Free group.

For set $S \neq \{\emptyset\}$, its elements are called letter
connect finite elements and get $a_n \cdots a_1$ is a word
define $(s_n' \cdots s_1')(s_n \cdots s_1) = s_n' \cdots s_1' s_n \cdots s_1$
denote $W(S) = \{1, s_n \cdots s_1 \mid s_i \in S\}$
$F(S) = \{1, s_n \cdots s_1 \mid s_i \in S \cup S^{-1}\}$

If $F(S)$ construt a group under connection of words
the group is called Free group.

define equivalent relation
$w_1 = f_1, f_2, f_3 \cdots f_r = w_2$ $\quad f_i \; f_{i+1}$ adjacent
then $w_1 \backsim w_2$

Defining relation:

any $G \cong F(S)/K$ call S generater sets of G
K defining relation see of G
elements in K are called defining ralation of G

denote $G = \langle s \in S \mid k \in K, k = 1 \rangle$ called expected performance
if K is finite generated call G finite performed

$P < K \triangleleft H$, if K is the smellest normal group contain P
denoted $K = \langle P \rangle$ normal, K is normal subgroup generated by P

K normal generated by $P \iff K = \bigcup_{g_\alpha} [g_\alpha P g_\alpha^{-1}]$

Performance of Group:

$P \subseteq F(S)$, $G \cong F(S)/\langle P \rangle_{normal}$

$G = \langle s \in S \mid p=1, \forall p \in P \rangle$ a performance of G

P is a defining relation set

Example for S_3:

set $S = \{a, b\}$ K normal generated by $\{aa, bb, ababab\}$

$F(S)/K = \langle a, b \mid aa=1 \quad bb=1 \quad ababab=1 \rangle$

thus $ba = abab$

thus $F(S)/K$ contains at most 6 elements:

1 , a, b , aba , $abab$, $ababa$

$S_3 = \langle (1\ 2), (1\ 3) \rangle$

thus $\exists$ epimorphism $\pi: F(S) \to S_3$. $a \mapsto (1\ 2)$ $b \mapsto (1\ 3)$

according to the First isomorphism Theorem:

$S_3 \cong F(S)/\ker(\pi)$

it can be proved that π is a isomorphism.

thus a performance of S_3:

$S_3 = \langle a, b \mid a^2=1, b^2=1, (ab)^3=1 \rangle$

Example of D_n (dihedral group)

$D_n = \langle a, b \mid a^n=1, b^2=1, (ab)^2=1 \rangle$

Example of quaternion group

a set of 8 elements $Q_8 = \{e, -e, i, -i, j, -j, k, -k\}$

where $i^2 = j^2 = k^2 = -e$ $ij = k = -ij$ $jk = i = -kj$ $ki = j = -ik$

$Q_8 = \langle a, b \mid a^4=1, a^2=b^2, ba=a^3b \rangle$

$(\ )\times(\cdot)$ \ $(\cdot)\times(\ )$	e	$-e$	i	$-i$	j	$-j$	k	$-k$
e	e	$-e$	i	$-i$	j	$-j$	k	$-k$
$-e$	$-e$	e	$-i$	i	$-j$	j	$-k$	k
i	i	$-i$	$-e$	e	k	$-k$	$-j$	j
$-i$	$-i$	i	e	$-e$	$-k$	k	j	$-j$
j	j	$-j$	$-k$	k	$-e$	e	i	$-i$
$-j$	$-j$	j	k	$-k$	e	$-e$	$-i$	i
k	k	$-k$	j	$-j$	$-i$	i	$-e$	e
$-k$	$-k$	k	$-j$	j	i	$-i$	e	$-e$