15: distribution of $\bar{x}$

16: confidence intervals

17: hypothesis tests

20: t-scores

21: distribution of $\bar{x}_1 - \bar{x}_2$

$\bar{x}$

p

Chapter 22: Inference about Proportions

Ex: 50% of Americans say that breakfast is the most important meal, but only ~30% eat breakfast regularly.

Let p be the proportion who eat breakfast regularly. $p = .3$

$$\bar{x} \longrightarrow \mu$$

$$\hat{p} \longrightarrow p$$

Def: In a sample, the proportion of individuals with a certain statistic is written $\hat{p}$ (read p-hat)

Thm: In a sample with n individuals, the

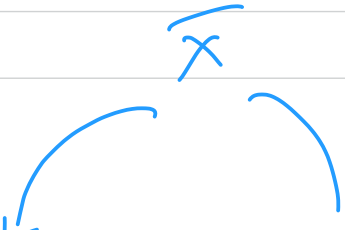
distribution of $\hat{p}$ is approximately $N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$. As with $\bar{x}$, this is a better approximation as n increases.

Ex: We take an SRS of 1500 Americans and find that 470 regularly eat breakfast. $\hat{p} = \frac{470}{1500} = .313$.

$\hat{p}$ is roughly $N\left(.3, \sqrt{\frac{(.3)(.7)}{1500}}\right)$

$N(.3, .012)$

Recall:



know σ → z^*

didn't know σ → t^*

In the case of $\hat{p}$, $\mu = p$ and $\sigma = \sqrt{\frac{p(1-p)}{n}}$

↑ involves p, the thing we want to approximate

$\Rightarrow$ we will never know σ beforehand.

Prop: the confidence interval for p given $\hat{p}$ and n is

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

(approximates $\hat{p} \pm z^* \sqrt{\frac{p(1-p)}{n}}$)

When we did this approximation to $\bar{x}$, we went from $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$ to $\bar{x} \pm \boxed{t^*} \frac{s}{\sqrt{n}}$

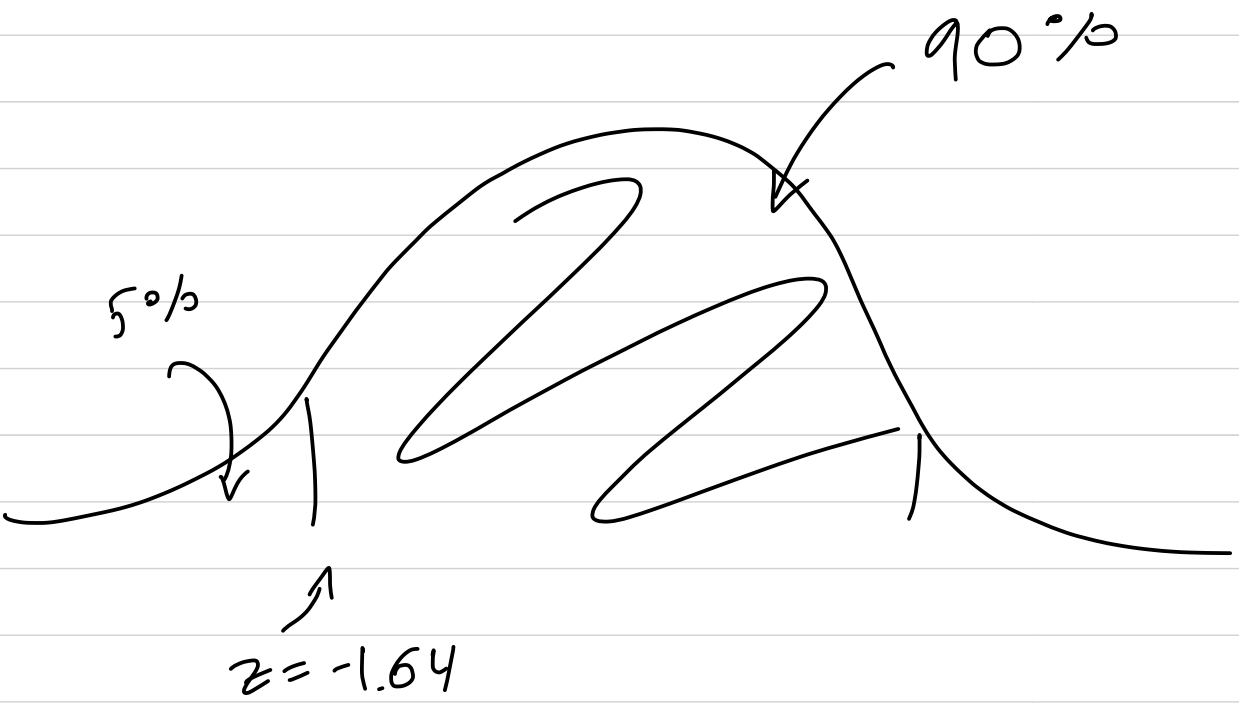
Why are we not using a t-score? It's because the mean is used in calculating the standard deviation. (Lots of complicated math going on behind the scenes that we don't need to worry about)

Ex: sample 30 Americans on whether they eat breakfast, and 9 do. Find a 90% confidence interval

for the proportion of all Americans that eat breakfast.

$\hat{p} = .3 \qquad n = 30$

$$CI: \quad .3 \pm z^* \sqrt{\frac{.3(1-.3)}{30}}$$



$z^* = 1.64$

$$.3 \pm 1.64\sqrt{\frac{.3 \cdot .7}{30}}$$

$$.3 \pm .137$$

Margins of error: $MoE = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$\hat{p}$ depends on n, so we need to approximate $\hat{p}$ before taking a sample.

Two ways to accomplish this:

① A previous sample was taken and approximated p.

② Take $\hat{p} \approx .5$ to find n. This is okay because $\hat{p} = .5$ maximizes MoE, so what $\hat{p}$ ends up being

your approximation of p will be at least as good as if $\hat{p}$ were .5.

Ex: Two candidates running for mayor. You take a SRS to find the proportion of the population voting for candidate #1. You want 90% confidence and a margin of error no larger than .03. How many people do you need to survey?

Assume $\hat{p} = .5$

$$MoE = z^* \sqrt{\frac{.5(1-.5)}{n}}$$

$$.03 = 1.64\sqrt{\frac{.25}{n}}$$

$$.03 = 1.64\,\frac{.5}{\sqrt{n}}$$

$$\sqrt{n} = \frac{(1.64)(.5)}{.03} = 27.33$$

$$n = 747.11 \Rightarrow 748$$

Thm. Suppose we have a proportion p_0 of the population that has a certain statistic and a subset of that population whose proportion is p. The null hypothesis that $p = p_0$ has test statistic $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$.

Want to use this test when

n is large enough that both np_0 and $n(1-p_0)$ are at least 10

Ex: 20 pairs of dogs and their humans per sheet, 2 sheets — 1 with dogs and humans matched, the other not. Students picked either sheet 1 or sheet 2 based on which they thought had a stronger resemblance. There were 61 students, and 49 chose correctly. If there were no correlation, we'd expect 50% of the students to guess right. Is this sufficient evidence to indicate that the students were doing better than

guessing?

$H_0 : p = .5$

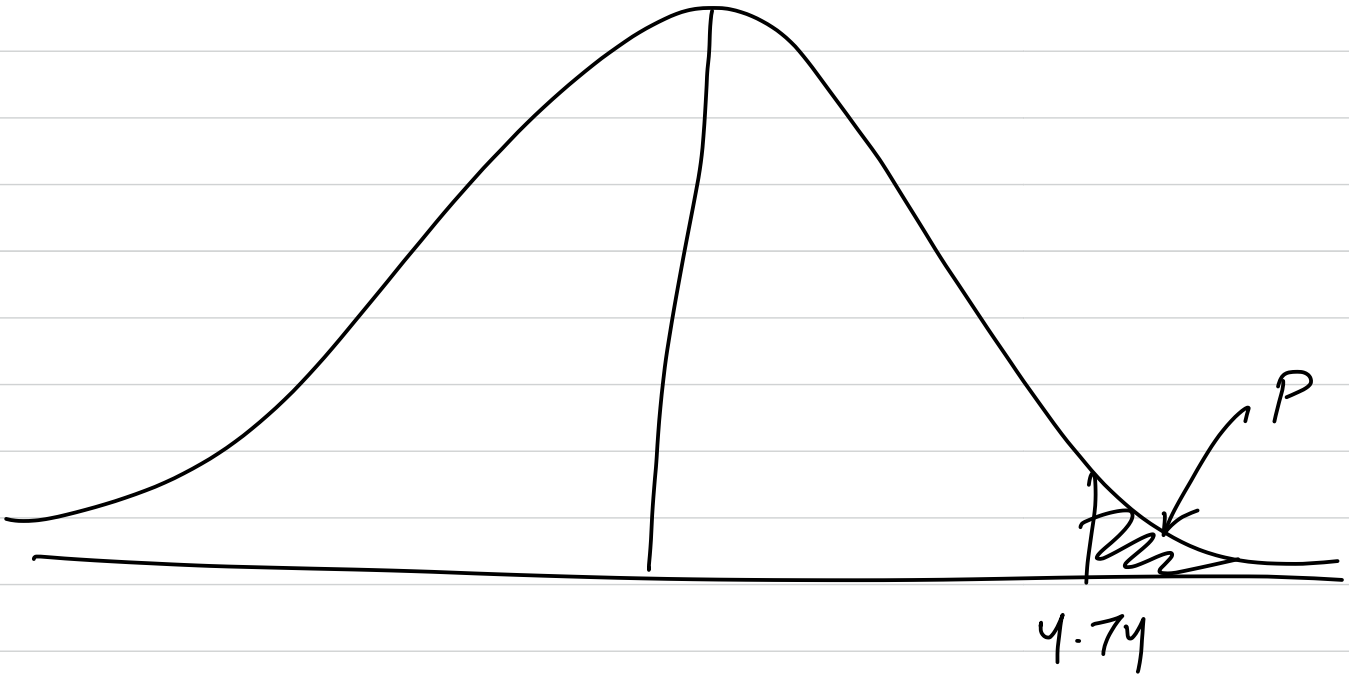
$H_a : p > .5$

$\hat{p} = 49/61 = .8033$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.8033 - .5}{\sqrt{\frac{.5(1-.5)}{61}}}$$

$z = 4.74$

Is this valid? $n p_0 = 61 \cdot .5 = 30.5$ and $61(1-.5) = 30.5$, both of which are ≥ 10, so yes.



p is so small that it is definitely less than .05 (we weren't given a value for α in the problem statement, so we take $\alpha = .05$). Therefore, we reject the null hypothesis.

Ex: Two sample of 29 and 35 people respectively, from two group measure heights

$\bar{x}_1 = 65$ inches

$\bar{x}_2 = 66$ inches

$s_1 = 2$

$s_2 = 3$

distribution $\bar{x}_1 - \bar{x}_2$ is

$$N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

If we want to get a 95% CI,

we take $\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

t^* ↓

$\min(28, 34) = 28$

$$(65 - 66) \pm 2.048 \sqrt{\frac{2^2}{29} + \frac{3^2}{35}}$$

$$-1 \pm 1.287$$

$\bar{x}_1 - \bar{x}_2$

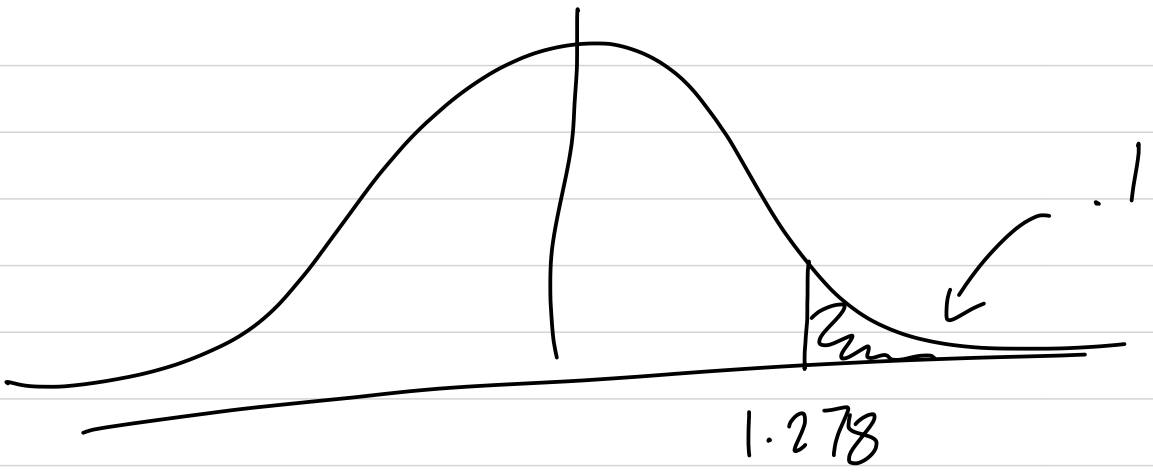
DOF: $\min(n_1 - 1, n_2 - 1)$

on campus ↓

$H_0: \mu = 2.7$ $\quad$ $H_a: \mu > 2.7$

$\bar{x} = 2.9$ $\quad$ $n = 20$

$$z = \frac{2.9 - 2.7}{.7/\sqrt{20}} = 1.278$$



$.1 \overset{?}{<} .05$ No

Fail to reject

$\bar{x} = 2.485$ $s = .819$

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

$$2.485 \pm t^* \frac{.819}{\sqrt{20}}$$

$C = 95\%$ $\quad$ $DOF = 19$

$$t^* = 2.093$$

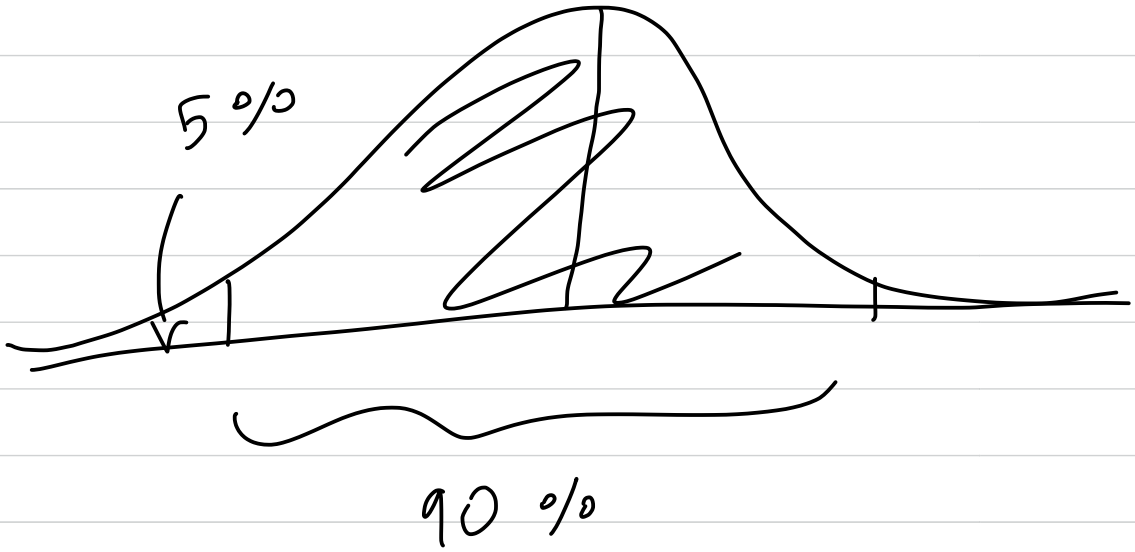
$$2.485 \pm 2.093 \frac{.819}{\sqrt{20}}$$

$$2.485 \pm .383$$

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

$$26.8 \pm z^* \frac{7.8}{\sqrt{654}}$$

$C = 90\%$



$z = -1.64$

$z = 1.64$

$$26.8 \pm 1.64 \frac{7.8}{\sqrt{654}}$$

Know $\mu = 2.7$

Hypothesis: First-year students have a lower GPA than overall average.

$H_0: \mu = 2.7$ (↑ 1st-year)

$H_a: \mu < 2.7$ (↑ one-sided)

Use $\alpha = .02$

$\bar{x} = 2.485$

$s = .819$

$n = 20$

$$t^* = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$t^* = \frac{2.485 - 2.7}{.819/\sqrt{20}} = -1.174$$

DOF = 19

Closest on table on row 19

is $t^* = 1.066$

One-sided p-value: .15

Not less than .02

fail to reject

Quiz 4

① $\bar{x}_1 - \bar{x}_2$ is roughly $N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$

$\min(n_1 - 1, n_2 - 1)$

② $\mu_0 = 2.7$ $\sigma_0 = .7$

one-sided p-value

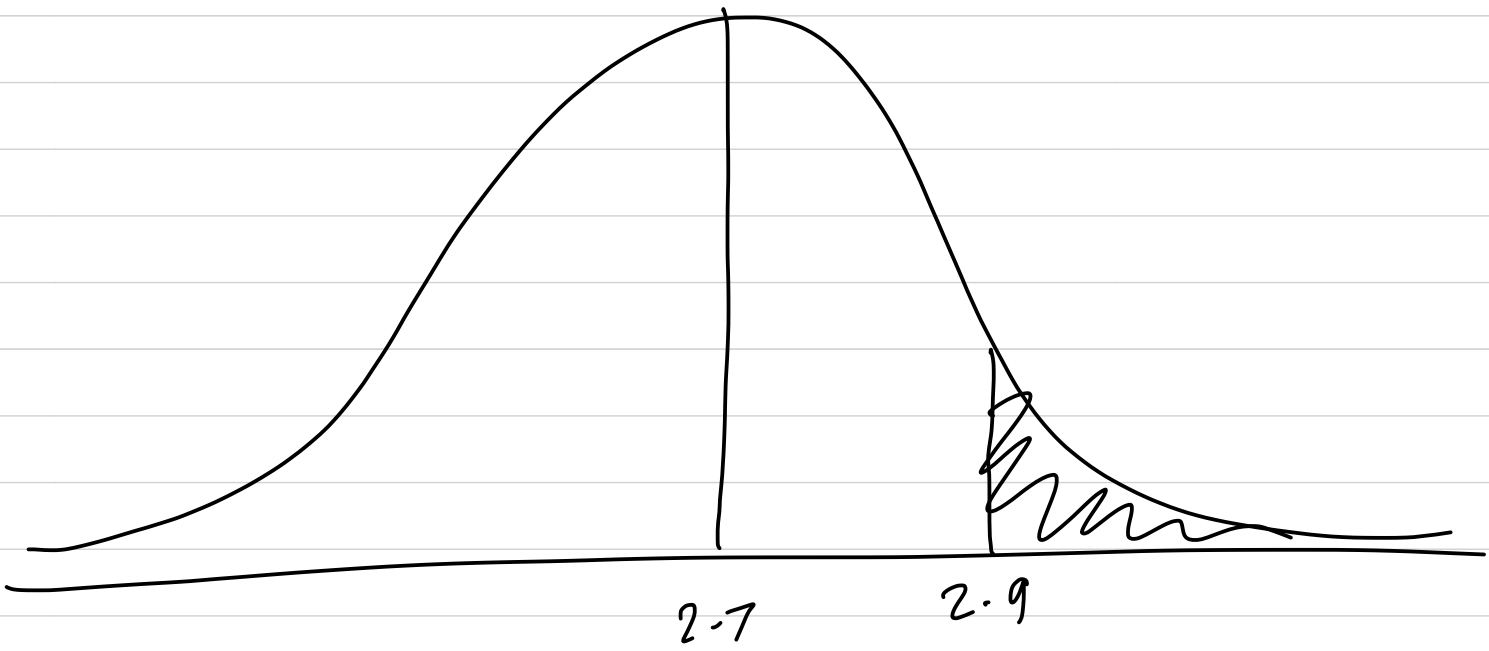
$H_0: \mu = 2.7$ $H_a: \mu > 2.7$

μ: students on campus

$\bar{x} = 2.9, \quad n = 20, \quad \alpha = .05$

$\bar{x}$ is roughly $N\left(2.7, \frac{.7}{\sqrt{20}}\right)$

$$z = \frac{2.9 - 2.7}{.7/\sqrt{20}} = \frac{.2}{.7/\sqrt{20}} = 1.278$$



from z-score table

$z = -1.278 \Rightarrow .1$

P-value : $p = .1$

Is $.1 < .05$? No, so we fail to reject the null hypothesis.

③ $n = 20$, $\bar{x} = 2.485$, $s = .819$

CI: $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$

t^*: $C = 95\%$, $DOF = 19$

$t^* = 2.093$ ← t-score table, row 19, column 95%

CI: $2.495 \pm 2.093 \cdot \frac{.819}{\sqrt{20}} = 2.495 \pm .383$

Chapter 23: Comparing Two Proportions

Recall: p : population proportion

$\hat{p}$: sample proportion

$\hat{p}$ is roughly $N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$, so approximately $N\left(p, \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$.

With p, we use z^* rather than t^*.

Ex: We want to study the difference in proportions of UO and OSU students who live on campus. We take a sample of 30 students from UO and 35 from OSU and find that

9 of the UO students and 12 of the OSU students live on campus. How do we estimate the difference in proportions among all students?

p_1 : proportion of UO students living on campus

p_2 : proportion of OSU students living on campus

Want to estimate $p_1 - p_2$ using $\hat{p}_1 - \hat{p}_2$.

Thm: the distribution of $\hat{p}_1 - \hat{p}_2$ is approximately $N\left(p_1 - p_2, \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}\right)$

n_1, n_2 = sample sizes of $\hat{p}_1$ and $\hat{p}_2$

Ex $n_1 = 30$
$n_2 = 35$
$\hat{p}_1 = 9/30 = .3$
$\hat{p}_2 = 12/35 = .343$

So the distribution of $\hat{p}_1 - \hat{p}_2$ is approximately

$$N\left(-.043, \sqrt{\frac{.3(1-.3)}{30} + \frac{.343(1-.343)}{35}}\right), \text{ so}$$

$$N(-.043, .116)$$

To take a confidence interval for $p_1 - p_2$, take the standard error

$$SE = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}.$$ Then the

confidence interval is $\hat{p}_1 - \hat{p}_2 \pm z^* SE$.

Ex: If we want a 99% CI for the difference in proportions of students living on campus between UO and OSU, we take $\hat{p}_1 - \hat{p}_2 = -.043$, $SE = .116$, and $z^* = 2.57$. So we have a

b/c -2.57 gives .005 on the table

CI of $-.043 \pm .116 \cdot 2.57 = -.043 \pm .298$.

Taking a hypothesis test of the difference in proportions: first, find the pooled sample proportion $\hat{p} = \frac{\#\text{ total successes}}{\#\text{ total individuals}}$ between both groups. Then the z-score (assuming

H_0 is true) is $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$.

What exactly is H_0 in this case? It's always that $p_1 = p_2$ (so $p_1 - p_2 = 0$)

Ex: Use the data about students living on campus to assess the claim that a different proportion of students live on campus at OSU compared to UO. Use a significance level of .1.

2-sided P-value

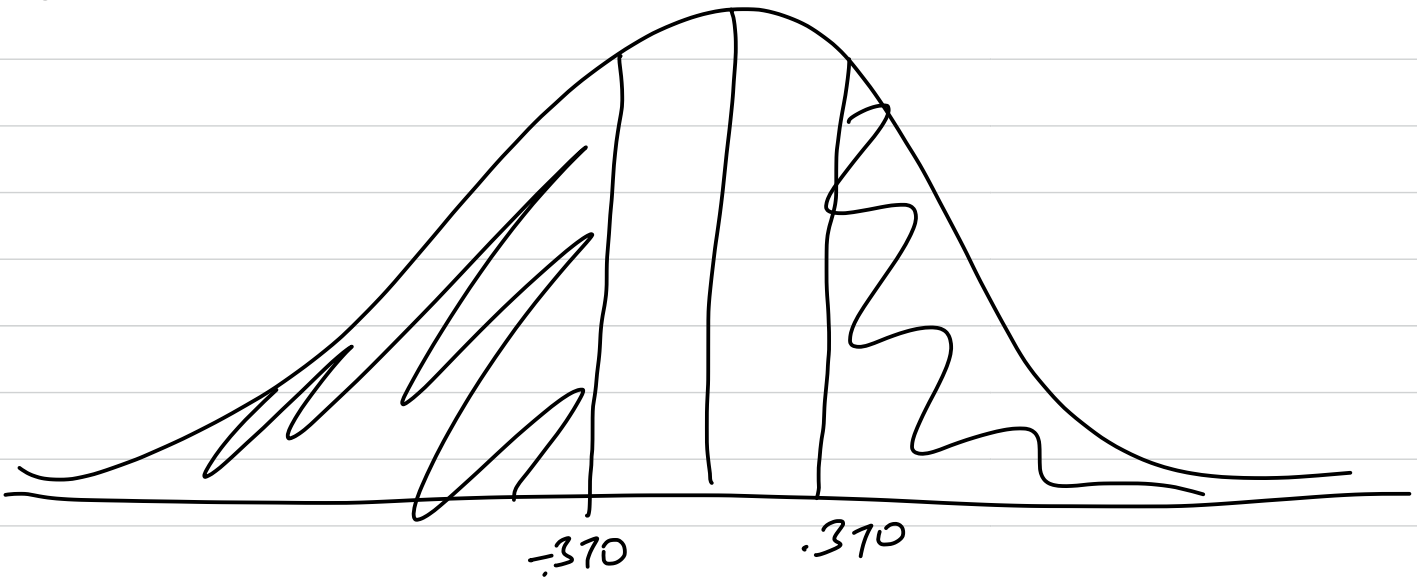
$H_0: p_1 = p_2$ $\quad H_a = p_1 \neq p_2$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{-.043}{\sqrt{.323(1-.323)\left(\frac{1}{30} + \frac{1}{35}\right)}}$$

$$\hat{p} = \frac{\#\text{ total students living on either campus}}{\#\text{ total students in sample}}$$

$$\hat{p} = \frac{9 + 12}{30 + 35} = .323$$

$z = -.370$



$z = -.37 \Rightarrow .3557$ by table

$P = 2(.3557) = .7114 \not< \alpha = .1$

So we fail to reject H_0.

Quiz 4

(1) $\bar{x}_1 - \bar{x}_2$ is approximately $N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$.

$\bar{x}$: DOF $= n - 1$

$\bar{x}_1 - \bar{x}_2$: DOF $= \min(n_1 - 1, n_2 - 1)$

(2)

$\mu = 2.7$

$\sigma = .7 \Rightarrow$ z-score

$\bar{x} = 2.9$

$s = ? \Rightarrow$ t-score

$n = 20$

$\alpha = .05$

In general, if you know σ, use a z-score b/c it gives a better

approximation. Otherwise, try to use a t-score.

Hypothesis test: find z/t-score, find p-value, compare against α.

$H_0: \mu = 2.7$ (μ: on-campus)

$H_a: \mu > 2.7$ (μ: on-campus)

$$z = \frac{\bar{x} - 2.7}{.7/\sqrt{20}} = \frac{2.9 - 2.7}{.7/\sqrt{20}} = 1.28$$

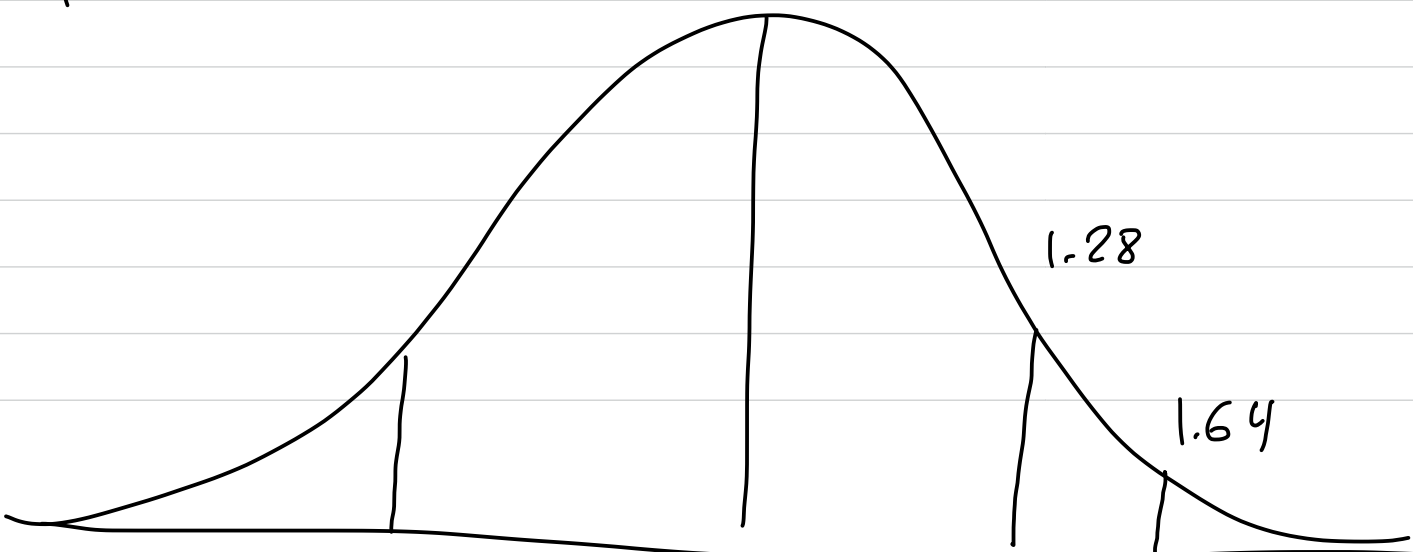
$z = -1.28 \Rightarrow .1003$

$p = .1003$

Is $p < \alpha$? No, so we fail to reject H_0.

Alternatively: $\alpha = .05 \Rightarrow z = -1.64$

We want $z = 1.64$ So, we can compare our z-value of 1.28 to 1.64



$\mu = ?$

$\sigma = ?$

$\bar{x} = 2.485$

$s = .819 \implies t\text{-score}$

$n = 20$

$C = 95\%$

don't know this

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Table: $t = 2.093$

$$2.495 \pm 2.093 \frac{.819}{\sqrt{20}}$$

$$2.495 \pm .383$$

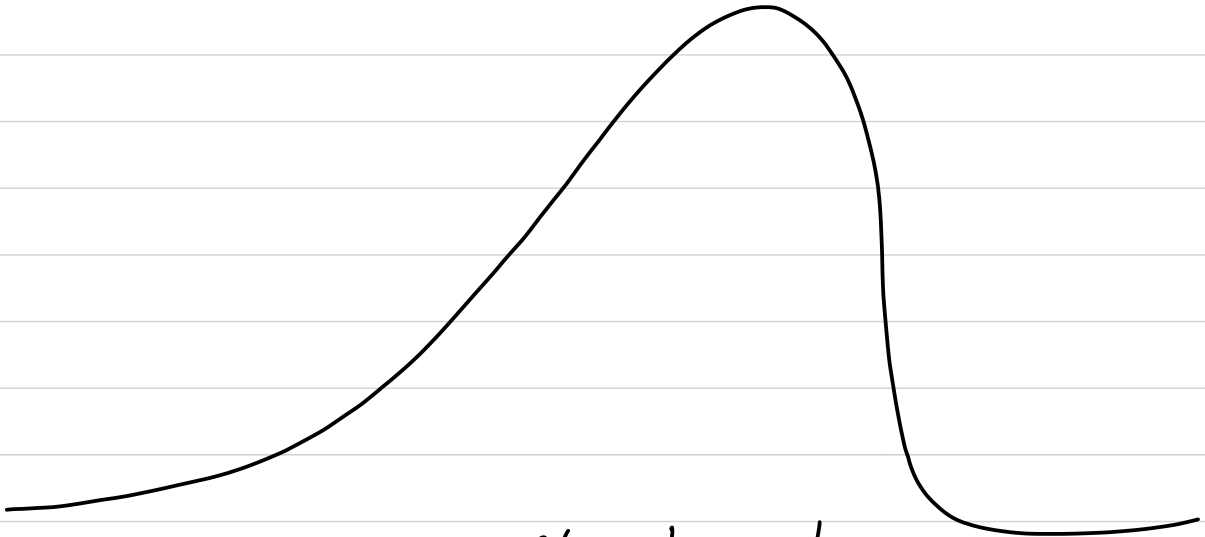
Course summary

Chapter 1: Defs of individuals, statistics, variables, etc.

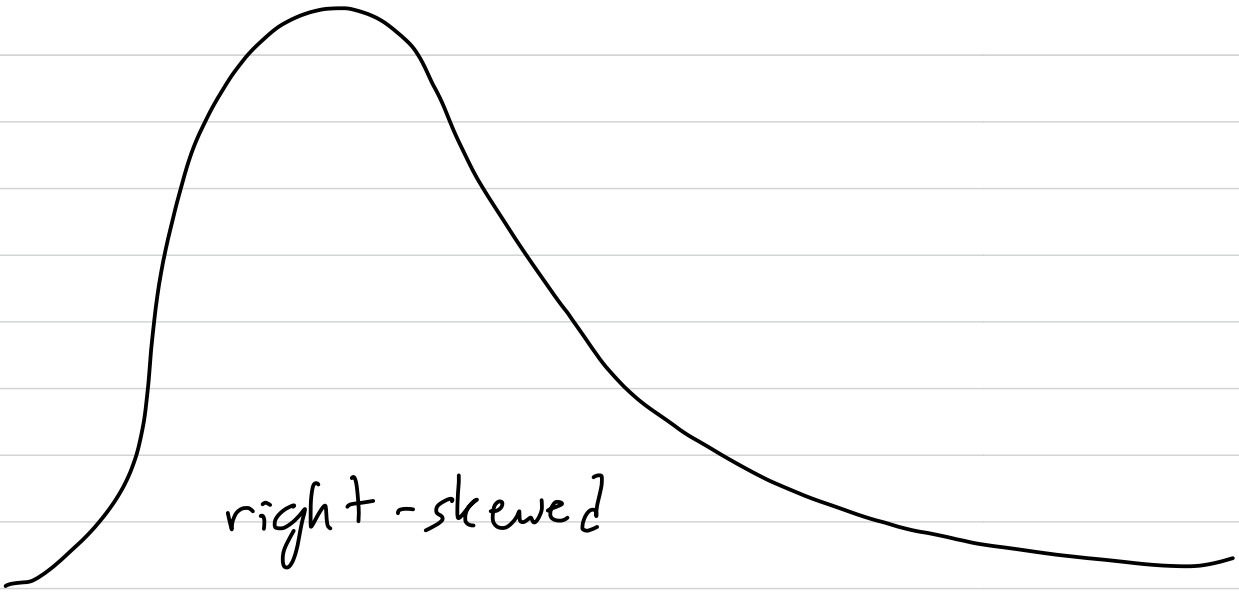
Distributions: give the information of what values a variable can take and how often it takes those values. Can be categorical or quantative (i.e. categories vs. measurements) and discrete or continuous (i.e. finite # of possible values vs. possible values along a line or curve).

Pie charts / bar graphs / histograms

Skews: the direction (if any) where the tail of a distribution extends



left-skewed



right-skewed

not resistant

resistant

Center: mean or median

Variability: standard deviation or 5-number summary

Measures are resistant when they aren't significantly affected by outliers/skews

Stem-and-leaf plots

Chapter 2

Def of mean, median, standard deviation, 5-number summary

Method to find outliers given 5-num summary

Box-plots and a modified version

with outliers marked as circles: once the outliers are removed and marked as dots, then the "whiskers" of the plot extend to the max and min non-outliers

Ex: 1, 2, 2, 5, -1, 20, 0, -2, -4, 0

-4, -2, -1, 0, 0, 1, 2, 2, 5, 20

-1.5 .5 3.5

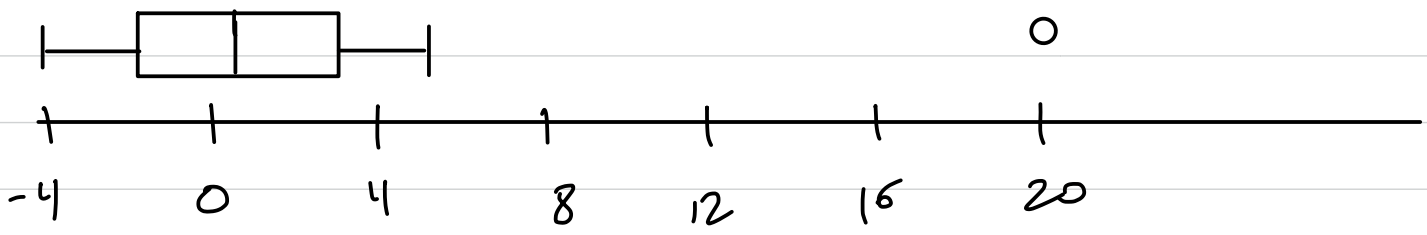
IQR: 3.5 + 1.5 = 5

$-1.5 - 1.5\,IQR \longrightarrow 3.5 + 1.5\,IQR$

$-9 \longrightarrow 11$

20 is an outlier

5-num: $-4, -1.5, -.5, 3.5, 20$



Chapter 3

z-scores: $z = \frac{x - \mu}{\sigma}$

	mean	std dev
pop	μ	σ
sample	$\bar{x}$	s

68-95-99.7

z-score table

Chapter 8

Population, sample, bias (i.e. one group being over/under represented)

SRS: equal chance for anyone to be chosen

Stratisfied Random Sample: split into groups of similar individuals, then take an SRS of each group

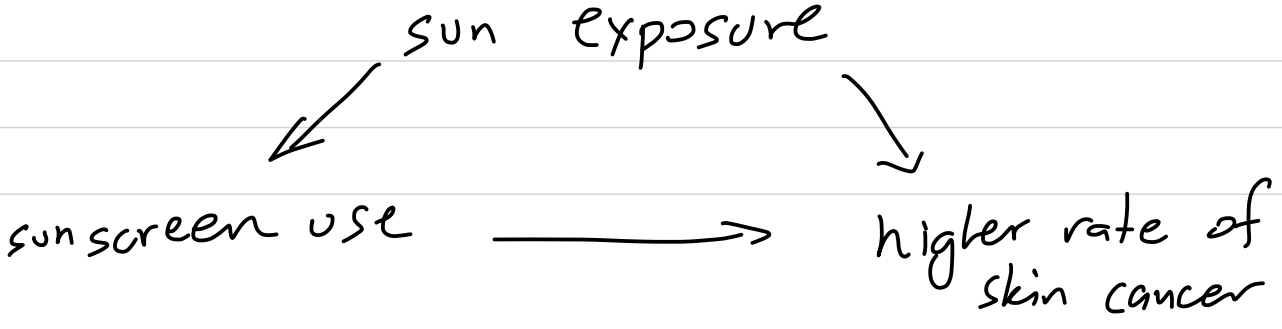
Clustered Random Sample: group by clusters like location and sample clusters

Chapter 9

Observational studies vs. experiments (i.e. whether you attempt to change variables)

Correlation ≠ causation

Lurking variables can make it look like an implication exists where it doesn't.



Explanatory and response variables: variables that we change and measure, respectively, in an experiment.

Control groups: groups of subjects that do not receive treatment

Randomization: using an SRS to divide subjects into groups

Replication: having the entire experiment replicated in other labs to verify the results.

Chapter 12

Sample space: the set S of all possible outcomes

Ex: 3 dice rolled, record the 3 numbers

$$S = \{(1,1,1), (1,1,2), \cdots\}$$

Event: any outcome or group of outcomes in the sample space

$$S = \{(a, b, c) \mid a, b, c \in \{1,2,3,4,5,6\}\}$$

Disjoint: two events that cannot occur simultaneously

Probability $P(A) = \frac{\#\text{ of outcomes with } A}{\#\text{ of outcomes total}}$

Disjoint probabilities add: if A and B are disjoint, then $P(A \text{ or } B) = P(A) + P(B)$.

Inverting probabilities: $P(\text{not } A) = 1 - P(A)$

Random variable: placeholder for the possible outcomes.

Ex: if X is the random variable corresponding to the result of rolling a 6-sided die, then we could write $P(X = 2) = 1/6$.

$P(X)$ ← meaningless

$P(X = x)$ ← a value that depends on X

Chapter 13

A and B are independent if knowing one occurs doesn't change the likelihood that other does.

Independent probabilities multiply: if A and B are independent, then $P(A \text{ and } B) = P(A)\,P(B)$

Ex: You flip a fair coin twice.

Let A be the event of flipping heads on the first flip, B the event of tails on the first flip, and C the event of tails on the second flip. Then A and B are disjoint, A and C are independent, and B and C are also independent.

$P(A) = .5$ $\quad$ $P(A \text{ or } B) = .5 + .5 = 1$

$P(B) = .5$ $\quad$ $P(A \text{ and } C) = (.5)(.5) = .25$

$P(C) = .5$ $\quad$ $P(B \text{ and } C) = .25$

A and C being independent means:

$P(A \mid C) = P(A)$ and $P(C \mid A) = P(C)$

$P(A) = .5$, $P(C) = .5$

Ex: if winning a lotteny has probability .1, what is the chance of winning at least once across 5 plays? Not disjoint, but are independent so reframe the question to use independence. P(not winning at least once) = P(losing 5 in a row) $= (.9)^5$

So P(winning at least 1) $= 1 - (.9)^5$.