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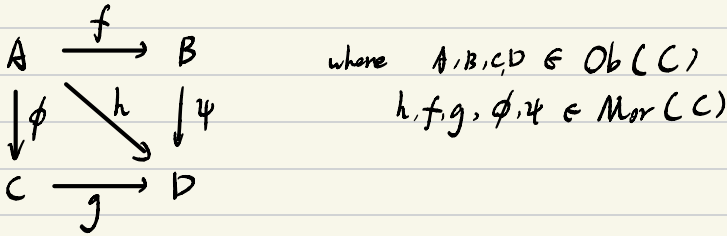
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图像 (Diagram)  $F$ , [有向 (Directed) 图]



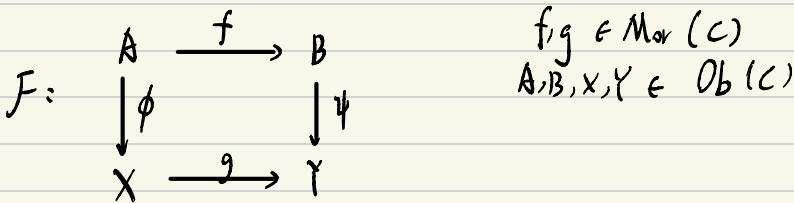
Call  $F$  commute (交换) iff  $h = \psi \circ f = \phi \circ g$

图像 [范畴 & 函子]

small  $C$

$I \xrightarrow{F} C$ 
commute iff  $\forall i, j \in I, \text{Hom}_C(i, j) \mapsto f \in \text{Mor}(C)$

态射  $\perp$  间的态射



$\exists g \circ \phi = \psi \circ f$  st.  $F$  commute  $\Leftrightarrow f, g$  同构

## 态射范畴

$$F: \begin{array}{ccc} A & \xrightarrow{f} & B \\ \alpha \downarrow & & \downarrow \beta \\ X & \xrightarrow{g} & Y \end{array}$$

$\text{Mor}(C)$  is a Category.

$$(\alpha, \beta) \in \text{Mor}(\text{Mor}(C))$$

$F$  commute.

## 自然同构 (natural isomorphism)

$$F: \begin{array}{ccc} F(X) & \xrightarrow{\alpha_X} & G(X) \\ \downarrow F(f) & & \downarrow G(f) \\ F(Y) & \xrightarrow{\alpha_Y} & G(Y) \end{array} \quad \begin{array}{l} \text{同构 } \alpha_X \in \text{Hom}_D(F(A), G(A)) \\ F \text{ commute.} \end{array}$$

$C, D$  equivalent  $\Leftrightarrow F: C \rightarrow D$  满忠实 & 本征满射

## 自然变换 (natural transformation)

$$F: \begin{array}{ccc} F(X) & \xrightarrow{\alpha_X} & G(X) \\ \downarrow F(f) & & \downarrow G(f) \\ F(Y) & \xrightarrow{\alpha_Y} & G(Y) \end{array} \quad \begin{array}{l} \text{态射 } \alpha_X \in \text{Hom}_D(F(A), G(A)) \\ F \text{ commute.} \end{array}$$

Category of functors.

$$\text{Ob}(\mathcal{D}^{\mathcal{C}}) = \text{Ob}([C, D]) = \text{Func}(C, D)$$

$$\forall F, G \in \text{Func}(C, D), \text{Hom}_{\mathcal{D}^{\mathcal{C}}}(F, G) = \text{Nat}(F, G)$$