Statistics is the science of data, and is used to evaluate claims.

Ex: I make 80% of free throws I shoot.

Chapter 1: Picturing Distributions with Graphs

Def: An individual is an object described by data.

Ex: Person, city, animal, company.

Def: A variable is a characteristic of an individual.

Ex: Age, population, species, profit.

Ex: We randomly select 4 people in the US and ask them to report their age and gender. We also ask them what state they're living in.

State	Age	Reported Gender
Kentucky	61	Female
Florida	27	Female
Wisconsin	27	Male
California	33	Female

catagorical quantative catagorical

4 individuals and 3 variables measured for each individual

Def: A variable is quantitative if it takes numerical values and arithmetic

makes sense.

Def: A variable is categorical if it is not quantative.

Now we ask for zip codes

State	Age	Reported Gender	Zip
Kentuchy	61	Female	41375
Florida	27	Female	93402
Wisconsin	27	Male	97403
California	33	Female	49102

categorical!

Ex: A study classifies bison in Yellowstone as young or adult. State the

individuals, variables, and the type of variable.

Bison, age, catagorical

Def: The distribution of a variable is the information of both its possible values and how often they occur.

Ex:

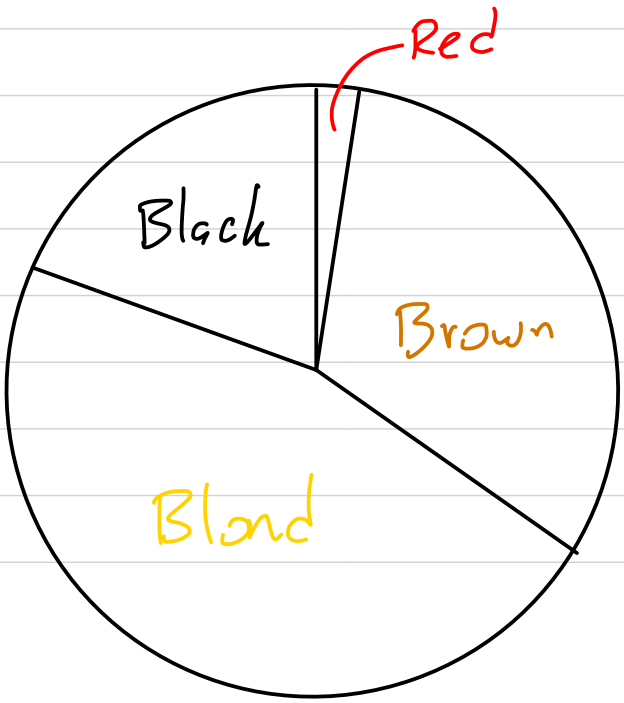
Student ID	Hair color
003	Red
005	Brown
035	Brown
089	Black

← not a distribution

Hair color	% of students w/ this color
Red	2 %
Brown	35 %
Blond	43 %
Black	20 %

distribution

Pie chart



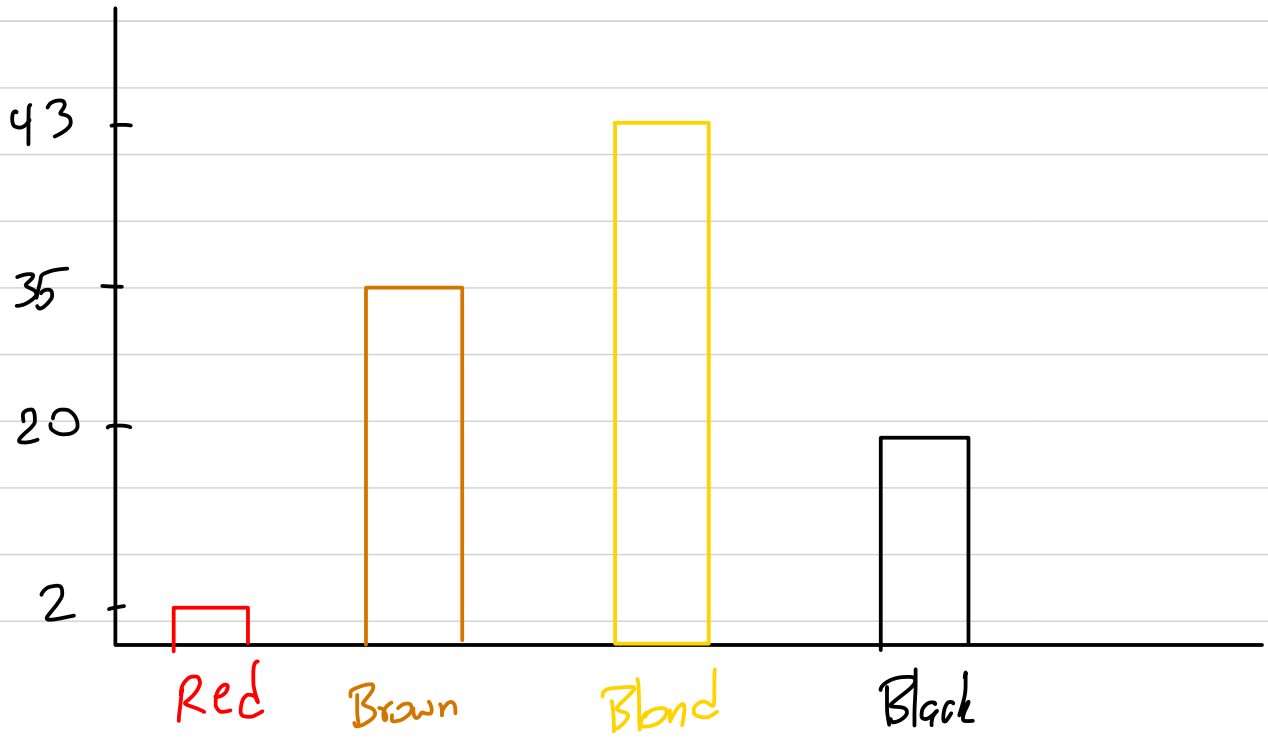
Comment: Only use pie charts when the values the variable can take are mutually exclusive — i.e. every individual has at most one value. Hair color is mutually exclusive since you can have at most one. A survey asking which types of soda you'd had in the past month would not be mutually exclusive since you could have had more than one type.

	% pop
Sprite	30 %
Dr. Pepper	25 %

this doesn't reflect the people who have had both

Hair color	% of students w/ this color
Red	2 %
Brown	35 %
Blond	43 %
Black	20 %

Bar graph:



Ex

Music source	% of 12-24 year olds who have used it
Radio	72
YouTube	77
iTunes	47

Don't use a pie chart, because the different music sources aren't mutually exclusive!

Histograms: when given a sample of individuals, you can make a histogram by dividing the data into ranges (called classes) and counting the number of individuals in each class. Then we make a bar graph of the result. This

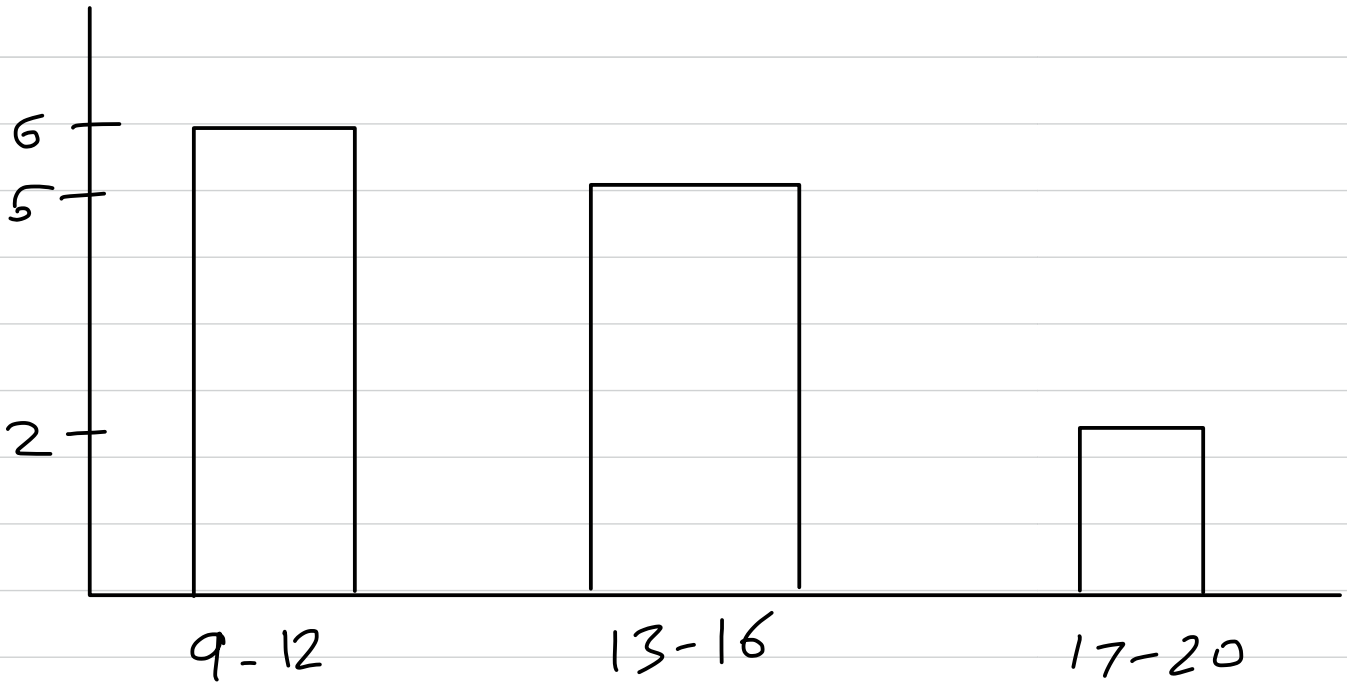
roughly approximates the distribution.

Ex: We get a set of ages:

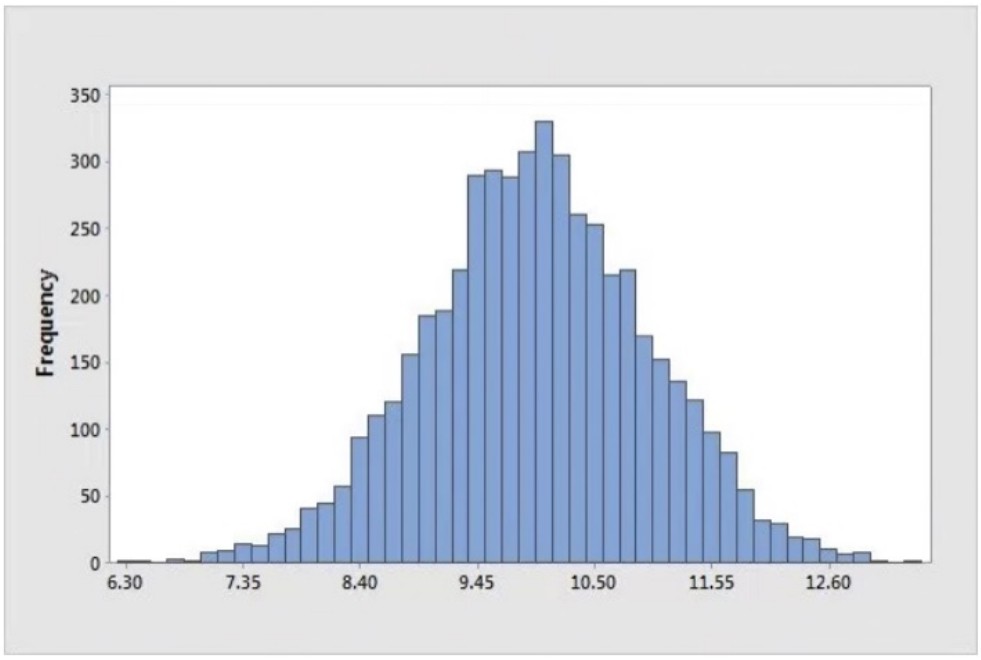
9 10 10 11 12 12 14 15 15
16 16 18 20

Classes: 9-12, 13-16, 17-20

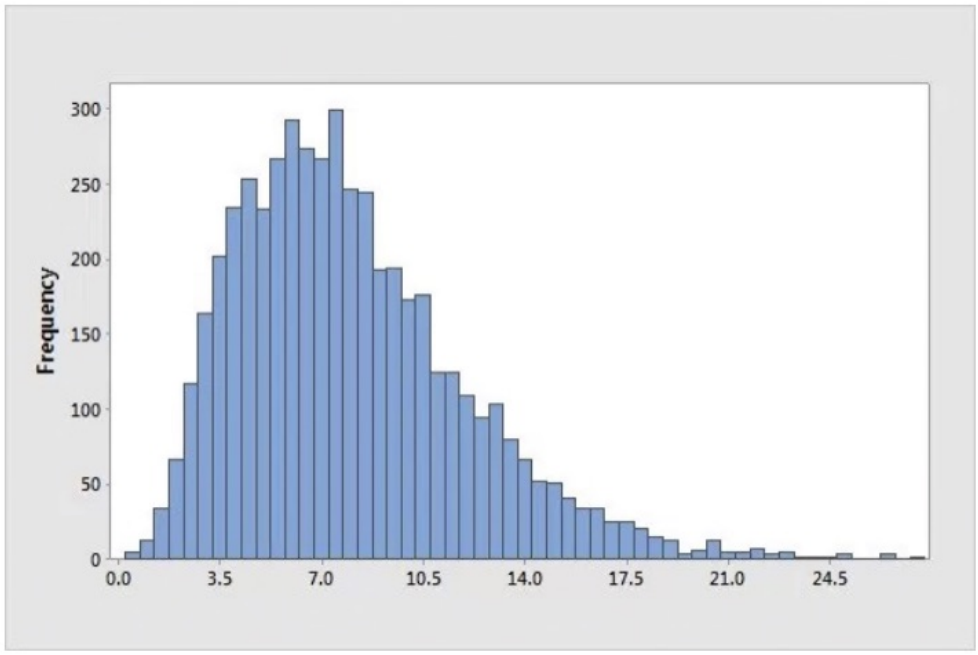
6 5 2



A **symmetric** distribution.
Ex: Heights of young women, Lengths of bird bills

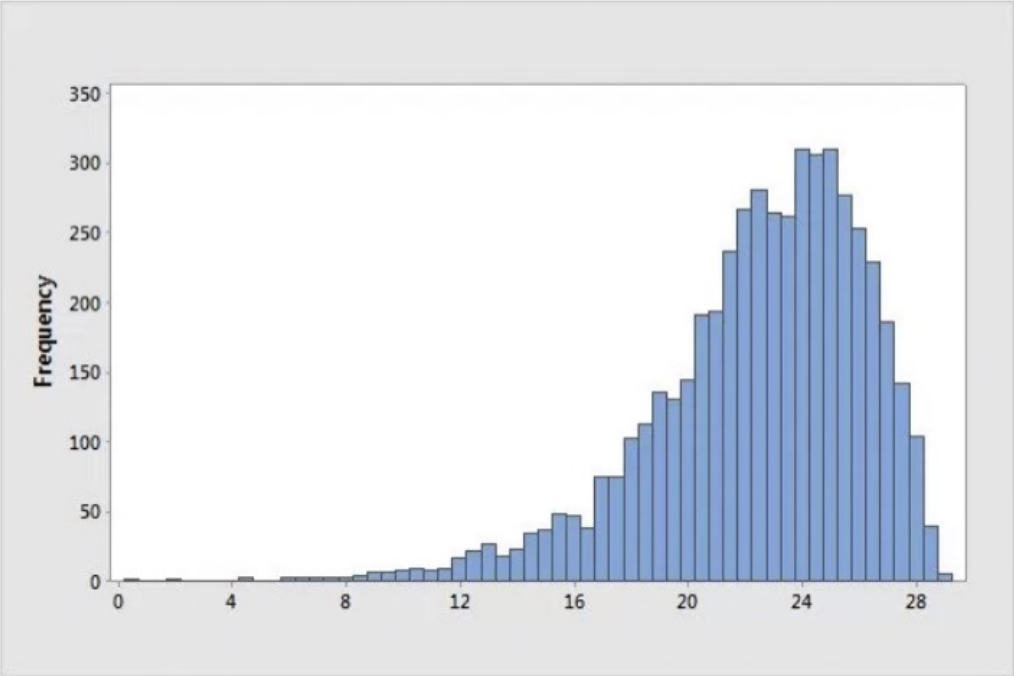


Right-skewed

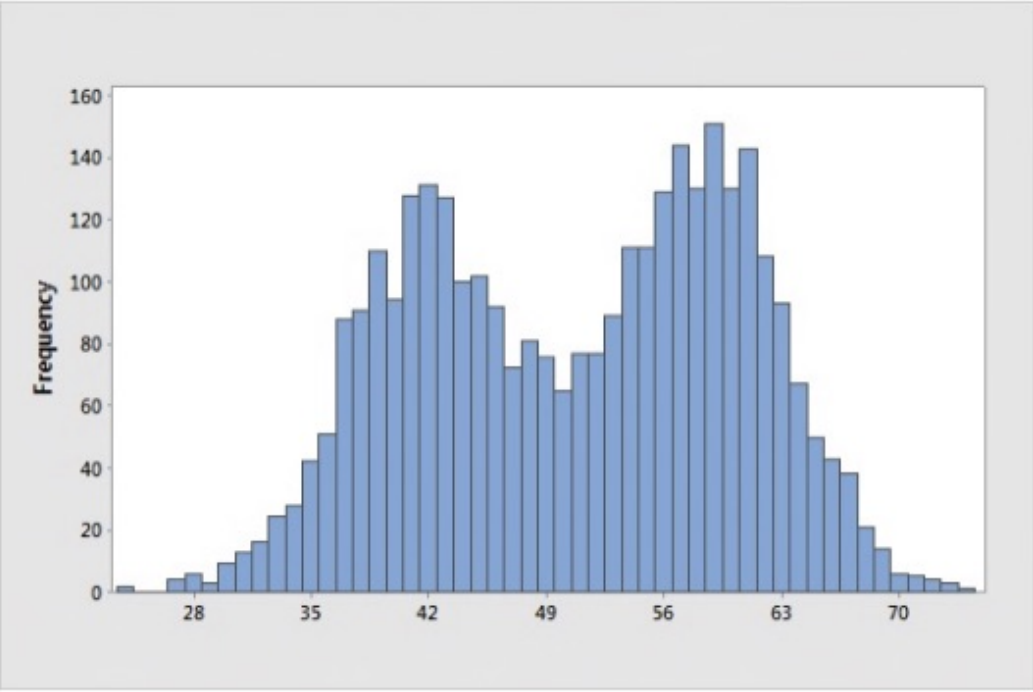


Ex: incomes

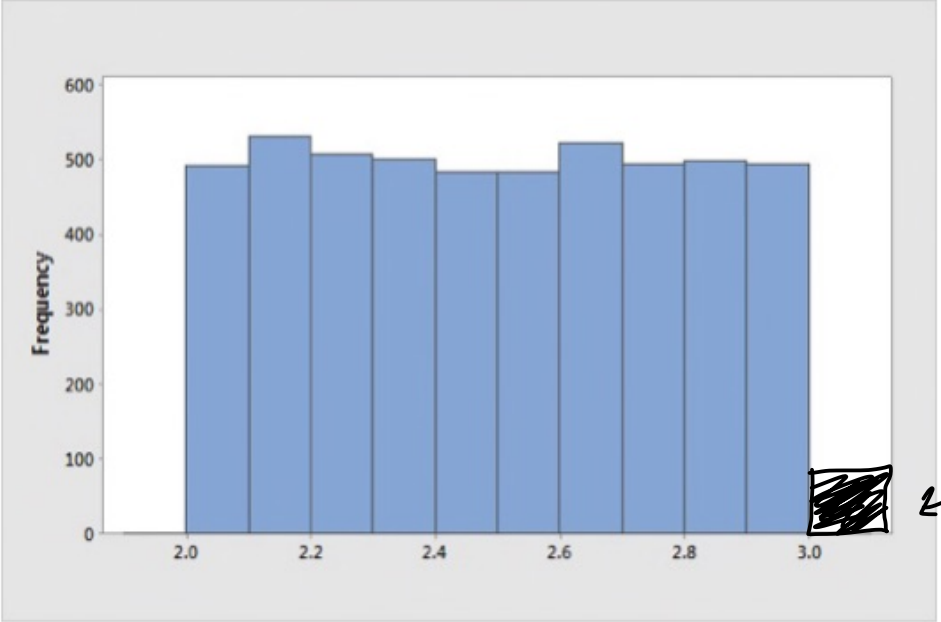
A **left-skewed** distribution.
Ex: Grades on an easy test



A **bimodal** distribution.
Ex: Exam scores when one group studied and another didn't



An **approximately uniform** distribution.
Ex: Rolling a die



Def: The center of the distribution is the mean or median. The variability is roughly how spread out the distribution is. Outliers are individuals who don't fit the pattern.

Def: Given a quantative set of data, we can form a stem-and-leaf plot: take all of the numbers and split them into the last digit and all the other digits. Then write the second piece (i.e. the prefix) and all the final digits with that prefix.

Ex: 9 10 10 11 12 12 14 15 15 16 16 18 20

0	9
1	0 0 1 2 2 4 5 5 6 6 8
2	0

Ex: | 5, 13, 18, 32, 91 | | 40, 45, 19, 60 |

0	5
1	3 8
3	2
9	1

← correct

Webwork + Textbook:

	0	5
9	1	38
	2	
	3	2
05	4	
	5	
0	6	
	7	
	8	
	9	1

Comment: we can also split the stems:

0	9
1	0 0 1 2 2 4 5 5 6 6 8
2	0

11

0	9
0	
1	0 0 1 2 2
1	4 5 5 6 6 8
2	0
2	

Chapter 2: Describing Distributions with Numbers

Ex: A list of travel times to work in North Carolina:

30, 20, 10, 40, 25, 20, 10, 60,
15, 40, 5, 30, 12, 10, 10

How to calculate center? One way is taking the average.

Def Given a set of data $x_1, \cdots, x_n$, the mean of the data is

$$\bar{x} = \frac{x_1 + \cdots + x_n}{n}.$$

Ex: $$\bar{x} = \frac{30 + 20 + 10 + \cdots + 12 + 10 + 10}{15} = 22.5$$

↑ 15 samples

Ex: 5, 10, 15, 200 ← Right-skewed

$$\bar{x} = \frac{5+10+15+200}{4} = \frac{230}{4} = 57.5$$

Comment: In a skewed distribution, the mean is drawn toward the skew (i.e. the tail). We say the mean is not a resistant measure of center.

Def: Let $x_1, \dots, x_n$ be a set of data. The median is M, defined by:

① if n is odd, then M is the data point such that as many x_i are greater than M as are less than M

② if n is even, M is the average of the two numbers with

as many x_i greater than them

as there are x_i less than them

Ex: 30, 20, 10, 40, 25, 20, 10, 60,

15, 40, 5, 30, 12, 10, 10

First arrange from smallest to largest

5, 10, 10, 10, 10, 12, 15, **20**, 20, 25, 30, 30,

40, 40, 60

↑ median

15 data points, which is odd, so we want the number "in the middle"

Ex: 5, **10, 15**, 200

↳ average is $\frac{10+15}{2} = 12.5$

Median : 12.5

Comment: The median is a resistant measure of center.

Ex: you roll a die. If you roll a 1-5, you get nothing. If you roll a 6, you get $100. What should you expect to get on average from rolling 6 times?

median: 0

mean: $\frac{100}{6}$ ← this is better for our purposes!

How do we measure variability?

Start small: min and max

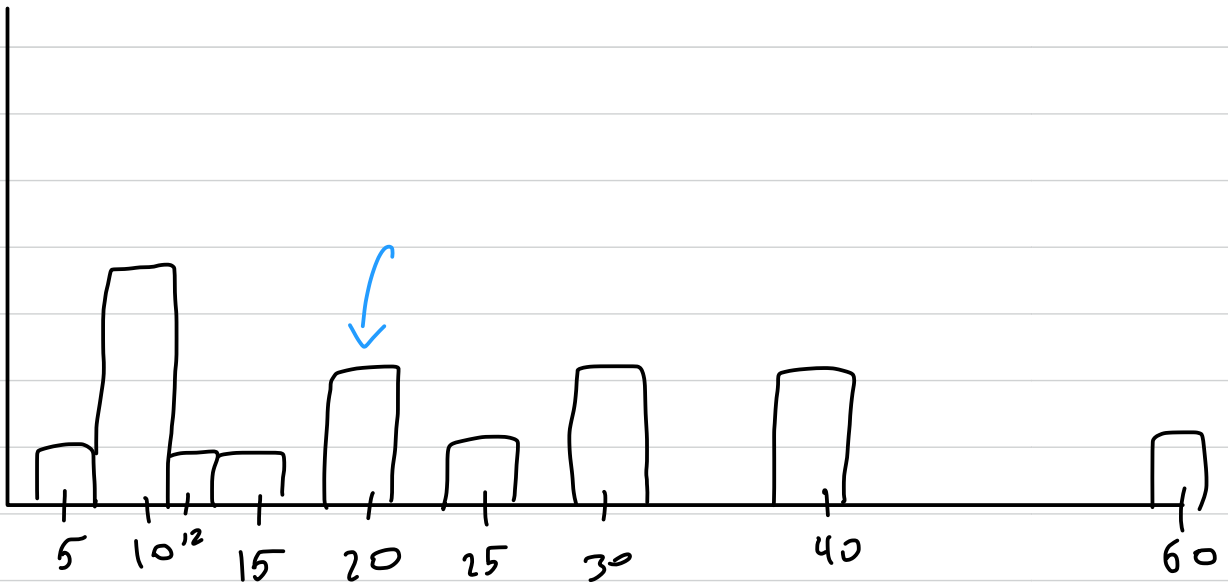
Ex. 5, 10, 10, 10, 10, 12, 15, 20, 20, 25, 30, 30, 40, 40, 60

5, 60

Better: min, median, max

5, 20, 60

gap indicates that this is a right-skewed distribution



Def: The first and third quartiles, Q_1 and Q_3, are the medians of the two halves of the data, not including the median of the whole data.

5, 10, 10, 10, 10, 12, 15, 20, 20, 25, 30, 30, 40, 40, 60

(you could say that $Q_2 = 20$)

$Q_1 = 10$

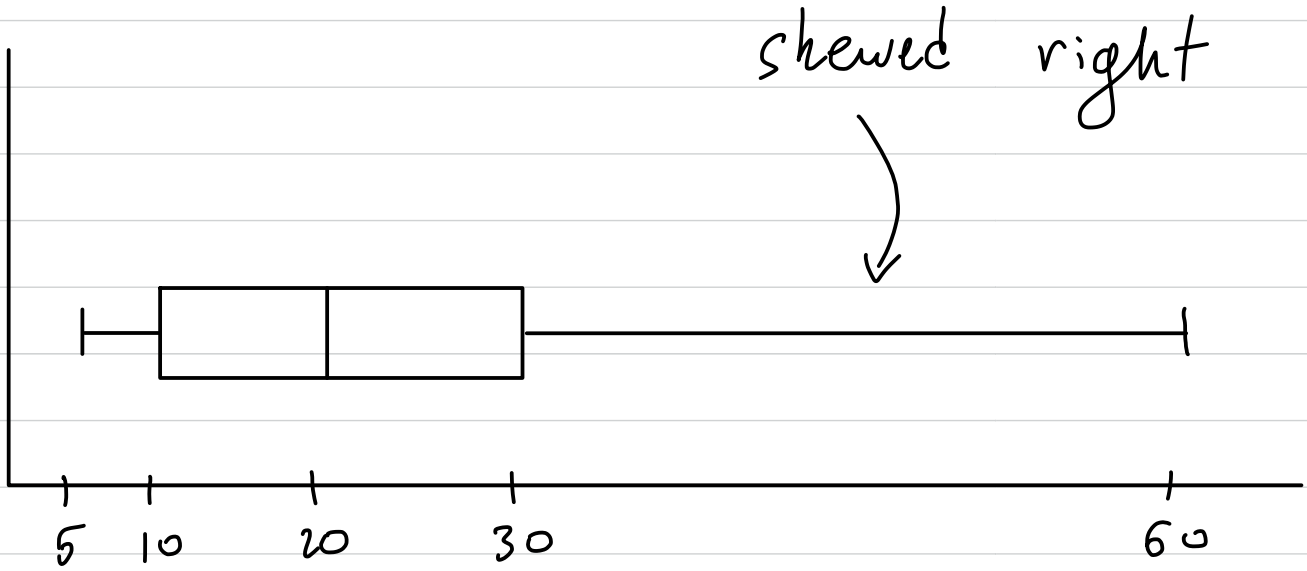
$Q_3 = 30$

Def: The 5-number summary of a set of data is min, Q_1, median, Q_3, max

Ex: 5, 10, 20, 30, 60

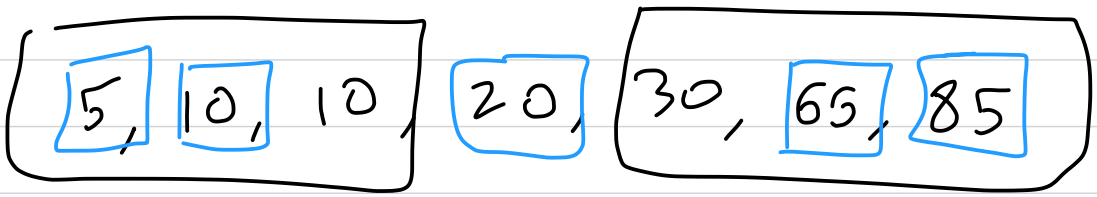
All 4 gaps have the same # of data points.

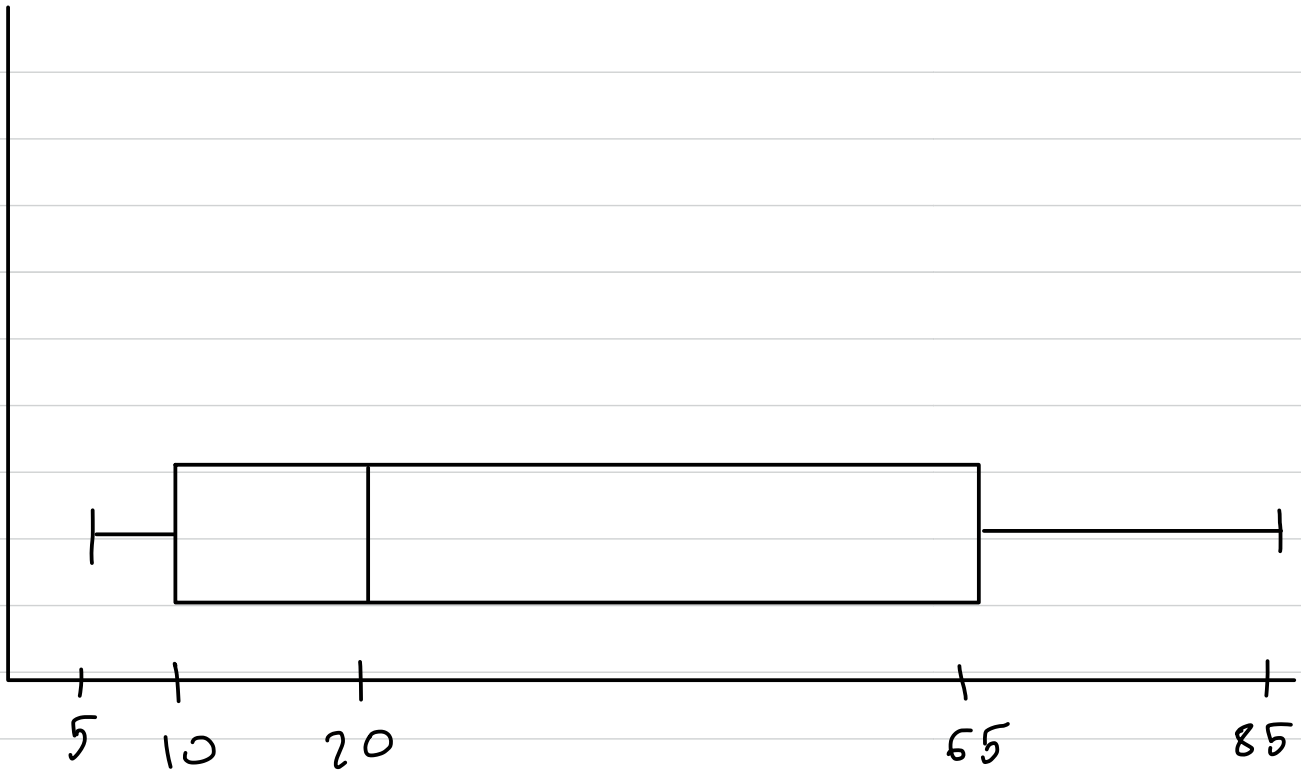
Box plots:



Ex: Draw a box plot of

10, 30, 5, 85, 65, 20, 10.





Def: The interquartile range, or IQR, is given by $IQR = Q_3 - Q_1$.

Def: An outlier in a data set is any point more than 1.5 IQR above Q_3 or below Q_1.

Ex: 10, 30, 5, 1000, 65, 20, 10.

5-num: 5, 10, 20, 65, 1000

$Q_1 = 10$

$Q_3 = 65$

$IQR = 65 - 10 = 55$

$1.5\ IQR = 82.5$

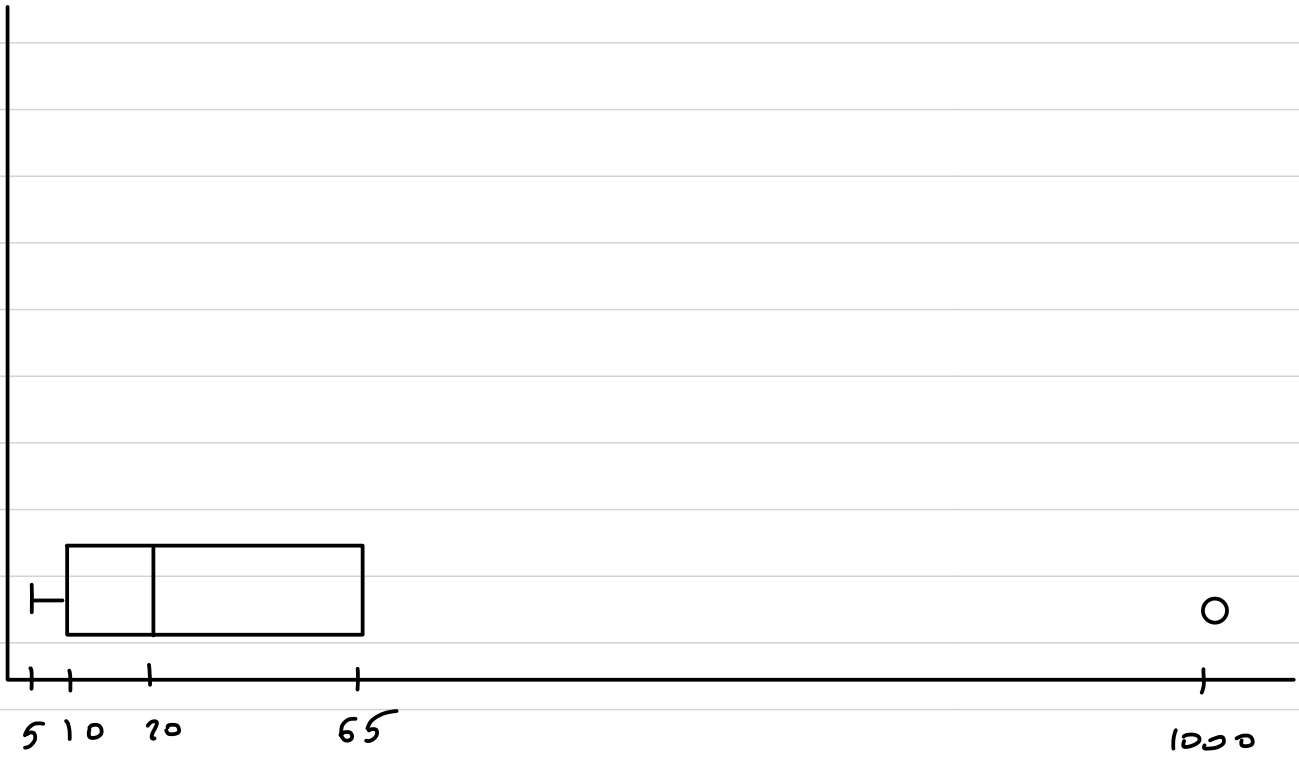
$Q_3 + 82.5 = 147.25$

$Q_1 - 82.5 = -72.5$

outliers are anything not between -72.5 and 147.25. So 1000 is an outlier.

Represent outliers by modifying the

box-plot: make the whiskers only reach the non-outliers.



The 5-num summary is a resistant measure of variability (but it's a little lacking)

How do we get a nonresistant measure of variability? Naive approach: take average distance to the mean

$x_1, \cdots, x_n$ mean: $\bar{x}$

$$\frac{(x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + (x_n - \bar{x})}{n}$$

This actually always is zero!

$$= \frac{x_1 + x_2 + \cdots + x_n - n\bar{x}}{n}$$

$$= \frac{x_1 + x_2 + \cdots + x_n}{n} - \frac{n\bar{x}}{n}$$

$$\underbrace{\qquad\qquad}_{\bar{x}} - \underbrace{\qquad}_{\bar{x}}$$

$$= 0$$

We can fix this issue by making the

distance to the mean always be positive.

Def: Let $x_1, \cdots, x_n$ be data with mean $\bar{x}$. The variance is

$$s^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n-1}$$

$n-1$ ← Bessel's correction (eliminates bias)

don't worry about this yet

Def: The standard deviation is s.

Ex: SAT math scores at Georgia

southern Ligh schod

490	580	450	570	650
x_1	x_2	x_3	x_4	x_5

$$\bar{x} = \frac{490+580+450+570+650}{5} = 548$$

$$s^2 = \frac{(490-548)^2 + (580-548)^2 + (450-548)^2 + (570-548)^2 + (650-548)^2}{5-1}$$

$$= 6220$$

$s = 78.87$ ← standard deviation

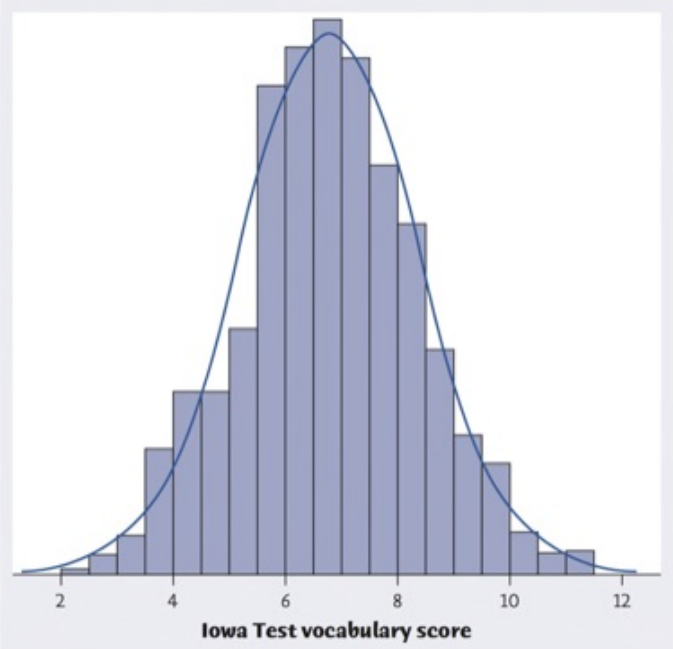
think of as: the average distance to mean among this data is 78.87

s is a good measure of variability, but only when $\bar{x}$ is a good measure of center.

Note: $\bar{x}$ and s do <u>not</u> give a complete description of data

Chapter 3: Normal Distributions

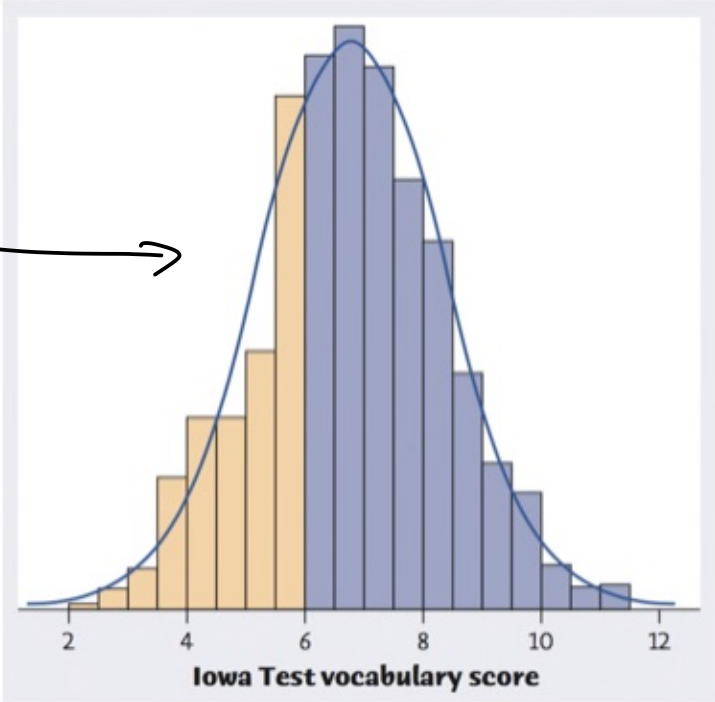
<u>Ex</u> 947 students in Gary, IN took the Iowa test. Here is a histogram of their scores. The histogram is roughly symmetric, has no large gaps or outliers



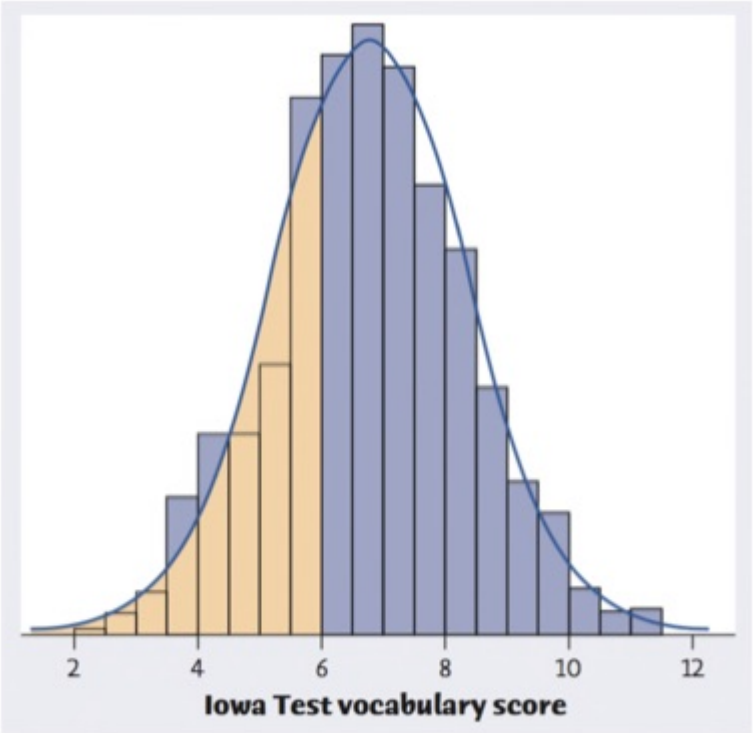
What is the proportion of students who scored 6 or lower?

curve: approximate distribution

bars: observation

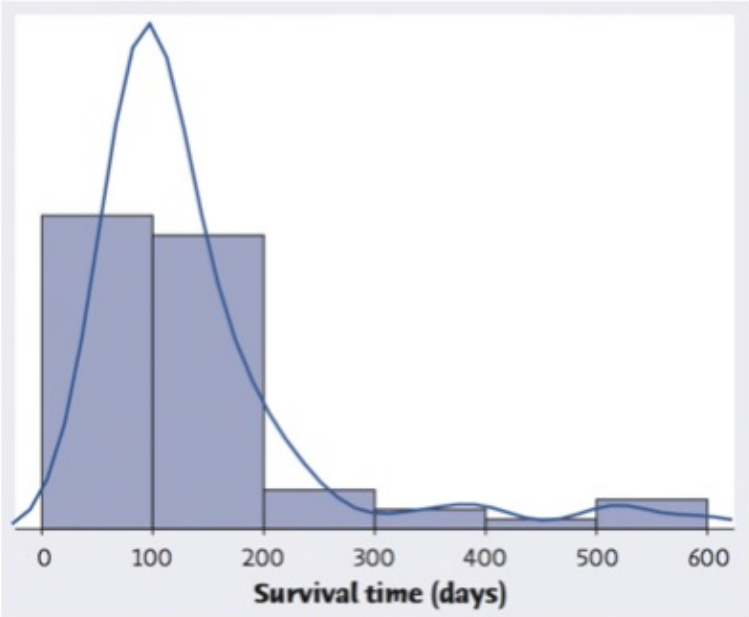


We want to find this proportion via the bell curve and not the histogram. Want the area under the curve less than 6

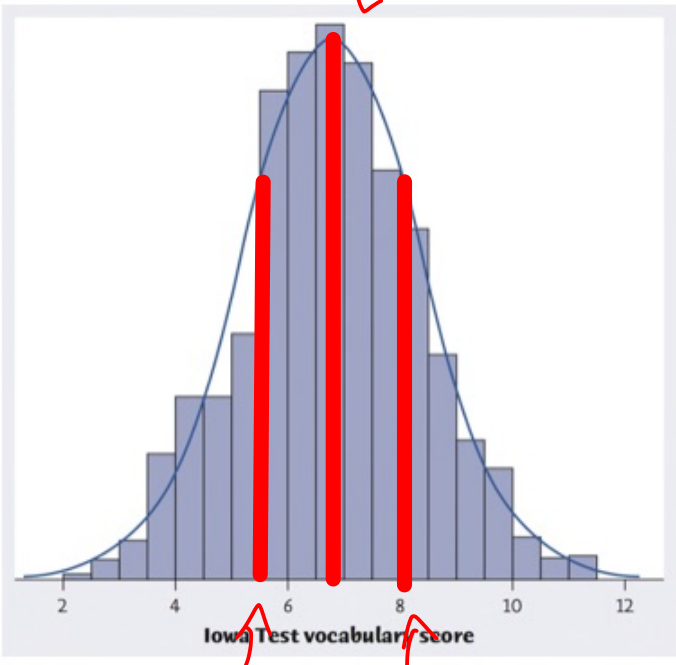


If we rescale the bell curve to have area 1, then the orange area will already be the proportion we want.

This defines something called a density curve: it's positive and has area 1. They come in many shapes: here's one that approximates a skewed dist:



Median + quartiles of density curves: just split the area into quarters.



median

Q_1

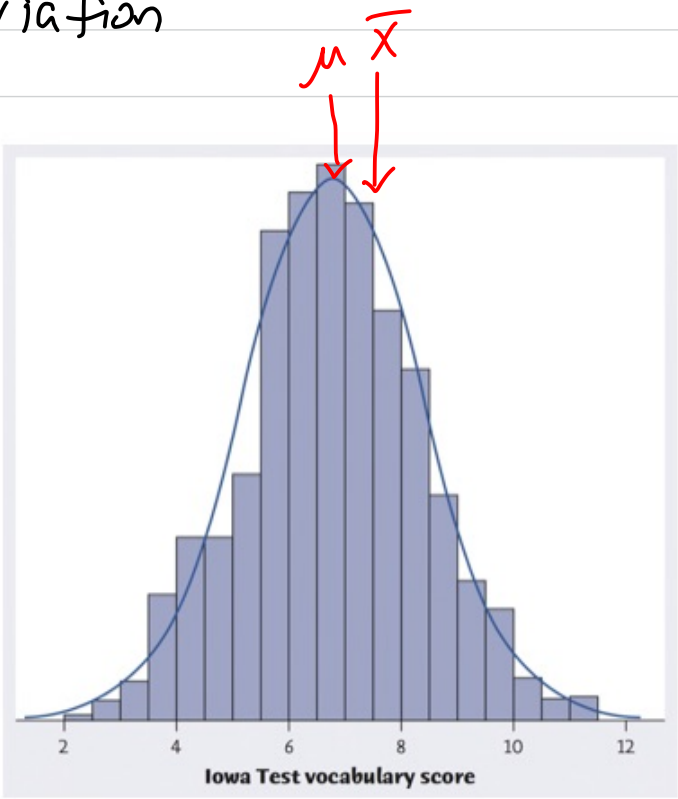
Q_3

no max b/c there is an asymptote at 0

Think of the mean as a weighted average:
It's the "balance point" of the density curve
For symmetric distributions, mean = median

Notational convention:

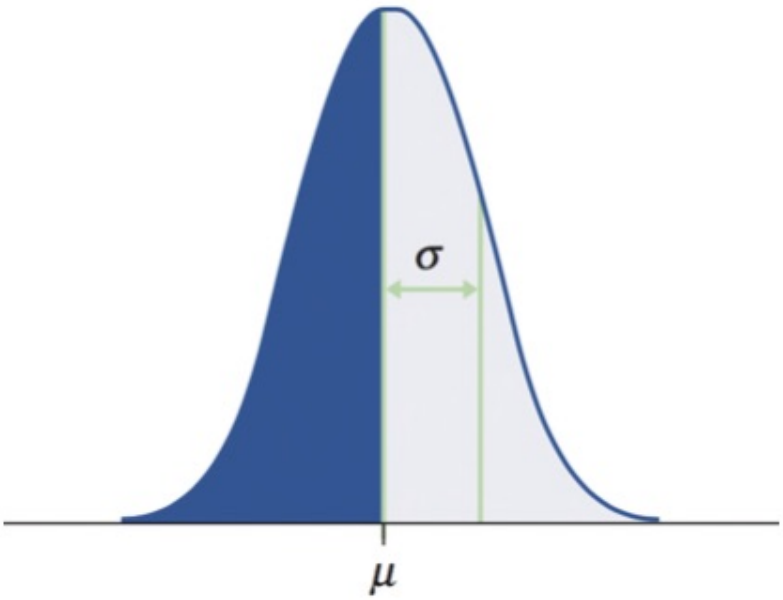
Observation (sample)	Distribution (population)
$\bar{x}$: mean	μ (mu): mean
s : standard deviation	σ (sigma): std dev



It turns out that normal distributions are completely determined by μ and σ.

Def: A normal distribution is one whose density curve is symmetric, single-peaked, and bell-shaped.

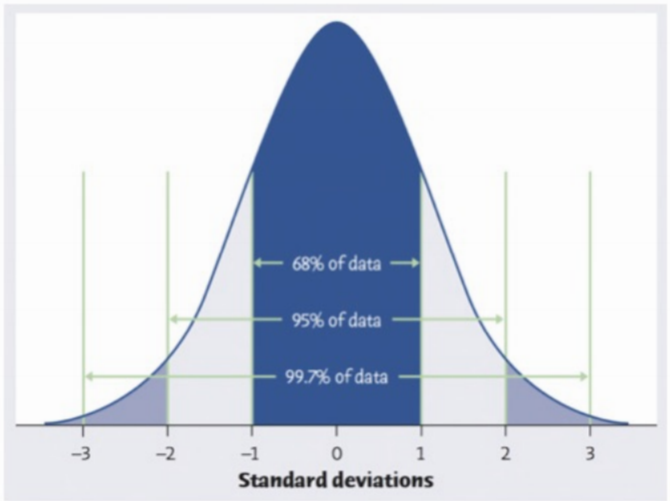
Eyeball σ: it's where the curve changes concavity: imagine skiing down the curve. The point when the slope stops getting steeper is the inflection point, and it's where σ is.



Prop: In a normal distribution with mean μ and standard deviation σ,

1. 68% of the observations lie within σ of μ.
2. 95% of the observations lie within 2σ of μ
3. 99.7% of the observations lie within 3σ of μ

This is called the 68-95-99.7 rule,



$\mu = 0$
$\sigma = 1$

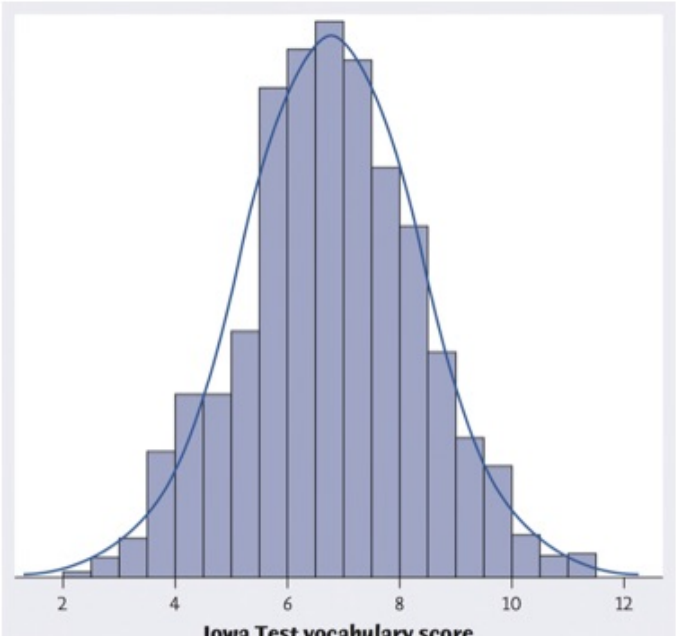
Ex: The # of heads obtained by flipping a coin 1000 times is roughly given by a normal distribution.

$\mu = 500$

can't get to this yet → $\sigma = \sqrt{250} \approx 15.8$

If you run many trials of flipping a coin 1000 times, roughly 68% of those trials will have between $\underbrace{500 - 15.8}_{484.2}$ and $\underbrace{500 + 15.8}_{515.8}$ heads.

Ex:



$\mu = 6.84$
$\sigma = 1.55$ } found by a computer

We expect 95% of the scores to be within 3.74 and 9.94

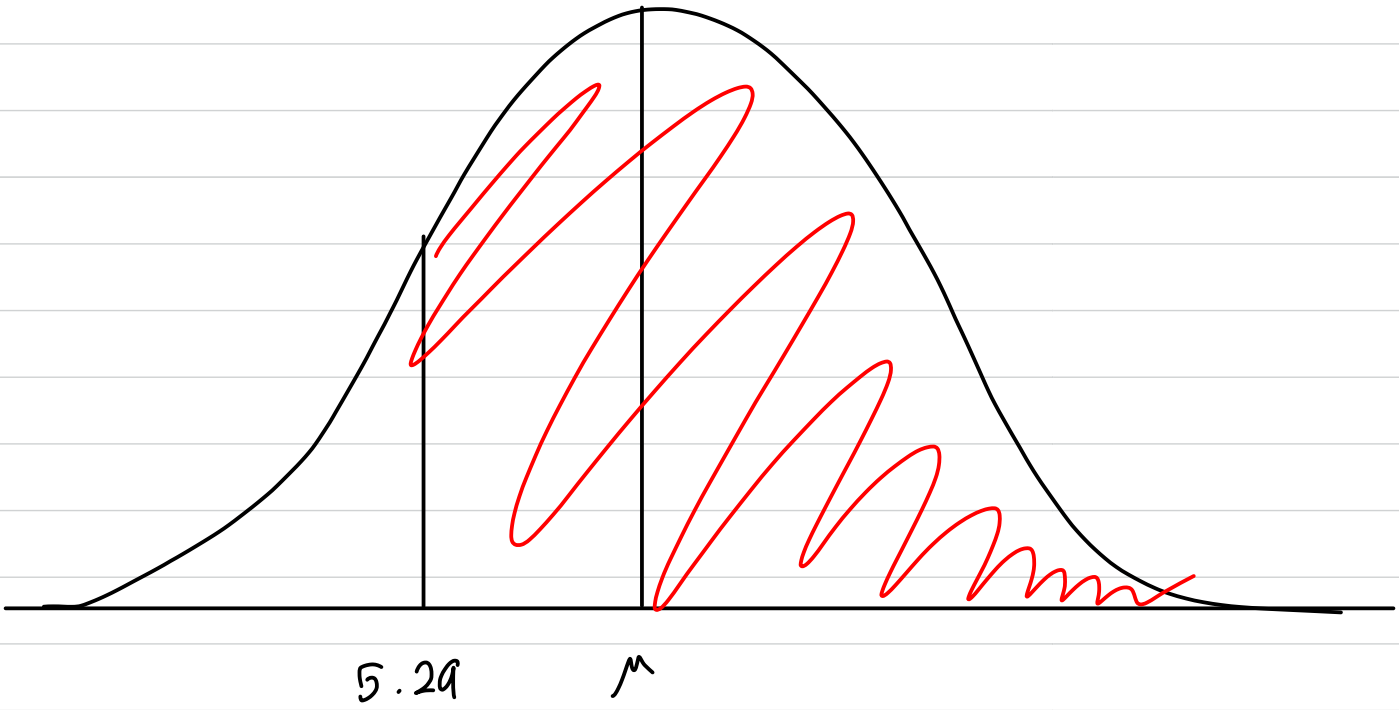
$6.84 - (2)(1.55)$ $\quad$ $6.84 + (2)(1.55)$

Actually: 900 between 3.74 and 9.94, (obtained by counting bars)

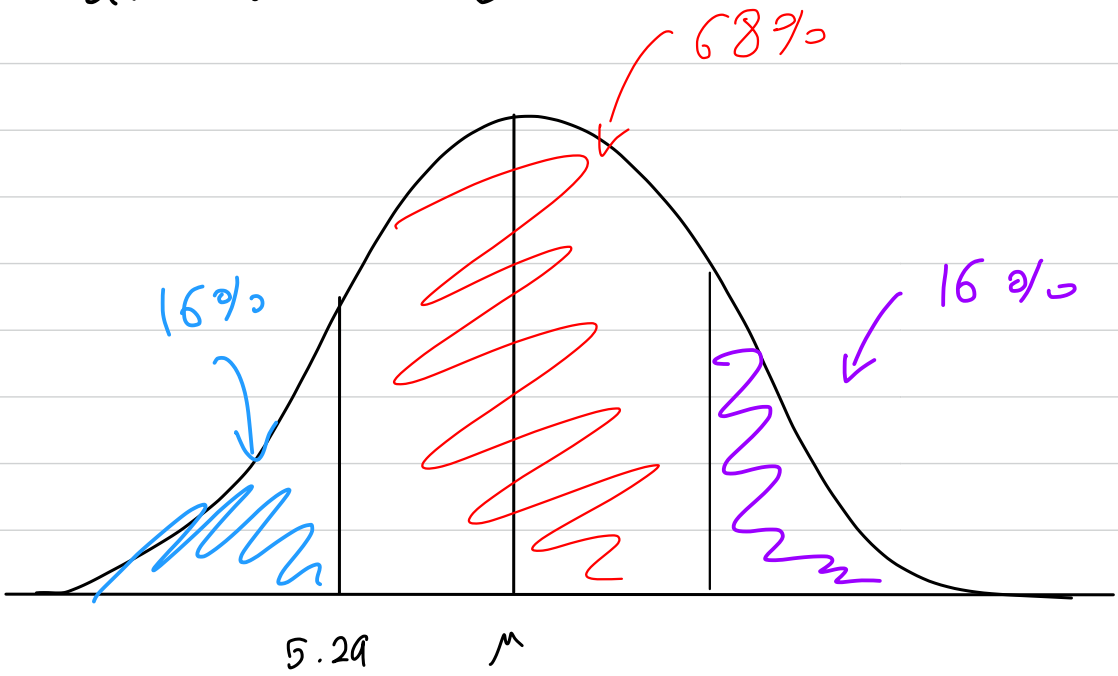
so $900/947 = .9504 = 95.04\%$. Really accurate!

Since the distribution is symmetric, we'd expect as many students to be above 9.94 as below 3.74. In reality, it's 27 and 20.

Ex:



In this distribution, 5.29 is one standard deviation below the mean. What percent of scores are above 5.29?



Since the bell curve is symmetric, the 32% that isn't red is split equally between the left and right tails. In particular, 16% is contained in the right one. So, a total of 68% + 16% = 84% is above 5.29.

Def: We denote a normal distribution with mean μ and standard deviation σ by $N(\mu, \sigma)$.

Ex: the distribution of test scores from the past example is approximately $N(6.84, 1.55)$.

Comment: All normal distributions are the

same up to μ and σ.

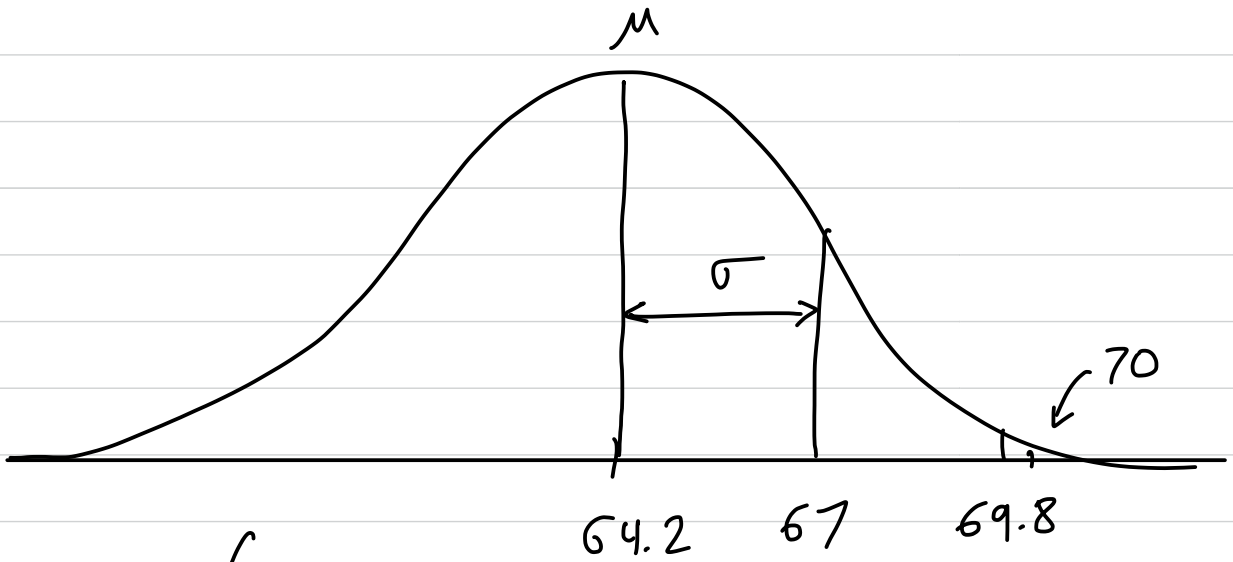
Def: A normal distribution is <u>standardized</u> if we measure in units of σ and center at μ. To standardize a value from a normal distribution, first subtract μ and then divide by σ. So if x is an observation from a distribution of type $N(\mu, \sigma)$, the standardized value of x is

$$z = \frac{x - \mu}{\sigma}.$$

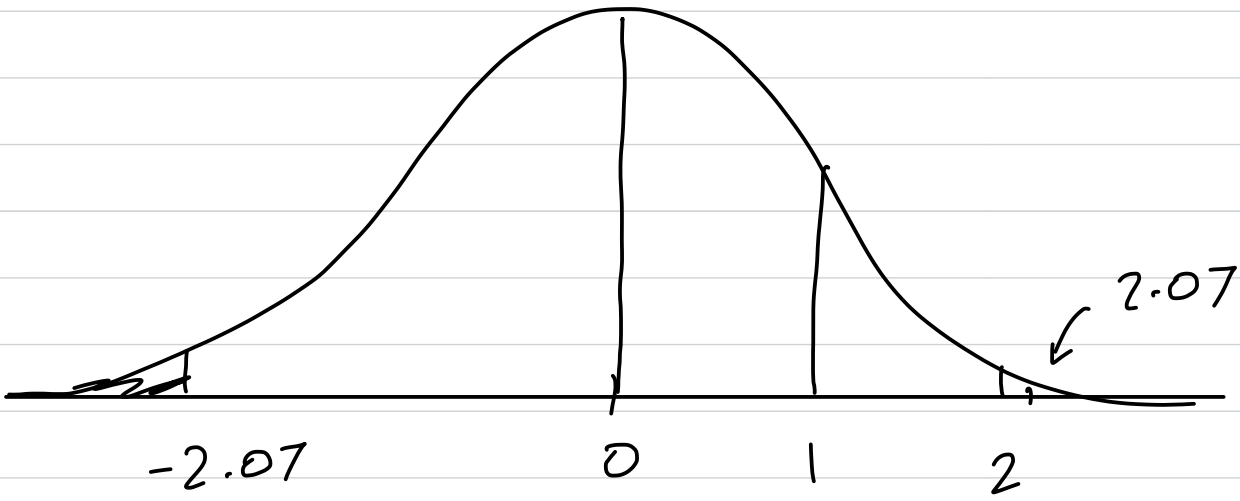
We call this value a z-score. It tells us how many standard deviations away from the mean x was.

Ex: the heights of women between 20 and 29 in the US is approximately $N(64.2, 2.8)$ in inches.

A woman between 20 and 29 in the US with a height of 70 inches has a z-score of $z = \frac{70 - 64.2}{2.8} = 2.07$, so her height is 2.07 standard deviations above the mean.



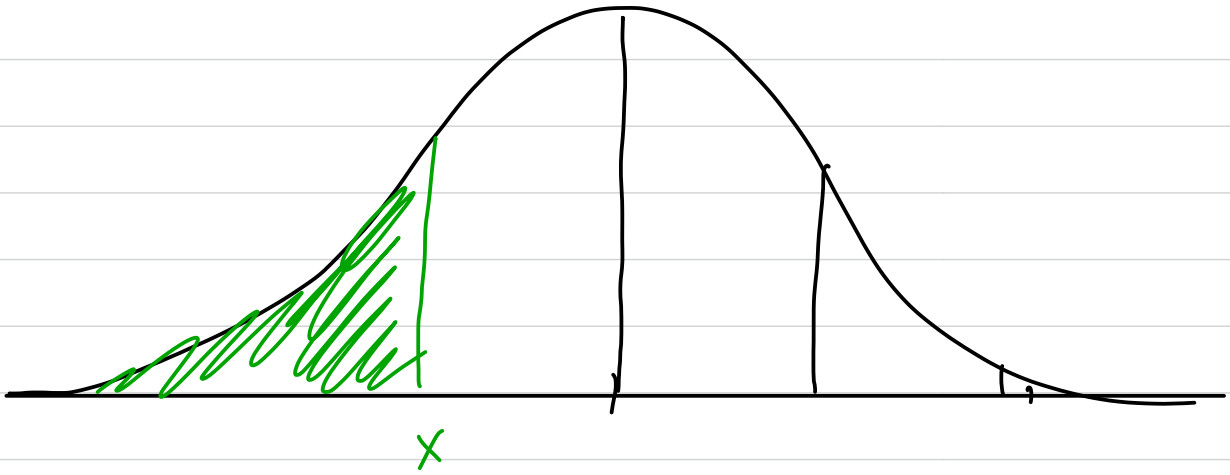
Standardize



Why do we care? z-scores let us compare across different bell curves. For example, men between 20 and 29 in US will have heights that follow a bell curve with a higher mean, but if we use the z-score, we can compare a given man's height to a woman's as a measure of how tall they are <u>relative to their populations</u>.

Although we don't have a formula for an arbitrary area under the standard bell curve, we can use a calculator or table to get a certain kind of area, called a cumulative proportion.

Def: A cumulative proportion for a value x is the proportion of observations in the distribution that are $\leq x$.



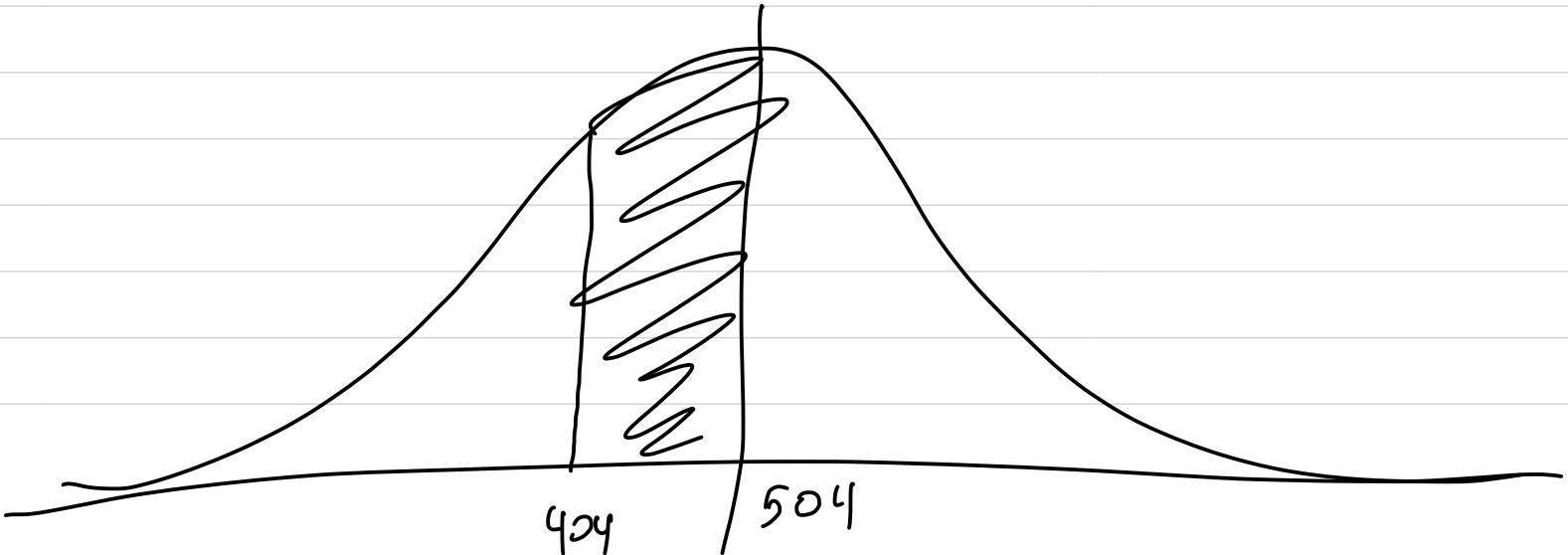
Key: standardizing doesn't change the proportion

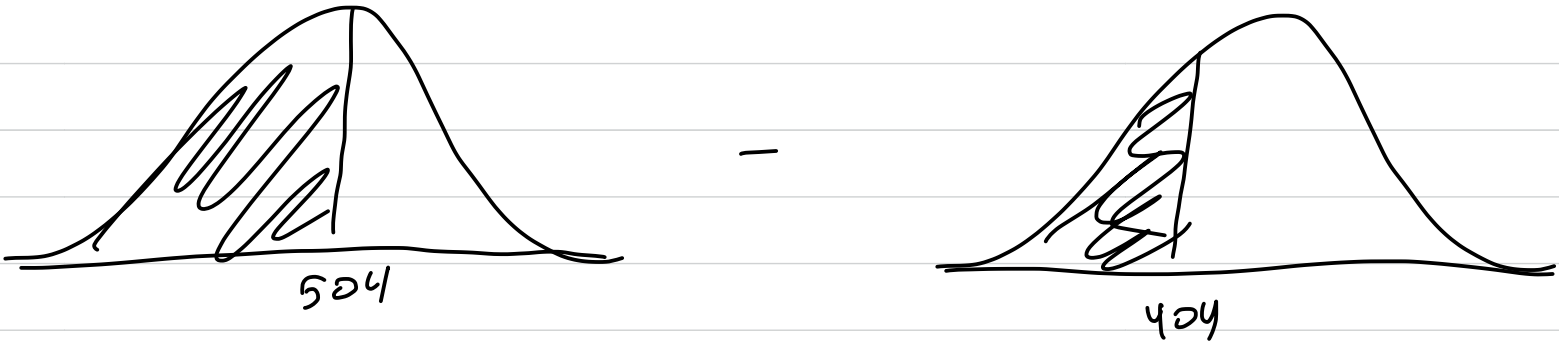
TABLE A Standard normal cumulative proportions

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

.0192 means that 1.92% of women have a z-score below -2.07 (so 1.92% have one above 2.07). Thus (100 - 1.92)% = 98.08% have a z-score below 2.07, so they are shorter than 70 inches.

Ex: Scores on the reading portion of the SAT are roughly N(504, 111). What proportion of students got scores between 404 and 504?





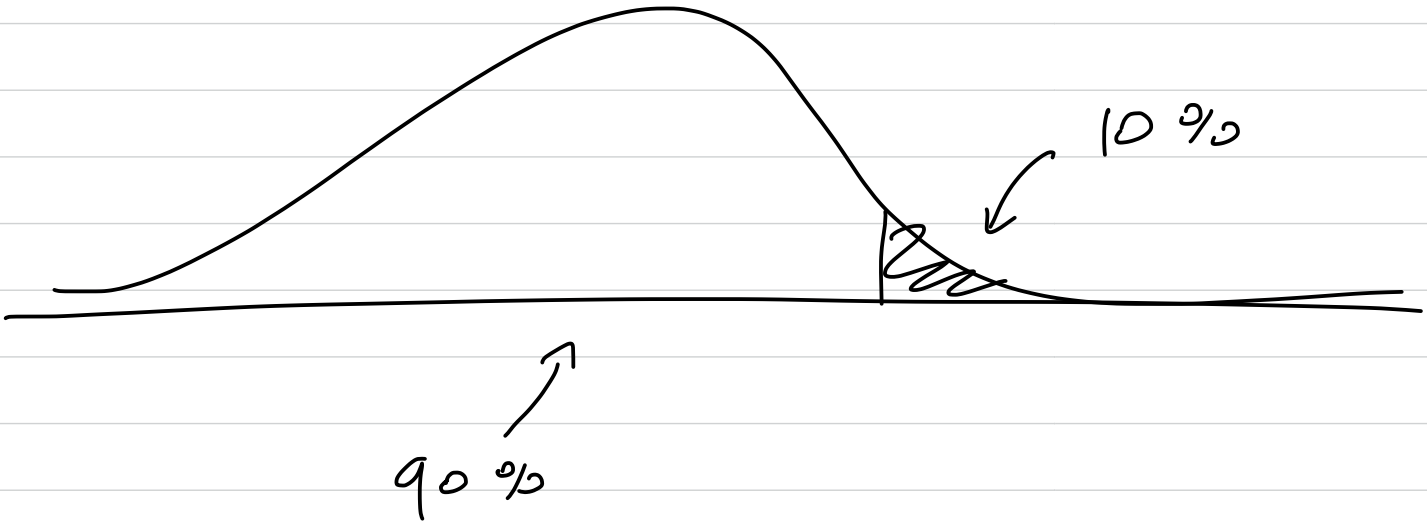
$504: \quad z = \frac{504 - 504}{111} = 0$

$404 = z = \frac{404 - 504}{111} = -\frac{100}{111} = -.901.$

$.5 - .1841 = .3159 \leftarrow$ proportion of students scoring between 404 and 504.

Ex: How high must a student score to

be in the top 10%?



want proportion below to be .9, so we want to find a z-score that corresponds to .9. Unfortunately, our table only goes to .5

A z-score of -1.28 corresponds to a proportion of .1003, so a z-score of 1.28 corresponds to one of .8997.

Let x be the SAT score with z-score of 1.28. Then $1.28 = \frac{x - 504}{111}$

$$x = 647.$$

Chapter 8: Sampling

Def: A population is an entire group we'd like to know something about.

A sample is a part of the

population that we actually collect information from.

Ex: A professor wants to know about students' opinions on social security. She finds the list of all 3465 undergrads at her university and sends a survey to 250 of them. 104 respond.

What is the population?

What is the sample?

Pop: students at this university.

Sample: 104 students who responded.

Why might it be a bad idea to conduct a survey only right outside of a shopping mall? This introduces bias into the sample.

Def: A sample is biased if it systematically favors certain outcomes. One particular way this can happen is when the sample is not representative.

Def: A convienience sample samples the individuals who the easiest to reach only. A voluntary response sample casts a wide net, but relies on individual motivation to get results.

Ex: A college student running for student government wants to know student opinion on college fees, so they survey the students on their floor of their dorm. This is an example of convenience sampling.

Ex: A news network asks all of their viewers to fill out a survey, then presents the results as the country's opinion. This is an example of voluntary response sampling.

Both of these two sampling methods are likely to introduce bias.

Def: A Simple Random Sample, or SRS, is a somewhat straightforward way to take a more unbiased sample. To take an SRS of size n: assign everyone in the population a different ID number, randomly select n ID numbers (can use random.org), and sample those n people.

Pros: avoids bias (larger samples do a better job of avoiding it).

Cons: incredibly inconvenient for large samples.

Def: A Stratisfied Random Sample is one taken by splitting the population into groups of similar individuals, then taking an SRS of each group.

Def: A Clustered Random Sample is one where we split the population into clusters that don't necessarily have similar properties, but that are grouped in some convenient way — for example, by location.

Ex: A consulting business has 6000 other businesses as clients. 5000

of those are small businesses and 1000 are large. We could take a stratified random sample by sampling 500 small businesses and 100 of the large ones, both with an SRS.

Ex: We want to sample from all elementary school teachers in the US, an SRS is nearly impossible.

Instead: cluster by school and take an SRS of schools. Then we survey teacher from the schools sampled.

Def: Undercoverage is the property of

leaving a group underrepresented in a sample. Nonresponse is when a contacted individual does not respond. Response bias is when an individual answers a question dishonestly or misleadingly. Wording bias is when the manner in which a survey is worded significantly affects the response.

Ex: wording a question as only "assistance to the poor" vs only "welfare".