# Polynomial

Def. $\mathbb{F}_2 = \mathbb{Z}_2$ is called Binary field

Def. $(R, +, \cdot)$ commutative $f$ is a polynomial on $R$

$f(x) = a_0 + a_1 x + \cdots + a_n x^n \quad (n \in \mathbb{N}_0, a_i \in R)$

$\deg(f) = a_n$

$f$ is a formal power series on $R$

$f(x) = \sum_{i=0}^{\infty} a_i x^i \quad a_i \in R$

Def. $(R, +, \cdot)$ commutative define Polynomial ring on $R$

denote as $(R[x], +, \cdot)$

$R[x] = \{ f : f \text{ is a polynomial on } R \}$

Prop. $(R, +, \cdot)$ commutative $\Rightarrow$ polynomial ring on $R$ is commutative

Prop. $\deg(fg) = \deg(f)\deg(g)$

Prop. $a \in R \quad \phi_a : R[x] \to R \quad \phi_a(f(x)) = f(a)$

is a ring homomorphism. called plug-in homomorphism

Prop. $f : R \to R'$ is a homomorphism

$\phi : R[x] \to R'[x]$

$\phi\left(\sum_{n=0}^{\infty} a_n x^n\right) = \sum_{n=0}^{\infty} f(a_n) x^n$ is a ring homomorphism

Def. $f$ is a multivariable polynomial about $x_1 \cdots x_n$ if

$f(x_1, \cdots x_n) = \sum_{\alpha_1=0}^{\infty} \cdots \sum_{\alpha_n=0}^{\infty} a_{\alpha_1, \alpha_2 \cdots \alpha_n} x_1^{\alpha_1} \cdots x_n^{\alpha_n}$

under multi-index $\alpha = (\alpha_1, \cdots \alpha_n)$

$f(x_1, \cdots x_n) = \sum_{\alpha} a_\alpha x^\alpha$

similar definition and proposition as single-variable