \epsilon f_i$, moving b_j from its boundary to a_i, apparently preserves a_i continuity.

Although the MSGM model moves only between virtual and real constituencies, rather than between two real regions, the scoring of each move is not isolated. Considering the two adjacent constituency a_i, and a_j borders on the shared channel $b_j \epsilon f_i \cap f_j$, non-shared rainfall is $b_j \epsilon f_i \cap f_j^c$. Accepting a move $M\big(b_j, a_i, a_j\big)$ means that the change in f_i and the associated movement score will also change the score of f_i and the associated movement. For example, it may result in that $M\big(b_j, a_i, a_j\big)$ chose b_i but not $M\big(b_j, a_i, a_j\big)$. Further, moving $M\big(b_j, a_i, a_j\big)$ will enlarge f_i and the size of f_i. Maybe the choice here is not the optimal move since there is no global optimization of the optimal value.

Of course, it is preferable to select the global optimum movement, but this will take a long operation time and therefore, it is not worth considering. The results of the MSGM are then improved using the POM (the program is in the appendix).

The score for all candidate moves can be scored based on the selected area rainfall and its consistency with Kariba. Rainfall scoring is aimed at minimizing rainfall and terrain biases in the regions, i.e., if the rainfall in the Zambezi is equal to $\bar{d}$, the highest score could be obtained. In addition, if the rainfall is biased, a penalty score is given. The maximum score is at $\bar{d}$, so the rainfall function is a concave curve.

Let $\mathrm{Q}(d_i)$ be the score of rainfall, population, terrain, and d_i for region a_i, and the regional segmentation function is denoted by

$$\mathrm{Q}(d_i) = \begin{cases} M\sqrt{\frac{d_i}{\bar{d}}} & d_i \ll \bar{d} \\ M - \frac{4M(d_i - \bar{d})}{d_i^2} & d_i > \bar{d} \end{cases} \tag{3}$$

Note that for $d_i > \bar{d}$, the $\mathrm{Q}(d_i)$ is more steep, because we must strictly control the regional scale, as shown in Figure 1

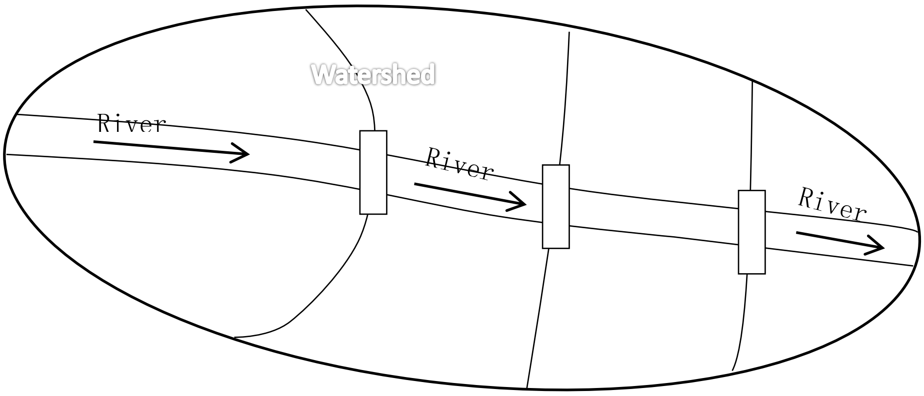


Figure 1

Boundary consistency valuation can be based on the Zambezi River along the rainfall, the proportion of the population, to measure the extent of the region to maintain the border of the dam. For region a_i, the score of the dam boundary consistency is as follows:

$$g(a_i) = \sum_{f_j \in A} \left(\frac{\sum_{b_l \in a_i \cap f_j} d(b_i)}{d(f_j)} \right)^2 \quad (4)$$

For example, if the precipitation of an area contains 30% of the precipitation of the adjacent area, the score is 0.18.[3]

3.4 Model Solving

Use Google Earth to draw and measure the middle Zambezi River elevation distribution (Figure 1), the lower Zambezi River elevation distribution (Figure 2).

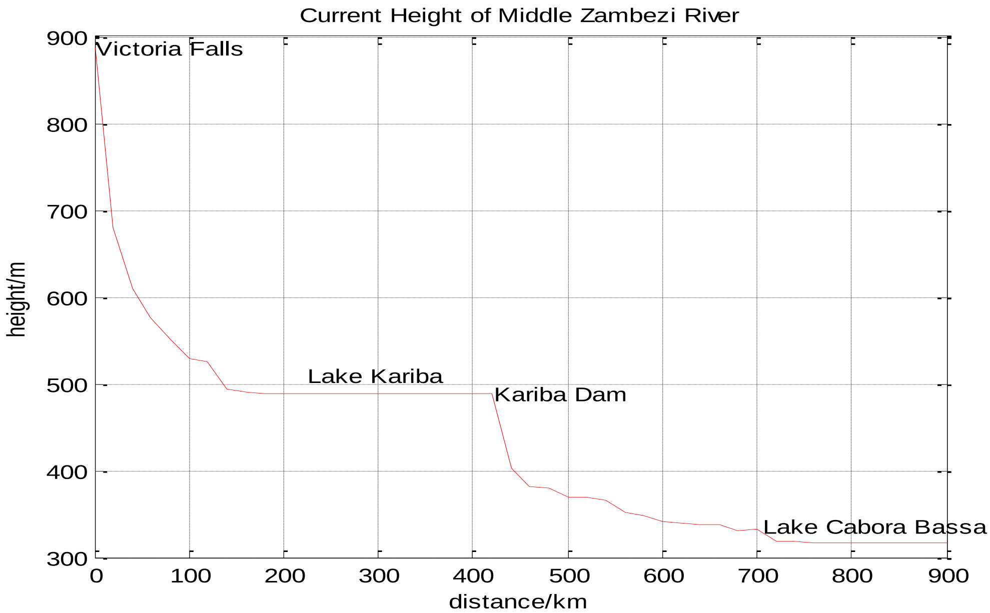


Figure 2 、Current terrain of middle Zambezi river

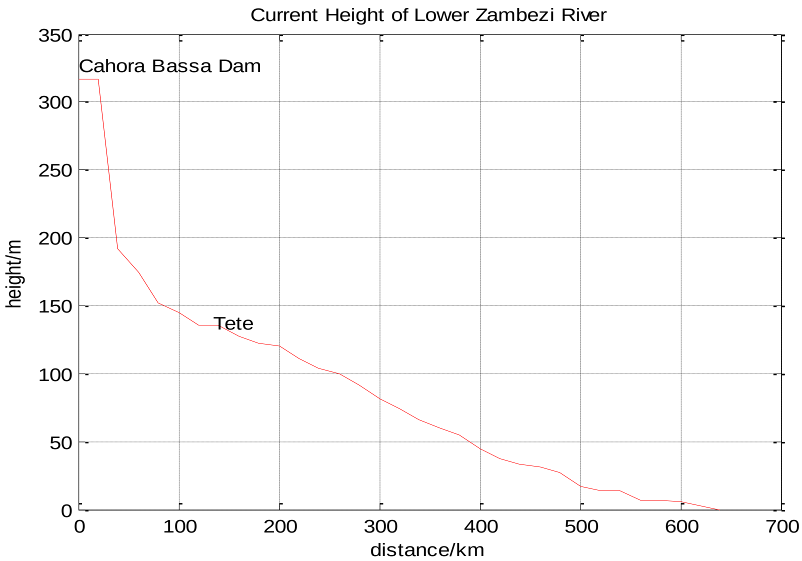


Figure 3、 Current terrain of low Zambezi river

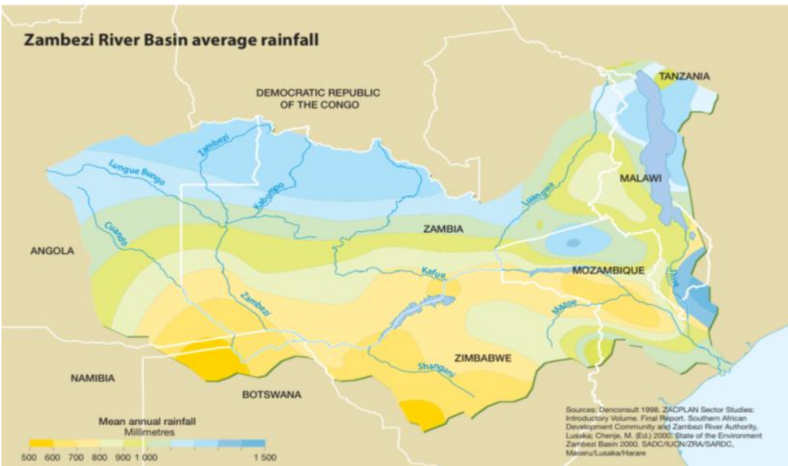


Figure 4、 Rainfall distribution along the Zambezi River

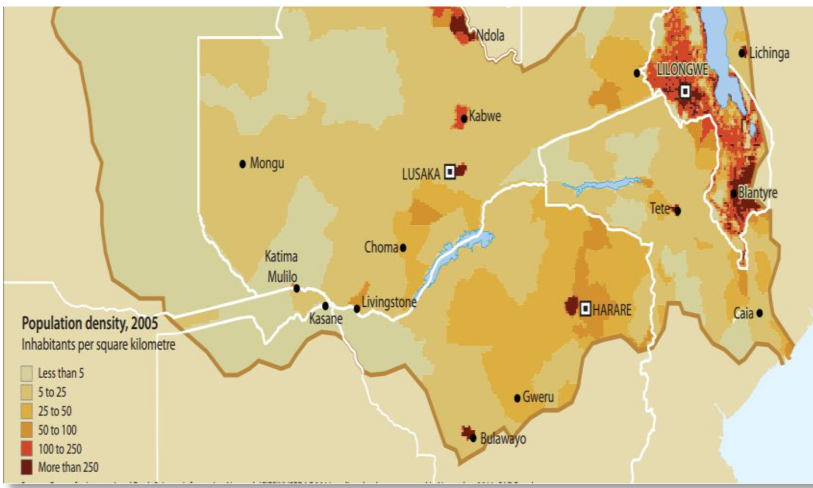


Figure 5、 Population distribution along the Zambezi River

Table 2, coordinates of some locations along the Zambezi River, runoff

Position	Longitude	Latitude	Gap	Min Temp(℃)	Max Temp (℃)	Annual Precipitation(mm)	Runoff（m^3/s）
1	17.56.59S	25.51.29E	-0.30%	11.9	26.1	755.2	1180
2	18.00.05S	25.57.57E	-0.20%	11.9	26.1	755.2	1197.92
3	17.55.13S	26.07.51E	-0.80%	11.9	26.1	755.2	1215.84
4	17.56.11S	26.18.36E	-0.10%	11.9	26.1	755.2	1251.68
5	17.57.49S	26.27.59E	-0.70%	12.9	28.3	560	1287.52
6	18.00.47S	26.34.24E	-0.50%	12.9	28.3	560	1341.28
7	17.59.07S	26.52.26E	0.20%	12.9	28.3	560	1359.2
8	17.57.54S	26.57.55E	-0.30%	12.9	28.3	560	1368.16
9	16.29.01S	28.48.59E	-0.30%	18.4	30.6	765.5	1386
10	15.38.09S	30.05.30E	-0.10%	14.9	26.4	842.7	1812
11	15.40.02S	30.12.46E	-0.30%	14.9	26.4	842.7	1920
12	15.38.45S	30.18.08E	-1.10%	20.7	32.7	629.9	2330

According to the formula and table (1) can be obtained from the dam position of the table 2

Table 3 、Fraction of the dam location

Pos.	1	2	3	4	5	6	7	8	9	10	11	12
Sco.	0. 053	0. 0535	0. 892	0. 103	0. 061	0. 085	0. 11	0. 991	0. 82	0. 055	0. 061	0. 471

According to the scores obtained from the middle reaches of the Zambezi River downstream dam location, the number of distribution as follows:

Figure 6、The middle low reaches of the Zambezi River, downstream dam location, and the distribution

There are 12 dams in the dam system, and then different types of dams will need to be built according to different locations. From the figure we can see the Zambezi River, the middle、downstream coastal dam distribution and specific.

4. Construction Types of Dams

4.1 Dam Types Analysis

4.1.1 Hydrological Data

The upper reaches of the Zambezi River mainly flows through the plateau, the river is more curved, longitudinal slope slow, slow water flow, combined with the two sides of the terrain is relatively gentle, widely distributed swamp along the river, basically do not have hydraulic development value. Most sections of the river cut through the sandstone layer, part of the river cut basalt layer, water flow according to river width and change, more waterfalls and canyon jet, drop is up to 670m. The length of the channel is about 70km long, and the width of the channel is about 100m ~ 430m, about 70m. The water flow is fast and the mountains are strong. The elevation difference between the two sides is generally 100 ~ 380m.

The types of dams are selected to meet safety and cost balances, to be able to handle emergencies such as (floods, droughts), maximum water storage expectations, and minimum expectations. In order to build such a dam we look up the Zambezi River along the relevant information, found for the middle reaches, downstream trapezoidal model to establish is more reasonable.

And because the Zambezi River, the lower reaches of the map is shown in Figure（See Appendix 1 for the procedure）

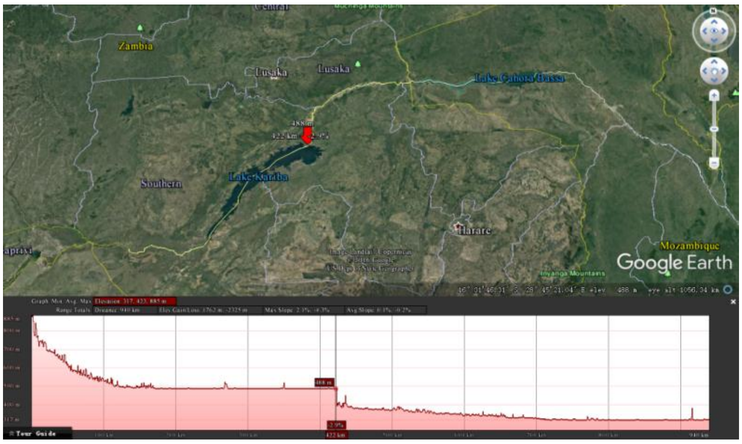


Figure 7、Topography along the middle Zambezi River

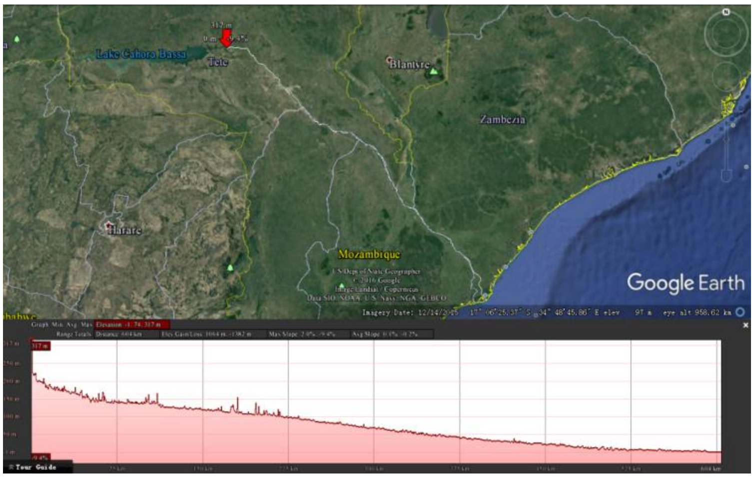


Figure 8、Topography along the lower Zambezi River

By searching for the database, the figure could be made as follows:

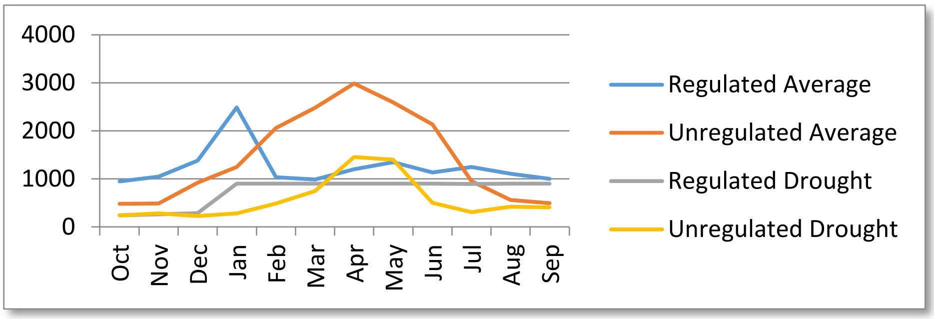


Figure 9、Middle Zambezi River flow diagram

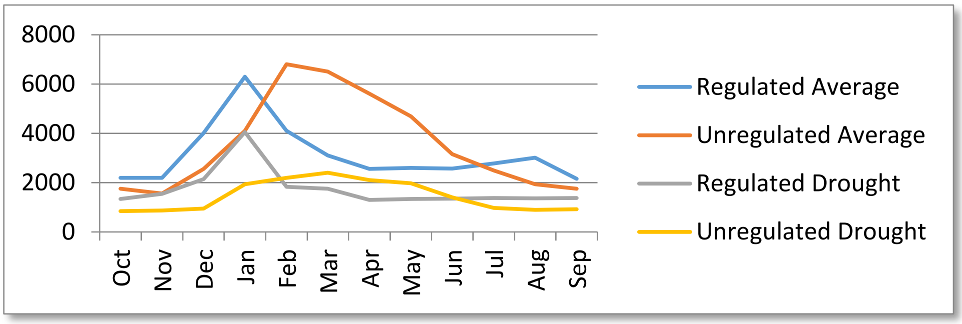


Figure 10、Lower Zambezi River flow diagram

From the figures above, we could see the middle and lower reaches of the terrain, water flow. From the map, the suitable areas for constructing trapezoidal dam system could be seen in the model shown below.

4.2 Trapezoidal Dam Model Establishment

In order to achieve the same overall water management capacity as the Kariba Dam, to satisfy the safety and cost balance, to be able to handle emergencies such as (floods, droughts), maximum water storage expectations and minimum expectations, the new dam capacity needs to be calculated before making sure of the safety level of the new dam system.

4.2.1 Calculation of Storage Capacity

Reservoir storage capacity refers to the volume of reservoir water. The application requirements and the reservoir level are known as the reservoir characteristics of water level, corresponding to a variety of characteristics of the reservoir capacity is called the characteristics of capacity. Characteristic water level and characteristic storage capacity of the most common there are mainly in three aspects: dead water level & dead storage capacity, the normal water level & irrigation capacity and the flood control capacity. Because the flood control limit water level is less than or equal to the normal water level, in order to facilitate the calculation, this paper assumes that the flood control limit water level is equal to the normal water level, so the total reservoir capacity is the sum of the three reservoir capacity. which is:

$$V_{Total} = V_{\text{Dead storage capacity}} + V_{\text{Irrigation}} + V_{\text{Flood control}} \quad (5)$$

In the formula,

V_{total}---- Total reservoir capacity, m^3;

$V_{\text{Dead storage capacity}}$----Dead storage capacity, m^3;

$V_{\text{Irrigation}}$----Irrigation capacity, m^3;

$V_{\text{Flood control}}$----Flood control capacity, m^3;

4.2.2 Determination of Dead Storage Capacity

In the normal use of reservoir, the allowing lowest water level, is called the dead water level. The storage capacity in below the dead water level is called the dead storage capacity.

To determine the size of dead storage capacity, the impact of local sedimentation should be taken into consideration:

$$V_{\text{Dead storage capacity}} = \frac{FS_0T}{\gamma}(1+E) \quad (6)$$

In the formula,

F---- The catchment area of the reservoir, km^2;

S_0---- The amount of sediment transport within the basin modulus, t/km^2;

T---- The reservoir siltation years;

E---- Bed load is the percentage of annual sediment discharge;

γ---- Sediment capacity, generally$1.2--1.4\,t/m^3$;

4.2.3 Determination of the Irrigation Storage Capacity

Generally, in order to meet the design requirements, the water level in the beginning of water supply, is called the normal water level, also known as the normal high water level. The capacity between the normal water level and the dead water level is the capacity of the reservoir to regulate the runoff. It is called the irrigation storage capacity. Its function is to adjust the natural water supply of the river to meet the needs of power generation, irrigation and water supply. It could be determined by the amount of water W_0.

$$W_0 = 1000Y_0F \quad (7)$$

$$V_{\text{Irrigation}} = \beta W_0 = 1000\beta Y_0 F \quad (8)$$

In the formula,

W_0---- Average annual runoff, m^3;

β---- Storage capacity adjustment coefficient, statistics are available through direct access;

Y_0---- Years average runoff depth, mm;

F---- Reservoir catchment area, km^2.

4.2.4 Determination of Flood Storage Capacity

When the reservoir meets the design standard of the downstream protection object, the highest water level that can be reached before the dam is called high level of flood control. The water level to the flood control limit between the reservoir volume is called flood storage capacity. Flood control limit water level is lower than or equal to the normal water level. To simplify the calculation, this paper assume that the flood control limit water level is equal to the normal water level, so flood control high water level to normal reservoir volume between the reservoir volume is called the flood storage capacity.

The peak discharge is:

$$Q_M = C_P F^n \qquad (9)$$

In the formula,

Q_M---- Peak discharge, m^3/s;

C_P---- Coefficient that is related to the natural geographical and frequency of the basin,

n---- Area index, generally use $\mathrm{n} = 2/3$ for calculation;

F---- Catchment area, km^2.

The relationship between peak discharge and maximum discharge flow is:

$$Q_M = \alpha Q_{\text{flood discharge}} \qquad (10)$$

α---- Empirical coefficient;

$Q_{\text{flood discharge}}$---- Discharge flow, km;

The formula for calculating the flood control dam height can be obtained by calculating the size of catchment area controlled by the dam：[4]

$$H_{\text{Flood control}}\left(\frac{C_p F^n}{aMb}\right) \qquad (11)$$

4.3 Solution of Trapezoidal Dam Model

It could be searched on the internet that the total capacity of Kariba Reservoir is 184 billion cubic meters, and the total generating capacity is 1626 MW, annual generating capacity is 6400GWh. To get the same or higher reservoir & power generation capacity than the Kariba Dam, By solving the model, the conclusion has come out that there are 12 dams in total, to achieve the same or higher overall water management capacity (reservoir capacity, power generation) than Kariba Dam. Their positions and scales are as follows:

A dam with capacity of 40 million and storage capacity of 18 billion m^3 will be built at about 50 in the upper reaches of Victoria Falls;

A reservoir with a capacity of 12 billion dams was built at Batoka gorge, 94 km downstream from Victoria Falls;

In the upstream and downstream of Kariba Dam plan to build three dams seperately with the power capacity of 120 million and the storage capacity of 19 billion m^3;

Three dams will be located at 60km, 110 km and 140 km downstream of the Chaora Bassa Dam, The planning installed capacity is 1.8 million 440 thousand and 660 thousand respectively;

The remaining one will be built in the most downstream with the capacity of 9 billion m^3.

Only by building such a dam system can the Riverside of Zambezi be safe. Then the safety cost of the new dam system will be analyzed and calculated.

5. Safety & Cost Model

The safety and costs are classified as follows:

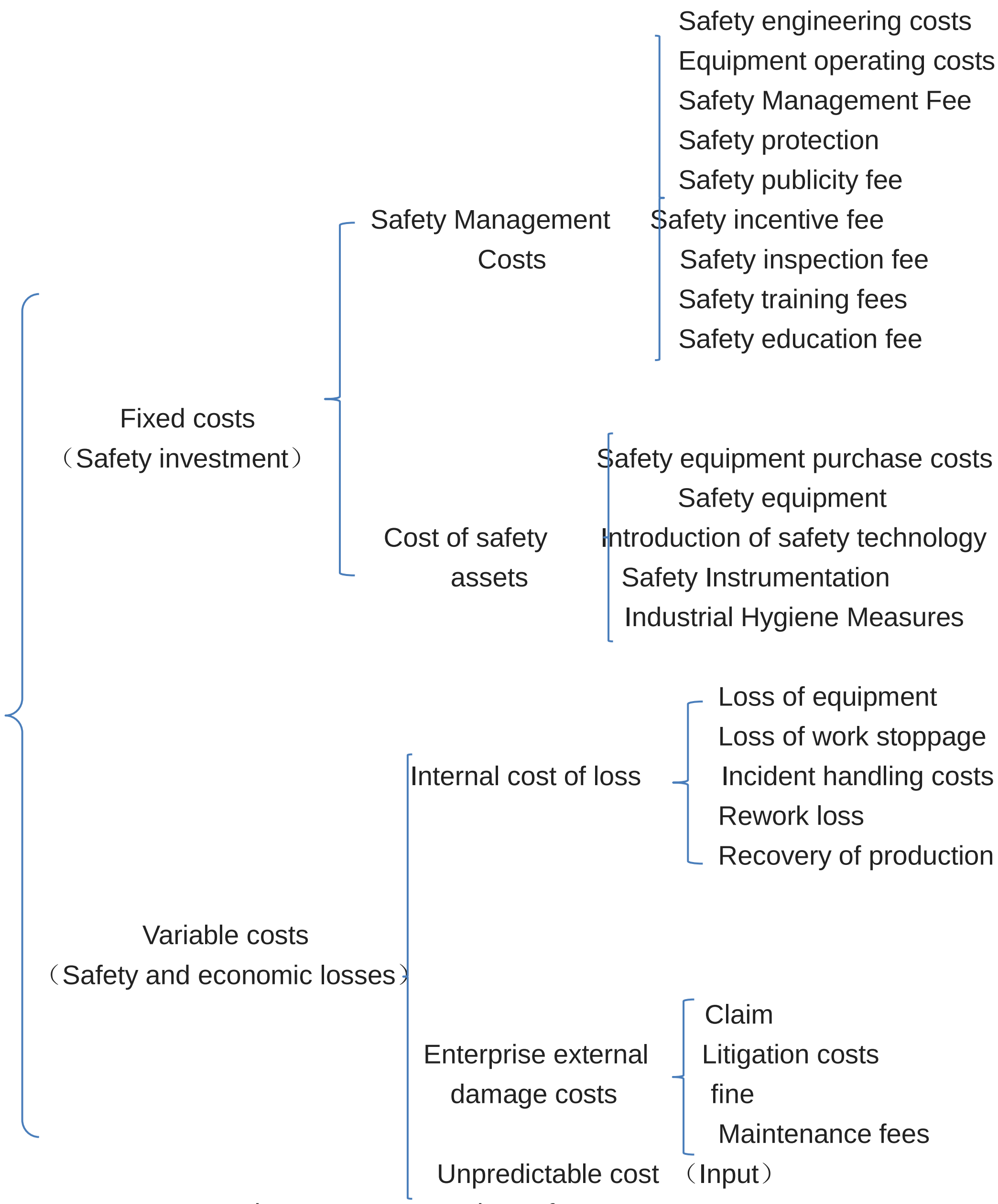


Figure 11、 Enterprise safety cost structure

5.1 Optimal Safety Cost

Safety costs constitute the tree, the total cost of safety can be expressed as

$$C_{total(x)} = C_h(x) + C_j(x) + C_k(x) \quad (12)$$

C_h is the safety asset cost;
C_j is the safety management cost;
C_k is the safety loss cost.

In practice, the internal structure of safety costs is dynamic and constantly changing. As long as the level of safety falls, failing to meet the national safety standards, accident costs will increase. To achieve a higher level of safety, management costs and asset costs will increase. Therefore, explore the change of the proportional relationship between safety management cost, safety asset cost and safety loss cost, the best safety cost is to seek the minimum of the sum of three sub-costs.

Table 3、Information on Safety Costs and Safeguards of a Dam in 1999-2004

Year	Safety Asset Costs/$M	Safety Management Cost/$M	Loss of safety costs/$M	Total Safety Cost/$M	Yield/Mton	Safety degree/(ton・dollar)
1999	2. 449825	1. 647176	1. 336754	5. 433755	2. 2602	0. 035824
2000	2. 526783	1. 757817	2. 071135	6. 355735	2. 284	0. 023365
2001	2. 634207	1. 939898	1. 238967	5. 813071	2. 61	0. 044633
2002	2. 847977	2. 240306	1. 217409	6. 305691	2. 7313	0. 047534
2003	2. 590306	1. 712475	1. 404309	5. 707089	2. 814	0. 042456
2004	2. 597889	1. 714905	1. 480349	5. 73492	3. 019	0. 04321

5.2 Safety Cost Model Solving

Use SPSS software to get the relationship among C_h, C_j, C_k (Figure)

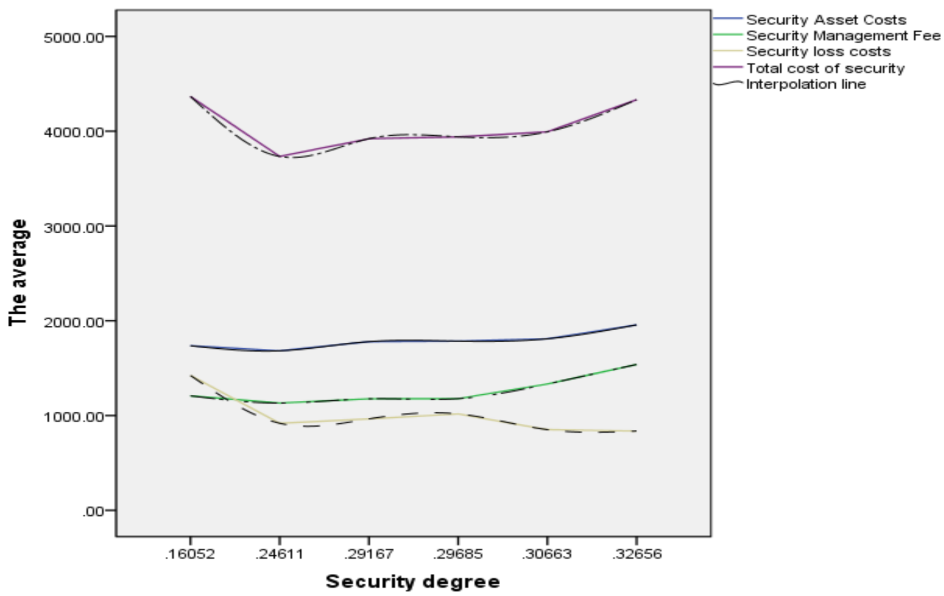


Figure 12.、Safety Costs and Safeguards

As can be seen from the figure, the safety cost curve is characterized by: the total cost of a safety level, there is a minimum value, this intersection is the best safety costs.

$$\frac{dC(x)}{dx} = 0, \text{That is: } \frac{dC_h}{dx} + \frac{dC_j}{dx} + \frac{dC_k}{dx} = 0 \qquad (13)$$

When $x = x_m$, achieved the best safety costs; (x_m total cost and safety of the lowest point of the curve)

If and only if $x \geq x_m$ (safety assurance standard value), the optimal level of safety will be possible.[5]

Use the SPSS software to get the safety and cost of the fitting curve and found that three times is better, the graphics are as follows:

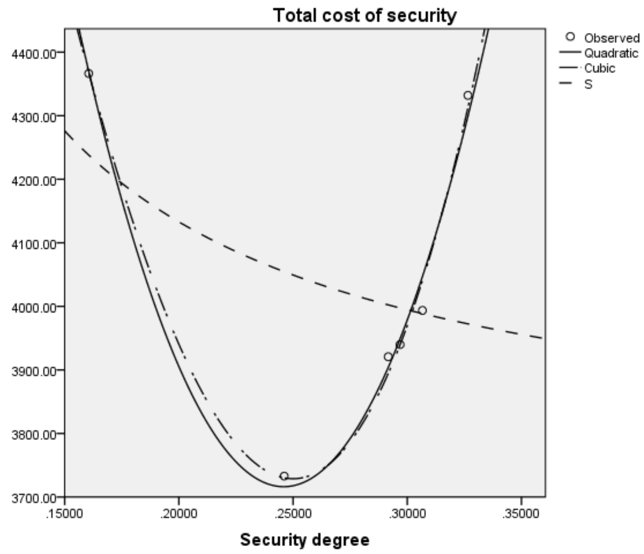


Figure 13、safety and the total cost of the fitting curve

Table 4、Model Summary and Parameter Estimates

Equation	Model Summary					Parameter Estimates			
	R Square	F	df1	df2	Sig.	Constant	b1	b2	b3
Quadratic	.985	101.362	2	3	.002	9138.470	-44099.856	89667.827	
Cubic	.989	141.325	2	3	.001	7502.056	-22636.266	.000	120692.433
S	.157	.745	1	4	.437	8.225	.020		

The independent variable is VAR00005、VAR00007

The fitting equation could be got by the safety function of the total cost and safety of the relationship and the fitting curve:

$$y = 120692.433x^3 - 22636.266x + 7502.056$$

Base on the established safety cost model, the relationship between safety and cost could be derived. Then adjust the construction of the new multi-dam system to provide a reasonable balance between safety and cost.

6. Measures to deal with Emergency Water

6.1 A Model for Handling Emergency Water Flows

As one of the important flood control works, dam system plays a flood storage flood detention to the role, so the dam system of flood control scheduling is an important non-engineering measures. In order to deal with the flood、drought - induced situation in time, we set up a large - scale system of decomposition and coordination with the GM (1,1) and dam joint scheduling model. We use the dam system optimization scheduling model and GM (1, 1) prediction model for the combination of the manager to provide solutions, to make flood、drought damage minimum.

6.1.1 The Establishment of the Grey Prediction Model

Remember $x = \left(x(1), x(2), \cdots x(n)\right)$, where $\mathrm{x}(i)$ represents the value of the i −th year.

Step1:Let $x^{(0)}$ be the GM(1,1) modeling sequence, denoting the gray derivative

$$x^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right) \tag{14}$$

Among them

$$x^{(0)}(k) = \sqrt{x(k)}, \quad k = 1,2,3 \cdots \tag{15}$$

Step2: For $x^{(1)}$ is the AGO sequence of $x^{(0)}$ when $x^{(0)}$ cumulation generation, get a new sequence $x^{(1)}$.

$$x^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right) \tag{16}$$

$$x^{(1)}(1) = x^{(0)}(1) \tag{17}$$

$$x^{(1)}(k) = \sum_{m=1}^{k} x^{(0)}(m) \tag{18}$$

Step3: Let $z^{(0)}$ is the $x^{(1)}$ the (mean) sequence, denote the white background value

$$\mathrm{z}^{(1)}(\mathrm{k}) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1) \tag{19}$$

$$z^{(1)} = \left(z^{(1)}(2), z^{(1)}(3), \cdots, z^{(1)}(n)\right) \tag{20}$$

The Gray differential equation model of GM(1,1)

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{21}$$

Formula(21):a 、 b for the parameters to be estimated, respectively, the development of gray and endogenous control gray.

$$a=\frac{\sum_{k=2}^{n} z^{(1)}(k)\sum_{k=2}^{n} x^{(0)}(k)-(n-1)\sum_{k=2}^{n} z^{(1)}(k)x^{(0)}(k)}{(n-1)\sum_{k=2}^{n} z^{(1)}(k)^2-(\sum_{k=2}^{n} z^{(1)}(k))^2}; \tag{22}$$

$$b=\frac{\sum_{k=2}^{n} z^{(1)}(k)\sum_{k=2}^{n} z^{(1)}(k)^2-\sum_{k=2}^{n} z^{(1)}(k)\sum_{k=2}^{n} z^{(1)}(k)x^{(0)}(k)}{(n-1)\sum_{k=2}^{n} z^{(1)}(k)^2-(\sum_{k=2}^{n} z^{(1)}(k))^2} \tag{23}$$

$$x^{(0)}(k)=b-az^{(1)}(k) \tag{24}$$

6.1.2 Gray GM (1,1) Predictive Model

In the formula (24) on both ends of multiplied by the e^{ak} too at the same time.

$$e^{ak}x^{(0)}(k)+e^{ak}z^{(1)}(k)=e^{ak}b \tag{25}$$

Namely

$$z^{(1)}(k)=e^{-ak}\int(be^{ak}d_t+C)$$

$$=Ce^{-ak}+\frac{b}{a} \tag{26}$$

Then we get the estimated value of the function $x^{(1)}(k+1)$

$$x^{(1)}(k+1)=\left[x^{(0)}(1)-\frac{b}{a}\right]e^{-ak}+\frac{b}{a} \tag{27}$$

The water flow of the Zambezi River is known from 1977 to 2016 as shown

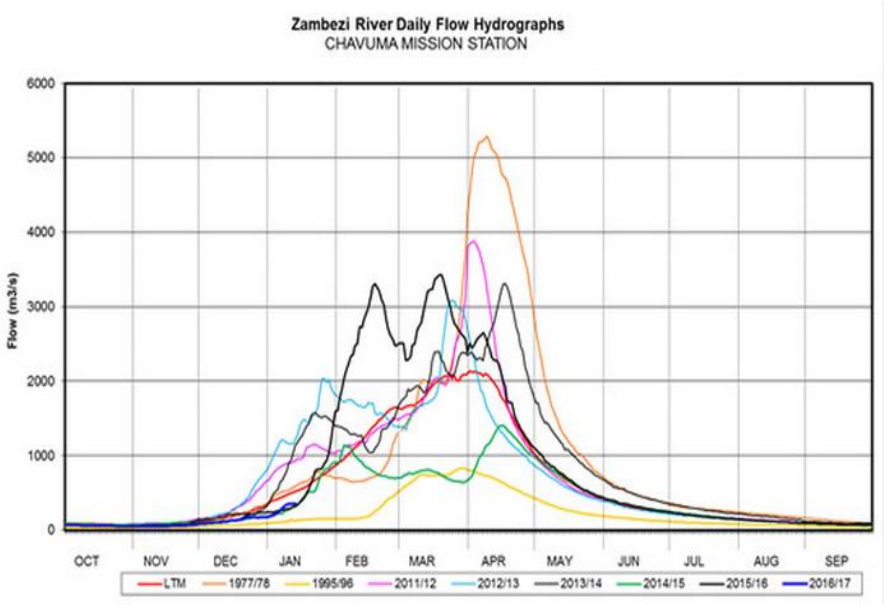


Figure 14、The Zambezi river1977-2016 water flow

The gray forecasting model can be used to predict the water flow of the Zambezi River Figure, the Zambezi River future peak load and demand power's forecast.

Prediction of water flow and power requirement of Zambezi River from 2016 to 2020. Then, draw with Excel.

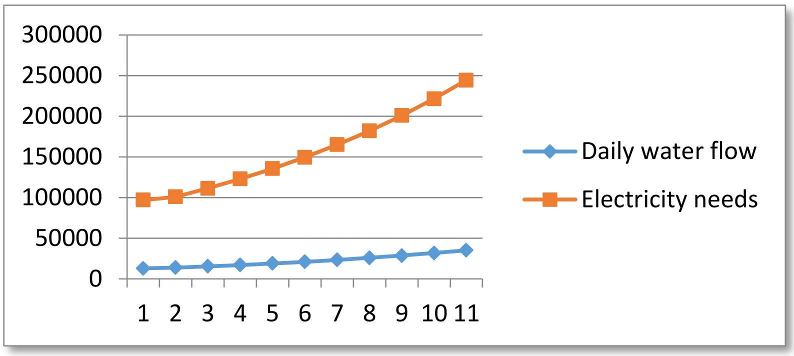


Figure 15、the Zambezi River future peak load and demand power's forecast

6.2 Dam joint scheduling model of coupling & gray prediction model

This paper built the flood control system main by the n series a flood regulating capacity of reservoir and downstream embankments composite. The optimal criterion is the maximum flood control safety certification standard, that could satisfy the requirement of flood control in the case, make every moment the reserved flood control reservoir capacity of weighted sum is the largest. The flow chart is as follows:

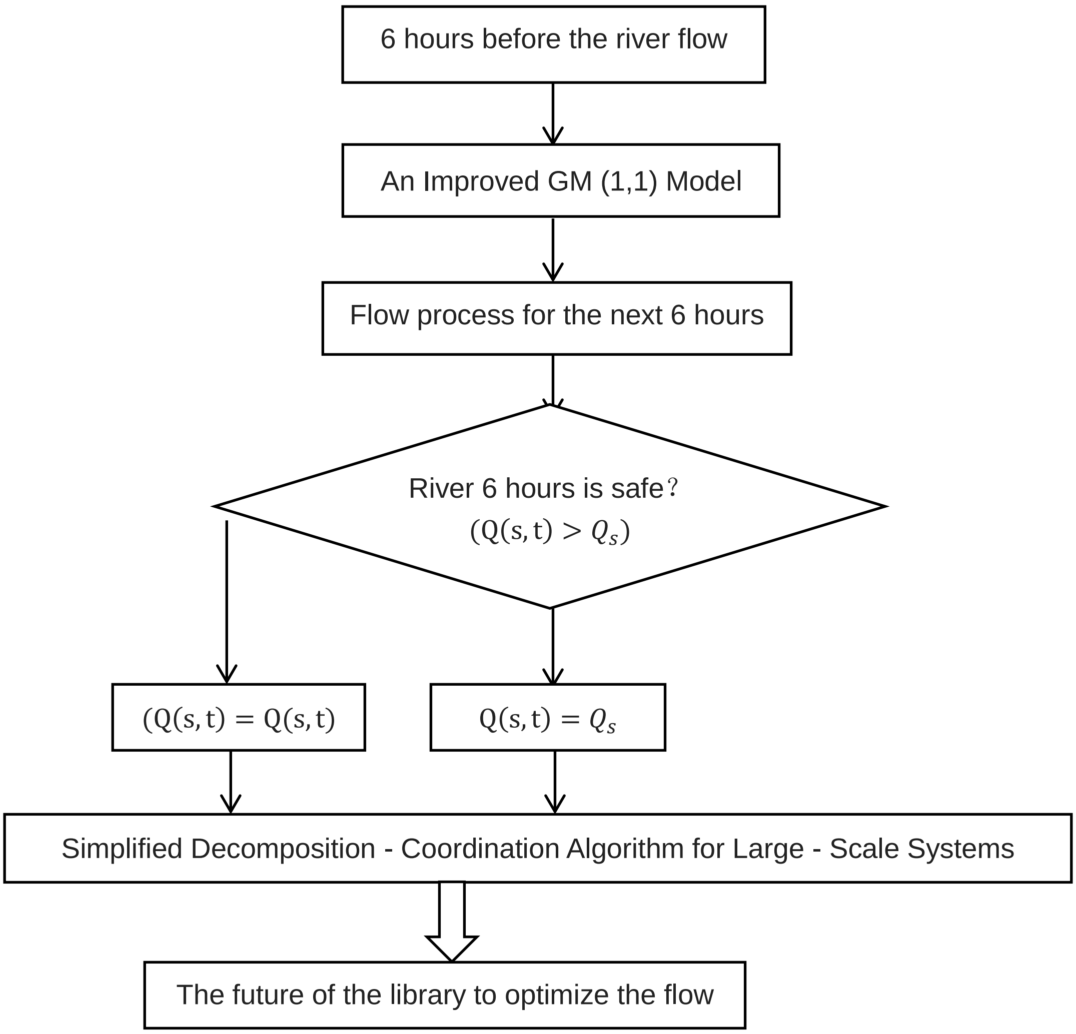


Figure 16、The flow chart of GM (1,1) dam joint transfer

Set n a series the reservoir, let i is the number of each reservoir, using the maximum value of their real-time flood storage capacity as the objective function.

$$\mathrm{Max(F)} = \sum_{i=1}^{n} V_{Fi(t)} \tag{28}$$

Among them $V_{Fi(t)}$ at the time t the i-th reservoir the flood storage capacity. In the same basin, the effectiveness of each reservoir in the reservoir flood control system is not necessarily the same, its efficiency depends on the reservoir of the local river basin and the size of rainfall, the basin area, geomorphic channel characteristics and run off. This paper uses the following formula to calculate its value:

$$\alpha_i = \frac{F_i P_i / S_i}{\sum_{i=1}^{n} F_i P_i / S_i} \tag{29}$$

F_i——the control area of the i-th reservoir.
P_i——the rainfall in the i-th reservoir area.
S_i——flood control capacity in the i-th reservoir.
Obviously, eachα_i should satisfy:

$$\sum_{i=1}^{n} \alpha_i = 1 \tag{30}$$

And then the following formula to do its normalized treatment:

$$\alpha_i = \frac{\alpha_i}{\sum_{i=1}^{n} \alpha_i} \tag{31}$$

Equation (4-4) is rewritten as follows：

$$\mathrm{Min(F)} = \sum_{i=1}^{n} \alpha_i \frac{V_{i(t)}}{V_{i(\text{flood season})}} \tag{32}$$

In the above formula：
$V_{i(t)}$ -- The capacity of reservoir i at time t.

$V_{i(\text{flood season})}$-- The first reservoir flood limit storage capacity.

The water balance equation 1 bring into 2, further derivation

$$\mathrm{Min(F)} = \sum_{i=1}^{n} \alpha_i \frac{V_{i(t)}}{V_{i(\text{flood season})}} = \sum_{i=1}^{n} \alpha_i \frac{V(i,t-1) + (I(i,t) - Q(i,t)) \times \Delta t}{V_{i(\text{flood season})}} \tag{33}$$

$V(i,t-1)$ -- The reservoir capacity at $t-1$ time of the i-th reservoir.

$I(i,t)$ -- The i-th reservoir storage time t flow.
$Q(i,t)$-- The outflow of the reservoir at time t.
Δt—Calculate the time period.

Formula() $V(i,t-1)$, $I(i,t)$, Δt, $V_{i(\text{flood season})}$ are known values, only $Q(i,t)$ is unknown, Substituting them into (4-5), further derivation, and omitting the constant term in the formula, can be simplified as follows:

$$\mathrm{Min(F)} = \sum_{i=1}^{n} \alpha_i Q(i,t) \tag{34}$$

Equation (4-8) is the objective function used in this paper.[6]

6.3 Joint Scheduling Model & Gray Prediction Model Analysis

Use the Gray Forecast Model to forecast the water flow and the precipitation in the future, the actual channel flow $Q(s,t)$ should be less than the safe flow $Q(s)$. The model is based on the principle that the total remaining flood control capacity of all reservoirs in the upper reaches should be the largest. Then use the predicted flow process $Q(s,t)$ to arrange the upstream reservoir group of the library flow distribution; when the range of large water and upstream gate outflow is large, that is, when the future flood control situation is in crisis, $Q(s)$ is the main principle. In the case that the upstream flood control capacity of the upstream reservoir group is the largest, the outflow of all the reservoirs is coordinated and distributed, so that the outflow of the reservoirs and the outflow of the upstream gates. The total flow is equal to $Q(s)$, so that both sides of the river dike could flood safely, to avoid the use of flood detention area to bring economic losses. In the coordination process, first use the model to determine the future flood control situation of the reservoir, to ensure the safety of the reservoir under the premise of trying to flood, and reduce the burden of downstream flood. If the river are still dangerous, the system will issue a warning message, showing the appropriate means of treatment.

7. Anticipate Emissions and Extreme Phenomena Analysis

7.1 Analysis of Expected Releases

According to the maximum daily storage capacity of $254m^3$or each dam, the maximum expected daily discharge is $254m^3$, and the minimum expected discharge is the dead water level of the dam system.

7.2 Extreme Situation Analysis

From downstream from Victoria Falls, along the Zambian and Zimbabwe border until the territory of Mozambique's Kaorla Bassa for the middle reaches of the river, about 870km, In turn, flows through the Battle of the Deokpal Canyon valley, the Kariba Gorge and the floodplain of the period, until the Caorabassa Valley. Most of the section of the channel cut through the sandstone layer, part of the section of cut through the basalt layer, water flow according to the width of the river change to change , waterfalls and canyon jet more, fall up to 670 m. On the other hand, from the Victoria Falls to the downstream flood peak is 4 to 5 months, mostly rain showers, rainfall and sometimes up to 150 mm (6 inches) per hour. So the Zambezi River in different parts of exposure to the extreme condition the most harmful position is the Victoria Falls downstream, the length of time for the peak period 4-5 months.

8. Sensitivity Analysis

The sensitivity analysis is the degree of the change of the actual and expected model when the main influencing factors of the model are changed. Factor sensitivity coefficient = evaluation index rate of change / the factor rate of change or the factor sensitivity coefficient. And then according to the sensitivity coefficient of the sort, the largest of which is a strong sensitive factor, which is the most influential factor on the model. The probability of dam accidents is predicted to be 0.0500. If the investment cost increases, the annual probability of accidents will decrease to 0.0001. In addition, at least $ 741,020 in economic loss was incurred for each such incident, and $ 480 million was invested in Cary Bar, with a project life span of about 60 years and a benchmark return of 10%. Annual economic loss reduction of such accidents (0.0500-0.0001) x 741,200 US dollars = 36,900 US dollars, technical and economic identity statistics as follows:

According to NPV formula: financial net present value = $ 129,800

According to IRR formula: Yield = 36.8%

Payback period = 3.2 years

The results show that the cost of investment is the most significant factor in the impact of safety, the sensitivity analysis results are as follows:

Table 5、Cost investment is a sensitivity analysis of safety

Amplitude of change / Uncertainties	-60%	-40%	-20%	Basic plan	20%	40%	60%
Accidental economic reduction	-0.56	3.92	8.41	12.94	17.45	21.96	26.43
cost of investment	18.77	16.87	14.84	12.94	11.02	9.08	7.16
Dam life	2.01	6.33	6.33	12.94	15.39	17.43	19.07

From table (1), it can be seen that if the economic loss of the dam accident is reduced by 58%, the investment cost is enough for safety, according to the data in the table, the sensitivity coefficient of economic loss reduction of dam accident is 0,743, The life sensitivity coefficient of the dam is 1,070, The investment cost sensitivity coefficient is 1.746, so the investment cost has the greatest impact on safety.

9. Strength and Weakness

9.1 Strength

The continuous seed growth model is used for regional partitioning, and we use this model to divide the distribution of dams the Zambezi River. Among them, make rainfall, topography as of the smallest units, rainfall, terrain can represent the Zambezi River along the dam need to consider the factors, more reliable.

The trapezoidal model dam is used to show the topography of the trapezoidal structure. It conforms to the middle and lower reaches of the Zambezi River, and the system connectivity is relatively strong. Therefore, the construction of the trapezoidal dam is better.

The concept of the coupling model of gray prediction and dam coupling is simple and clear, and the calculation steps are few. The key is to predict the flow of the river to be accurate, followed by the ability to study how to coordinate the library outflow, this paper uses the improved GM (1,1) model for forecasting, to achieve better accuracy.

9.2 Weakness

The continuous seed growth model is the most important consideration for rainfall and terrain, but dam construction may also be related to the economic conditions of the area.

Trapezoidal model dams are primarily concerned with topography, and rainfall may ignore other influences

10. Reference

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[9]Population distribution in the Zambezi River Basin.2013.
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11. Appendix

Appendix 1

```
clear
clc
load mid
load midh
plot(mid, midh,'r')
grid on
text(0,887,'Victoria Falls')
text(225,510,'Lake Kariba')
text(423,488,'Kariba Dam')
text(707,336,'Lake Cabora Bassa')
xlabel('distance/km')
ylabel('height/m')
title('Current Height of Middle Zambezi River')
```

Appendix 2

```
clear
clc
load low
load lowh
plot(low, lowh,'r')
grid on
text(0,327,'Cahora Bassa Dam')
text(134,138,'Tete')
xlabel('distance/km')
ylabel('height/m')
title('Current Height of Lower Zambezi River')
```

Appendix 3（Daily water flow）

```
clear
syms a b;
c=[a b]';
A=[1425,1537,1697,1929];
B=cumsum(A);
n=length(A);
for i=1:(n-1)
    C(i)=(B(i)+B(i+1))/2;
end
D=A;D(1)=[];
D=D';
E=[-C;ones(1,n-1)];
c=inv(E*E')*E*D;
c=c'
a=c(1);b=c(2);
F=[];F(1)=A(1);
for i=2:(n+10)
    F(i)=(A(1)-b/a)/exp(a*(i-1))+b/a;
end
G=[];G(1)=A(1);
  for i=2:(n+10)
      G(i)=F(i)-F(i-1);
  end
  t1=2013:2016;
  t2=2013:2026;
  G
  plot(t1,A,'o',t2,G)
```

Appendix 4（Electricity needs）

```
clear
syms a b;
```

```
c=[a b]';
A=[9694,10177,11029,12359];
B=cumsum(A);
n=length(A);
for i=1:(n-1)
        C(i)=(B(i)+B(i+1))/2;
end
D=A;D(1)=[];
D=D';
E=[-C;ones(1,n-1)];
c=inv(E*E')*E*D;
c=c'
a=c(1);b=c(2);
F=[];F(1)=A(1);
for i=2:(n+10)
        F(i)=(A(1)-b/a)/exp(a*(i-1))+b/a;
end
G=[];G(1)=A(1);
  for i=2:(n+10)
          G(i)=F(i)-F(i-1);
  end
  t1=2013:2016;
  t2=2013:2026;
  G
  plot(t1,A,'o',t2,G)
```