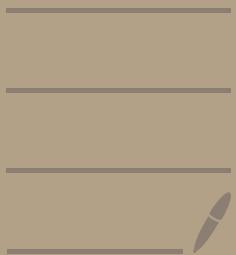


Irreducible polynomial



Prop. (Eisenstein test)

$$(R, +, \cdot) u-f-d \quad F = \text{Frac}(R) \quad f(x) \in R[x] \quad \deg(f) \neq 0$$

$$f(x) = a_n x^n + \dots + a_0$$

if $\exists p$ prime $\in R$ s.t.

$$p \mid a_n \& p \nmid a_{n-1} \dots p \nmid a_0 \& p^2 \nmid a_0$$

$\Rightarrow f(x)$ irreducible on $F[x]$

furthermore, if $\text{cont}(f) = 1$, $f(x)$ irreducible on $R[x]$

Proof. If $f(x)$ irreducible on $F[x]$

$\exists g(x), h(x) \in F[x]$, s.t.

$$g(x) = b_0 + \dots + b_m x^m$$

$$h(x) = c_0 + \dots + c_l x^l$$

$$f(x) = g(x) \cdot h(x)$$

suppose $p \mid b_0 \quad p \nmid c_0 \quad p \nmid b_k \quad \text{for } 0 < k < l \leq n-1$

when $k=l$

$$p \mid a_l = b_0 c_l + \dots + b_l c_0$$

thus $p \mid b_l c_0 \Rightarrow p \mid b_l$

because $p \nmid a_n \quad a_n = b_0 c_n + \dots + b_n c_0$

thus $p \nmid b_n$ thus $m=n$

$$h(x) = c \in F \setminus \{0\}$$

$$f(x) = c g(x)$$

however, c is an unit of F and $F[x]$

thus $f(x)$ irreducible on $F[x]$ contradiction

Example $f(x) = x^3 + 20x^2 - 6x + 2$

$R = \mathbb{Z}, p=2, f(x)$ irreducible on $\mathbb{Q}[x] = \text{Frac}(R)$

$\Rightarrow f(x)$ irreducible on $\mathbb{Z}[x]$