P.1

$$(x-1)(a_0 + a_1 x + \cdots + x^r)(b_0 + b_1 x + \cdots + x^s) = x^{r+s+1} - 1$$

$$\alpha_k = e^{\frac{2\pi \cdot k}{r+s+1} \cdot i} \quad (k = 1, 2, \cdots, r+s)$$

$$F_a \cup F_b = \{\alpha_k \mid k \in \{1, 2, \cdots, r+s\}\} \qquad F_a \cap F_b = \phi$$

$$a_0 + a_1 x + \cdots + x^r = \prod_{\alpha_i \in F_a} (x - \alpha_i)$$

$$b_0 + b_1 x + \cdots + x^s = \prod_{\alpha_i \in F_b} (x - \alpha_i)$$

$$a_0 = b_0 = (-1)^r \prod \alpha_i = (-1)^s \prod \alpha_i = 1$$

r and s are both even.

$$a_1 = (-1)^{r-1} \sum \prod a_i$$

2.

let $a_n = 2\sin\theta_n$

$$a_{n+1} = \sqrt{2 - 2\cos^2\theta_n}$$

$$= 2\sin\frac{\theta_n}{2}$$

thus

$$a_n = 2\sin\frac{\pi}{2^{n+2}}$$

let $b_n = 2\tan\theta_n$

$$b_{n+1} = \frac{4\tan\theta_n}{2 + 2\cot\theta}$$

$$= 2\frac{\sin\theta}{\cos\theta + 1} = 2\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$= 2\tan\frac{\theta}{2}$$

thus

$$b_n = 2\tan\frac{\pi}{2^{n+2}}$$

there fore

(a) (b) 13 obivious

for (c)

$$b_n - a_n = 2\left(\tan\frac{\pi}{2^{n+2}} - \sin\frac{\pi}{2^{n+2}}\right)$$

$$< 2\left[\frac{\pi}{2^{n+2}} - \left(\frac{\pi}{2^{n+2}} - \frac{1}{3!}\cdot\left(\frac{\pi}{2^{n+2}}\right)^3\right)\right]$$

$$= \frac{1}{3}\cdot\frac{\pi^3}{4^3\cdot 8^n}$$

P3

$n<k$

set $A = (a_1 \ a_2 \ \cdots \ a_k)$ where $a_i \in \mathbb{R}^n$

$$A^T \cdot A = \begin{pmatrix} 1 & & & \\ & 1 & <0 & \\ & <0 & 1 & \\ & & & \ddots \\ & & & & 1 \end{pmatrix}$$

$$\text{rank}(A) = \text{rank}(A^T A) \le k$$