

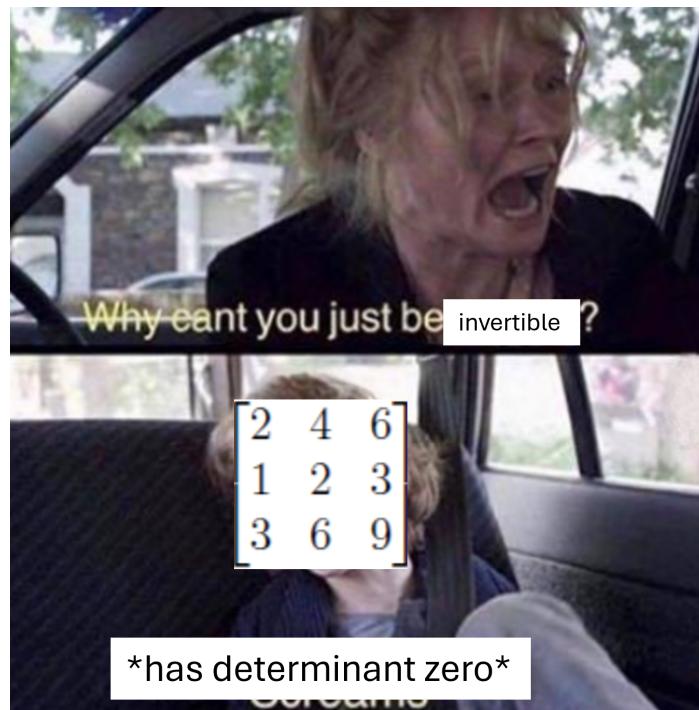
## Introduction to Functions of Several Variables Exam 1

Please sign the following Honor Code statement.

**I affirm that I have upheld the highest principles of honesty and integrity in my academic work and have not witnessed a violation of academic integrity.**

**Signature:** \_\_\_\_\_

- There are 75 total points on this exam.
- If you need extra space, you may write on the backs of the pages. In that case, please indicate that you have extra work on another page.
- You are not permitted to use any resources besides a “cheat sheet” of one side of an  $8.5 \times 11$  inch (or smaller) sheet of paper. Please make sure your name is on your cheat sheet and turn it in with your exam. I will return your cheat sheet at a later date.
- Please give exact answers (e.g.,  $\sqrt{2}$  instead of 1.41) unless explicitly instructed not to.
- **Please read all instructions carefully and ask for clarification if you are unsure about anything.**
- Take a deep breath and trust yourself. **I believe in you and am proud of you no matter how you perform on this exam.**



**1. (15 points) Short Answer.** Briefly show your work (unless explicitly told not to).

(a) **(3 points)** For what value(s) of  $a$  are  $\langle -4, a, -3 \rangle$  and  $\langle -3, 2, -4 \rangle$  orthogonal?

(b) **(3 points)** Write down the matrix representing the linear transformation  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  satisfying  $T(\vec{i}) = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$ ,  $T(\vec{j}) = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ , and  $T(\vec{k}) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ . No justification necessary.

(c) **(3 points)** If  $M$  is a  $4 \times 10$  matrix and  $\text{rank}(M) = 3$ , then what is the dimension of  $\text{Nul}(M)$ ? In other words, how many vectors are in any basis for  $\text{Nul}(M)$ ? No justification necessary.

(d) **(3 points)** Suppose  $A$  is a  $2 \times 2$  matrix such that  $A^{-1} = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$ . Find a vector  $\vec{x}$  in  $\mathbb{R}^2$  such that  $A\vec{x} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$ , or explain why no such  $\vec{x}$  exists.

(e) **(3 points)** For what value of  $b$  does the linear transformation represented by  $A = \begin{bmatrix} 3 & b \\ 1 & -4 \end{bmatrix}$  stretch space by a factor of 7 and reverse (or flip) the orientation of  $\mathbb{R}^2$ ?

**2. (16 points)** For each statement, write exactly one of the following. You do not need to justify your answers.

- Write “I” if the given set of vectors **must be** linearly independent.
- Write “D” if the given set of vectors **must be** linearly dependent.
- Write “N” if there is not enough information to determine whether the set of vectors is linearly independent or dependent.

**(a) (2 points)** The columns of a  $3 \times 3$  matrix  $M$  such that  $M \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} = \vec{0}$ .

**(b) (2 points)** A set of vectors that forms a basis for  $\mathbb{R}^5$ .

**(c) (2 points)** The columns of a  $7 \times 7$  matrix  $A$ , where  $B$  is an invertible  $7 \times 7$  matrix and  $\det(AB) = 0$ .

**(d) (2 points)** The columns of a  $4 \times 2$  matrix of rank 2.

**(e) (2 points)** The set of vectors  $\{\vec{v}, \vec{w}, \vec{0}\}$  in  $\mathbb{R}^3$ , where  $\vec{v}$  and  $\vec{w}$  are nonzero vectors.

**(f) (2 points)** The set  $\left\{ \begin{bmatrix} a \\ c \end{bmatrix}, \begin{bmatrix} b \\ d \end{bmatrix} \right\}$ , where  $ad = bc$ .

**(g) (2 points)** A set of vectors in  $\mathbb{R}^2$  whose span is all of  $\mathbb{R}^2$ .

**(h) (2 points)** The set  $\left\{ \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix} \right\}$ , where  $a, b$ , and  $c$  are all nonzero scalars.

**3. (15 points)** Find the line  $y = a_0 + a_1x$  that best fits the data below. **Show your work.** If the columns of the matrix that you attain are linearly independent, you may use this fact without explicitly justifying it. **Hint:** You may use the following fact without proof: For any nonzero scalars  $a$  and  $d$ ,

$$\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}^{-1} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/d \end{bmatrix}.$$

$x$	$y$
-1	-6
0	8
1	4

- 4. (14 points)** For the following system of linear equations: Write down the augmented matrix for the system (1 point; no justification necessary), perform elementary row operations to row-reduce your matrix **until it is in reduced row echelon form** (10 points; show your work by labeling your elementary row operations), and interpret your reduced matrix from to find all solutions of the system, or determine that no solutions exist (3 points; no justification necessary).

$$\begin{array}{rclcl} x & - & 3z & = & 8 \\ 2x & + & 2y & + & 9z & = & 7 \\ & & y & + & 5z & = & -2 \end{array}$$

**5. (15 points)** Consider the points  $P = (1, -2, 0)$  and  $Q = (5, 0, -4)$  in  $\mathbb{R}^3$ .

(a) **(2 points)** Find the displacement vector  $\overrightarrow{PQ}$ . No justification necessary.

(b) **(3 points)** Find a set of parametric equations for the line through  $P$  and  $Q$  in  $\mathbb{R}^3$ . No justification necessary.

(c) **(4 points)** Find the equation of the form  $ax+by+cz = d$  for the plane containing the point  $R = (6, -4, -5)$  and orthogonal to  $\overrightarrow{PQ}$ . Briefly show your work.

(d) **(6 points)** Find the point of intersection of the line from (b) and the plane from (c), or determine that they do not intersect. **Show your work.**