Field

Def. F is a Field $\iff$ $(F, +, \cdot)$ commutative
& every element of F is an unit (not 0)

Def. F, F' fields $f: F \to F'$ field homomorphism $\iff$
$f(1) = 1'$
$\forall x, y \in F,\ F(x+y) = F(x) + F(y)$
$\forall x, y \in F,\ F(xy) = F(x)F(y)$
$\Bigg\}$ f is a ring homomorphism

Lemma. $f: F \to R$ a ring homomorphism F is a field
R is not $\{0\}$ $\Rightarrow$ f injective

Def. $(F, +, \cdot)$ field $E \subset F$ E is subfield of F
if E forms a field under addition and multiplication.

Prop. $(F, +, \cdot)$ field $E \subset F$ $E < F \iff$
$1 \in E$
$\forall a, b \in E,\ a+b,\ a \cdot b \in E$
$\forall b \in E \setminus \{0\},\ \frac{1}{b},\ -b \in E$

Def. $(F, +, \cdot)$ field $A \subset F$ subfield generated by A
denoted as (A)

$$(A) = \bigcap_{A \subset E < F} E$$

Def. $(F, +, \cdot)$ field
if $\exists\ n \in \mathbb{N}_0$ s.t. $n \cdot 1 = 1 + \cdots + 1 = 0$
call n is the characteristics of F, denote as $\operatorname{char}(F)$
if n doesn't exist $\operatorname{char}(F) = 0$