Prop.

X a simply-ordered set of at least two elements
for $a \in X$, denote $(-\infty, a) = \{x \in X \mid x < a\}$ $(a, +\infty) = \{x \in X \mid x > a\}$
then $S = \{(-\infty, a), (a, +\infty) \mid a \in X\}$ is a topological subbase of X

Def.

Topology generated by S is called Order Topology

Lemma.

(a, b) open & $[a, b)$ close in T
for M the greatest element (if exist) $(a, M]$ open in T
all the sets above is the base of the topology

Def.

$Y \subset$ fully ordered X, call Y convex iff
$\forall\ y_1, y_2 \in Y$, $(y_1, y_2) \subset Y$

Theorem

if Y is a convex subset of X. then the subtopology = ordered topology on Y
otherwise the subtopology $\supset$ ordered topology

Prop.

(I) Ordered topology space is T_2 (Hausdorff)
(II) Product of T_i $(i = 1, 2)$ space is T_i
(III) Subspace of T_i $(i = 1, 2)$ is T_i
(IV) all metric spaces are T_4 space

Def.

if $\exists$ open neighbourhood $\{U_n\}_{n=1}^{+\infty}$ of x, $\forall$ open neighbourhood of x
$\exists\ n \in N^*$ s.t. $U_n \subset U$, call X satisfies The First Countability Axiom at x
if X satisfies FCA everywhere, call X satisfies A_1

Def.

$\succeq$ is a binary relation on D, satisfying:

(I) if $m \succeq n$, $n \succeq p$, then $m \succeq p$

(II) $\forall\, m \in D$. $m \succeq m$

(III) if $m \succeq n$, $n \succeq m$, then $n = m$

(IV) if $\forall\, m, n \in D$, $\exists\, p \in D$

then call D directed set.

for X a set, $S: D \to X$ a mapping, call S is the net defined on X

for $A \subset X$, if $\exists\, m \in D$ s.t. when $n \succeq m$, $S(n) \in A$, call S end in A

(Moore-Smith convergence)

for X a topology space, if $\forall$ neighbourhood U of x, S ends in U, call S converge to x.

Prop.

$A \subset X$ topology space, then

(I) $x \in \bar{A} \iff \exists\, S$ in A converge to x

(II) $x \in A' \iff \exists\, S$ in $A \setminus \{x\}$ converge to x