1、

$$A^{-1}+B^{-1}=(A+B)^{-1}$$

$$(A+B)(A^{-1}+B^{-1})=I \quad \& \quad (A^{-1}+B^{-1})(A+B)=I$$

$$I+AB^{-1}+BA^{-1}+I=I \quad \& \quad I+A^{-1}B+B^{-1}A+I=I$$

$$-(A+B)=BA^{-1}B=AB^{-1}A$$

$$-(A+B)^{-1}=B^{-1}AB^{-1}=A^{-1}BA$$

$$(B^{-1}A)^3=I$$

$$\det A=\det B$$

If complex

$$\det(B^{-1}A)=x \qquad x^3=1 \quad x=1 \text{ or } \tfrac{1}{2}(-1\pm i\sqrt{3})$$

2、 $f(n)+n=2^{i+1}-1 \quad i=\sup\{2^i\le n\}$

for $2^{j-1}-1\le n<2^j-1$

$$\sum_{k=1}^{n}f(k)=\sum_{k=1}^{j-1}2^{k-1}(2^k-1)+(n-2^{j-1}+1)(2^j-1)-\frac{n(n+1)}{2}$$

$$\frac{n^2}{4}-\sum_{k=1}^{n}f(k)=\frac{n^2}{4}-\left((2^j-1)n-\tfrac{1}{3}4^j+2^j+2^j-\tfrac{2}{3}\right)$$

$$=\frac{3}{4}n^2-(2^j-\tfrac{3}{2})n+\tfrac{1}{3}4^j-2^j+\tfrac{2}{3}$$

$$=\frac{3}{4}\left(n^2-\tfrac{4}{3}(2^j-\tfrac{3}{2})n+\tfrac{4}{9}4^j-\tfrac{4}{3}2^j+\tfrac{8}{9}\right)$$

$$= \frac{3}{4}\left(n - \frac{2^{j+1}-2}{3}\right)\left(n - \frac{2^{j+1}-4}{3}\right) \geq 0$$

when $n = \frac{2^{j+1}-2}{3}$ or $n = \frac{2^{j+1}-4}{3}$

3、 $2x^2 - 5x + 2 = 0$

$x_1 = 2 \quad x_2 = \frac{1}{2}$

$$F(n) = 2^n - \left(\tfrac{1}{2}\right)^n$$

$$\sum_{n=0}^{\infty} \frac{1}{F(2^n)} = \sum_{n=0}^{\infty} \frac{1}{2^{2^n} - 2^{-2^n}} = \sum_{n=0}^{\infty} \frac{2^{2^n}}{2^{2^{n+1}} - 1}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2^{2^n}-1} - \frac{1}{2^{2^{n+1}}-1}$$

$$= 1$$

4. $i \tan x = \dfrac{e^{ix} - e^{-ix}}{e^{ix} + e^{ix}}$

$$\arctan x = \frac{1}{2i} \ln \frac{i-x}{i+x}$$

$$\ln \frac{i-16}{i+16} = \ln \left(\frac{i-1}{i+1}\right)^{m_1} \cdot \left(\frac{i-2}{i+2}\right)^{m_2} \cdots\cdots \left(\frac{i-15}{i+15}\right)^{m_{15}}$$

$$\frac{1}{1+16^2}(16+i) = \frac{1}{(1+1^2)^{m_1}} \frac{1}{(1+2^2)^{m_2}} \cdots \frac{1}{(1+15^2)^{m_{15}}} (1+i)^{m_1} (2+i)^{m_2} \cdots\cdots (15+i)^{m_{15}}$$

$$\frac{(1+i)^{m_1} \cdots (15+i)^{m_{15}}}{16+i} = \frac{1+16^2}{(1+1^2)^{m_1} \cdots (1+15^2)^{m_{15}}}$$ is not an integer

不太行

contradiction.

5. $a_i = \overrightarrow{BA_i} \in \mathbb{R}^n$

because B is inside of the convex hull of $A_1 A_2 \cdots A_{n+1}$

$$c_1 a_1 + c_2 a_2 + c_3 a_3 \cdots + c_{n+1} a_{n+1} = 0$$

$$c_1, c_2, \cdots, c_{n+1} > 0$$

$$\alpha = (c_1 a_1 \quad c_2 a_2 \quad \cdots \quad c_{n+1} a_{n+1})$$

$$A = \alpha^T \alpha = \begin{pmatrix} (c_1 a_1)^2 & & c_i a_i c_j a_j \\ & (c_2 a_2)^2 & \\ c_j a_j c_i a_i & & \ddots \\ & & & (c_{n+1} a_{n+1})^2 \end{pmatrix}$$

$$A \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = 0 \cdot \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$