# 泊松公式

# 泰勒展开余项

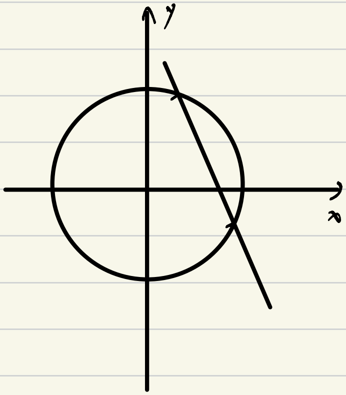
1. Possion Formula

$$\int \cdots \int_{x_1^2+x_2^2+\cdots x_n^2 \le r^2} f(a_1x_1+\cdots a_nx_n)\, g(x_1^2+\cdots x_n^2)\, dx_1 \wedge dx_2 \cdots \wedge dx_n$$

$$= \int \cdots \int_{x_1^2+x_2^2+\cdots x_n^2 \le r^2} f\left(\sqrt{a_1^2+\cdots+a_n^2}\, x_i\right) g(x_1^2+\cdots x_n^2)\, dx_1 \wedge \cdots \wedge dx_n$$

$$\oint_{x_1^2+\cdots x_n^2 = r^2} f(a_1x_1+\cdots+a_nx_n)\, g(x_1^2+\cdots+x_n^2)\, d\Omega$$

$$= \oint_{x_1^2+\cdots+x_n^2 = r^2} f\left(\sqrt{a_1^2+\cdots+a_n^2}\, x_i\right) g(x_1^2+\cdots+x_n^2)\, d\Omega$$




obvious ...

2. **Taylor Expansion.**

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k + R_n(x)$$

$R_n$ :

**Peano Reminder:**

for $f$ differentiable at $x=x_0$ to the $n$ th order

$$R_n(x) = o[(x-x_0)^n]$$

for $f$ differentiable at $x=x_0$ to the $n+1$ th order

$$R_n(x) = O[(x-x_0)^n]$$

**Lagrange Reminder:**

$$f \in C^{n+1}(a,b) \text{ , } x_0 \in (a,b) \text{ , } \exists\, \xi \in (x, x_0)$$

s.t. $$R_n = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}$$

**Cauchy Reminder:**

$$f \in C^{n+1}(a,b) \text{ , } x_0 \in (a,b) \text{ , } \exists\, \theta \in (0,1)$$

s.t. $$R_n = \frac{f^{n+1}(x_0+\theta(x-x_0))}{n!}(1-\theta)^n (x-x_0)^{n+1}$$

**Schlomilch-Roche Reminder.**

$$f \in C^{n+1}(a,b) \text{ , } x_0 \in (a,b) \text{ , } \exists\, \xi \in (x, x_0)$$

s.t. $$R_n = \frac{f^{(n+1)}(\xi)}{n!\,p}(x-\xi)^{n+1-p}(x-x_0)^p$$

$p \in \mathbb{R}^+$ , $p=n+1 \to$ Lagrange , $p=1 \to$ Cauchy.

## Integral Reminder

$$f \in C^{n+1}[a,b], \quad x_0 \in [a,b]$$

$$R_n = \frac{(x-x_0)^{n+1}}{n!} \int_0^1 (1-y)^n f^{(n+1)}[x_0 + y(x-x_0)]\, dy$$

$$= \frac{1}{n!} \int_{x_0}^{x} f^{(n+1)}(y)\, (x-y)^n\, dy$$