分式泰勒方法

$$P(x) = a_k x^k + a_{k+1} x^{k+1} + \cdots \quad a_k \neq 0$$

$k, p \in N$

$$Q(x) = b_p x^p + b_{p+1} x^{p+1} + \cdots \quad b_p \neq 0$$

$$\frac{P(x)}{Q(x)} = \frac{P(x)}{1} \frac{1}{Q(x)}$$

$$= (a_k x^k + a_{k+1} x^{k+1} + \cdots) \frac{1}{b_p x^p} \frac{1}{1 + \frac{b_{p+1}}{b_p} x + \frac{b_{p+2}}{b_p} x^2 + \cdots}$$

$$= (a_k x^k + a_{k+1} x^{k+1} + \cdots) \frac{1}{b_p x^p} \sum_{n=0}^{\infty} (-1)^n \left(\frac{b_{p+1}}{b_p} x + \frac{b_{p+2}}{b_p} x^2 + \cdots \right)^n$$

帕德近似

$f(x)$ 在 x_0 处的 $[m,n]$ 阶帕德近似 $m, n \in Z$

$$g(x) = \frac{a_0 + \cdots + a_n x^n}{b_0 + \cdots + b_n x^n}$$

且满足
$$f(x_0) = g(x_0)$$
$$f'(x_0) = g'(x_0)$$
$$\vdots$$
$$f^{(m+n)}(x_0) = g^{(m+n)}(x_0)$$

e.g. e^x 在 0 处 $[3,3]$ 阶 $\sim \dfrac{x^2 + 6x + 12}{x^2 - 6x + 12}$

$\ln(x+1)$ 在 0 处 $[2,2]$ 阶 $\sim \dfrac{3x^2 + 6x}{6x^2 + x + 6}$

实洛朗级数近似.

e.g. $$\frac{1}{e^x - x - 1} \sim \frac{1}{\frac{x^2}{2} + \frac{x^3}{6}}$$

$$= \frac{2}{x^2} \cdot \frac{1}{1+\frac{x}{3}}$$

$$\sim \frac{2}{x^2} \left(1 - \frac{x}{3} + \frac{x^2}{9}\right)$$

$$e^x \sim \frac{1}{\frac{2}{x^2}\left(1-\frac{x}{3}+\frac{x^2}{9}\right)} + x + 1 = \frac{2x^3+5x^2+12x+18}{2x^2-6x+18}$$