1. Laplace方法估阶

2. $\int_0^1 x^k \, dx = \frac{1}{k+1}$

转换，用于解题

3. 逐步构造

1. $\lim_{x\to\infty} x^5 \left[e^{x^2}\left(\int_0^x e^{-y^2}\,dy - \frac{\sqrt{\pi}}{2} \right) + \frac{a}{x} + \frac{b}{x^3} \right] = c$, $a, b, c = ?$

$$\int_0^x e^{-y^2}\,dy = \frac{\sqrt{\pi}}{2} - \int_x^{\infty} e^{-y^2}\,dy$$

$$I = e^{x^2}\int_x^{\infty} e^{-y^2}\,dy = \int_x^{+\infty} e^{x^2-y^2}\,dy$$

$$= -x\int_1^{+\infty} e^{-x^2(t^2-1)}\,dt \qquad y = xt$$

$$= -x\int_0^{+\infty} e^{-x^2u}\,\frac{1}{2\sqrt{u+1}}\,du \qquad t = \sqrt{u+1}$$

$$\frac{1}{2\sqrt{u+1}} = \frac{1}{2} - \frac{u}{4} + \frac{3}{16}u^2 + o(u^2)$$

$$I = -\frac{1}{2x} + \frac{1}{4x^3} - \frac{3}{8x^5} + o\left(\frac{1}{x^5}\right)$$

$$a = \frac{1}{2} \qquad b = \frac{1}{4} \qquad c = -\frac{1}{8}$$

2. prove that $\forall\, m, n \in \mathbb{N}_+$

$$\sum_{k=0}^{m} \frac{(-1)^k C_m^k}{1+k+n} = \sum_{k=0}^{n} \frac{(-1)^k C_n^k}{1+k+m}$$

Proof.

$$\sum_{k=0}^{m} (-1)^k C_m^k \frac{1}{1+k+n} = \sum_{k=0}^{m} (-1)^k C_m^k \int_0^1 x^{k+n}\,dx$$

$$= \int_0^1 x^n \left[\sum_{k=0}^{m} (-1)^k \cdot C_m^k\, x^k \right] dx$$

$$= \int_0^1 x^n (1-x)^m \, dx$$
$$= B(n+1, m+1)$$

similarly for RHS.

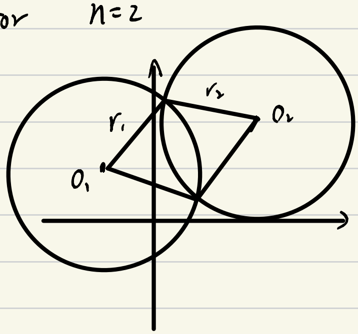
Q.E.D.

3. a drill can drill all the point whose distance to the centre is a irrational number
How many time does the drill need to drill the whole $\mathbb{R}^2$?

Solution:

for $n=1$ obviously can't

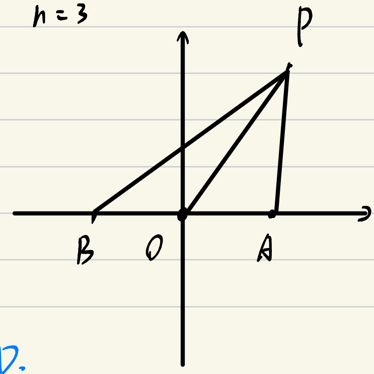
for $n=2$



O_1 O_2 arbitrary drill point
r_1 r_2 rational number

then $n=2$ can't

for $n=3$



A, B symmetric about O

for P an arbitrary point

$PB^2 + PA^2 - 2PO^2 = 2OA^2$ $|OA|$ irrational

then $\forall P$, PB PA PO can't be both rational

$n=3$ works.

Q.E.D.