Ring homomorphism.

Def. similarly to group homomorphism
$f(1) = 1'$

Def. (to def. a similar concept of distinguished subgroup)
$(R,+,\cdot)$ ring, $I \subset R$, I is left ideal of $R \Leftrightarrow$
$(I,+) < (R,+)$
$\forall r \in R, \forall a \in I.\ ra \in I$
similarly for right ideal of R
$(I,+) < (R,+)$
$\forall r \in R, \forall a \in I,\ ar \in I$
if I is both r & l ideal, then call I an ideal of R, $I \triangleleft R$

Lemma. $I \triangleleft R$, $I < R \Leftrightarrow I = R$

Prop. $(R,+,\cdot)$ is a commutative Ring then
I is a right ideal $\Leftrightarrow$ I is a left ideal $\Leftrightarrow$ I is a ideal

Prop. $n \in \mathbb{N}_1$, $n\mathbb{Z} \triangleleft \mathbb{Z}$

Prop. $f: (R,+,\cdot) \to (R',+,\cdot)$ a ring homomorphism $\Rightarrow$
$\ker(f) \triangleleft R$
$\mathrm{im}(f) < R'$

Def. $I \triangleleft R$, quotient Ring of R by I is defined as $(R/I,+,\cdot)$
$R/I = \{a+I,\ a \in R\}$
addition and multiplication is defined as
$(a+I)+(b+I) = (a+b)+I$
$(a+I)(b+I) = ab+I$

Compare to group quotient Ring $(R/I,+,\cdot)$ quotient group $(G/N,\cdot)$
Ring Ideal Group normal subgroup

Theorem (first theorem of ring isomorphism)

$R/\ker(f) \cong \operatorname{im}(f)$ $\quad(\bar{f}(a+\ker(f)) = f(a))$

Theorem (second theorem of Ring isomorphism)

$S < R,\ I \triangleleft R \Rightarrow$

$S+I < R,\quad S\cap I \triangleleft S,\quad I \triangleleft S+I$

&

$S/(S\cap I) \cong (S+I)/I$

Theorem (third theorem of Ring isomorphism)

$I, J \triangleleft R,\ I \subset J \Rightarrow$

$J/I \triangleleft R/I$

&

$(R/I)/(J/I) \cong R/J$ $\quad(f(a+I) = a+J)$