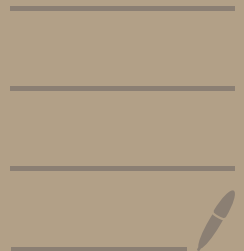


Distinguished subgroup.



Def. $\forall a \in G, aN = Na \Leftrightarrow N \triangleleft G, N$ is a distinguished subgroup

Prop. $N \triangleleft G, (aN) \cdot (bN) = (ab)N$ is well-defined.

Prop. $N \triangleleft G \Rightarrow (G/N, \cdot)$ is a group
whose identity is N , the inverse of aN is $a^{-1}N$

Lemma. $N < G,$
 $\forall a \in G, aNa^{-1} \subset N$
or equivalently
 $\forall a \in G, \forall n \in N, ana^{-1} \in N \} \Rightarrow N \triangleleft G$

Prop. $(N_i)_{i \in I}$ is a family of distinguished subgroup

$$\bigcap N_i \triangleleft G$$

Prop. $\{e\} \triangleleft G, G \triangleleft G$

all subgroup of a Abelian group is distinguished

Theorem : First theorem of group isomorphism

$$f: G \rightarrow G' \Rightarrow \ker(f) \triangleleft G, G/\ker(f) \cong \text{im}(f)$$

specially, f is epimorphism $\Rightarrow G/\ker(f) \cong G'$
 f is homomorphism $\Rightarrow G/\{e\} \cong G \cong \text{im}(f)$
 G is finite group $\Rightarrow \frac{|G|}{|\ker(f)|} = |\text{im}(f)|$

Theorem Second theorem of group isomorphism

$$N \triangleleft G, H < G \Rightarrow H \cap N \triangleleft H, N \triangleleft HN, H/(H \cap N) \cong HN/N$$

Theorem Third theorem of group isomorphism.

$$N \triangleleft G, M \triangleleft G, M < N \Rightarrow N/M \triangleleft G/M \\ (G/M)/(N/M) \cong G/N$$