

Predictive systems

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Predictive Systems.

Unit 1.

Assignment 1.

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1 Exercises

Exercise 1

The proportions of blood phenotypes: A , B , AB , and O , in the population of all caucasians in the US are approximately 0.41, 0.10, 0.04, and 0.45, respectively. A single caucasian is chosen at random from the population.

- 1 List the sample space for this experiment.
- 2 Make use of the information given above to assign probabilities to each of the simple events.
- 3 What is the probability that the person chosen at random has either type A or type AB blood?

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(a) $S = \{A, B, AB, O\}$

(b) $P(S) = 1, P(A) = 0.41, P(B) = 0.10, P(AB) = 0.04, P(O) = 0.45.$

(c) Therefore: (3) $P(A \cup AB) = P(A) + P(AB) = 0.41 + 0.04 = 0.45$

Exercise 2

A sample space consists of five simple events: E_1, E_2, E_3, E_4 , and E_5 .

- If $P(E_1) = P(E_2) = 0.15$, $P(E_3) = 0.4$ and $P(E_4) = 2P(E_5)$ find the probabilities of E_4 and E_5
- If $P(E_1) = 3P(E_2) = 0.3$, find the probabilities of the remaining simple events if you know they are equally probable.

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(a) $P(E1) = 0.15, P(E2) = 0.15, P(E3) = 0.4, P(E4) = 2P(E5)$.
 $S = \{E1, E2, E3, E4, E5\}$ $P(S) = P(E1) + P(E2) + P(E3) + P(E4) + P(E5) = 1 = 0.15 + 0.15 + 0.4 + P(E4) + P(E5)$
Then: $0.3 = P(E4) + P(E5)$. $P(E4) = 2P(E5)$ $0.3 = 3P(E5)$ $P(E5) = 0.3/3$
Therefore: $P(E4) = 2P(E5) = 0.1 * 2 = 0.2$
So, $P(E4) = 0.2$ and $P(E5) = 0.1$

(b)

$$\begin{aligned} P(E1) &= 3P(E2) = 0.3 \\ P(E3) &= P(E4) = P(E5) \\ P(E1) &= 0.3 \\ P(E2) &= 0.1 \\ P(E3) &=? \\ P(E4) &=? \\ P(E5) &=? \end{aligned}$$

$$S = \{E1, E2, E3, E4, E5\}$$

$$3P(E2) = 0.3$$

$$P(E2) = 0.3/3 = 0.1$$

$$P(S) = P(E1) + P(E2) + P(E3) + P(E4) + P(E5)$$

$$1 = 0.3 + 0.1 + P(E3) + P(E4) + P(E5)$$

$$P(E3) = P(P(E4)) = P(E5)$$

$$P(E3) + P(E4) + P(E5) = 3P(E3)$$

$$1 = 0.4 + 3P(E3)$$

$$\frac{1 - 0.4}{3} = P(E3)$$

$$P(E3) = 0.2$$

$$P(E3) = P(E4) = P(E5)$$

$$P(E4) = 0.2$$

$$(E5)$$

Exercise 3

An oil prospecting firm hits oil or gas on 10% of its drillings. If the firm drills two wells, the four possible simple events and three of their associated probabilities are as given in the accompanying table. Find the probability that the company will hit oil or gas:

- on the first drilling and miss on the second
- on at least of the two drillings

Simple Event	Outcome of First Drilling	Outcome of Second Drilling	Probability
E_1	Hit (oil or gas)	Hit (oil or gas)	.01
E_2	Hit	Miss	?
E_3	Miss	Hit	.09
E_4	Miss	Miss	.81



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- (a) $S = \{E1, E2, E3, E4\}$
 $P(E1) = 0.01, P(E2) = ?, P(E3) = 0.09, P(E4) = 0.81$
 Therefore
 $P(E2) = P(S) - P(E1) - P(E3) - P(E4)$
 $P(E2) = 1 - 0.1 - 0.09 - 0.81 = 0.09$
- (b) $P(E1 \cup E2 \cup E3) = 0.01 + 0.09 + 0.09 = 0.19$

Exercise 4

A survey classified a large number of adults according to whether they were diagnosed as needing eyeglasses and whether they use eyeglasses when reading. The proportions of the resulting categories are given in the following table:

Needs glasses	Uses Eyeglasses for Reading	
	Yes	No
Yes	.44	.14
No	.02	.40

If a single adult is selected from the large group, find the probabilities of the events defined below. The adult:

- needs glasses
- needs glasses but does not use them
- uses glasses whether the glasses are needed or not



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- (a) According to the table, we have that:
 $P(\text{need glasses}) = 0.44 + 0.14 = 0.58$
- (b) $P(\text{need glasses but does not use them}) = 0.14$
- (c) $P(\text{uses glasses whether the glasses are needed or not}) = 0.44 + 0.02 = 0.46$

Exercise 5

Hydraulic landing assemblies coming from an aircraft rework facility are each inspected for defects. Historical records indicate that 8% have defects in shafts only, 6% have defects in bushings only, and 2% have defects in both shafts and bushings. One of the hydraulic assemblies is selected randomly. What is the probability that the assembly has:

- a bushing defect
- a shaft or bushing defect
- exactly one of the two types of defects
- neither type of defect



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Event	Defects in shafts	Defects in bushings	Probability
E1	YES	YES	0.02
E2	YES	NO	0.08
E3	NO	YES	0.06
E4	NO	NO	0.84

- (a) $P(\text{a bushing effect}) = P(E1 \cup E3) = 0.02 + 0.06 = 0.08$
- (b) $P(\text{a shaft or bushing effect}) = 1 - P(E4) = 1 - 0.84 = 0.16$
- (c) $P(\text{exactly one of the two types of defects}) = P(E2 \cup E3) = 0.08 + 0.06 = 0.14$
- (d) $P(\text{neither type of defect}) = P(E4) = 0.84$

Exercise 6

Four equally qualified runners John, Bill, Ed, and Dave, run a 100-meter sprint, and the order of finish is recorded.

- 1 How many simple events are in the sample space?
- 2 If the runners are equally qualified, what probability should you assign to each simple event?
- 3 What is the probability that Dave wins the race?
- 4 What is the probability that Dave wins and John places second?
- 5 What is the probability that Ed finishes last?



- 6.
- (a) $4! = 24$
 - (b) $P(E_i) = 1/24 = 0.0416$ - simple event
 - (c)
$$P(\text{Dave wins}) = \frac{1}{24} * \frac{3!}{(3-3)!} = \frac{1}{4}$$
 - (d)
$$P(\text{Dave wins and John places second}) = \frac{1}{24} * \frac{2!}{(2-2)!} = \frac{1}{12}$$
 - (e)
$$P(\text{Ed finishes last}) = \frac{1}{24} * \frac{3!}{(3-3)!} = \frac{1}{4}$$

Exercise 7

The Bureau of the Census reports that the median family income for all families in the United States during the year 2003 was \$43,318. That is, half of all American families had incomes exceeding this amount, and half had incomes equal to or below this amount. Suppose that four families are surveyed and that each one reveals whether its income exceeded \$43,318 in 2003.

- List all the points in the sample space
- Identify the simple events in each of the following events:
 - A: At least two had incomes exceeding \$43,318
 - B: Exactly two had incomes exceeding \$43,318
 - C: Exactly one had incomes exceeding \$43,318
- Which are the probabilities of the events A, B and C?

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We will say that

M = More than

B = equal to or below

$$n^r = 2^4 = 16$$

E1 = MMMM E2 = BMMM

E3 = MBMM

E4 = MMBM

E5 = MMMB

E6 = BBMM

E7 = BMBM

E8 = MBBM

E9 = MMBB

E10 = MBMB

E11 = BMMB

E12 = BBBM

E13 = BBMB

E14 = BMBB

E15 = MBBB

E16 = BBBB

A = {E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11}

B = {E6, E7, E8, E9, E10, E11}

C = {E12, E13, E14, E15}

$$P(E1) = \frac{1}{16}$$

$$P(A) = 11 * \frac{1}{16} = \frac{11}{16}$$

$$P(B) = 6 * \frac{1}{16} = \frac{6}{16}$$

$$P(C) = 4 * \frac{1}{16} = \frac{4}{16} = \frac{1}{4}$$

Exercise 8

A boxcar contains six complex electronic systems. Two of the six are to be randomly selected for thorough testing and then classified as defective or not defective.

- If two of the six systems are actually defective, find the probability that at least one of the two systems tested will be defective. Find the probability that both are defective.
- If four of the six systems are actually defective, find the probabilities indicated in the previous point.



$$(a) P(\text{both not defective}) =$$

$$\frac{\binom{4}{2} \binom{2}{0}}{\binom{6}{2}} = \frac{6}{15}$$

$$P(\text{at least one defective}) =$$

$$1 - \frac{6}{15} = \frac{9}{15}$$

Therefore

$$P(\text{both are defective}) =$$

$$\frac{\binom{2}{2} \binom{4}{0}}{\binom{6}{2}} = \frac{1}{15}$$

$$(b) P(\text{at least one defective}) =$$

$$1 - \frac{\binom{2}{2} \binom{4}{0}}{\binom{6}{2}} = \frac{14}{15}$$

Therefore

$$P(\text{both defective}) =$$

$$\frac{\binom{4}{2} \binom{2}{0}}{\binom{6}{2}} = \frac{6}{15}$$

Exercise 9

For a certain population the percentage passing or failing a competency exam, listed according to sex, were as shown in the accompanying table. An individual is to be selected randomly from this population. Let A be the event that the individual scores a passing grade on the exam and let M be the event that a male is selected.

Outcome	Sex		<i>Total</i>
	Male (M)	Female (F)	
Pass (A)	24	36	60
Fail (\bar{A})	16	24	40
<i>Total</i>	40	60	100

- Are the events A and M independent?
- Are the events A^c and F independent?



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(a) By looking at the table, we can see that

$$P(A) = \frac{60}{100} = 0.6$$

$$P(M) = \frac{40}{100} = 0.40$$

$$P(A \cap M) = \frac{24}{100} = 0.24$$

By the formula of independent events, we have that:

$$P(A \cap B) = P(A)P(B)$$

Therefore

$$P(A)P(M) = 0.6 * 0.4 = 0.24 = P(A \cap M)$$

We conclude that events A and M are independent.

(b)

$$P(\bar{A}) = \frac{40}{100} = 0.4$$

$$P(F) = \frac{60}{100} = 0.6$$

$$P(\bar{A} \cap F) = \frac{24}{100} = 0.24$$

Therefore by using the same formula, we can say that:

$$P(\bar{A})P(F) = P(\bar{A} \cap F) = 0.4 * 0.6 = 0.24$$

Therefore,

$$\bar{A} \text{ and } F$$

are independent.

Exercise 10

A study of the posttreatment behavior of a large number of drug abusers suggests that the likelihood of conviction within a two-year period after treatment may depend upon the offenders education. The proportions of the total number of cases falling in four education–conviction categories are shown in the following table:

Education	Status within 2 Years after Treatment		
	Convicted	Not Convicted	Total
10 years or more	.10	.30	.40
9 years or less	.27	.33	.60
<i>Total</i>	.37	.63	1.00

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Exercise 10

Suppose that a single offender is selected from the treatment program. Define the events:

- A: The offender has 10 or more years of education.
- B: The offender is convicted within two years after completion of treatment.

Find the following:

- $P(A)$
- $P(B)$
- $P(A \cap B)$
- $P(A^c)$
- $P(A \cup B)$
- $P(A|B)$
- $P(B|A)$

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(a) By looking at the table, we can say that

$$P(A) = 0.10 + 0.30 = 0.40$$

(b)

$$P(B) = 0.10 + 0.27 = 0.37$$

(c)

$$P(A \cap B) = 0.10$$

(d)

$$P(\text{complement of } A) = 1 - 0.4 = 0.6$$

(e)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.40 + 0.37 - 0.1 = 0.67$$

(f)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.10}{0.37} = 0.27$$

(g)

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.10}{0.40} = 0.25$$