1. (15 points) Short Answer. Briefly show your work (unless explicitly told not to).

(a) (3 points) For what value(s) of a are $\langle -4, a, -3\rangle$ and $\langle -3, 2, -4\rangle$ orthogonal?

(b) (3 points) Write down the matrix representing the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ satisfying $T\left(\vec{i}\right) = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$, $T\left(\vec{j}\right) = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$, and $T\left(\vec{k}\right) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$. No justification necessary.

(c) (3 points) If M is a 4×10 matrix and $\operatorname{rank}(M) = 3$, then what is the dimension of $\operatorname{Nul}(M)$? In other words, how many vectors are in any basis for $\operatorname{Nul}(M)$? No justification necessary.

(d) (3 points) Suppose A is a 2×2 matrix such that $A^{-1} = \begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$. Find a vector $\vec{x}$ in $\mathbb{R}^2$ such that $A\vec{x} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$, or explain why no such $\vec{x}$ exists.

(e) (3 points) For what value of b does the linear transformation represented by $A = \begin{bmatrix} 3 & b \\ 1 & -4 \end{bmatrix}$ stretch space by a factor of 7 and reverse (or flip) the orientation of $\mathbb{R}^2$?

2. (16 points) For each statement, write exactly one of the following. You do not need to justify your answers.

- Write "I" if the given set of vectors **must be** linearly independent.
- Write "D" if the given set of vectors **must be** linearly dependent.
- Write "N" if there is not enough information to determine whether the set of vectors is linearly independent or dependent.

(a) (2 points) The columns of a 3×3 matrix M such that $M \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} = \vec{0}$.

(b) (2 points) A set of vectors that forms a basis for $\mathbb{R}^5$.

(c) (2 points) The columns of a 7×7 matrix A, where B is an invertible 7×7 matrix and $\det(AB) = 0$.

(d) (2 points) The columns of a 4×2 matrix of rank 2.

(e) (2 points) The set of vectors $\left\{\vec{v}, \vec{w}, \vec{0}\right\}$ in $\mathbb{R}^3$, where $\vec{v}$ and $\vec{w}$ are nonzero vectors.

(f) (2 points) The set $\left\{ \begin{bmatrix} a \\ c \end{bmatrix}, \begin{bmatrix} b \\ d \end{bmatrix} \right\}$, where $ad = bc$.

(g) (2 points) A set of vectors in $\mathbb{R}^2$ whose span is all of $\mathbb{R}^2$.

(h) (2 points) The set $\left\{ \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix} \right\}$, where a, b, and c are all nonzero scalars.

3. (15 points) Find the line $y = a_0 + a_1 x$ that best fits the data below. **Show your work**. If the columns of the matrix that you attain are linearly independent, you may use this fact without explicitly justifying it. **Hint:** You may use the following fact without proof: For any nonzero scalars a and d,

$$\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}^{-1} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/d \end{bmatrix}.$$

x	y
-1	-6
0	8
1	4

4. (14 points) For the following system of linear equations: Write down the augmented matrix for the system (1 point; no justification necessary), perform elementary row operations to row-reduce your matrix **until it is in reduced row echelon form** (10 points; show your work by labeling your elementary row operations), and interpret your reduced matrix from to find all solutions of the system, or determine that no solutions exist (3 points; no justification necessary).

$$\begin{array}{ccccccc} x & & & - & 3z & = & 8 \\ 2x & + & 2y & + & 9z & = & 7 \\ & & y & + & 5z & = & -2 \end{array}$$

5. (15 points) Consider the points $P = (1, -2, 0)$ and $Q = (5, 0, -4)$ in $\mathbb{R}^3$.

(a) (2 points) Find the displacement vector $\overrightarrow{PQ}$. No justification necessary.

(b) (3 points) Find a set of parametric equations for the line through P and Q in $\mathbb{R}^3$. No justification necessary.

(c) (4 points) Find the equation of the form $ax+by+cz = d$ for the plane containing the point $R = (6, -4, -5)$ and orthogonal to $\overrightarrow{PQ}$. Briefly show your work.

(d) (6 points) Find the point of intersection of the line from (b) and the plane from (c), or determine that they do not intersect. **Show your work**.