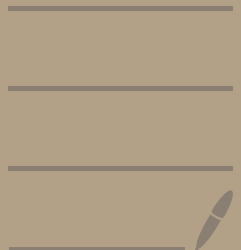


Field



Def. F is a **Field** $\Leftrightarrow (F, +, \cdot)$ commutative
 & every element of F is an unit (not 0)

Def. F, F' fields $f: F \rightarrow F'$ field homomorphism \Leftrightarrow

$$f(1) = 1'$$

$$\forall x, y \in F, F(x+y) = F(x) + F(y),$$

$$\forall x, y \in F, F(xy) = F(x)F(y)$$

} f is a ring homomorphism

Lemma. $f: F \rightarrow K$ a ring homomorphism F is a field
 R is not $\{0\} \Rightarrow f$ injective

Def. $(F, +, \cdot)$ field $E \subset F$ E is subfield of F
 if E forms a field under addition and multiplication.

Prop. $(F, +, \cdot)$ field $E \subset F$ $E < F \Leftrightarrow$
 $1 \in E$

$$\forall a, b \in E, a+b, a \cdot b \in E$$

$$\forall b \in E \setminus \{0\}, \frac{1}{b}, -b \in E$$

Def. $(F, +, \cdot)$ field $A \subset F$ **subfield** generated by A
 denoted as (A)

$$(A) = \bigcap_{A \subseteq E \subset F} E$$

Def. $(F, +, \cdot)$ field

$$\text{if } \exists n \in \mathbb{N}_0 \text{ s.t. } n \cdot 1 = \underbrace{1 + \dots + 1}_n = 0$$

call n is the characteristics of F , denote as $\text{char}(F)$

if n doesn't exist $\text{char}(F) = 0$

