$$df = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\partial f}{\partial X_{ij}} dX_{ij} = tr\left(\frac{\partial f}{\partial X}^{T} dX \right)$$

$$d(AB) = (dX)Y + X\,dY$$

$$d|X| = tr(X^{\#} dX)$$
$$= |X|\, tr(X^{-1} dX) \quad \text{if } X \text{ invertible.}$$

Proof:

$$\det A = \sum_{\sigma} (-1)^{r(\sigma)} a_{\sigma}$$

$$(\det A)' = \sum_{\sigma} (-1)^{r(\sigma)} \sum_{i=1}^{n} a_{\sigma}'$$

$$= \begin{vmatrix} r_1' \\ r_2 \\ \vdots \\ r_n \end{vmatrix} + \cdots + \begin{vmatrix} r_1 \\ r_2 \\ \vdots \\ r_k' \\ \vdots \\ r_n \end{vmatrix} + \cdots + \begin{vmatrix} r_1 \\ \vdots \\ r_n' \end{vmatrix}$$