Theorem.

A 可逆 $m \times n$ D $n \times n$ B $m \times n$ C $n \times m$

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A|\,|D - CA^{-1}B|$$

A B 都可逆时

$$|A|\,|D - CA^{-1}B| = |D|\,|A - BD^{-1}C|$$

Lemma I.

$$\lambda^n \cdot |\lambda I_m - AB| = \lambda^m |\lambda I_n - BA|$$

Lemma II.

$$AC = CB \qquad r(C) = r$$

$\Rightarrow AB$ has at least $r$ same eigenvalues.