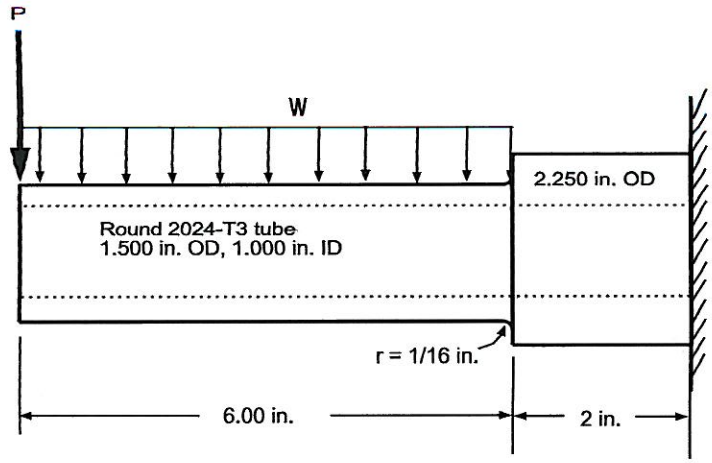
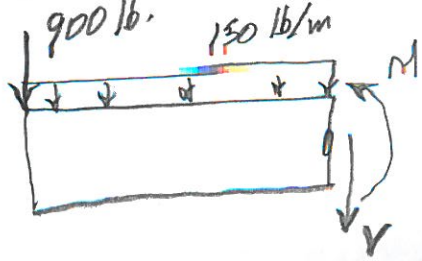
KEY

Design Homework 4

The cantilevered aluminum tube shown below is loaded with a steady w = 150 lb/in. and P which cycles between 900 lb. and zero. It is suspected that a crack of length 2a = 0.020 in. is present in radius region at the top of the tube. Determine the remaining fatigue lifetime in cycles using fracture mechanics and the Paris Law. Material properties: UTS = 68 ksi, S_Y = 47 ksi, K_{1c} = 33.7 ksi-in^1/2, β= 1.03, Paris Law parameters C = 5.4 x 10^{-11} and m = 4, and notch sensitivity factor q = 0.75. The stress concentration factor chart for solid shafts given below will suffice for determining K_t, allowing you to find K_f.





$M_{max} = -\{900(6)+900(3)\}$

$M_{max} = -8100$ lb·in

$M_{min} = -900(3) = -2700$ lb·in

$\sigma_{max} = -\frac{Mc}{I} = \frac{8100(.75)}{\pi(1.5^4-1^4)/64}$

$\sigma_{max} = 30{,}463.73$ psi

$\sigma_{min} = \frac{2700(.75)}{I} = 10154.58$ psi

$\Delta\sigma = 20{,}309.15$ psi

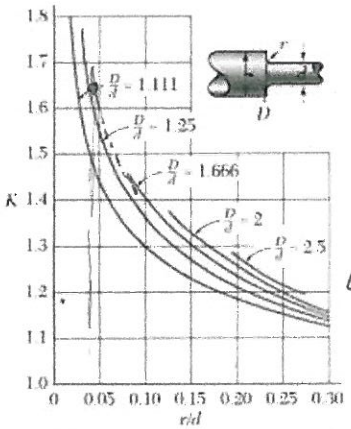


Fig. 3.32 Stress-concentration factors for fillets in circular shafts.†

$\frac{D}{d} = \frac{2.25}{1.5} = 1.5$

$\frac{r}{d} = \frac{.0625}{1.5} = .04167$

$K_t \approx 1.65$

$0.75 = \frac{K_f - 1}{1.65 - 1}$

$K_f = 1.4875$

$33.7 = 1.03(30.46373)\sqrt{\pi a_f}$

$a_f = 0.36717$ in

$a_i = 0.010$ in

$$\int_0^{N_f} dN = N_f = \frac{1}{5.4\times10^{-11}} \int_{.010}^{.36717} \frac{da}{[(1.03)1.4875(20.30915)\sqrt{\pi a}]^4}$$

σ in ksi

$$N_f = \frac{2001.53}{a}\Big|_{.01}^{.36717} = 194{,}702 \text{ cycles} \approx \boxed{194{,}700 \text{ cycles}}$$