图像 (Diagram) F, [有向 (Directed) 图]

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "\phi"] \arrow[dr, "h"] & B \arrow[d, "\psi"] \\ C \arrow[r, "g"'] & D \end{tikzcd}$$

where $A, B, C, D \in Ob(C)$
$h, f, g, \phi, \psi \in Mor(C)$

Call F commute (交换) iff $h = \psi \circ f = \phi \circ g$

图像 [范畴 & 函子]

small C

$$I \xrightarrow[F]{} C$$

commute iff $\forall i, j \in I$, $Hom_I(i,j) \mapsto f \in Mor(C)$

态射之间的态射

$$F: \begin{tikzcd} A \arrow[r, "f"] \arrow[d, "\phi"] & B \arrow[d, "\psi"] \\ X \arrow[r, "g"] & Y \end{tikzcd}$$

$f, g \in Mor(C)$
$A, B, X, Y \in Ob(C)$

$\exists\ g \circ \phi = \psi \circ f$ s.t. F commute $\Leftrightarrow$ f, g 同构

态射范畴

$$\mathcal{F}: \begin{tikzcd} A \arrow[r, "f"] \arrow[d, "\alpha"'] & B \arrow[d, "\beta"] \\ X \arrow[r, "g"'] & Y \end{tikzcd}$$

$Mor(C)$ is a Category.

$(\alpha, \beta) \in Mor(Mor(C))$

$\mathcal{F}$ commute.

自然同构 (natural isomorphism)

$$\mathcal{F}: \begin{tikzcd} F(X) \arrow[r, "\alpha_X"] \arrow[d, "F(f)"'] & G(X) \arrow[d, "G(f)"] \\ F(Y) \arrow[r, "\alpha_Y"'] & G(Y) \end{tikzcd}$$

同构 $\alpha_X \in Hom_D(F(A), G(A))$

$\mathcal{F}$ commute.

C, D equivalent $\iff$ $F: C \to D$ 满忠实 & 本质满射

自然变换 (natural transformation)

$$\mathcal{F}: \begin{tikzcd} F(X) \arrow[r, "\alpha_X"] \arrow[d, "F(f)"'] & G(X) \arrow[d, "G(f)"] \\ F(Y) \arrow[r, "\alpha_Y"'] & G(Y) \end{tikzcd}$$

态射 $\alpha_X \in Hom_D(F(A), G(A))$

$\mathcal{F}$ commute.

Category of functors.

$Ob(\mathcal{D}^{\mathcal{C}}) = Ob([\mathcal{C}, \mathcal{D}]) = Func(\mathcal{C}, \mathcal{D})$

$\forall\ F, G \in Func(\mathcal{C}, \mathcal{D})$, $Hom_{\mathcal{D}^{\mathcal{C}}}(F, G) = Nat(F, G)$