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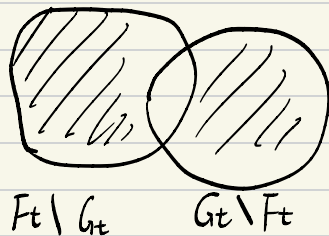


1.  $X, Y$  独立同分布, 证  $E(|X-Y|) \leq E(|X+Y|)$

Proof.

先证  $E(|X-Y|) = \int_{-\infty}^{\infty} P[(F_t \setminus G_t) \cup (G_t \setminus F_t)] dt$

where  $F_t = [X > t]$   $G_t = [Y > t]$



$$\begin{aligned} \text{RHS} &= \int_{-\infty}^{\infty} \int_{\mathcal{N}} \mathbb{1}_{t \in [\min(X,Y), \max(X,Y)]} dP dt \\ &= \int_{\mathcal{N}} \int_{-\infty}^{\infty} \mathbb{1}_{t \in [\min(X,Y), \max(X,Y)]} dt dP \\ &= \int_{\mathcal{N}} [\max(X,Y) - \min(X,Y)] dP \end{aligned}$$

where  $\max(X,Y) = \frac{x+y+|x-y|}{2}$   $\min(X,Y) = \frac{x+y-|x-y|}{2}$

$$\text{RHS} = \int_{\mathcal{N}} |x-y| dP = E(|X-Y|)$$

$$\text{则 } E(|X-Y|) = \int_{-\infty}^{\infty} P(F_t) P(F_{-t}) + [1-P(F_t)][1-P(F_{-t})] dt$$

$$\begin{aligned} E(|X+Y|) &= 2 \int_{-\infty}^{\infty} P(F_t) - [P(F_t)]^2 dt \\ &= 2 \int_{-\infty}^{\infty} P(F_{-t}) - [P(F_{-t})]^2 dt \end{aligned}$$

$$E(|X-Y|) - E(|X+Y|)$$

$$= \int_{-\infty}^{\infty} - [P(F_t) + P(F_{-t}) - 1]^2 dt \leq 0$$

Q.E.D.