the group of units is called the multiplication group of R denoted $R^{\times}$

R is called a division ring $\Leftrightarrow$ $\forall\, p \neq 0$, p is a unit
Moreover, R is a commutative ring $\Leftrightarrow$ R is a field

Centre $\quad Z(R) = \{x \in R : \forall y \in R,\ xy = yx\}$

centralizer of x

$C_R(x) = \{y \in R : xy = yx\}$

A finite division ring is a field

If F is a field, F^{*} is cyclic

S is called a multiplicative subset
$\Leftrightarrow$ (I) $S \subset R^{*}$ (R commutative)
(II) S is closed under multiplication.

R is called left (resp., right) Noetherian
$\Leftrightarrow$ $\forall$ ascending chain of left (resp., right) ideals terminates in finitely many steps.

Moreover, R is Noetherian $\Leftrightarrow$ it's both right & left Noetherian

Every PID is Noetherian

R is Noetherian $\Leftrightarrow$ every ideal is finitely generated

Hilbert's basis theorem

The polynomial ring over a Noetherian ring is Noetherian.

R is called left (resp. right) Artinian
$\Leftrightarrow$ $\forall$ descending chain of left (r.) ideals terminates in a finitely many steps.

R is Artinian $\Leftrightarrow$ it's both right & left Artinian

An Aritinian domain is a field

Eisenstein's Criterion.

suppose R is a UFD, $F = \mathrm{Frac}(R)$
if f is not constant, $\deg(f) = n$
p prime in R s.t. $p \mid a_0, \cdots, a_{n-1}$ $p \nmid a_n$ $p^2 \nmid a_0$
$\Rightarrow$ $f(x)$ irreducible in $F[x]$

The nilradical of R, $\mathrm{Nil}(R)$, consists of all nilponent element of R

The nilradical is always an ideal

$\mathrm{Nil}(R) = \bigcap$ All prime Ideal

The radical of an ideal I. $\mathrm{Rad}(I)$
$\Leftrightarrow$ $\mathrm{Rad}(I) = \sqrt{I} = \{a \in R : \exists n, a^n \in I\}$

The radical is an ideal, since it's the preimage of the nilradical of R/I under canonical homomorphism.

Ideal I is called primary $\Leftrightarrow \forall\, a, b \in R$, $(ab \in I \Rightarrow a \in I$ or $b \in Rad(I))$

I is prime $\Leftrightarrow I = Rad(I)$ & I primary

Jacobson radical of R, $J(R)$, is the intersection of all maximum ideas

$J(R) \supset Nil(R)$