P6. let $f: \mathbb{R}_{2\times 2} \to \mathbb{R}_{2\times 2}$ $f(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = \begin{bmatrix} \ln a & \ln b \\ \ln c & \ln d \end{bmatrix}$

thus by taking steps $\det f(\begin{bmatrix} a & b \\ c & d \end{bmatrix})$ holds still

$$\ln 2 \ln 4 - \ln 3 \ln 2 = \det f(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) \neq \det f(\begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix})$$

$$= \ln 2 \ln 3 - \ln 2 \ln 4$$

thus it's impossible to achieve such matrix by finite steps

P7. $$\frac{d}{dx} f(x) - f(x) = \alpha \qquad P(x) = -1 \quad Q(x) = \alpha$$

$$f(x) = e^{-\int -1\, dx} \left(\int \alpha e^{\int -1\, dx} dx + C \right)$$

$$= e^{x} \left(-\alpha e^{-x} + c \right)$$

$$= -\alpha + C \cdot e^{x}$$

$$e^{-x} (f(x) + \alpha) = C = g(x)$$

$$g'(x) = \frac{f'(x) e^{x} - (f(x)+\alpha) e^{x}}{e^{2x}} = \frac{f'(x) - f(x) - \alpha}{e^{x}}$$

$$g(0) = \alpha \qquad g(1) = \frac{1+\alpha}{e}$$

iff $g(0) = g(1) \iff \alpha = \frac{1}{e-1}$

then $\exists\ g'(x) = 0$ (Rolle's theorem)

Problem 8.

let $k = \binom{n}{2}$ $x_1 \le x_2 \cdots \le$

P9. use extreme principle

P10 consider n is prime