$A \in M_{n\times n}$ $x \in Vec_n$

Rayleigh quotient $R(A, x) = \dfrac{x^T A x}{x^T x}$

有以下性质：

(I) $R(A, kx) = R(A, x)$

(II) 引 λ Lagrange multiplier function: $L(x, \lambda) = x^T A x - \lambda(x^T x - 1)$

$\nabla L(x, \lambda) = 2Ax - 2\lambda x \overset{equ}{\sim} Ax = \lambda x$

因此 $\lambda_{min} \le R(A, x) \le \lambda_{max}$

广义瑞利商：

$$R(A, B, x) = \frac{x^T A x}{x^T B x}$$

for B positive definite (正定)

$\exists$ Cholesky Decomposition $B = C \cdot C^T$

令 $x = (C^T)^{-1} y$

$$R(A, B, x) = \frac{y^T C^{-1} A (C^T)^{-1} y}{y^T y} \in [\lambda_{min}, \lambda_{max}]$$

利用：求 $(x, y > 0)$，$a = \dfrac{5x + 12\sqrt{xy}}{x+y}$ 取值

Sol.

$$a = R(A, a) = \frac{\sqrt{x}(5\sqrt{x} + t\sqrt{y}) + \sqrt{y}(12 - t)\sqrt{x}}{(\sqrt{x})^2 + (\sqrt{y})^2}$$

$$A = \begin{pmatrix} 5 & t \\ 12 - t & 0 \end{pmatrix} \qquad \lambda_{max} \le 9$$

□.