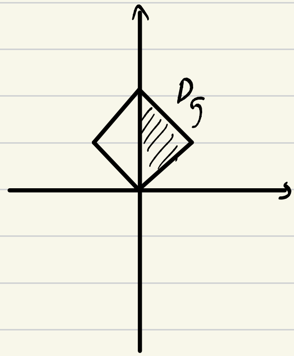
1、积分分区

2、韦达定理

不等式极值

1. Calculate $\iint_{[0,1]^2} \frac{1}{1-xy}\,dx\,dy$



Sol.

$$\begin{cases} u = \dfrac{x+y}{\sqrt{2}} \\ v = \dfrac{x-y}{\sqrt{2}} \end{cases} , \quad \begin{cases} x = \dfrac{u+v}{\sqrt{2}} \\ y = \dfrac{u-v}{\sqrt{2}} \end{cases}$$

$$\mathrm{Ori} = \iint_{[0,1]^2} \frac{1}{1-xy}\,dx\,dy = 4\iint_D \frac{1}{2-u^2+v^2}\,du\,dv$$

$$= 4\Big(\underbrace{\int_0^{\frac{\sqrt{2}}{2}}\int_0^{u} \frac{1}{2-u^2+v^2}\,dv\,du}_{I} + \underbrace{\int_{\frac{\sqrt{2}}{2}}^{\sqrt{2}}\int_0^{\sqrt{2}-u} \frac{1}{2-u^2+v^2}\,du\,dv}_{J}\Big)$$

$$I = \int_0^{\frac{\sqrt{2}}{2}} \frac{1}{\sqrt{2-u^2}} \arctan\frac{u}{\sqrt{2-u^2}}\,du \qquad u = \sqrt{2}\sin\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{\cancel{\sqrt{2}\cos\theta}} \arctan(\tan\theta)\cdot \cancel{\sqrt{2}\cos\theta}\,d\theta$$

$$= \frac{\pi^2}{36}$$

$$J \overset{类似}{=} \frac{\pi^2}{24}$$

$$\mathrm{Ori} = 4\cdot\frac{\pi^2}{24} = \frac{\pi^2}{6}$$ （巴萨尔问题解法之一）

2. z_i $(1\le i\le n)$ 为模长 ≤ 1 的 n 个复数，σ_r 为其 r 次对称多项式

求最大的 $\lambda\in\mathbb{R}$ s.t. $\sum_{i=0}^{n}|\sigma_i| \ge \lambda\sum_{i=1}^{n}|z_i|$ $\sigma_0=1$

Sol.

$\prod_{i=1}^{n}(|z_i|+x) = \sum_{i=0}^{n}|z_i|x^{n-i}$

$x^n+|\sigma_1|x^{n-1}+\cdots+|\sigma_n|=0$ 对 $x\in\{-|z_i|\}_{i\in I}$

$|x|^n = \sum_{i=1}^{n}|\sigma_i|x^{n-i} \le \sum_{i=1}^{n}|\sigma_i|$

$|z|^n+1 \le \sum_{i=0}^{n}|\sigma_i|$ (1)

同时，有 $\sum_{i=1}^{n}|z_i| \le n|z_{max}|$

Or $|z|^n+1 \ge n|z|\lambda$

$\lambda\le(n-1)^{-\frac{n-1}{n}}$

□.