

Laplace-transzformáció

Függvények és Laplace-transzformáltjaik

1. (a) $e^{2t} \circ \bullet \frac{1}{s-2}$

(b) $e^{3t+1} \circ \bullet \frac{e}{s-3}$

(c) $e^{-5t} \circ \bullet \frac{1}{s+5}$

(d) $\sin 2t \circ \bullet \frac{2}{s^2+4}$

(e) $\cos 3t \circ \bullet \frac{s}{s^2+9}$

(f) $\operatorname{sh} 3t \circ \bullet \frac{3}{s^2-9}$

(g) $\operatorname{ch} 6t \circ \bullet \frac{s}{s^2-36}$

(h) $t^2 \circ \bullet \frac{2}{s^3}$

(i) $5t^4 \circ \bullet \frac{120}{s^5}$

(j) $-8t^6 \circ \bullet -8 \frac{6!}{s^7}$

2. (a) $e^{3t} - 5^{-t} + 2e^t \circ \bullet \frac{1}{s-3} - \frac{1}{s+\ln 5} + \frac{2}{s-1}$

(b) $3 \sin 4t - 2 \cos 2t + \operatorname{sh} t + 8 \operatorname{ch} 4t \circ \bullet 3 \frac{4}{s^2+16} - 2 \frac{s}{s^2+4} + \frac{1}{s^2-1} + 8 \frac{s}{s^2-16}$

(c) $4t^3 - 2t^2 + 7t - 3 \circ \bullet 4 \frac{3!}{s^4} - 2 \frac{2!}{s^3} + 7 \frac{1}{s^2} - 3 \frac{1}{s}$

BKSS 10.1.1.:

(d) $2e^{9t} + 3t^7 - 2 \circ \bullet 2 \frac{1}{s-9} + 3 \frac{7!}{s^8} - \frac{2}{s}$

(e) $9e^{-12t} + 3 \sin 7t - \frac{1}{2}t \circ \bullet \frac{9}{s+12} + 3 \frac{7}{s^2+49} - \frac{1}{2} \frac{1}{s^2}$

(f) $3 - 2 \cos 8t + 3e^{-\frac{t}{2}} \circ \bullet \frac{3}{s} - 2 \frac{s}{s^2+64} + 3 \frac{1}{s+\frac{1}{2}}$

(g) $\frac{1}{3} \operatorname{sh} 6t - 5 \operatorname{ch} 3t \circ \bullet \frac{1}{3} \frac{6}{s^2-36} - 5 \frac{s}{s^2-9}$

(h) $\frac{R}{L} e^{-\omega t} + \frac{1}{C} \sin \omega t \circ \bullet \frac{R}{L} \frac{1}{s+\omega} + \frac{1}{C} \frac{\omega}{s^2+\omega^2}$

(R, L, ω, C pozitív állandók)

(i) $\sin^2 t \circ \bullet \frac{1}{2s} - \frac{s}{2s^2+8}$

(j) $\sin t \cos t \circ \bullet \frac{1}{s^2+4}$

3. (a) $\frac{e^{4t} - 3e^{2t} - 4e^{-t}}{2e^t} \circ \bullet \frac{1}{2} \frac{1}{s-3} - \frac{3}{2} \frac{1}{s-1} - 2 \frac{1}{s+2}$

(b) $\frac{\sin 2t + \sin^3 t}{\sin t} \circ \bullet 2 \frac{s}{s^2+1} + \frac{1}{2} \frac{1}{s} - \frac{1}{2} \frac{s}{s^2+4}$

$$(c) \frac{2t^2 - 7t + 6}{t - 2} \circ \bullet 2 \frac{1}{s^2} - 3 \frac{1}{s}$$

$$4. (a) e^{2t} \sin 3t \circ \bullet \frac{3}{(s - 2)^2 + 9}$$

$$(b) e^{3t} \cos 7t \circ \bullet \frac{s - 3}{(s - 3)^2 + 49}$$

$$(c) e^{3t}(2 \sin t - 3 \cos 4t) \circ \bullet \frac{2}{(s - 3)^2 + 1} - \frac{3(s - 3)}{(s - 3)^2 + 16}$$

$$(d) 3e^{6t}(4t^3 - 3t^2 + 2t - 4) \circ \bullet 3 \left[\frac{24}{(s - 6)^4} - \frac{6}{(s - 6)^3} + \frac{2}{(s - 6)^2} - \frac{4}{s - 6} \right]$$

BKSS 10.1.1.:

$$(e) 2e + e^{-t} \operatorname{ch} 5t \circ \bullet \frac{2e}{s} + \frac{s + 1}{(s + 1)^2 - 25}$$

$$(f) e^{3t} t^4 \circ \bullet \frac{24}{(s - 3)^5}$$

$$(g) e^{3t} \operatorname{sh} \frac{2t}{3} + 2e^{-\frac{t}{3}} \operatorname{ch} 5t \circ \bullet \frac{6}{9(s - 3)^2 - 4} + 2 \frac{s + \frac{1}{3}}{(s + \frac{1}{3})^2 - 25}$$

$$(h) e^{5t} \cos 2t + 4e^{-t} \sin 9t \circ \bullet \frac{s - 5}{(s - 5)^2 + 4} + 4 \frac{9}{(s + 1)^2 + 81}$$

$$(i) 8 \operatorname{sh} t \cos 3t + \sin^2 t + \cos^2 t \circ \bullet 4 \frac{s - 1}{(s - 1)^2 + 9} - 4 \frac{s + 1}{(s + 1)^2 + 9} + \frac{1}{s}$$

$$5. (a) t \sin t \circ \bullet \frac{2s}{s^4 + 2s^2 + 1}$$

$$(b) 3t \cos 2t \circ \bullet \frac{3s^2 - 12}{(s^2 + 4)^2}$$

$$(c) t \operatorname{ch} 3t \circ \bullet \frac{s^2 + 9}{(s^2 - 9)^2}$$

$$(d) t^2 \operatorname{sh} 3t \circ \bullet \frac{18s^2 + 54}{(s^2 - 9)^3}$$

$$(e) t^2(2 \sin 3t - \cos t) \circ \bullet 36 \frac{s^2 - 3}{(s^2 + 9)^3} - 2 \frac{s^3 - 3s}{(s^2 + 1)^3}$$

6. BKSS 10.1.2.:

$$(a) f(t) = \begin{cases} 0 & \text{ha } t < 5 \\ (t - 5)^3 & \text{ha } t \geq 5 \end{cases} \quad f(t) \circ \bullet e^{-5s} \frac{6}{s^4}$$

$$(b) f(t) = \begin{cases} 0 & \text{ha } t < 3 \\ \sin(t - 3) & \text{ha } t \geq 3 \end{cases} \quad f(t) \circ \bullet e^{-3s} \frac{1}{s^2 + 1}$$

$$(c) f(t) = \begin{cases} 0 & \text{ha } t < 2 \\ \cos(3t - 6) & \text{ha } t \geq 2 \end{cases} \quad f(t) \circ \bullet e^{-2s} \frac{s}{s^2 + 9}$$