牛顿多项式

牛顿公式

Define. $$p_k = \sum x_i^k$$
$$\sigma_k = \sum \prod_k x_i$$

对多项式 $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$，$\{x_i\}_{i\in I}$ 为其根

则 $$\sigma_k = (-1)^k \frac{a_{n-k}}{a_n}$$

$$k\sigma_k = \sum_{i=1}^{k} (-1)^{k+1} \sigma_k p_{k-i}$$

$$p_k = (-1)^{k-1} k\sigma_k + \sum_{i=1}^{k-1} (-1)^{k+i+1}$$

$$p_k = \sum_{j=1}^{k} (-1)^{j+1} \sigma_j p_{k-j} \quad (k > n)$$

多项式判别式：

$$D = \prod_{\substack{1\le i,j\le n \\ i\ne j}} (x_i - x_j) = \prod_{1\le i<j\le n} (x_i - x_j)^2$$

则有 Vandermonde 行列式：

$$V = \begin{pmatrix} 1 & \cdots & 1 \\ x_1 & & x_n \\ \vdots & & \vdots \\ x_1^{n-1} & \cdots & x_n^{n-1} \end{pmatrix}$$

$$D = [\det(V)]^2 = \det(V\cdot V^T)$$
$$= \begin{vmatrix} p_0 & \cdots & p_{n-1} \\ \vdots & & \vdots \\ p_{n-1} & \cdots & p_{2n-2} \end{vmatrix}$$