4、 there are at least $n+1$ set containing 1 element

denote them as $A_{1,1}$ $A_{1,2}$ --- $A_{1,n+1}$

so there must be $\binom{n+1}{n}$ sets containing n elements

5、

$$\sum_{\pi \in S_n} x^{inv(\pi)} = 1\ (1+x) \cdots (1+x+x^2+\cdots x^{n-1})$$

$(1+x+\cdots x^{j-1})$ 表示在 j 前比 j 大的数的数目

$$G_n(x) = \sum_{\pi \in S_n} x^{inv(\pi)}$$

$\varepsilon_1 \cdots\cdots \varepsilon_{n+1}$ is the root of $x^{n+1}=1$ where $\varepsilon_{n+1}=1$

$$G_n(\varepsilon_1) = C_0 + C_1\varepsilon_1 + C_2\varepsilon_1^2 + \cdots + C_{n+1}\varepsilon_1^{n+1} + \cdots$$

$$G_n(\varepsilon_2) = C_0 + C_1\varepsilon_2 + C_2\varepsilon_2^2 + \cdots + C_{n+1}\varepsilon_2^{n+1} + \cdots$$

$\vdots$

$$G_n(\varepsilon_{n+1}) = C_0 + C_1\varepsilon_{n+1} + \cdots + C_{n+1}\varepsilon_{n+1}^{n+1} + \cdots$$

$$\sum_{i=1}^{n+1} G_n(\varepsilon_i) = (n+1)(C_0 + C_{n+1} + \cdots)$$

$$f(n) = \frac{1}{n+1} \sum_{i=1}^{n} G_n(\xi_i)$$

$$f(n) - \frac{n!}{n+1} = \sum_{i=1}^{n} G_n(\xi_i)$$