



LINEAR CLASSIFIERS IN PYTHON

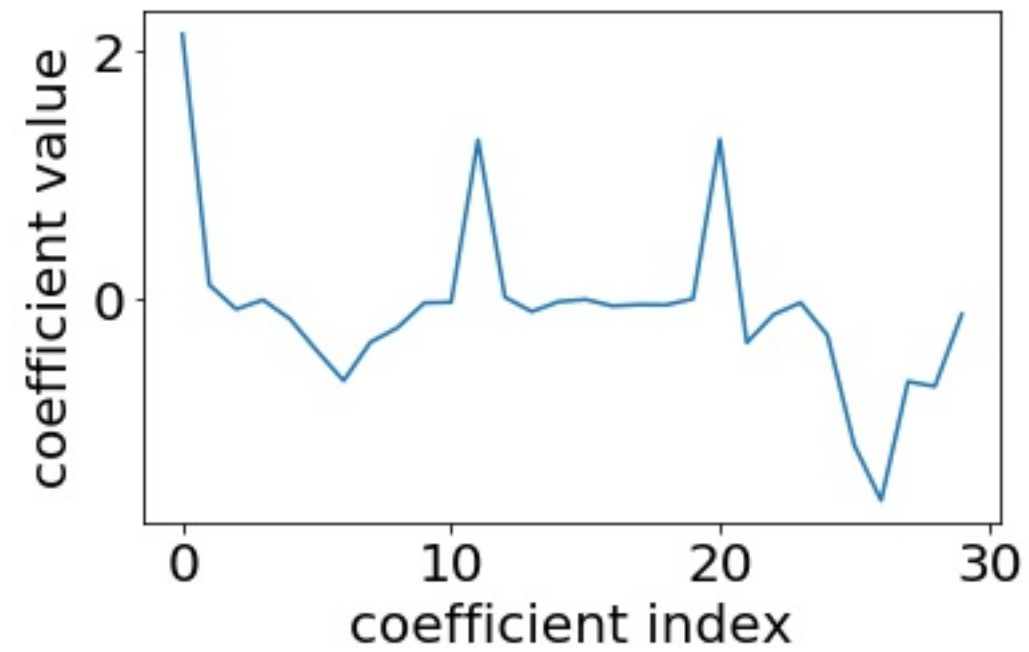
Logistic regression and regularization

Michael (Mike) Gelbart

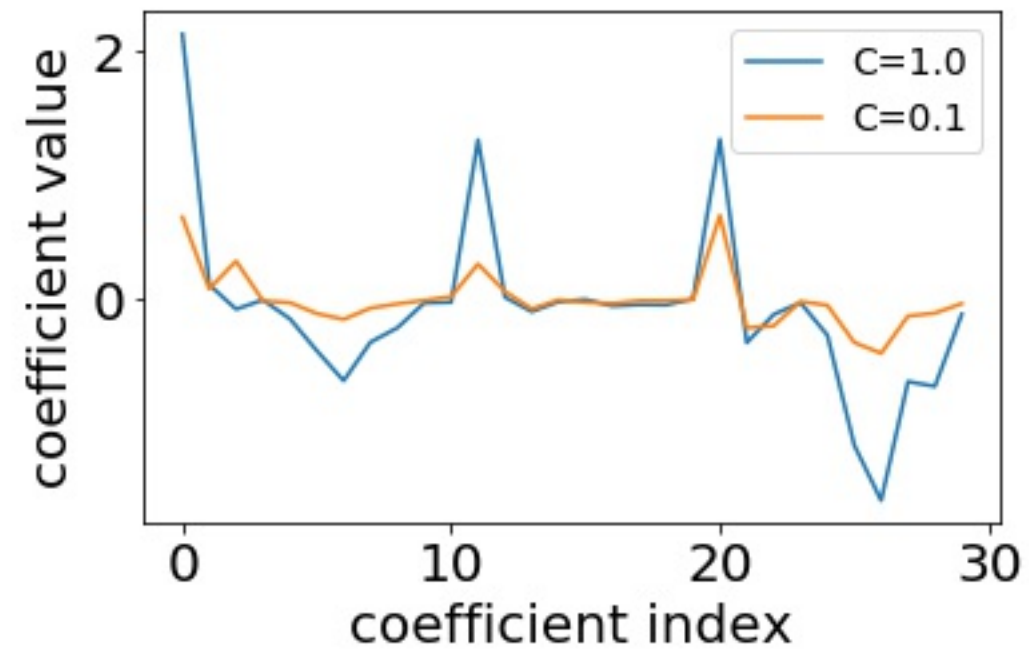
Instructor

The University of British Columbia

Regularized logistic regression



Regularized logistic regression



How does regularization affect training accuracy?

```
In [1]: lr_weak_reg = LogisticRegression(C=100)
```

```
In [2]: lr_strong_reg = LogisticRegression(C=0.01)
```

```
In [3]: lr_weak_reg.fit(X_train, y_train)
```

```
In [4]: lr_strong_reg.fit(X_train, y_train)
```

```
In [3]: lr_weak_reg.score(X_train, y_train)
```

```
Out[3]: 1.0
```

```
In [4]: lr_strong_reg.score(X_train, y_train)
```

```
Out[4]: 0.92
```

- regularized loss = original loss + large coefficient penalty
- more regularization: lower training accuracy

How does regularization affect test accuracy?

```
In [5]: lr_weak_reg.score(X_test, y_test)
Out[5]: 0.86
```

```
In [6]: lr_strong_reg.score(X_test, y_test)
Out[6]: 0.88
```

- regularized loss = original loss + large coefficient penalty
- more regularization: lower training accuracy
- less regularization: (almost always) higher test accuracy

L1 vs. L2 regularization

- Lasso = linear regression with L1 regularization
- Ridge = linear regression with L2 regularization
- For other models like logistic regression we just say L1, L2, etc.

```
In [1]: lr_L1 = LogisticRegression(penalty='l1')
```

```
In [2]: lr_L2 = LogisticRegression() # penalty='l2' by default
```

```
In [3]: lr_L1.fit(X_train, y_train)
```

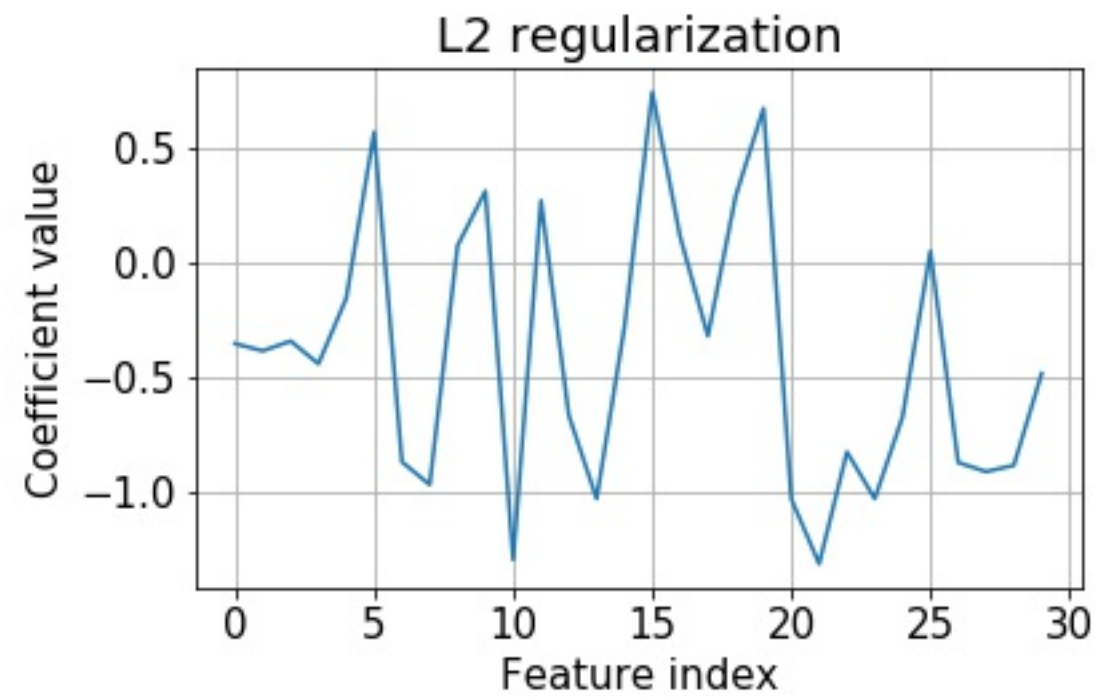
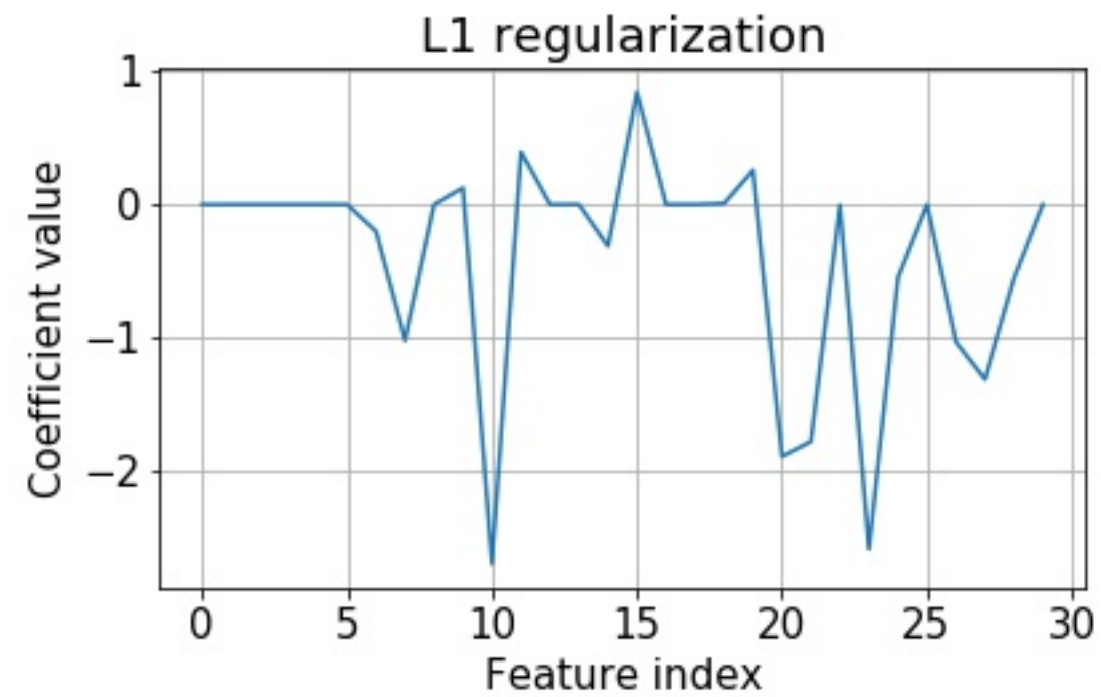
```
In [4]: lr_L2.fit(X_train, y_train)
```

```
In [5]: plt.plot(lr_L1.coef_.flatten())
```

```
In [6]: plt.plot(lr_L2.coef_.flatten())
```



L2 vs. L1 regularization





LINEAR CLASSIFIERS IN PYTHON

Let's practice!



LINEAR CLASSIFIERS IN PYTHON

Logistic regression and probabilities

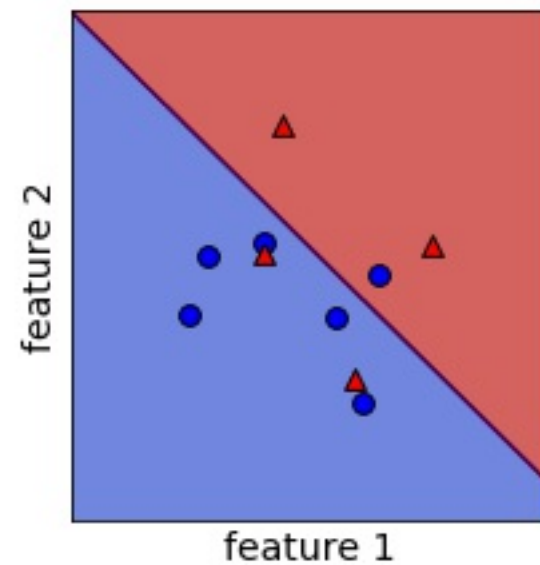
Michael (Mike) Gelbart

Instructor

The University of British Columbia

Logistic regression probabilities

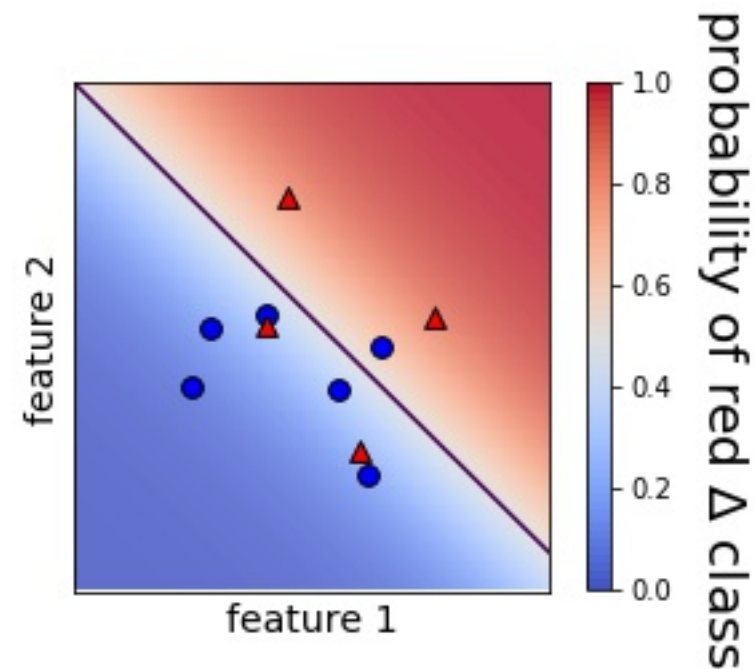
Without regularization ($C = 10^8$):



- model coefficients: $\begin{bmatrix} 1.55 & 1.57 \end{bmatrix}$
- model intercept: $[-0.64]$

Logistic regression probabilities

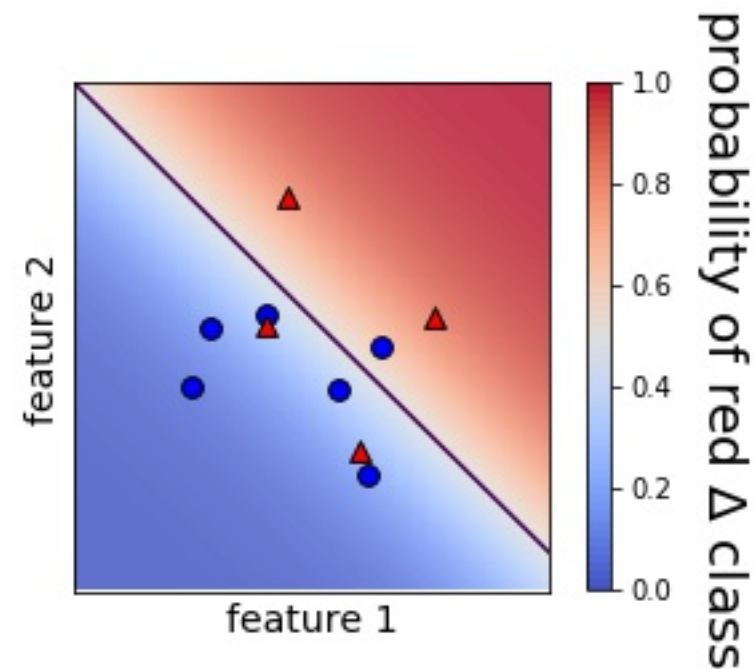
Without regularization ($C = 10^8$):



- model coefficients: `[[1.55 1.57]]`
- model intercept: `[-0.64]`

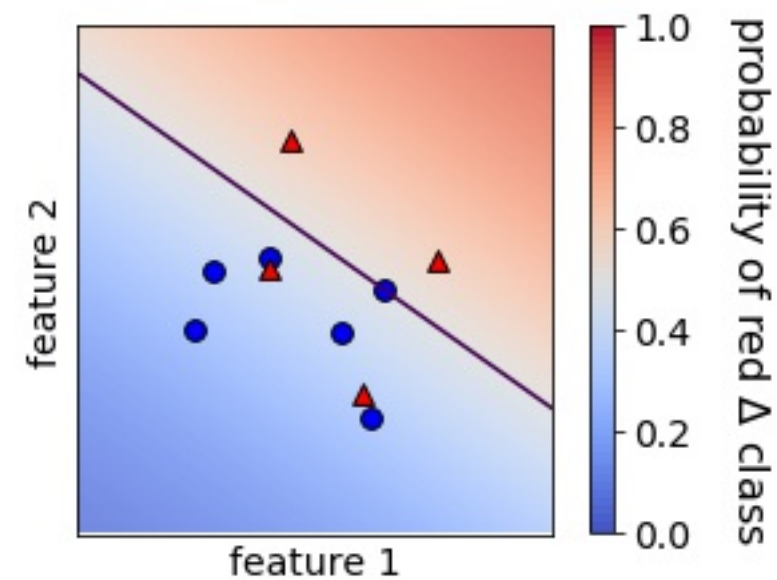
Logistic regression probabilities

Without regularization ($C = 10^8$):



- model coefficients: $\begin{bmatrix} 1.55 & 1.57 \end{bmatrix}$
- model intercept: $[-0.64]$

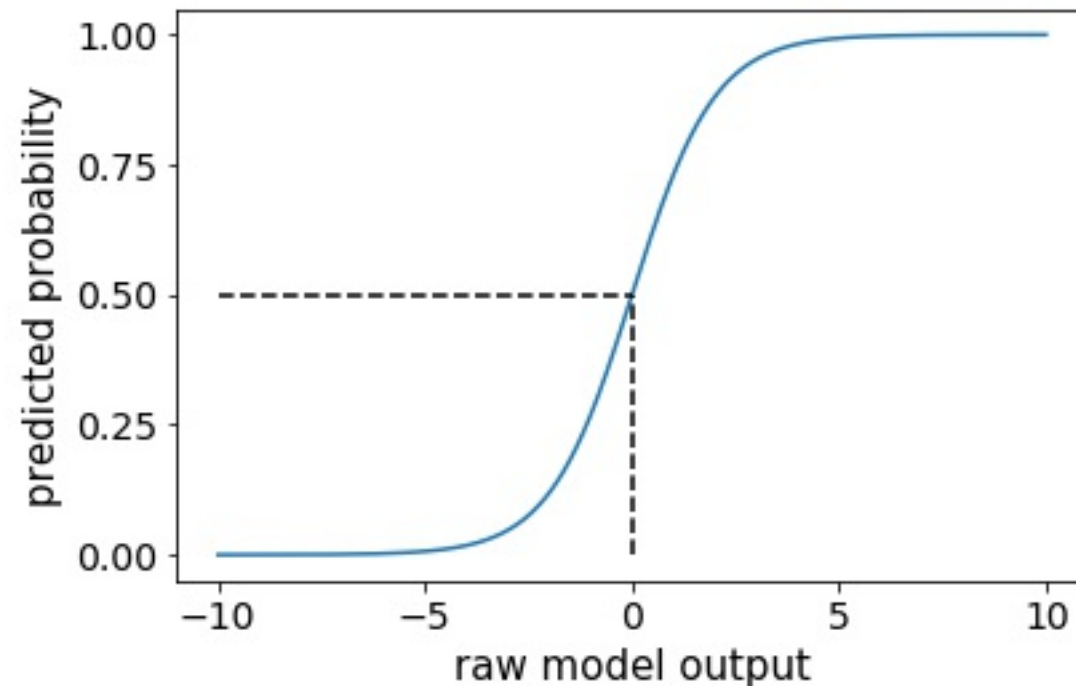
With regularization ($C = 1$):



- model coefficients: $\begin{bmatrix} 0.45 & 0.64 \end{bmatrix}$
- model intercept: $[-0.26]$

How are these probabilities computed?

- logistic regression predictions: sign of raw model output
- logistic regression probabilities: "squashed" raw model output





LINEAR CLASSIFIERS IN PYTHON

Let's practice!



LINEAR CLASSIFIERS IN PYTHON

Multi-class logistic regression

Michael (Mike) Gelbart

Instructor

The University of British Columbia



Combining binary classifiers with one-vs-rest

```
In [1]: lr0.fit(X, y==0)
```

```
In [2]: lr1.fit(X, y==1)
```

```
In [3]: lr2.fit(X, y==2)
```

```
In [4]: lr0.decision_function(X)[0] # get raw model output
```

```
Out[4]: 6.124
```

```
In [5]: lr1.decision_function(X)[0]
```

```
Out[5]: -5.429
```

```
In [6]: lr2.decision_function(X)[0]
```

```
Out[6]: -7.532
```

```
In [7]: lr.fit(X, y)
```

```
In [8]: lr.predict(X)[0]
```

```
Out[8]: 0
```




One-vs-rest vs. multinomial/softmax

One-vs-rest:

- fit a binary classifier for each class
- predict with all, take largest output
- pro: simple, modular
- con: not directly optimizing accuracy
- common for SVMs as well
- can produce probabilities

"Multinomial" or "softmax":

- fit a single classifier for all classes
- prediction directly outputs best class
- con: more complicated, new code
- pro: tackle the problem directly
- possible for SVMs, but less common

Model coefficients for multi-class

```
In [1]: lr_ovr = LogisticRegression() # one-vs-rest by default
```

```
In [2]: lr_ovr.fit(X,y)
```

```
In [3]: lr_ovr.coef_.shape
```

```
Out[3]: (3,13)
```

```
In [4]: lr_ovr.intercept_.shape
```

```
Out[4]: (3,)
```

```
In [5]: lr_mn = LogisticRegression(multi_class="multinomial",solver="lbfgs")
```

```
In [6]: lr_mn.fit(X,y)
```

```
In [7]: lr_mn.coef_.shape
```

```
Out[7]: (3,13)
```

```
In [8]: lr_mn.intercept_.shape
```

```
Out[8]: (3,)
```



LINEAR CLASSIFIERS IN PYTHON

Let's practice!