1. 质数倒数和发散

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2. 含参积分

数列递推思路

1. Prove that $\sum \frac{1}{p} = \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \cdots$ diverge

Proof.

for $n \in \mathbb{N}^+$ $p$ prime. $\lfloor \frac{n}{p} \rfloor$ denotes the number of numbers that can divide $p$ in $\{1, 2, \cdots n\}$

suppose $p_i$ means the $i$-th prime.

then $\sum_{i>k} \frac{n}{p_i}$ denotes the upper bound of the number of the numbers that can divide at least one $p_n$

If $\sum \frac{1}{p}$ converge, $\exists k \in \mathbb{N}^+$ $\sum_{i>k} \frac{1}{p_i} < \frac{1}{2}$

then $\sum \frac{n}{p} < \frac{n}{2}$, meening that the proportion of numbers coprime to $\forall p_i, i>k$ in $\{1, \cdots n\}$ is bigger then $\frac{1}{2}$

Such numbers can be expressed as $p_1^{e_1} \cdots p_k^{e_k}$
$p^{e_1} \cdots p^{e_k}$ can be expressed as $A \cdot B^2$, where $A = p_1^{\delta_1} \cdots p_k^{\delta_k}$ $\delta = 0$ or $1$

then $A$ has $2^k$ choice, $B < \sqrt{n}$, so such numbers are less than $2^k\sqrt{n}$
the proportion $\frac{2^k\sqrt{n}}{n}$ can be arbitrary small, contradiction,

Q.E.D.

2 calculate $\int_0^{\pi} \frac{\sin nx}{\sin x} dx$

$$I(n) = \int_0^{\pi} \frac{\sin nx}{\sin x} dx$$

$$I(n+2) - I(n) = \int_0^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x} dx$$

$$= \int_0^{\pi} \frac{2 \sin x \cos (n+1) x}{\sin x} dx = 0$$

[ $\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \sin\beta \cos\alpha$ ]

$I(\text{even}) = I(0) = 0$

$I(\text{odd}) = I(1) = \pi$