A **lie group** is a group which is also a manifold that satisfying the following two conditions:

(I) multiplicative mapping $G \times G \to G$ is C^∞

(II) inverse mapping $G \to G$ is C^∞

Simply connected manifold

$\forall\ p, q \in M$ $\exists$ curve $t \in M$ that connects p & q

n connected manifold

$\forall$ n-dimension closing surface can be reduced to a point by continuous deformation.

Dimention of a lie group equals that of corresponding manifold

Lie Algebra as the tangent space of Identity, its dimension equals that of Lie group.

Special group

$$SL(n, R) \equiv \{T \mid T \in GL(n,R),\ |T| = 1\}$$
$$SL(n, C) \equiv \{T \mid T \in GL(n,C),\ |T| = 1\}$$
$$SO(n) \equiv \{Q \mid Q \in O(n),\ |Q| = 1\}$$
$$SU(n) \equiv \{U \mid U \in U(n),\ |U| = 1\}$$

where $O(p+q) \equiv \{\Lambda \mid \Lambda \in GL(p+q, \mathbb{R}) \mid \Lambda \eta \Lambda = \eta\}$

$O(n) \equiv \{\Lambda \mid \Lambda \in GL(n, \mathbb{R}) \mid Q^T Q = I_{n\times n}\}$

(orthogonal group & indefinite orthogonal group)

$U(n) \equiv \{U \mid U \in GL(n, C) \mid U^\dagger U = I_{n\times n}\}$

(unitary group)

From $SU(2)$:

$$SU(2) \equiv \{ U \mid U \in GL(2, \mathbb{C}),\ U^\dagger U = I_{2\times 2},\ |U| = 1 \}$$

$$= \left\{ \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix} \,\middle|\, a, b \in \mathbb{C},\ |a|^2 + |b|^2 = 1 \right\}$$