Lemma $\forall$ subset family $\mathcal{C}$ of X
$\exists$ the most coarse topology contain $\mathcal{C}$

Def. subset family $\mathcal{C}$ of X is a topological basis of X
where element of $\mathcal{C}$ is called basis element.

$\Leftrightarrow$

(I) $\forall x \in X \; \exists C \in \mathcal{C}$ s.t. $x \in C$

(II) $\forall C_1, C_2 \in \mathcal{C} \;\; \forall x \in C_1 \cap C_2 \; \exists C_3 \in \mathcal{C}$
s.t. $x \in C_3 \subset C_1 \cap C_2$

Prop. $\mathcal{C}$ is a topo-basis of X
$\Rightarrow (U \in \mathcal{T}_{\mathcal{C}} \Leftrightarrow \forall x \in U, \exists C \in \mathcal{C}$ s.t. $x \in C \subset U$

Prop. $\mathcal{T} = \{ \bigcup_{\alpha \in J} C_\alpha \mid C_\alpha \in \mathcal{C} \}$
$\Rightarrow$ ($\mathcal{T}$ is a topo of $X \Leftrightarrow \mathcal{C}$ is a topo-basis of X)
where $\mathcal{T} = \mathcal{T}_{\mathcal{C}}$

Def. $\mathcal{T}$ is a topo of X topo-basis $\mathcal{C}$ of X is a basis of X
$\Leftrightarrow \mathcal{T}_{\mathcal{C}} = \mathcal{T}$

Lemma. $\mathcal{C} \subset \mathcal{T} \Rightarrow$
($\mathcal{C}$ is a basis of $X \Leftrightarrow \forall U \in \mathcal{T} \; \forall x \in U \; \exists C \in \mathcal{C}$ s.t. $x \in C \subset U$)

Def. $\mathcal{S} = \{ C_\alpha \in \mathcal{P}(X) \mid \alpha \in J \} \;\; \bigcup \mathcal{S} = X$
$\hat{\mathcal{S}} = \{ \bigcap_{i=1}^{k} U_i \mid U_i \in \mathcal{S}, k \geq 1 \}$ is a topo-basis generated by $\mathcal{S}$
where $\mathcal{T}_{\mathcal{S}} = \mathcal{T}_{\hat{\mathcal{S}}}$

Def. $\mathcal{C}_\varepsilon = \{ B(x,r) \mid x \in X, r \in (0,\varepsilon] \}$

$\mathcal{T}_{\mathcal{C}_\varepsilon}$ is the topo induced by metric

Def. $D = \sup \{ d(x,y) \mid x,y \in X \}$

if $D < +\infty$. call d finite metric

D is the diameter of X denoted $\operatorname{diam}(X,d)$

Def. $\bar{d}(x,y) = \min \{ d(x,y), 1 \}$

$\bar{d}$ is called standard bounded metric of d

$\mathcal{T}_{\mathcal{C}_d} = \mathcal{T}_{\mathcal{C}_{\bar{d}}}$

Def. $A \subset X$ $\quad \mathcal{T}_A = \{ U \cap A \mid U \in \mathcal{T} \}$

$\mathcal{T}_A$ is a topo of A called subspace topo of A

Lemma. $\mathcal{C}$ is basis of $\mathcal{T}$

$\mathcal{C}_A = \{ C \cap A \mid C \in \mathcal{C} \}$ $\quad \mathcal{C}_A$ is basis of $\mathcal{T}_A$