

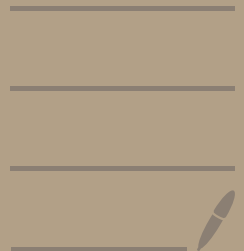
1. 积分求最佳思想

在复数不等式中的运用

+ Poission 公式利用

2. 分子 分母 化简

求和 换序



1. $\alpha_i \in \mathbb{C}$, $i = 1, \dots, m$, prove $\exists J \subset \{1, \dots, m\}$ s.t. $|\sum_{i \in J} \alpha_i| \geq \frac{1}{\pi} \sum_{i=1}^m |\alpha_i|$

Proof.

$$a_k = x_k + iy_k \quad x_k, y_k \in \mathbb{R}, \quad k = 1, 2, \dots, m$$

$$f(a) := \sum_{k=1}^m \max \{ x \cdot x_k + y \cdot y_k, 0 \} \quad a = x + iy, \quad x, y \in \mathbb{R}$$

$$\frac{1}{2\pi} \int_{|a|=1} f(a) dS = \frac{1}{2\pi} \sum_{k=1}^m \int_{|a|=1} \max \{ x \cdot x_k + y \cdot y_k, 0 \} dS$$

$$[\text{Poisson's formula}] = \frac{1}{2\pi} \sum_{k=1}^m \int_{|a|=1} \max \{ y \sqrt{x_k^2 + y_k^2}, 0 \} dS$$

$$= \frac{1}{2\pi} \sum_{k=1}^m \int_{x^2 + y^2 = 1, y \geq 0} \sqrt{x_k^2 + y_k^2} \cdot y dS$$

$$= \frac{1}{\pi} \sum_{k=1}^m \sqrt{x_k^2 + y_k^2}$$

by mean value theorem of integral.

$$\exists |a_0|=1 \text{ s.t. } f(a_0) = \frac{1}{\pi} \sum_{k=1}^m |a_k| \quad a_0 = x_0 + iy_0 \quad x_0, y_0 \in \mathbb{R}$$

Define $J = \{k \mid x_k \cdot x_0 + y_k \cdot y_0 \geq 0, k = 1, 2, \dots, m\}$

$$\text{then } f(a_0) = \sum_{k=1}^m \max \{ x_k \cdot x_0 + y_k \cdot y_0, 0 \}$$

$$= \sum_{k=1}^m (x_k \cdot x_0 + y_k \cdot y_0)$$

$$= \left(\sum_{k \in J} x_k \right) \cdot x_0 + \left(\sum_{k \in J} y_k \right) \cdot y_0 \leq \sqrt{\left(\sum_{k \in J} x_k \right)^2 + \left(\sum_{k \in J} y_k \right)^2} \cdot \sqrt{x_0^2 + y_0^2} \quad (\text{Cauchy})$$

$$= \left| \sum_{k \in J} a_k \right|$$

Q.E.D.

2. calculate
$$\frac{\sqrt{10+\sqrt{1}} + \sqrt{10+\sqrt{2}} + \dots + \sqrt{10+\sqrt{99}}}{\sqrt{10-\sqrt{1}} + \sqrt{10-\sqrt{2}} + \dots + \sqrt{10-\sqrt{99}}}$$

$$\text{Numerator} = \sum_{k=1}^{n^2-1} \sqrt{n+\sqrt{k}} \quad \text{Denominator} = \sum_{k=1}^{n^2-1} \sqrt{n-\sqrt{k}}$$

$$(\sqrt{n+\sqrt{k}} - \sqrt{n-\sqrt{k}})^2 = 2n - 2\sqrt{n^2-k}$$

$$\sum_{k=1}^{n^2-1} \sqrt{n+\sqrt{k}} - \sum_{k=1}^{n^2-1} \sqrt{n-\sqrt{k}} = \sqrt{2} \sum_{k=1}^{n^2-1} \sqrt{n-\sqrt{n^2-k}} = \sqrt{2} \sum_{k=1}^{n^2-1} \sqrt{n-\sqrt{k}}$$

thus $LHS = \sqrt{2} + 1$