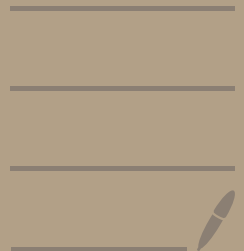


# 斐波那契数列的性质

Millin 级数

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欲吴桂花同载酒，终不似，少年游？

此处定义  $F_0 = 1$ ,  $F_1 = F_2 = 1$

$$F_n = \frac{1}{\sqrt{5}} \left[ \left( \frac{1+\sqrt{5}}{2} \right)^n - \left( \frac{1-\sqrt{5}}{2} \right)^n \right]$$

有以下性质：

$$(1) \lim_{n \rightarrow \infty} \frac{F_n}{F_{n+1}} = \frac{\sqrt{5}-1}{2}$$

$$(2) F_1 + \dots + F_n = F_{n+2} - 1$$

$$(3) F_1 + F_3 + \dots + F_{2n-1} = F_{2n}$$

$$(4) F_2 + F_4 + \dots + F_{2n} = F_{2n+1} - 1$$

$$(5) F_1^2 + F_2^2 + \dots + F_n^2 = F_n \cdot F_{n+1}$$

$$(6) \text{母函数 } f(x) = \sum_{i=0}^{\infty} F_i x^i \\ = 0 + x + x^2 + 2x^3 + 3x^4 + \dots$$

$$f(x) = x f(x) + x^2 f(x)$$

$$f(x) = \frac{1}{1-x-x^2}$$

$$(7) \sum_{i+j=n} \binom{i}{j} = F_{n+1}, \quad \sum_{i \geq 0} \sum_{j \geq 0} \binom{n-i}{j} \binom{n-j}{i} = F_{2n+1}$$

再有再补充罢

Millin 恒数:

$$\sum_{n=0}^{\infty} \frac{1}{F_{2^n}} = \frac{7-\sqrt{5}}{2}$$

Proof.

有以下引理:

Lemma.

$$1. F_{n+1} F_{n-1} - F_n^2 = (-1)^n$$

$$2. F_{m+n} = F_m F_n + F_{m-1} F_{n+1}$$

assert that  $\sum_{n=0}^k 1/F_{2^n} = 3 - \frac{F_{2^{k+1}}}{F_{2^k}}$

when  $k=1$ , 成立

$$\begin{aligned} 3 - \frac{F_{2^k}}{F_{2^k}} + \frac{1}{F_{2^{k+1}}} &= 3 - \frac{\frac{F_{2^k-1} F_{2^{k+1}}}{F_{2^k}} - 1}{F_{2^{k+1}}} \\ &\stackrel{\text{lemma 2.}}{=} 3 - \frac{F_{2^k-1} (F_{2^k-1} + F_{2^k}) - 1}{F_{2^{k+1}}} \\ &\stackrel{\text{lemma 1.}}{=} 3 - \frac{F_{2^k-1}^2 + F_{2^k}^2}{F_{2^{k+1}}} \\ &\stackrel{\text{lemma 2.}}{=} 3 - \frac{F_{2^{k+1}} - 1}{F_{2^{k+1}}} \end{aligned}$$

then by Prop. (1) the equation is proved-  
Q.E.D.