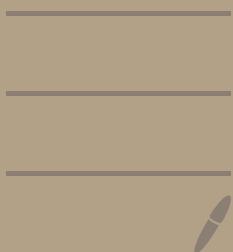


Distinguished subgroup.



Def.  $\forall a \in G, aN = Na \Leftrightarrow N \triangleleft G$ ,  $N$  is a distinguished subgroup

Prop.  $N \triangleleft G$ ,  $(aN) \cdot (bN) = (ab)N$  is well-defined.

Prop.  $N \triangleleft G \Rightarrow (G/N, \cdot)$  is a group  
whose identity is  $N$ , the inverse of  $aN$  is  $a^{-1}N$

Lemma.  $N \triangleleft G$ ,  
 $\forall a \in G, aNa^{-1} \subset N$   
or equivalently  
 $\forall a \in G, \forall n \in N, ana^{-1} \in N$  }  $\Rightarrow N \triangleleft G$

Prop.  $(N_i)_{i \in I}$  is a family of distinguished subgroup

$$\bigcap N_i \triangleleft G$$

Prop.  $\{e\} \triangleleft G$ ,  $G \triangleleft G$

all subgroup of a Abelian group is distinguished

Theorem : First theorem of group isomorphism

$$f: G \rightarrow G' \Rightarrow \ker(f) \triangleleft G, G / \ker(f) \cong \text{im}(f)$$

specially .  $f$  is epimorphism  $\Rightarrow G / \ker(f) \cong G'$   
 $f$  is homomorphism  $\Rightarrow G / \{e\} \cong G \cong \text{im}(f)$   
 $G$  is finite group  $\Rightarrow \frac{|G|}{|\ker(f)|} = |\text{im}(f)|$

Theorem Second theorem of group isomorphism

$$N \triangleleft G, H < G \Rightarrow H \cap N \triangleleft H, N \triangleleft HN, H/(H \cap N) \cong HN/N$$

Theorem Third theorem of group isomorphism.

$$N \triangleleft G, M \triangleleft G, M < N \Rightarrow N/M \triangleleft G/M \\ (G/M)/(N/M) \cong G/N$$