Def.

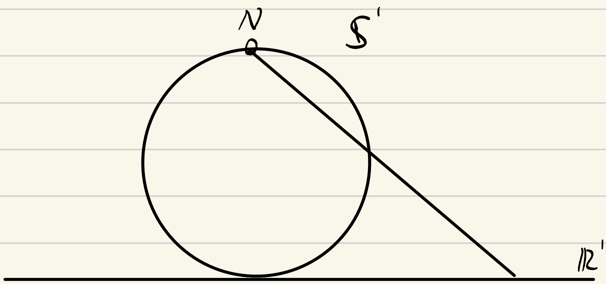
Y compact T_2, a dense $A \subset Y$ homeomorphic to X, call Y compactification of X
if $Y \setminus A$ is single-point, call Y single point compactification of X.

Prop.

(I) if $\exists$ single point compactification of $X \Rightarrow X$ local compact T_2
(II) $\exists$ single point compactification of $X \Leftrightarrow X$ non-compact local compact T_2

Example:

Riemann Sphere is a single point compactification from $\mathbb{R}^n \to S^n$



Lemma (Lebesgue number lemma)

set (X, d_X) compact metric space, $\mathcal{A}$ open covering of $X \Rightarrow$
$\exists\ \delta > 0$ s.t. subsets with diameters less that $\delta \subset$ some element of $\mathcal{A}$
call such δ a Lebesgue Number of $\mathcal{A}$.

Def.

(X, d_X) (Y, d_Y) metric spaces. $f: X \to Y$
if $\forall\ \varepsilon > 0$, $\exists\ \delta > 0$, s.t. when $d_X(x_1, x_2) < \delta$, $d_Y(f(x_1), f(x_2)) < \varepsilon$
call f uniform continuous.

Theorem (Uniform Continuity Theorem)

(X, d_X) compact , $f: X \to Y$ continous $\Rightarrow$ f uniform $\sim$

Def.

if $\forall$ infinite $A \subset X$, $A' \neq \phi$, call X limit point compact

if $\forall$ sequence on X has converge subsequence , call X sequential compact.

Prop.

(I) X compact $\Rightarrow$ X limit point compact ,

X sequential compact $\Rightarrow$ X limit point compact.

(II) (X, d_X) sequential compact $\mathcal{A}$ open covering of X $\Rightarrow$

$\exists\ \delta > 0$ s.t. $\forall$ sub sets with diameter $< \delta$ $\subset$ some element of $\mathcal{A}$

(III) (X, d_X) sequential compact $\forall\ \varepsilon > 0$ $\{B(x, \varepsilon) \mid x \in X\}$ has finite subcovering.

(IV) for (X, d) , the following equivalent:

① X compact

② X limit point compact

③ X sequential compact.

Def.

$\forall\ x, y \in X$, $x \neq y$ $d(f(x), f(y)) < d(x, y)$, f is called Tighten map.

Prop.

if X compact f Tighten , f has unique fixed point.