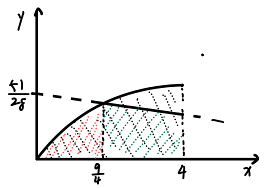
$\mathbb{E}(Y|X) = \int_{\mathbb{R}} f_{Y|X}(y|x) \cdot y \, dy$



$\mathbb{E}(Y) = \int_{\mathbb{R}} f_Y(y) \cdot y \, dy$

$= \int_{\mathbb{R}} \left(\int_{\mathbb{R}} f_{Y|X}(y|x) f_X(x) dx \right) \cdot y \, dy$

$= \int_{\mathbb{R}} \left(\int_{\mathbb{R}} f_{Y|X}(y|x) \cdot y \, dy \right) f_X(x) \, dx$

$= \mathbb{E}[\mathbb{E}(Y|X)]$

$Var(Y) = \int_{\mathbb{R}} f_Y(y) \cdot [y - \mathbb{E}(y)]^2 dy$

$= \int_{\mathbb{R}} \int_{\mathbb{R}} f_{Y|X}(y|x) \, f_X(x) dx \; [y - \mathbb{E}(y)]^2 dy$

$= \iint_{\mathbb{R}^2} f_{Y|X}(y|x) \cdot f_X(x) \cdot [y - \mathbb{E}(y)]^2 ds = \int_{\mathbb{R}} f_X(x) dx \int_{\mathbb{R}} f_{Y|X}(y|x) \left([y - \mathbb{E}(y|x)]^2 + 2[y - \mathbb{E}(y) - \mathbb{E}(y|x)] \cdot [\mathbb{E}(y|x) - \mathbb{E}(x)] \right) dy$

$= \mathbb{E}(Var(Y|X)] + \int_{\mathbb{R}} f_X(x) [\mathbb{E}(y|x) - \mathbb{E}(x)]^2 dx dy$

$\mathbb{E}[Var(Y|X)] = \int_{\mathbb{R}} f_X(x) \cdot \int_{\mathbb{R}} f_{Y|X}(y|x) [y - \mathbb{E}(y|x)]^2 dy \, dx$

$Var[\mathbb{E}(Y|X)] = \int_{\mathbb{R}} f_X(x) \cdot [\mathbb{E}(Y|X) - \mathbb{E}(\mathbb{E}(Y|X))]^2 dx$

$= \int_{\mathbb{R}} f_X(x) [\mathbb{E}(Y|X) - \mathbb{E}(y)]^2 dx$

$y_{n+1}(x) = y_0 + \int_{x_0}^{x} f(t, y_n(t)) \, dt.$

$y_{n+1}(x) - y_n(x) = \int_{x_0}^{x_0} [f(t, y_n(t)) - f(t, y_{n-1}(t))] \, dt$

$\| y_1 - y_0 \| \leq M |x - x_0|$

$|x_n + \cdots x_{n+m}|$

suppose $\mathbb{E}[X_i] = 0$

set $Y_i = X_i \mathbb{1}_{\{|X_i| \leq i\}}$

$\mathbb{E}[X_i] = \int_{-\infty}^{\infty} x \, \mathbb{P}(X = x) \, dx$

$= \int_0^{\infty} x \, \mathbb{P}(|X| = x) \, dx$

$= \int_0^1 x \, \mathbb{P} \, dx + \int_1^2 x \, \mathbb{P} \, dx + \cdots$

$\geq 0 \cdot \int_0^1 \mathbb{P} \, dx + 1 \int_0^1 \mathbb{P}(|x| > 1) \, dx$

$= \sum \mathbb{P}(|X_i| > i)$