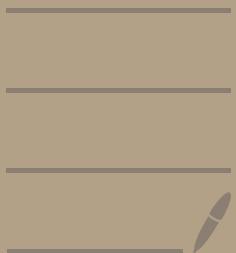


Ring homomorphism.



Def. similarly to group homomorphism
 $f(1) = 1'$

Def. (to def. a similar concept of distinguished subgroup)
 $(R, +, \cdot)$ ring, $I \subset R$, I is left ideal of $R \Leftrightarrow$
 $(I, +) \subset (R, +)$

$$\forall r \in R, \forall a \in I, ra \in I$$

similarly for right ideal of R
 $(I, +) \subset (R, +)$

$$\forall r \in R, \forall a \in I, ar \in I$$

if I is both r & l ideal, then call I an ideal of R , $I \triangleleft R$

Lemma. $I \triangleleft R$, $I \subset R \Leftrightarrow I = R$

Prop. $(R, +, \cdot)$ is a commutative ring then

I is a right ideal $\Leftrightarrow I$ is a left ideal $\Leftrightarrow I$ is a ideal

Prop. $n \in \mathbb{N}$, $n\mathbb{Z} \triangleleft \mathbb{Z}$

Prop. $f: (R, +, \cdot) \rightarrow (R', +, \cdot)$ a ring homomorphism \Rightarrow

$$\ker(f) \triangleleft R$$

$$\text{im}(f) \triangleleft R'$$

Def. $I \triangleleft R$, quotient ring of R by I is defined as $(R/I, +, \cdot)$
 $R/I = \{a+I, a \in R\}$

addition and multiplication is defined as

$$(a+I) + (b+I) = (a+b)+I$$

$$(a+I)(b+I) = ab+I$$

Compare to group quotient ring $(R/I, +, \cdot)$ quotient group $(G/N, \cdot)$

Ring Ideal

Group normal subgroup

Theorem (first theorem of ring isomorphism)
 $R/\ker(f) \cong \text{im}(f)$ ($f(a + \ker(f)) = f(a)$)

Theorem (second theorem of Ring isomorphism)

$$S < R, I < R \Rightarrow$$

$$S+I < R, S \cap I < S, I \cap S+I$$

&

$$S/(S \cap I) \cong (S+I)/I$$

Theorem (third theorem of Ring isomorphism)

$$I, J < R, I \subset J \Rightarrow$$

$$J/I \supset R/I$$

&

$$(R/I)/(J/I) \cong R/J \quad (f(a+I) = a+J)$$