设 $a_n = e^{i\frac{nk}{m}\cdot 2\pi}$

$$b_{k,m} = \sum_{q=1}^{m} e^{i\frac{qk}{m}\cdot 2\pi}$$

when $\frac{k}{n} \in \mathbb{Z}^+$, $b_{k,m} = m$

otherwise , $b_{k,m} = 0$ （韦达定理）

则 $$\frac{1}{m}\sum_{q=1}^{m}\left(1+e^{i\frac{q}{m}2\pi}\right)^n = \frac{1}{m}\sum_{q=1}^{m}\sum_{k=0}^{n}\binom{n}{k}e^{i\frac{qk}{m}2\pi}$$

$$= \sum_{h=0}^{[\frac{n}{m}]}\binom{n}{hm}$$

where:

$$1+e^{i\frac{q}{m}2\pi} = 1+\cos\frac{q}{m}\cdot 2\pi + i\sin\frac{q}{m}\cdot 2\pi$$

$$= 2\cos\left(\frac{q}{m}\pi\right)\cdot e^{i\frac{q}{m}\pi}$$

则 $$\sum_{h=0}^{[\frac{n}{m}]}\binom{n}{hm} = \frac{2^n}{m}\sum_{q=1}^{m}\left(\cos\frac{q}{m}\pi\right)^n e^{i\frac{nq}{m}\pi}$$

$$\sum_{h=0}^{[\frac{n}{m}]}\binom{n}{hm} = \frac{2^n}{m}\left[1+2\sum_{q=1}^{[\frac{m-1}{2}]}\left(\cos\frac{q}{m}\pi\right)^n\cos\left(\frac{nq}{m}\pi\right)\right]$$