Differenciálszámítás és alkalmazásai

1. Deriváltfüggvény képzése

$f(x) = 3x^2 + 5x - 7 \qquad f'(x) = 6x + 5$

Röviden: $(3x^2 + 5x - 7)' = 6x + 5$

(a) $1' = 0 \qquad (-3)' = 0 \qquad \left(\frac{1}{2}\right)' = 0 \qquad x' = 1 \qquad (x^2)' = 2x \qquad (x^4)' = 4x^3$

$$(x^{-1})' = -x^{-2} = -\frac{1}{x^2} \qquad (x^{-4})' = -4x^{-5} = -\frac{4}{x^5}$$

$$\left(x^{\frac{1}{2}}\right)' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \qquad \left(x^{\frac{4}{3}}\right)' = \tfrac{4}{3}x^{\frac{1}{3}} = \tfrac{4}{3}\sqrt[3]{x} \qquad \left(x^{-\frac{2}{5}}\right)' = -\frac{2}{5}x^{-\frac{7}{5}} = -\frac{2}{5\sqrt[5]{x^7}}$$

$$\left(x^{-\frac{5}{3}}\right)' = -\frac{5}{3}x^{-\frac{8}{3}} = -\frac{5}{3\sqrt[3]{x^8}} \qquad \left(\sqrt{x}\right)' = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2\sqrt{x}} \qquad \left(\sqrt[3]{x}\right)' = \left(x^{\frac{1}{3}}\right)' = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$$

$$\left(\sqrt[4]{x^3}\right)' = \left(x^{\frac{3}{4}}\right)' = \frac{3}{4}x^{-\frac{1}{4}} = \frac{3}{4\sqrt[4]{x}} \qquad \left(\sqrt[3]{x^7}\right)' = \left(x^{\frac{7}{3}}\right)' = \frac{7}{3}x^{\frac{4}{3}} = \frac{7}{3}\sqrt[3]{x^4}$$

$$\left(\frac{1}{x}\right)' = \left(x^{-1}\right)' = -\frac{1}{x^2} \qquad \left(\frac{1}{x^2}\right)' = \left(x^{-2}\right)' = -\frac{2}{x^3} \qquad \left(\frac{1}{\sqrt{x}}\right)' = \left(x^{-\frac{1}{2}}\right)' = -\frac{1}{2\sqrt{x^3}}$$

$$\left(\frac{1}{\sqrt[3]{x}}\right)' = \left(x^{-\frac{1}{3}}\right)' = -\frac{1}{3\sqrt[3]{x^4}} \qquad \left(\frac{4}{\sqrt[3]{x^5}}\right)' = \left(4x^{-\frac{5}{3}}\right)' = -\frac{20}{3\sqrt[3]{x^8}}$$

$$\left(\frac{-2}{\sqrt[5]{x^4}}\right)' = \left(-2x^{-\frac{4}{5}}\right)' = \frac{8}{5\sqrt[5]{x^9}}$$

$$\left(\frac{1}{\sqrt[3]{x^2}} + \sqrt{x}\right)' = -\frac{2}{3\sqrt[3]{x^5}} + \frac{1}{2\sqrt{x}} \qquad \left(\frac{1}{2}x^{\frac{2}{3}} + x^{-3}\right)' = \frac{1}{3\sqrt[3]{x}} - \frac{3}{x^4}$$

$$\left(\frac{\sqrt{x} - 3\sqrt[3]{x} + 4\sqrt[5]{x^2}}{x}\right)' = \left(x^{-\frac{1}{2}} - 3x^{-\frac{2}{3}} + 4x^{-\frac{3}{5}}\right)' = -\frac{1}{2}x^{-\frac{3}{2}} + 2x^{-\frac{5}{3}} - \frac{12}{5}x^{-\frac{8}{5}}$$

(b) $\left(3x^4 - 5x^3 + x^2 - \pi\right)' = 12x^3 - 15x^2 + 2x$

$$(x + \sin x - e^x + 2\log_2 x)' = 1 + \cos x - e^x + 2\frac{1}{x\ln 2}$$

(c) $\left(x^2 \sin x\right)' = 2x\sin x + x^2\cos x \qquad (3x2^x)' = 3\cdot 2^x + 3x\cdot 2^x \ln 2$

$$\left(\sqrt{x}\,\mathrm{ch}\, x\right)' = \frac{1}{2\sqrt{x}}\,\mathrm{ch}\, x + \sqrt{x}\,\mathrm{sh}\, x \qquad \left(\frac{1}{x}\,\mathrm{tg}\, x\right)' = -\frac{1}{x^2}\,\mathrm{tg}\, x + \frac{1}{x}\frac{1}{\cos^2 x}$$

$$(xe^x\cos x)' = e^x\cos x + xe^x\cos x - xe^x\sin x$$

(d) $\left(\dfrac{2x^2 + 3x}{1 + 2x}\right)' = \dfrac{(4x+3)(1+2x) - (2x^2+3x)2}{(1+2x)^2}$

$$\left(\frac{5 - x - x^2}{1 + \sqrt{x}}\right)' = \frac{(-1-2x)(1+\sqrt{x}) - (5 - x - x^2)\frac{1}{2\sqrt{x}}}{(1+\sqrt{x})^2}$$

$$\left(\frac{\sin x}{3^x}\right)' = \frac{(\cos x)3^x - (\sin x)3^x \ln 3}{3^{2x}}$$

$$\left(\frac{x\ln x}{x+\ln x}\right)' = \frac{(\ln x + 1)(x+\ln x) - (x\ln x)(1+\frac{1}{x})}{(x+\ln x)^2}$$

$$\left(\frac{1}{x}\right)' = \frac{0\cdot x - 1\cdot 1}{x^2} = -\frac{1}{x^2} \qquad \left(\frac{\operatorname{sh} x}{x\operatorname{ch} x}\right)' = \frac{(\operatorname{ch} x)(x\operatorname{ch} x) - (\operatorname{sh} x)(\operatorname{ch} x + x\operatorname{sh} x)}{(x\operatorname{ch} x)^2}$$

(e) $\left(\sin^2 x - \sin x^2\right)' = 2\sin x\cos x - 2x\cos x^2$

$$\left(\frac{\sqrt{2x+1}}{x^2-1}\right)' = \frac{(\frac{2}{2\sqrt{2x+1}})(x^2-1) - 2x\sqrt{2x+1}}{(x^2-1)^2}$$

$$\left(xe^{-x}\cos 2x\right)' = \left(\left(xe^{-x}\right)\cos 2x\right)' = (e^{-x} - xe^{-x})\cos 2x - 2xe^{-x}\sin 2x =$$

$$= e^{-x}\cos 2x - xe^{-x}\cos 2x - 2xe^{-x}\sin 2x$$

$$\left(x^2\operatorname{arctg}\sqrt{x}\right)' = 2x\cdot\operatorname{arctg}\sqrt{x} + x^2\cdot\frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}$$

$$\left(\frac{1}{\cos x}\right)' \overset{\text{I.mo}}{=} \left((\cos x)^{-1}\right)' = -(\cos x)^{-2}(-\sin x) = \frac{\sin x}{\cos^2 x}$$

$$\left(\frac{1}{\cos x}\right)' \overset{\text{II.mo}}{=} \frac{0\cdot\cos x + \sin x}{\cos^2 x} = \frac{\sin x}{\cos^2 x}$$

$$\left(\operatorname{tg}^2\sqrt{x} + \frac{1}{\cos\pi}\right)' = 2\operatorname{tg}\sqrt{x}\cdot\frac{1}{\cos^2\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}$$

$$\left(\operatorname{arctg}\frac{x}{x^2+1}\right)' = \frac{1}{1+\left(\frac{x}{x^2+1}\right)^2}\cdot\frac{x^2+1-x\cdot 2x}{(x^2+1)^2}$$

$$\left(\frac{\sqrt[3]{x^2}}{2^{\cos^2 x}} + \operatorname{tg}\frac{\pi}{4}\right)' = \frac{\frac{2}{3}\cdot\frac{1}{\sqrt[3]{x}}\cdot 2^{\cos^2 x} - \sqrt[3]{x^2}\cdot 2^{\cos^2 x}\cdot(\ln 2)\cdot 2(\cos x)(-\sin x)}{2^{2\cos^2 x}}$$

$$(\ln f(x))' = \frac{1}{f(x)}\cdot f'(x)$$

$$\left(\ln(x^2+x+1)\right)' = \frac{1}{x^2+x+1}\cdot(2x+1) \qquad \left(\log_3\sin x\right)' = \frac{1}{\sin x}\cdot\frac{1}{\ln 3}\cdot\cos x$$

$$\left(\ln\ln^2(x^3)\right)' = \frac{1}{\ln^2(x^3)}\cdot 2\ln(x^3)\cdot\frac{1}{x^3}\cdot 3x^2$$

$$\left(\log_2\left(\frac{\operatorname{ch} 3x}{\sin^2 x}\right) + e^3\right)' = \frac{\sin^2 x}{\operatorname{ch} 3x}\cdot\frac{1}{\ln 2}\cdot\frac{(3\operatorname{sh} 3x)(\sin^2 x) - (\operatorname{ch} 3x)(2\sin x\cos x)}{\sin^4 x}$$

$$\left(\operatorname{ch}\ln\sin\sqrt{e^x}\right)' = \operatorname{sh}\ln\sin\sqrt{e^x}\cdot\frac{1}{\sin\sqrt{e^x}}\cdot\cos\sqrt{e^x}\cdot\frac{1}{2\sqrt{e^x}}\cdot e^x$$

$$\left(e^{f(x)}\right)' = e^{f(x)} \cdot f'(x)$$

$$\left(e^{x^2+x+\sqrt{x}}\right)' = e^{x^2+x+\sqrt{x}} \cdot \left(2x+1+\frac{1}{2\sqrt{x}}\right) \qquad \left(e^{3x-\sin x}\right)' = e^{3x-\sin x} \cdot (3-\cos x)$$

$$\left(e^{\sqrt{\sin\ln\operatorname{ch} x}}\right)' = e^{\sqrt{\sin\ln\operatorname{ch} x}} \cdot \frac{1}{2\sqrt{\sin\ln\operatorname{ch} x}} \cdot \cos\ln\operatorname{ch} x \cdot \frac{1}{\operatorname{ch} x} \cdot \operatorname{sh} x$$

$$\left(2^{\operatorname{arctg} x}\right)' = 2^{\operatorname{arctg} x} \cdot (\ln 2) \cdot \frac{1}{1+x^2} \qquad \left(5^{\sqrt{\sin x}}\right)' = 5^{\sqrt{\sin x}} \cdot (\ln 5) \cdot \frac{1}{2\sqrt{\sin x}} \cdot \cos x$$

2. Logaritmikus deriválás

(a) $f(x) = \sqrt[x]{x^2-1}$

$$f(x) = (x^2-1)^{\frac{1}{x}}$$

$$e^{\ln f(x)} = e^{\ln(x^2-1)^{\frac{1}{x}}}$$

$$e^{\ln f(x)} = e^{\frac{1}{x}\cdot\ln(x^2-1)}$$

$$\ln f(x) = \frac{1}{x} \cdot \ln(x^2-1)$$

$$\frac{1}{f(x)} \cdot f'(x) = -\frac{1}{x^2} \cdot \ln(x^2-1) + \frac{1}{x} \cdot \frac{1}{x^2-1} \cdot 2x$$

$$f'(x) = (x^2-1)^{\frac{1}{x}} \cdot \left[-\frac{1}{x^2} \cdot \ln(x^2-1) + \frac{1}{x} \cdot \frac{1}{x^2-1} \cdot 2x\right]$$

(b) $f(x) = (x^2+2x+2)^x$

$$f'(x) = \left(x^2+2x+2\right)^x \cdot \left[\ln(x^2+2x+2) + x \cdot \frac{1}{x^2+2x+2} \cdot (2x+2)\right]$$

(c) $f(x) = (2x)^{\frac{1}{x}}$

$$f'(x) = (2x)^{\frac{1}{x}} \cdot \left[-\frac{1}{x^2} \cdot \ln 2x + \frac{1}{x} \cdot \frac{1}{2x} \cdot 2\right]$$

(d) $f(x) = (\ln x)^{\sqrt{x}}$

$$f'(x) = (\ln x)^{\sqrt{x}} \cdot \left[\frac{1}{2\sqrt{x}} \cdot \ln\ln x + \sqrt{x} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}\right]$$

(e) $f(x) = \left(\frac{1+x}{1-x}\right)^{\frac{1-x}{1+x}}$

$$f'(x) = \left(\frac{1+x}{1-x}\right)^{\frac{1-x}{1+x}} \cdot \left[\frac{-2}{(1+x)^2} \cdot \ln\left(\frac{1+x}{1-x}\right) + \frac{2}{(1+x)^2}\right]$$

3. Érintő egyenes egyenlete

(a) $f(x) = \sqrt{x} \qquad x_0 = 4$

$$e(x) = \frac{1}{4}x + 1$$

(b) $f(x) = \sin x \qquad x_0 = 0$

$$e(x) = x$$

(c) $f(x) = x^2 - 6x + 5 \qquad x_0 = 2$

$$e(x) = -2x + 1$$

(d) $f(x) = \cos 2x \qquad x_0 = \frac{3\pi}{4}$

$$e(x) = 2x - \frac{3}{2}\pi$$

(e) $f(x) = \ln x \qquad x_0 = 2$

$$e(x) = \frac{1}{2}x + \ln 2 - 1 \approx \frac{1}{2}x - 0,3$$

4. L'Hospital-szabály

(a) $\lim\limits_{x\to\infty} \dfrac{100x^2}{x^4 - 100} \overset{\text{L'H}}{=} \lim\limits_{x\to\infty} \dfrac{200x}{4x^3} \overset{\text{L'H}}{=} \lim\limits_{x\to\infty} \dfrac{200}{12x^2} = 0$

(b) $\lim\limits_{x\to\infty} \dfrac{e^x}{\ln x} \overset{\text{L'H}}{=} \lim\limits_{x\to\infty} \dfrac{e^x}{\frac{1}{x}} = \lim\limits_{x\to\infty} xe^x = \infty$

(c) $\lim\limits_{x\to 0} \dfrac{e^x - e^{-x}}{\sin x} \overset{\text{L'H}}{=} \lim\limits_{x\to 0} \dfrac{e^x + e^{-x}}{\cos x} = \dfrac{1+1}{1} = 2$

(d) $\lim\limits_{x\to 0} \dfrac{\sin x}{\arcsin x} \overset{\text{L'H}}{=} \lim\limits_{x\to 0} \dfrac{\cos x}{\frac{1}{\sqrt{1-x^2}}} = 1$

(e) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{\operatorname{tg} 3x} \overset{\text{L'H}}{=} \lim\limits_{x\to 0} \dfrac{2\cos 2x}{\frac{3}{\cos^2 3x}} = \dfrac{2}{3}$

(f) $\lim\limits_{x\to 0^+} 2x \operatorname{ctg} 3x = \lim\limits_{x\to 0^+} \dfrac{2x}{\operatorname{tg} 3x} \overset{\text{L'H}}{=} \lim\limits_{x\to 0^+} \dfrac{2}{\frac{3}{\cos^2 3x}} = \dfrac{2}{3}$

(g) $\lim\limits_{x\to 0^+} x \ln x = \lim\limits_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x}} \overset{\text{L'H}}{=} \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim\limits_{x\to 0^+} (-x) = 0^-$

(h) $\lim\limits_{x\to 0} (1 + 3x^2)^{\frac{1}{x^2}} = \lim\limits_{x\to 0} e^{\ln(1+3x^2)^{\frac{1}{x^2}}} = \lim\limits_{x\to 0} e^{\frac{1}{x^2}\ln(1+3x^2)} = e^3$

msz.: $\quad \lim\limits_{x\to 0} \dfrac{1}{x^2} \ln(1 + 3x^2) = \lim\limits_{x\to 0} \dfrac{\ln(1 + 3x^2)}{x^2} \overset{\text{L'H}}{=} \lim\limits_{x\to 0} \dfrac{\frac{6x}{1+3x^2}}{2x} = \lim\limits_{x\to 0} \dfrac{3}{1 + 3x^2} = 3$

(i) $\lim\limits_{x\to 1^+} (2 - x)^{\frac{1}{x-1}} = \lim\limits_{x\to 1^+} e^{\ln(2-x)^{\frac{1}{x-1}}} = \lim\limits_{x\to 1^+} e^{\frac{1}{x-1}\ln(2-x)} = e^{-1}$

msz.: $\quad \lim\limits_{x\to 1^+} \dfrac{1}{x - 1} \ln(2 - x) = \lim\limits_{x\to 1^+} \dfrac{\ln(2 - x)}{x - 1} \overset{\text{L'H}}{=} \lim\limits_{x\to 1^+} \dfrac{\frac{-1}{2-x}}{1} = -1$

5. Teljes függvényvizsgálat

(a) $f(x) = \dfrac{4-4x}{(x+1)^2}$

(b) $f(x) = \ln\left(\dfrac{1+x}{1-x}\right)$

(c) $f(x) = x \ln \dfrac{1}{x}$

(d) $f(x) = x^2 e^{-x}$

(e) $f(x) = x(x-2)^2$

(f) $f(x) = x e^{\frac{1}{x}}$

(g) $f(x) = \dfrac{x-2}{x^3}$

(h) $f(x) = x \ln^2 x$

Mo.:

$D_f = \mathbb{R}^+$

A függvény egyetlen zérushelye: $x = 1$. A grafikon nem metszi az y-tengelyt.

D_f alapján f nem páros, nem páratlan, nem periodikus.

$\lim\limits_{x\to\infty} x\ln^2 x = \infty$

$$\lim_{x\to 0^+} x\ln^2 x = \lim_{x\to 0^+} \frac{\ln^2 x}{\frac{1}{x}} \overset{\text{L'H}}{=} \lim_{x\to 0^+} \frac{2(\ln x)\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} (-2(\ln x)x) = \lim_{x\to 0^+} \frac{-2\ln x}{\frac{1}{x}} \overset{\text{L'H}}{=}$$

$$\overset{\text{L'H}}{=} \lim_{x\to 0^+} \frac{-\frac{2}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} 2x = 0^+$$

$f'(x) = \ln^2 x + 2x(\ln x)\dfrac{1}{x} = \ln^2 x + 2\ln x = \ln x(\ln x + 2) = 0 \Leftrightarrow$

$\Leftrightarrow x = 1$ vagy $x = e^{-2} \approx 0,135$

	$0 < x < e^{-2}$	e^{-2}	$e^{-2} < x < 1$	1	$1 < x$
f'	$+$	0	$-$	0	$+$
f	$\nearrow$	lok. max.	$\searrow$	lok. min.	$\nearrow$

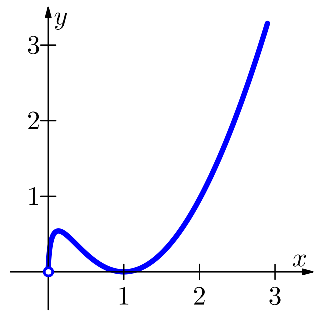
f-nek lokális maximuma van az $x = e^{-2}$ helyen, és ennek értéke: $f(e^{-2}) = \dfrac{4}{e^2} \approx 0,541$

f-nek lokális minimuma van az $x = 1$ helyen, és ennek értéke: $f(1) = 0$

$f''(x) = \dfrac{1}{x}(\ln x + 2) + \dfrac{1}{x}\ln x = \dfrac{2}{x}(\ln x + 1) = 0 \Leftrightarrow x = e^{-1} \approx 0,37$

	$0 < x < e^{-1}$	e^{-1}	$e^{-1} < x$
f''	$-$	0	$+$
f	$\frown$	infl.	$\smile$

A függvénygrafikon:



$R_f = [0, \infty[$

f-nek globális minimuma 0.

(i) $f(x) = \dfrac{3x+6}{(x-2)^2}$

Mo.:

$D_f = \mathbb{R} \setminus \{2\}$

zérushely: $x = -2$ $\qquad$ y-tengelyen a metszet: $f(0) = \dfrac{3}{2}$

f nem páros, nem páratlan, nem periodikus, mert 1db zérushelye van

$$\lim_{x\to\infty} \frac{3x+6}{(x-2)^2} = 0^+ \qquad \lim_{x\to-\infty} \frac{3x+6}{(x-2)^2} = 0^- \qquad \lim_{x\to2} \frac{3x+6}{(x-2)^2} = \infty$$

$$f'(x) = \frac{-3(x+6)}{(x-2)^3} = 0 \Leftrightarrow x = -6$$

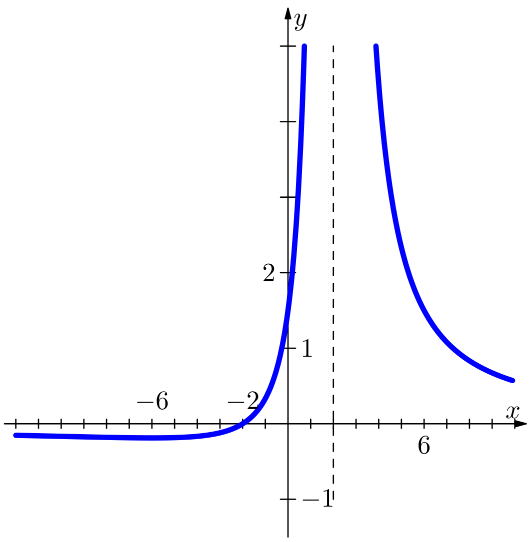
	$x < -6$	-6	$-6 < x < 2$	2	$2 < x$
f'	$-$	0	$+$	$*$	$-$
f	$\searrow$	lok. min.	$\nearrow$	$*$	$\searrow$

f-nek az $x = -6$ helyen lokális minimuma van, aminek értéke: $f(-6) = -\dfrac{3}{16}$

$$f''(x) = \frac{6(x+10)}{(x-2)^4} = 0 \Leftrightarrow x = -10$$

	$x < -10$	-10	$-10 < x < 2$	2	$2 < x$
f''	$-$	0	$+$	$*$	$+$
f	$\frown$	infl.	$\smile$	$*$	$\smile$

A grafikon:



$$R_f = \left[-\frac{3}{16}, \infty \right[$$

f-nek globális minimuma $-\frac{3}{16}$.

(j) $f(x) = xe^{-x}$

Mo.:

$D_f = \mathbb{R}$

$$\left.\begin{array}{l} f(x)=0 \quad \Leftrightarrow \quad x=0 \\ f(0)=0 \end{array}\right\} \text{ f grafikonja átmegy az origón.}$$

$$\left.\begin{array}{l} f(1)=\dfrac{1}{e} \\ \\ f(-1)=-e \end{array}\right\} \begin{array}{l} f(1) \neq f(-1) \quad \Rightarrow \quad f \text{ nem páros;} \\ \\ f(1) \neq -f(-1) \quad \Rightarrow \quad f \text{ nem páratlan.} \end{array}$$

f nem periodikus, pl. mert 1 db zérushelye van.

$$\lim_{x\to\infty} xe^{-x} \overset{\infty\cdot 0}{=} \lim_{x\to\infty} \frac{x}{e^x} \overset{\text{L'H}}{=} \lim_{x\to\infty} \frac{1}{e^x} = \frac{1}{\infty} = 0$$

$$\lim_{x\to-\infty} xe^{-x} = -\infty \cdot \infty = -\infty$$

$$f'(x) = e^{-x} - xe^{-x} = e^{-x}(1-x)$$

$$f'(x) = 0 \quad \Leftrightarrow \quad x = 1$$

	$x<1$	1	$1<x$
f'	$+$	0	$-$
f	$\nearrow$	lok. max.	$\searrow$

$$f(1) = \frac{1}{e} \qquad \text{lokális maximum.}$$

$$f''(x) = -e^{-x} - (1-x)e^{-x} = e^{-x}(x-2)$$

$$f''(x) = 0 \quad \Leftrightarrow \quad x = 2$$

	$x<2$	2	$2<x$
f''	$-$	0	$+$
f	$\frown$	infl.	$\smile$

$x = 2$ helyen inflexió.

$R_f = \left]-\infty\,,\frac{1}{e}\right]$ $\qquad f(1) = \frac{1}{e}$ globális maximum.

A függvénygrafikon ábrázolását az olvasóra bízzuk.

(k) $f(x) = (x-1)\cdot e^x$

Mo.:

$D_f = \mathbb{R}$

$f(x) = 0 \quad \Leftrightarrow \quad x = 1$

$f(0) = -1$

$$\left.\begin{array}{l} f(1) = 0 \\ \\ f(-1) = -\dfrac{2}{e} \end{array}\right\} \begin{array}{l} f(1) \neq f(-1) \quad \Rightarrow \quad f \text{ nem páros;} \\ \\ f(1) \neq -f(-1) \quad \Rightarrow \quad f \text{ nem páratlan.} \end{array}$$

f nem periodikus, pl. mert 1 db zérushelye van.

$$\lim_{x\to-\infty}(x-1)e^x \overset{(-\infty)\cdot 0}{=} \lim_{x\to-\infty}\frac{x-1}{e^{-x}} \overset{\text{L'H}}{=} \lim_{x\to-\infty}\frac{1}{-e^{-x}} = \frac{1}{-\infty} = 0^{(-)}$$

$$\lim_{x\to\infty}(x-1)e^x = \infty\cdot\infty = \infty$$

$$f'(x) = e^x + (x-1)e^x = xe^x$$

$$f'(x) = 0 \quad \Leftrightarrow \quad x = 0$$

	$x<0$	0	$0<x$
f'	$-$	0	$+$
f	$\searrow$	lok. min.	$\nearrow$

$f(0) = -1$ lokális minimum.

$$f''(x) = e^x + xe^x = (1+x)e^x$$

$$f''(x) = 0 \quad \Leftrightarrow \quad x = -1$$

	$x<-1$	-1	$-1<x$
f''	$-$	0	$+$
f	$\frown$	infl.	$\smile$

$x = -1$ helyen inflexió.

$R_f = \left[-1\,,\infty\right[$ $\qquad f(0) = -1$ globális minimum.

A függvénygrafikon ábrázolását az olvasóra bízzuk.

(l) $f(x) = x + \dfrac{1}{x}$

Mo.:

$D_f = \mathbb{R} \setminus \{0\}$

$$\left.\begin{array}{l} \nexists x \ f(x) = 0 \\ 0 \notin D_f \end{array}\right\} \quad \Rightarrow \quad \text{f grafikonja nem metszi sem az } x \text{ sem az } y \text{ tengelyt.}$$

$x = 0$ szakadási hely

$$\left.\begin{array}{l} x \in D_f \ \Rightarrow \ -x \in D_f \\ f(-x) = -x + \frac{1}{-x} = -x - \frac{1}{x} = -\left(x + \frac{1}{x}\right) = -f(x) \end{array}\right\} \quad \Rightarrow \quad f \text{ páratlan függvény.}$$

f nem periodikus, pl. mert 1 db szakadási helye van.

$$\lim_{x\to\infty}\left(x + \frac{1}{x}\right) = \infty + 0 = \infty$$

$$\lim_{x\to-\infty}\left(x + \frac{1}{x}\right) = -\infty + 0 = -\infty$$

$$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$$

$f'(x) = 0 \quad \Leftrightarrow \quad x = \pm 1$

	$x < -1$	-1	$-1 < x < 0$	$*$	$0 < x < 1$	1	$1 < x$
f'	$+$	0	$-$	$*$	$-$	0	$+$
f	$\nearrow$	lok. max.	$\searrow$	$*$	$\searrow$	lok. min.	$\nearrow$

$f(1) = 2$ lokális minimum.

$f(-1) = -2$ lokális maximum.

$$f''(x) = \frac{2}{x^3}$$

$\nexists x \ f''(x) = 0 \quad \Leftrightarrow \quad f$-nek nincs inflexiója

	$x < 0$	0	$0 < x$
f''	$-$	$*$	$+$
f	$\frown$	$*$	$\smile$

$$R_f = \Big]-\infty\,, -2\Big] \cup \Big[2\,, \infty\Big[$$

A függvénygrafikon ábrázolását az olvasóra bízzuk.

6. Nem x-változójú fv-ek deriválása (Nem kell zh-ra!)

(a) $s(t) = v_0 t + \frac{a}{2} t^2 \quad v_0, a \in \mathbb{R}$

$s'(t) = v_0 + at$

(b) $r(\phi) = 10(1 + \cos\phi)$

$r'(\phi) = -10 \sin\phi$

(c) $u(v) = \frac{1+v^2}{1-v^2}$

$$u'(v) = \frac{2v(1-v^2) + (1+v^2)2v}{(1-v^2)^2}$$

(d) $u(v) = \dfrac{1+t^2}{1-v^2} \quad t \in \mathbb{R}$

$$u'(v) = \frac{(1+t^2)2v}{(1-v^2)^2}$$

(e) $u(v) = \dfrac{1+v^2}{1-t^2} \quad t \in \mathbb{R}$

$$u'(v) = \frac{2v}{1-t^2}$$

(f) $P(r) = \frac{U^2 r}{(R+r)^2} \quad U, R \in \mathbb{R}^+$

$$P'(r) = \frac{U^2(R+r)^2 - 2U^2 r(R+r)}{(R+r)^4}$$