1、切比雪夫处理三角函数

泰勒折分级数

2、待总结！！

1. Calculate $\int_0^{\pi} \sin^2[3t+\cos^4(5t)]\,dt$

Sol.

$$\int_0^{\pi} \sin^2[3t+\cos^4(5t)] = \int_0^{\pi} \frac{1}{2} - \frac{1}{2}\cos(6t+2\cos^4 5t)\,dt$$

$$= \frac{\pi}{2} - \frac{1}{2} I$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\left(6\left(t+\frac{\pi}{2}\right) + 2\sin^4 5x\right) dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos 6x \cdot \cos(2\sin^4 5x) - \underbrace{\sin 6x \, \sin(2\sin^4 5x)}_{\text{odd}}\, dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos 6x \sum_{i=0}^{\infty} (-1)^i \frac{1}{(2i)!} (2\sin^4 5x)^i \, dx$$

set $J_k = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos 6x \, \sin^{4k} 5x \, dx$

notice J_k even

$$J_k = \int_0^{\frac{\pi}{2}} \cos 6x \, \sin^{4k} 5x \, dx$$

$$(1 \quad \cos x \quad \cdots \quad \cos nx) = (1 \quad \cos x \quad \cdots \quad \cos^n x) \underbrace{\begin{pmatrix} 1 & 0 & * & & \\ & 1 & * & & * \\ & & 2 & & \\ & & & \ddots & \\ & 0 & & & 2^{n-1} \end{pmatrix}}_{= A}$$

A invertible, thus

$$(1 \quad \cos x \quad \cdots \quad \cos nx) \cdot A^{-1} = (1 \quad \cos x \quad \cdots \quad \cos^n x)$$

thus $\cos^i x$ is a linear combination of $1, \cos x \cdots \cos ix$

thus $J_k = 0$, origin $= \frac{\pi}{2}$

$\square$.

2、 $a, b, c > 0$

calculate $m = \min\left[\max\{a, \frac{1}{b}\} + \max\{b, \frac{2}{c}\} + \max\{c, \frac{3}{a}\} \right]$

Sol.

$m = \min M$

$$M \geq a + \frac{2}{c} + \left(\max\{c, \frac{3}{a}\}\right)$$

$$\Updownarrow$$

$$\geq a + \frac{2}{c} + \left(\lambda c + (1-\lambda)\frac{3}{a}\right) \qquad \text{equal when } ac = 3$$

$$= \left(a + 3\frac{1-\lambda}{a}\right) + \left(\lambda c + \frac{2}{c}\right)$$

$$\geq 2\left(\sqrt{3(1-\lambda)} + \sqrt{2\lambda}\right) \qquad \text{equal when } a = \sqrt{3(1-\lambda)} \qquad c = \sqrt{\frac{2}{\lambda}}$$

$\lambda = \frac{2}{5}$

$M \geq 2\sqrt{5}$