1. A, B has n eignvector that are linearly independent

for any x

$|Ax| > |x|$

$|Bx| > |x|$

So

$|ABx| > |x|$

$|\lambda| > 1$

2. $g(x) = f(x)\cos x$

$g(-\frac{\pi}{2}) = g(0) = g(\frac{\pi}{2}) = 0$

$g'(\xi_1) = g'(\xi_2) = 0 \quad -\frac{\pi}{2} < \xi_1 < 0, \; 0 < \xi_2 < \frac{\pi}{2}$

$h(x) = \frac{g'(x)}{\cos^2 x}$

$h'(x) = \frac{g''(x)\cos^2 x + 2\cos x \sin x\, g'(x)}{\cos^4 x}$

$= \frac{1}{\cos x}\left(f''(x) - f(x)(1 + 2\tan^2 x)\right)$

$\exists\ \xi \in (\xi_1, \xi_2),\ h'(x) = 0$

3、

4.

$$A = \sigma_1$$

$$\sigma_1 = \sum_{i=1}^{n} x_i$$

$$B = \sigma_1^2 - 2\sigma_2$$

$$\sigma_2 = \sum_{\substack{i,j=1 \\ i \neq j}}^{n} x_i x_j$$

$$C = \sigma_1 B - \sigma_2 \cdot A + 3\sigma_3$$

$$\sigma_3 = \sum_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} x_i x_j x_k$$

$$= \sigma_1^3 - 3\sigma_1\sigma_2 + 3\sigma_3$$

$$(n+1)\sigma_1^2(\sigma_1^2 - 2\sigma_2) + (n-2)(\sigma_1^4 - 4\sigma_1^2\sigma_2 + 4\sigma_2^2) - \sigma_1^4 - (2n-2)\sigma_1(\sigma_1^3 - 3\sigma_1\sigma_2 + 3\sigma_3)$$

$$= 4(n-2)\sigma_2^2 - (2n-2)\cdot 3\sigma_3$$

$$= 2\cdot\left(2\cdot(n-2)\sigma_2^2 - 3(n-1)\sigma_3\right)$$

$$= 2n^2(n-1)^2(n-2)\left(\frac{2^2\cdot\sigma_2^2}{n^2(n-1)^2} - \frac{2\cdot 3\ \sigma_3}{n^2(n-1)(n-2)}\right)$$

where $$\left(\frac{\sigma_2}{\binom{n}{2}}\right)^2 > \frac{1}{n}\frac{\sigma_3}{\binom{n}{3}}$$

5.