


庄家 n 个筹码, p 概率赢一个, q 输一个 ($p+q=1$)

设 $f(n)$ $n=0, 1, \dots, \infty$ 为庄家把 n 个筹码输掉的概率

$$f(0) = 0 \quad f(\infty) = 1$$

$$f(n) = p f(n+1) + q f(n-1)$$

$$f(n+1) - f(n) = \left(\frac{q}{p}\right)^n (f(1) - f(0))$$

$$f(n) = \begin{cases} 1 + \frac{1 - \left(\frac{q}{p}\right)^n}{1 - \frac{q}{p}} (f(1) - 1) & \text{if } \frac{q}{p} \neq 1 \\ 1 + n(f(1) - 1) & \text{if } \frac{q}{p} = 1 \end{cases}$$

$$f(\infty) = 0$$

$$f(n) = \begin{cases} \left(\frac{q}{p}\right)^n & p > q \\ 1 & p \leq q \end{cases}$$

另一个问题是关于赌徒“输完”或“达到 N ”时结束

$f(n)$ 为赌徒赢得 n 个筹码的概率.

$$f(0) = 0 \quad f(N) = 1$$

$$f(n) = \begin{cases} \frac{1 - \left(\frac{q}{p}\right)^n}{1 - \left(\frac{q}{p}\right)^N} & \frac{q}{p} \neq 1 \\ \frac{n}{N} & \frac{q}{p} = 1 \end{cases}$$