Berge's Lemma (贝尔引理)

M is a Maximam match $\Leftrightarrow \nexists$ augmenting path of M

To find a maximum match

By Recursion.

1. - Input arbitrary match M
2. - if $\exists$ M- augmenting path, set $M^* = M$

$$M^* = M \triangle P := (P-M) \cup (M-P)$$

3. - repeat 2. until $\nexists$ new M^*

$$\beta(G) \geq \gamma(G)$$

[minimal vertex cover $\geq$ maximal matching]

(二分图)

König (考尼格)'s Lemma alleges that if G is bipartite graph

$$\beta(G) = \gamma(G)$$

Hall's Marriage Theorem (霍尔婚配定理)

G a bipartite graph

then there $\exists$ a matching covers $A \iff$

$|N(S)| \geq |S| \quad \forall S \subset A$

where $N(S) = \bigcup_{s \in S} N(s)$ is the set of all neighbors

k-coloring (k 种颜色染色)

k-coloring is a function $K: V(G) \to \{1, \cdots k\}$

s.t. $K(u) \neq K(v)$

the number of k that G is k-colorable is called

$\Downarrow$

Chromatic Number

(图色数) $\chi(G)$

$\chi(G) \leq \Delta(G) + 1$

and Brooks' Theorem states that:

if G is connected & not a odd cycle & not complete.

$\Rightarrow \chi(G) \leq \Delta(G)$