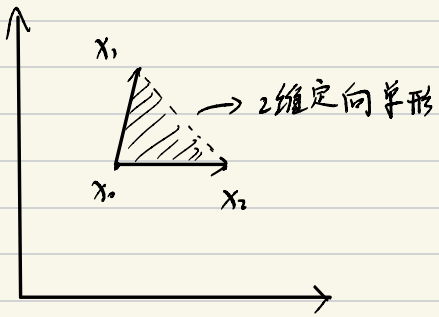
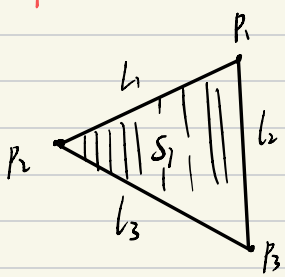
单形.




边缘映射 (Edge mapping)

$$\partial_n [x_0, \cdots, x_n] = \sum_{i=0}^{n} (-1)^i [x_0, \cdots, x_{i-1}, x_{i+1}, \cdots x_n]$$

复形 (complex)



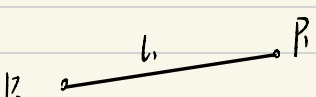

$K = \{P_1, P_2, P_3, l_1, l_2, l_3, S_1\}$

$|K| = P_1 \cup P_2 \cup P_3 \cup l_1 \cup l_2 \cup l_3 \cup S_1$

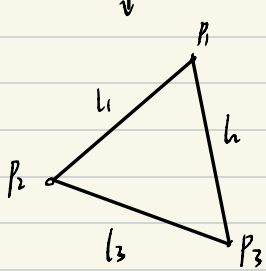
称为 K 的 多面体 (polyhedron)

K 称为 |K| 的 三角剖分 (Triangulation)

子复形 (subcomplex)




l 维骨架 (l-dimension skeleton)



## p 维链群 (p-dimensional chain group)

$$C_p(K, \mathbb{R}) = \left\{ \sum_{\sigma \in K,\ \dim\sigma = p} a_\sigma [\sigma] \,\middle|\, a_\sigma \in \mathbb{R} \right\}$$

$c \in C_p(K)$ 为 链

$$0 \xrightarrow{\partial_{n+1}} C_n(K) \xrightarrow{\partial_n} C_{n-1}(K) \xrightarrow{\partial_{n-1}} \cdots \xrightarrow{\partial_1} C_0(K) \xrightarrow{\partial_0} 0$$

由 $\partial_p \circ \partial_{p+1} = 0 \Rightarrow \operatorname{im}\partial_{p+1} \subset \ker \partial_p \subset C_p(K)$

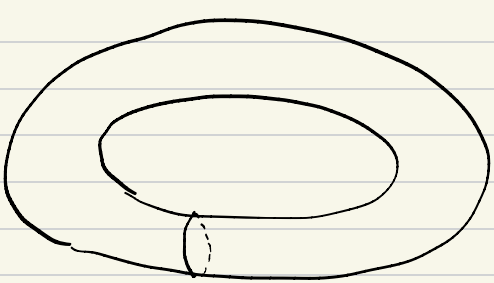
$B_p(K)$ / $Z_p(K) \Rightarrow H_p(K)$

p维 同调 (Homology) 群.

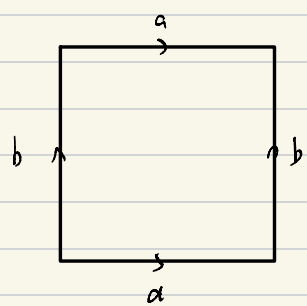
$\dim H_p(K) = b_p(K)$ 为第 p 个 Betti 数.

## 基本多边形

$T^2$



$= S^1 \times S^1$



Klein Bottle