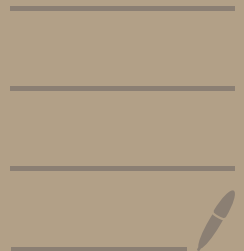


Ring homomorphism.



Def. similarly to group homomorphism
 $f(1) = 1'$

Def. (to def. a similar concept of distinguished subgroup)

$(R, +, \cdot)$ ring, $I \subset R$, I is left ideal of $R \Leftrightarrow$

$$(I, +) \subset (R, +)$$

$$\forall r \in R, \forall a \in I, ra \in I$$

similarly for right ideal of R

$$(I, +) \subset (R, +)$$

$$\forall r \in R, \forall a \in I, ar \in I$$

if I is both r & l ideal, then call I an ideal of R , $I \triangleleft R$

Lemma: $I \triangleleft R$, $I \subset R \Leftrightarrow I = R$

Prop. $(R, +, \cdot)$ is a commutative ring then

I is a right ideal $\Leftrightarrow I$ is a left ideal $\Leftrightarrow I$ is an ideal

Prop. $n \in \mathbb{N}$, $n\mathbb{Z} \triangleleft \mathbb{Z}$

Prop. $f: (R, +, \cdot) \rightarrow (R', +, \cdot)$ a ring homomorphism \Rightarrow

$$\ker(f) \triangleleft R$$

$$\text{im}(f) \subset R'$$

Def. $I \triangleleft R$, quotient ring of R by I is defined as $(R/I, +, \cdot)$

$$R/I = \{a + I, a \in R\}$$

addition and multiplication is defined as

$$(a + I) + (b + I) = (a + b) + I$$

$$(a + I)(b + I) = ab + I$$

Compare to group

quotient ring $(R/I, +, \cdot)$
 \uparrow \uparrow
 Ring Ideal

quotient group $(G/N, \cdot)$
 \uparrow \uparrow
 Group normal subgroup

Theorem (first theorem of ring isomorphism)

$$R / \ker(f) \cong \operatorname{im}(f)$$

$$(\tilde{f}(a + \ker(f)) = f(a))$$

Theorem (second theorem of Ring isomorphism)

$$S \leq R, I \triangleleft R \Rightarrow$$

$$S+I \leq R, S \cap I \triangleleft S, I \triangleleft S+I$$

&

$$S / (S \cap I) \cong (S+I) / I$$

Theorem (third theorem of Ring isomorphism)

$$I, J \triangleleft R, I \subset J \Rightarrow$$

$$J/I \triangleleft R/I$$

&

$$(R/I) / (J/I) \cong R/J \quad (f(a+I) = a+J)$$