9.10.2

$$\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}k^2 = \frac{1}{2^n}\sum_{k=0}^{n}\binom{n-1}{k-1}kn$$

$$\underline{\binom{n}{k}k = \binom{n-1}{k-1}n}$$

$$= \frac{1}{2^n}n\sum_{k=0}^{n}\binom{n-1}{k-1}(k-1) + \binom{n-1}{k-1}$$

$$= \frac{n(n-1)}{2^n}\sum\binom{n-2}{k-2} + \frac{n}{2^n}\sum\binom{n-1}{k-1}$$

$$= \frac{n(n-1)}{4} + \frac{n}{2}$$

$$= \frac{n(n+1)}{4}$$

$$\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}k^3 = \frac{1}{2^n}\sum_{k=0}^{n}\binom{n-1}{k-1}k^2 n$$

$$= \frac{n}{2^n}\sum_{k=0}^{n}\binom{n-1}{k-1}\left[(k-1)^2+2(k-1)+1\right]$$

$$= \frac{n(n-1)}{2^n}\sum_{k=0}^{n}\binom{n-2}{k-2}(k-1) + \frac{2n(n-1)}{2^n}\sum_{k=0}^{n}\binom{n-2}{k-2}$$

$$+ \frac{n}{2^n}\sum_{k=0}^{n}\binom{n-1}{k-1}$$

$$= \frac{n(n-1)}{2^n}\sum_{k=0}^{n}\binom{n-3}{k-3}(k-2) + \frac{3n(n-1)}{2^n}2^{n-2} + \frac{n}{2}$$

$$= \frac{n(n-1)(n-2)}{2^n}2^{n-3} + \frac{3n(n-1)}{2^n}2^{n-2} + \frac{n}{2}$$

$$= \frac{n(n-1)(n-2)}{6} + \frac{3n(n-1)}{4} + \frac{n}{2}$$

9.10.8

(1) $E[1/x] = \int_{-\infty}^{\infty} \frac{1}{x} f_x(x)\, dx$

$$= \int_0^1 \frac{1}{x} \cdot 1 \, dx$$

$$= \ln 1 - \ln 0 \text{ ? } \quad \times$$

(2) $E[1/x] = \int_0^{+\infty} \frac{1}{x} \cdot e^{-x}\, dx$

$$= -\int_0^{+\infty} \frac{1}{x}\, d e^{-x}$$

$$= -\frac{1}{x} \cdot e^{-x} \Big|_0^{+\infty} + \int_0^{+\infty} e^{-x}\, d\frac{1}{x}$$

$$= -\frac{1}{xe^{x}} \Big|_0^{+\infty} \text{ ?}$$

(3) $E[1/X] = \int_{-\infty}^{\infty} \frac{1}{x} \cdot \frac{e^{-|x|}}{2}\, dx = 0$

$E[X] = \int_{-\infty}^{\infty} x \cdot e^{-|x|}\, dx = 0$?

(4) $E[1/X] = \int_0^{\infty} x^{\frac{v}{2}-2} e^{-\frac{x}{2}} \cdot 2^{-\frac{v}{2}} \int_0^{+\infty} x^{\frac{v}{2}-1} e^{-x}\, dx\, dx$

$$= \iint_0^{\infty} x^{v-3} e^{-\frac{3}{2}x} \cdot 2^{-\frac{v}{2}}\, dx\, dx$$

$=$

9.10.10/11

$$\mu_4 \cdot \sigma^2 - \mu_3^2 - \sigma^6$$

$$= \int_{-\infty}^{\infty} (X-\mu)^4 \cdot f(x)dx \cdot \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$$

$$-\left[\int_{-\infty}^{\infty} (x-\mu)^3 \cdot f(x) dx\right]^2 - \left[\int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx\right]^2 \cdot \sigma^2$$

$$= \iint (x-\mu)^4 (y-\mu)^2 f(x) f(y) dx\,dy - \iint (x-\mu)^3 (y-\mu)^3 f(x) f(y) dx\,dy$$

$$-\sigma^2 \iint (x-\mu)^2 (y-\mu)^2 f(x) f(y) dx\,dy$$

$$= \iint f(x) f(y) (x-\mu)^2 (y-\mu)^2 \left[(x-\mu)^2 - (x-\mu)(y-\mu) - \sigma^2\right] dx\,dy$$

$$= \iint f(x) f(y) (x-\mu)^2 (y-\mu)^2 dx\,dy$$