Name: _____

Midterm 1

Math 256

Spring 2023

You have 50 minutes to complete this exam and turn it in. You may use a 3x5 inch two-sided handwritten index card and a scientific calculator, but not a graphing one, and you may not consult the internet or other people. If you have a question, don't hesitate to ask — I just may not be able to answer it. **Enough work should be shown that there is no question about the mathematical process used to obtain your answers.**

You should expect to spend about one minute per question per point it's worth — there are 50 points possible on the exam and 50 minutes total.

Part I (9 points) Multiple choice. You don't need to show your work.

1. (3 points) The DE $\frac{y}{t}y' = 1$ is

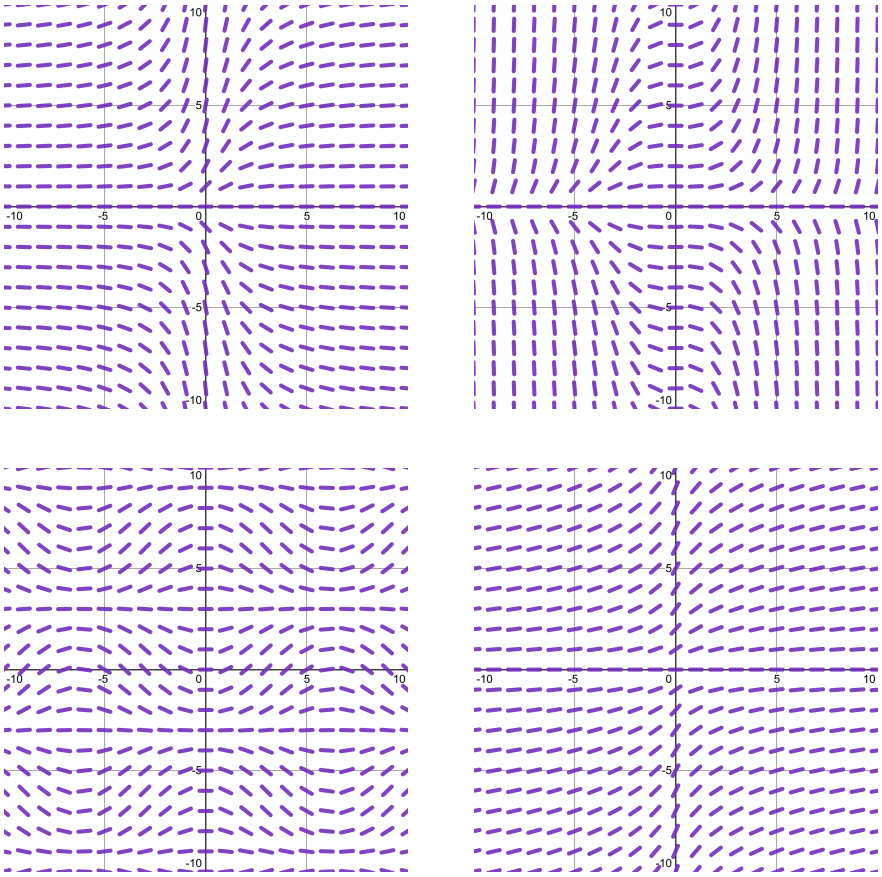
   A) Linear.

   B) Separable.

   C) Exact.

   D) None of the above.

2. (3 points) Which of the following direction fields corresponds to $(1 + x^2)y' = y$? (Circle it)



3. (3 points) A second-order DE has a Wronskian of $W[y_1, y_2] = t^2e^{2t}$. Which of the following initial conditions would **not** have a solution?

   A) $y(0) = 1$, $y'(0) = 4$.

   B) $y(2) = 1$, $y'(2) = 4$.

   C) $y(-2) = 1$, $y'(-2) = 4$.

   D) None of the above.

Part II (9 points) Short-answer. Explain your reasoning and show your work for each question.

1. (3 points) Evaluate $\dfrac{\partial}{\partial x}\left[\log(\cos(xy))\right]$.

2. (6 points) One fundamental solution to $t^2y'' - 4ty' + 6y = 0$ is $y = t^2$. What is the other fundamental solution?

Part III (32 points) More involved questions with multiple parts.

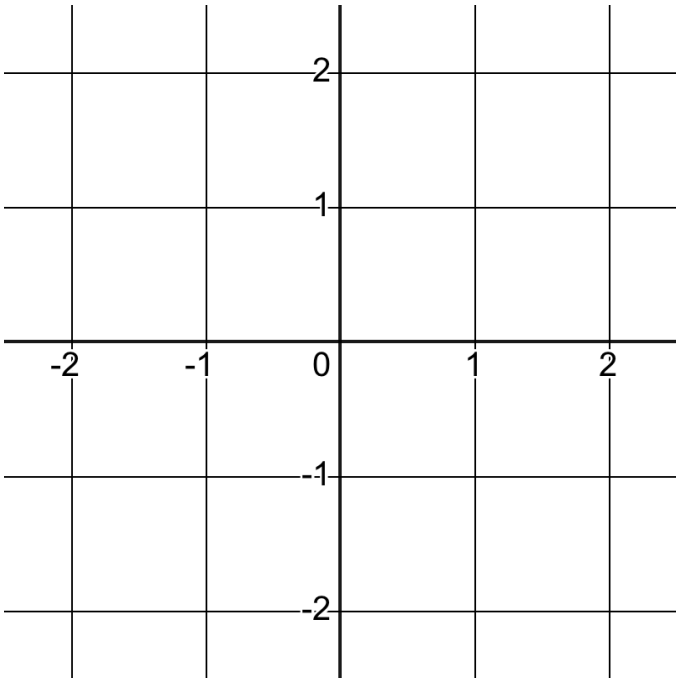
1. (22 points) The population of rabbits on an island *measured in thousands* after t years is given by the function $P(t)$, which satisfies the DE $P' + t^2P = t^2$. There are initially 500 rabbits when we start measuring (i.e. at time $t = 0$).

a) (6 points) Solve for the function P using integrating factors.

b) (6 points) Solve for P using separation of variables.

c) (4 points) What will the rabbit population settle down to over time?

d) (6 points) Sketch a direction field for the DE below for $-2 \le t \le 2$ and $-2 \le P \le 2$. Draw your solution curve from parts a) and b).



2. (12 points) By changing the coefficients in a DE just slightly, we can produce wildly different solutions — let's see this in action.

a) (4 points) Find the general solution to $y'' + 6y' + 8y = 0$.

b) (4 points) Find the general solution to $y'' + 6y' + 9y = 0$.

c) (4 points) Find the general solution to $y'' + 6y' + 10y = 0$.

d) (2 points extra credit) Solve all of the previous DEs for the particular solutions with $y(0) = 0$ and $y'(0) = 6$. What are the values of $y(1)$ for all three?