P_1 $f(7x+1) = 49 f(x)$

thus

$\cancel{49} f''(7x+1) = \cancel{49} f''(x)$

suppose $x = t - \frac{1}{6}$

$f''(7t - \frac{1}{6}) = f''(t - \frac{1}{6})$

therefore for any $x \in R$

$f''(x) = f''(x + \frac{1}{6} - \frac{1}{6}) = f''(\frac{x+\frac{1}{6}}{7^n} - \frac{1}{6})$

when $n \to \infty$

$f''(x) = f''(-\frac{1}{6}) = C_0$

thus

$f'(x) = C_0 x + C_1$

$f(x) = \frac{1}{2} C_0 x^2 + C_1 x + C_2$

where $f'(7x+1) = 7 f'(x)$

$7\cancel{C_0 x} + C_0 \quad + C_1 = 7(\cancel{C_0 x} + C_1)$

$C_0 = 6 C_1$

and $f(7x+1) = 49 f(x)$

$\frac{C_0}{2}(7x+1)^2 + C_1(7x+1) + C_2 = 49(\frac{C_0}{2} x^2 + C_1 x + C_2)$

$C_1 = 12 C_2$

thus

$$f(x) = C(6x+1)^2$$

P2

$$\begin{cases} A^2B = ABC + 2I \\ BC^2 = ABC + 2I \end{cases}$$

$$A(AB - BC) = 2I$$

$$(AB - BC)(-C) = 2I$$

$$A = -C$$

$$B^3 = -ABA + 2I$$

$$B^4 = -(AB)^2 + 2B = -(BA)^2 + 2B$$

$$(AB + BA)(AB - BA) = 0$$

$$AB = BA$$

$$B^3 = I$$

$$A^6 = (B^2)^3 = (B^3)^2 = I$$

P_3. if P_1, P_2 satisfied the condition.

then $P_1(x,y)P_2(x,y)P_1(z,t)P_2(z,t) = P_1(xz-yt, xt+yz)P_2(xz-yt, xt+yz)$

then P_1P_2 satisfied the condition, same as $\frac{P_1}{P_2}$

suppose $v_1 = x+yi$ $\quad v_2 = z+ti$

$v_1 v_2 = xz - yt + (xt+yz)i = v_3$

thus $p(x,y) = x+yi$ is a complex polynomial that fits the needs

so do $p(x,y) = x-yi$

thus $p(x,y) = (x^2+y^2)^n$ $\quad n \in \mathbb{N}$ satisfy the identity

P4. $a_0 = 0$ $a_1 = 2$

$$a_{p-1} = (p-1)^k + p-1 \equiv (-1)^k - 1 \equiv 0 \text{ or } p-2 \pmod p$$

a_{p-1} cannot equal a_0 mod p

so $a_{p-1} \equiv p-2$, k is odd

$$\prod_{i=0}^{p-1} (i^k + i) = \prod_{i=1}^{p-1} i$$

p

$i=0$	$i=1$	$i=2$	$i=\frac{p-1}{2}$	$i=\frac{p+1}{2}$	$i=p-1$
0	1	2^k+2			$(p-1)^k+p-1$
$\downarrow$	$\downarrow$	$\downarrow$			$\downarrow$
0	2	$?$			$p-2$

$p-1$

$$\left(\frac{p-1}{2}\right)^k + \frac{p-1}{2} \equiv$$

Ps barely have time to look at it.