Policies

- Due 9 PM, March 8th, via Gradescope.
- You are free to collaborate on all of the problems, subject to the collaboration policy stated in the syllabus.
- You should submit all code used in the homework. We ask that you use Python 3.6+ for your code, and that you comment your code such that the TAs can follow along and run it without any issues.

Submission Instructions

- **In the report, include any images generated by your code along with your answers to the questions.** For instructions specifically pertaining to the Gradescope submission process, see https://www.gradescope.com/get_started#student-submission.
- Since we are only using Gradescope this year, we ask that you upload both your code and solution pdf together as a single pdf and mark each page with the problem number. There are various free online tools to combine pdfs. To download your code as a pdf from Jupyter notebook, you can navigate to File -> Download as -> ”PDF via Latex” or alternatively download the Latex file and compile it yourself.

1 Probability Refresher [0 points]

This problem is a probability refresher and is worth 0 points. It should take you no more than 2-4 minutes to solve for each part. If you find that you are taking more time, you should probably watch the Probability recitation.

Problem A [0 points]: If $X, Y \sim \mathcal{N}(0, 1)$, compute $\Pr[X < \frac{Y}{\sqrt{2}}]$.

Solution A:

Problem B [0 points]: If $X, Y \sim \mathcal{N}(0, 1)$, compute $\Pr[X + Y > 0 | X > 0]$.

Solution B:

Problem C [0 points]: Prove that σ-algebras are closed under finitely many intersections. Hint: Recall that they are closed under finitely many unions and complements.

Solution C:

Problem D [0 points]: If X is a continuous random variable that takes on values in the range $(0, \infty)$, and Y is another random variable such that the marginal density of X and Y is given by

$$f_{X,Y}(x, y) = \frac{1}{y} e^{-x/y} e^{-y}$$

What is the probability distribution function of Y?

Solution D:

2 Class-Conditional Densities for Binary Data [25 Points]

This problem will test your understanding of probabilistic models, especially Naive Bayes. Consider a generative classifier for C classes, with class conditional density $p(x|y)$ and a uniform class prior $p(y)$. Suppose all the D features are binary, $x_j \in \{0, 1\}$. If we assume all of the features are conditionally independent, as in Naive Bayes, we can write:

$$p(x \mid y = c) = \prod_{j=1}^{D} P(x_j \mid y = c)$$

This requires storing DC parameters.

Now consider a different model, which we will call the 'full' model, in which all the features are fully *dependent*.

Problem A [5 points]: Use the chain rule of probability to factorize $p(x \mid y)$, and let $\theta_{xjc} = P(x_j|x_{1,\ldots,j-1}, y = c)$. Assuming we store each θ_{xjc}, how many parameters are needed to represent this factorization? Use big-O notation.

Solution A:

$$\begin{aligned} p(x \mid y = c) &= P(x_1, \ldots, x_D \mid y = c) \\ &= \theta_{xDc} P(x_1, \ldots, x_{D-1} \mid y = c) \\ &= \prod_{j=1}^{D} \theta xjc \end{aligned}$$

There are DC terms in the product, but θ_{xjc} depends on $j - 1$ other x values, so it uses 2^{j-1} parameters. So the number of parameters is $C \sum_{j=1}^{D} 2^{j-1} = C(2^D - 1)$, which is $O(C \cdot 2^D)$.

Problem B [5 points]: Assume we did no such factorization, and just used the conditional probability $p(x \mid y = c)$. How many parameters would we need to estimate in order be able to compute $p(x|y = c)$ for arbitrary x and c? How does this compare to your answer from the previous part? Again, use big-O notation.

Solution B: *Given 2^D possible values for x and C values for c, we have $O(C \cdot 2^D)$ again.*

Problem C [2 points]: Assume the number of features D is fixed. Let there be N training cases. If the sample size N is very small, which model (Naive Bayes or full) is likely to give lower test set error, and why?

Solution C: *Naive Bayes is likely to give the lower test error, because as long as $N < 2^D$ the full model can just memorize the entire training set, so it will overfit.*

Problem D [2 points]: If the sample size N is very large, which model (Naive Bayes or full) is likely to give lower test set error, and why?

Solution D: *With very large N the full model is likely to give lower test error, because it has higher capacity and can thus learn more from the training set.*

Problem E [11 points]: Assume all the parameter estimates have been computed. What is the computational complexity of making a prediction, i.e. computing $p(y \mid x)$, using Naive Bayes for a single test case? What is the computation complexity of making a prediction with the full model? For the full-model case, assume that converting a D-bit vector to an array index is an $O(D)$ operation. Also, recall that we have assumed a uniform class prior.

Solution E: *Assuming a uniform class prior,*

$$p(y \mid x) = \frac{p(y, x)}{p(x)} = \frac{p(x \mid y)p(y)}{p(x)} \propto p(x \mid y)$$

For Naive Bayes, we have to look up and multiply D parameters for each of the C classes, so the complexity is $O(DC)$.

For the full model, we have to convert x into an array index (which is $O(D)$) for each of the C classes, so the complexity is $O(DC)$ again.

3 Sequence Prediction [75 Points]

In this problem, we will explore some of the various algorithms associated with Hidden Markov Models (HMMs), as discussed in lecture. For these problems, make sure you are **using Python 3** to implement the algorithms. Please see the HMM notes posted on the course website—they will be helpful for this problem!

Sequence Prediction

These next few problems will require extensive coding, so be sure to start early! We provide you with eight different files:

- You will write all your code in `HMM.ipynb`, within the appropriate functions where indicated by ”TODO” in the comments. There is no need to modify anything else aside from what is indicated. There should be no need to write additional functions or use NumPy in your implementation, but feel free to do so if you would like.

The supplementary data folder contains 6 files titled `sequence_data0.txt`, `sequence_data1.txt`, ... , `sequence_data5.txt`. Each file specifies a **trained** HMM. The first row contains two tab-delimited numbers: the number of states Y and the number of types of observations X (i.e. the observations are $0, 1, ..., X - 1$). The next Y rows of Y tab-delimited floating-point numbers describe the state transition matrix. Each row represents the current state, each column represents a state to transition to, and each entry represents the probability of that transition occurring. The next Y rows of X tab-delimited floating-point numbers describe the output emission matrix, encoded analogously to the state transition matrix. The file ends with 5 possible emissions from that HMM.

The supplementary data folder also contains one additional file titled `ron.txt`. This is used in problems 2C and 2D and is explained in greater detail there.

Problem A [10 points]: For each of the six trained HMMs, find the max-probability state sequence for each of the five input sequences at the end of the corresponding file. To complete this problem, you will have to implement the Viterbi algorithm. Write your implementation well, as we will be reusing it in a later problem. See the end of problem 2B for a big hint!

Show your results on the 6 files. (Copy-pasting the results of under the ”Part A” heading of `HMM.ipynb` suffices.)

Solution A: *Colab notebook*

```
File #0:
Emission Sequence              Max Probability State Sequence
######################################################################
25421                          31033
```

```
01232367534                   22222100310
5452674261527433              1031003103222222
7226213164512267255           1310331000033100310
0247120602352051010255241     2222222222222222222222103

File #1:
Emission Sequence             Max Probability State Sequence
######################################################################
77550                         22222
7224523677                    2222221000
505767442426747               222100003310031
7213413164553611226 7         1031031000031033310 0
4733667771450051060253041     2221000003222223103222223

File #2:
Emission Sequence             Max Probability State Sequence
######################################################################
60622                         11111
4687981156                    2100202111
815833657775062               021011111111111
2131022251596350501 5         0202011111111111021
6503199452571274006320025     1110202111111102021110211

File #3:
Emission Sequence             Max Probability State Sequence
######################################################################
13661                         00021
2102213421                    3131310213
166066262165133               133333133133100
5316466211216263415 6         2000002131313100213 3
1523541005123230226306256     1310021333133133313133133

File #4:
Emission Sequence             Max Probability State Sequence
######################################################################
23664                         01124
3630535602                    0111201112
350201162150142               011244012441112
0021400540201514636 2         1120111241244401111 2
2111266524665143562534450     2012012424124011112411124
```

```
File #5:
Emission Sequence              Max Probability State Sequence
######################################################################
68535                          10111
4546566636                     1111111111
638436858181213                110111010000011
13240338308444514688           00010000000111111100
0111664434441382533632626      2111111111111100111110101
```

Problem B [17 points]: For each of the six trained HMMs, find the probabilities of emitting the five input sequences at the end of the corresponding file. To complete this problem, you will have to implement the Forward algorithm and the Backward algorithm. You may assume that the initial state is randomly selected along a uniform distribution. Again, write your implementation well, as we will be reusing it in a later problem.

Note that the probability of emitting an input sequence can be found by using either the α vectors from the Forward algorithm or the β vectors from the Backward algorithm. You don't need to worry about this, as it is done for you in `probability_alphas()` and `probability_betas()`.

Implement the Forward algorithm. Show your results on the 6 files.
Implement the Backward algorithm. Show your results on the 6 files.

After you complete problems 2A and 2B, you can compare your results for the file titled `sequence_-data0.txt` with the values given in the table below:

Dataset	Emission Sequence	Max-probability State Sequence	Probability of Sequence
0	25421	31033	4.537e-05
0	01232367534	22222100310	1.620e-11
0	5452674261527433	1031003103222222	4.348e-15
0	7226213164512267255	1310331000033100310	4.739e-18
0	0247120602352051010255241	2222222222222222222222103	9.365e-24

Solution B: *Forward algorithm:*

```
File #0:
Emission Sequence              Probability of Emitting Sequence
######################################################################
```

```
25421                         4.537e-05
01232367534                   1.620e-11
5452674261527433              4.348e-15
7226213164512267255           4.739e-18
0247120602352051010255241     9.365e-24

File #1:
Emission Sequence             Probability of Emitting Sequence
######################################################################
77550                         1.181e-04
7224523677                    2.033e-09
505767442426747               2.477e-13
72134131645536112267          8.871e-20
4733667771450051060253041     3.740e-24

File #2:
Emission Sequence             Probability of Emitting Sequence
######################################################################
60622                         2.088e-05
4687981156                    5.181e-11
815833657775062               3.315e-15
21310222515963505015          5.126e-20
6503199452571274006320025     1.297e-25

File #3:
Emission Sequence             Probability of Emitting Sequence
######################################################################
13661                         1.732e-04
2102213421                    8.285e-09
166066262165133               1.642e-12
53164662112162634156          1.063e-16
1523541005123230226306256     4.535e-22

File #4:
Emission Sequence             Probability of Emitting Sequence
######################################################################
23664                         1.141e-04
3630535602                    4.326e-09
350201162150142               9.793e-14
00214005402015146362          4.740e-18
2111266524665143562534450     5.618e-22
```

```
File #5:
Emission Sequence              Probability of Emitting Sequence
######################################################################
68535                          1.322e-05
4546566636                     2.867e-09
638436858181213                4.323e-14
13240338308444514688           4.629e-18
0111664434441382533632626      1.440e-22
```

Backward algorithm:

```
File #0:
Emission Sequence              Probability of Emitting Sequence
######################################################################
25421                          4.537e-05
01232367534                    1.620e-11
5452674261527433               4.348e-15
7226213164512267255            4.739e-18
0247120602352051010255241      9.365e-24

File #1:
Emission Sequence              Probability of Emitting Sequence
######################################################################
77550                          1.181e-04
7224523677                     2.033e-09
505767442426747                2.477e-13
72134131645536112267           8.871e-20
4733667771450051060253041      3.740e-24

File #2:
Emission Sequence              Probability of Emitting Sequence
######################################################################
60622                          2.088e-05
4687981156                     5.181e-11
815833657775062                3.315e-15
21310222515963505015           5.126e-20
6503199452571274006320025      1.297e-25

File #3:
```

```
Emission Sequence              Probability of Emitting Sequence
######################################################################
13661                          1.732e-04
2102213421                     8.285e-09
166066262165133                1.642e-12
53164662112162634156           1.063e-16
1523541005123230226306256      4.535e-22

File #4:
Emission Sequence              Probability of Emitting Sequence
######################################################################
23664                          1.141e-04
3630535602                     4.326e-09
350201162150142                9.793e-14
00214005402015146362           4.740e-18
2111266524665143562534450      5.618e-22

File #5:
Emission Sequence              Probability of Emitting Sequence
######################################################################
68535                          1.322e-05
4546566636                     2.867e-09
638436858181213                4.323e-14
13240338308444514688           4.629e-18
0111664434441382533632626      1.440e-22
```

HMM Training

Ron is an avid music listener, and his genre preferences at any given time depend on his mood. Ron's possible moods are happy, mellow, sad, and angry. Ron experiences one mood per day (as humans are known to do) and chooses one of ten genres of music to listen to that day depending on his mood.

Ron's roommate, who is known to take to odd hobbies, is interested in how Ron's mood affects his music selection, and thus collects data on Ron's mood and music selection for six years (2190 data points). This data is contained in the supplementary file `ron.txt`. Each row contains two tab-delimited strings: Ron's mood and Ron's genre preference that day. The data is split into 12 sequences, each corresponding to half a year's worth of observations. The sequences are separated by a row containing only the character `-`.

Problem C [10 points]: Use a single M-step to train a supervised Hidden Markov Model on the data in `ron.txt`.

After you implement 2C, please compare your output transition and observation matrices with the state transition and output emission matrices below. You should make sure your outputs match before continuing.

$$\text{Transition matrix} = \begin{pmatrix} 2.833\text{e-}01 & 4.714\text{e-}01 & 1.310\text{e-}01 & 1.143\text{e-}01 \\ 2.321\text{e-}01 & 3.810\text{e-}01 & 2.940\text{e-}01 & 9.284\text{e-}02 \\ 1.040\text{e-}01 & 9.760\text{e-}02 & 3.696\text{e-}01 & 4.288\text{e-}01 \\ 1.883\text{e-}01 & 9.903\text{e-}02 & 3.052\text{e-}01 & 4.075\text{e-}01 \end{pmatrix}$$

Observation Matrix =

$$\begin{pmatrix} 1.486\text{e-}01 & 2.288\text{e-}01 & 1.533\text{e-}01 & 1.179\text{e-}01 & 4.717\text{e-}02 & 5.189\text{e-}02 & 2.830\text{e-}02 & 1.297\text{e-}01 & 9.198\text{e-}02 & 2.358\text{e-}03 \\ 1.062\text{e-}01 & 9.653\text{e-}03 & 1.931\text{e-}02 & 3.089\text{e-}02 & 1.699\text{e-}01 & 4.633\text{e-}02 & 1.409\text{e-}01 & 2.394\text{e-}01 & 1.371\text{e-}01 & 1.004\text{e-}01 \\ 1.194\text{e-}01 & 4.299\text{e-}02 & 6.529\text{e-}02 & 9.076\text{e-}02 & 1.768\text{e-}01 & 2.022\text{e-}01 & 4.618\text{e-}02 & 5.096\text{e-}02 & 7.803\text{e-}02 & 1.274\text{e-}01 \\ 1.694\text{e-}01 & 3.871\text{e-}02 & 1.468\text{e-}01 & 1.823\text{e-}01 & 4.839\text{e-}02 & 6.290\text{e-}02 & 9.032\text{e-}02 & 2.581\text{e-}02 & 2.161\text{e-}01 & 1.935\text{e-}02 \end{pmatrix}$$

Solution C: *Our outputs match the matrices given.*

```
Transition Matrix:
##################################################################
2.833e-01   4.714e-01   1.310e-01   1.143e-01
2.321e-01   3.810e-01   2.940e-01   9.284e-02
1.040e-01   9.760e-02   3.696e-01   4.288e-01
1.883e-01   9.903e-02   3.052e-01   4.075e-01


Observation Matrix:
##################################################################
1.486e-01   2.288e-01   1.533e-01   1.179e-01   4.717e-02   5.189e-02   2.830e-02   1.297
1.062e-01   9.653e-03   1.931e-02   3.089e-02   1.699e-01   4.633e-02   1.409e-01   2.394
1.194e-01   4.299e-02   6.529e-02   9.076e-02   1.768e-01   2.022e-01   4.618e-02   5.096
1.694e-01   3.871e-02   1.468e-01   1.823e-01   4.839e-02   6.290e-02   9.032e-02   2.581
```

Problem D [15 points]: Now suppose that Ron has a third roommate who is also interested in how Ron's mood affects his music selection. This roommate is lazier than the other one, so he simply steals the first roommate's data. Unfortunately, he only manages to grab half the data, namely, Ron's choice of music for each of the 2190 days.

In this problem, we will train an unsupervised Hidden Markov Model on this data. Recall that unsupervised HMM training is done using the Baum-Welch algorithm and will require repeated EM steps. For this problem, we will use 4 hidden states and run the algorithm for 1000 iterations. The transition and observation matrices are initialized for you in the helper functions `supervised_learning()` and `unsupervised_learning()` such that they are random and normalized.

What are the learned state transition and output emission matrices?

Below are the transition and observation matrices for the unsupervised Hidden Markov Model at the zero-th iteration.

$$\text{Transition Matrix} = \begin{pmatrix} 2.003\text{e-}01 & 3.720\text{e-}01 & 5.642\text{e-}02 & 3.713\text{e-}01 \\ 1.581\text{e-}01 & 2.147\text{e-}01 & 4.197\text{e-}01 & 2.075\text{e-}01 \\ 2.941\text{e-}01 & 1.475\text{e-}02 & 4.032\text{e-}01 & 2.880\text{e-}01 \\ 1.759\text{e-}01 & 4.205\text{e-}01 & 1.617\text{e-}01 & 2.419\text{e-}01 \end{pmatrix}$$

Observation Matrix =

$$\begin{pmatrix} 2.585\text{e-}02 & 7.773\text{e-}02 & 3.923\text{e-}02 & 5.058\text{e-}02 & 1.447\text{e-}01 & 5.407\text{e-}02 & 9.356\text{e-}02 & 1.891\text{e-}01 & 1.854\text{e-}01 & 1.398\text{e-}01 \\ 9.821\text{e-}02 & 5.024\text{e-}02 & 2.915\text{e-}02 & 1.760\text{e-}01 & 9.364\text{e-}02 & 2.102\text{e-}02 & 1.131\text{e-}01 & 1.409\text{e-}01 & 1.112\text{e-}01 & 1.664\text{e-}01 \\ 8.272\text{e-}03 & 1.104\text{e-}01 & 9.597\text{e-}02 & 1.303\text{e-}02 & 1.340\text{e-}01 & 1.781\text{e-}01 & 1.239\text{e-}01 & 5.434\text{e-}02 & 1.755\text{e-}01 & 1.065\text{e-}01 \\ 1.028\text{e-}01 & 1.516\text{e-}01 & 2.977\text{e-}02 & 1.650\text{e-}01 & 1.375\text{e-}01 & 1.584\text{e-}01 & 3.856\text{e-}02 & 1.615\text{e-}01 & 3.851\text{e-}02 & 1.641\text{e-}02 \end{pmatrix}$$

Below are the transition and observation matrices for the unsupervised Hidden Markov Model at the first iteration.

$$\text{Transition Matrix} = \begin{pmatrix} 1.887\text{e-}01 & 3.750\text{e-}01 & 5.354\text{e-}02 & 3.828\text{e-}01 \\ 1.516\text{e-}01 & 2.204\text{e-}01 & 4.083\text{e-}01 & 2.196\text{e-}01 \\ 2.832\text{e-}01 & 1.556\text{e-}02 & 3.924\text{e-}01 & 3.089\text{e-}01 \\ 1.647\text{e-}01 & 4.254\text{e-}01 & 1.559\text{e-}01 & 2.540\text{e-}01 \end{pmatrix}$$

Observation Matrix =

$$\begin{pmatrix} 6.272\text{e-}02 & 5.705\text{e-}02 & 8.345\text{e-}02 & 5.738\text{e-}02 & 1.380\text{e-}01 & 5.191\text{e-}02 & 8.730\text{e-}02 & 1.572\text{e-}01 & 2.127\text{e-}01 & 9.231\text{e-}02 \\ 2.152\text{e-}01 & 3.537\text{e-}02 & 5.388\text{e-}02 & 1.786\text{e-}01 & 8.065\text{e-}02 & 1.888\text{e-}02 & 8.992\text{e-}02 & 1.053\text{e-}01 & 1.168\text{e-}01 & 1.054\text{e-}01 \\ 1.988\text{e-}02 & 8.040\text{e-}02 & 1.846\text{e-}01 & 1.378\text{e-}02 & 1.243\text{e-}01 & 1.626\text{e-}01 & 1.081\text{e-}01 & 4.324\text{e-}02 & 1.937\text{e-}01 & 6.944\text{e-}02 \\ 2.222\text{e-}01 & 1.001\text{e-}01 & 5.614\text{e-}02 & 1.652\text{e-}01 & 1.169\text{e-}01 & 1.376\text{e-}01 & 3.193\text{e-}02 & 1.205\text{e-}01 & 3.943\text{e-}02 & 9.930\text{e-}03 \end{pmatrix}$$

Below are the transition and observation matrices for the unsupervised Hidden Markov Model at the final iteration (iteration 1000).

$$\text{Transition Matrix} = \begin{pmatrix} 4.345\text{e-}01 & 1.559\text{e-}01 & 9.612\text{e-}02 & 3.134\text{e-}01 \\ 5.904\text{e-}15 & 2.996\text{e-}01 & 7.004\text{e-}01 & 1.328\text{e-}11 \\ 5.219\text{e-}01 & 3.828\text{e-}18 & 2.259\text{e-}01 & 2.522\text{e-}01 \\ 5.016\text{e-}03 & 3.679\text{e-}01 & 7.971\text{e-}03 & 6.191\text{e-}01 \end{pmatrix}$$

Observation Matrix =

$$\begin{pmatrix}
2.235\text{e-}01 & 8.353\text{e-}09 & 7.603\text{e-}02 & 9.263\text{e-}02 & 2.315\text{e-}02 & 1.023\text{e-}02 & 1.273\text{e-}01 & 3.294\text{e-}01 & 1.288\text{e-}10 & 1.177\text{e-}01 \\
2.260\text{e-}01 & 3.339\text{e-}05 & 7.427\text{e-}14 & 2.039\text{e-}01 & 3.226\text{e-}02 & 1.151\text{e-}01 & 1.575\text{e-}01 & 3.954\text{e-}30 & 8.256\text{e-}02 & 1.828\text{e-}01 \\
3.403\text{e-}02 & 1.351\text{e-}01 & 1.900\text{e-}01 & 5.006\text{e-}02 & 2.247\text{e-}01 & 2.150\text{e-}05 & 6.462\text{e-}02 & 2.926\text{e-}02 & 2.723\text{e-}01 & 3.988\text{e-}65 \\
8.931\text{e-}02 & 1.173\text{e-}01 & 1.035\text{e-}01 & 9.344\text{e-}02 & 1.505\text{e-}01 & 2.098\text{e-}01 & 9.588\text{e-}07 & 7.679\text{e-}02 & 1.594\text{e-}01 & 1.312\text{e-}07
\end{pmatrix}$$

Please use these outputs to check your code.

Solution D: *Our outputs match the matrices given.*

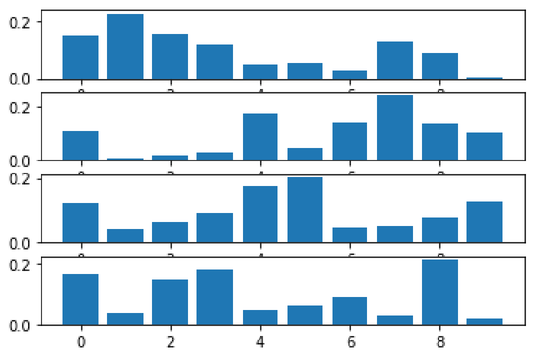
```
Transition Matrix:
######################################################################
4.345e-01   1.559e-01   9.612e-02   3.134e-01
5.904e-15   2.996e-01   7.004e-01   1.328e-11
5.219e-01   3.828e-18   2.259e-01   2.522e-01
5.016e-03   3.679e-01   7.971e-03   6.191e-01


Observation Matrix:
######################################################################
2.235e-01   8.353e-09   7.603e-02   9.263e-02   2.315e-02   1.023e-02   1.273e-01   3.294
2.260e-01   3.339e-05   7.427e-14   2.039e-01   3.226e-02   1.151e-01   1.575e-01   3.954
3.403e-02   1.351e-01   1.900e-01   5.006e-02   2.247e-01   2.150e-05   6.462e-02   2.926
8.931e-02   1.173e-01   1.035e-01   9.344e-02   1.505e-01   2.098e-01   9.588e-07   7.679
```

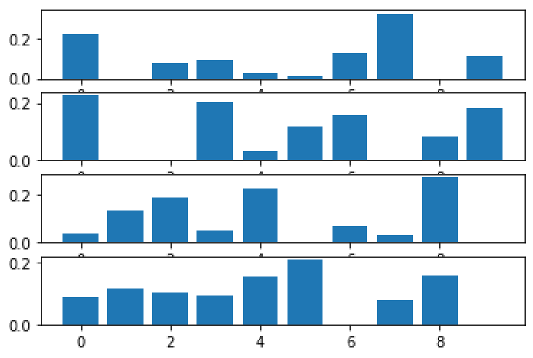
Problem E [5 points]: How do the transition and emission matrices from 2C and 2D compare? Which do you think provides a more accurate representation of Ron's moods and how they affect his music choices? Justify your answer. Suggest one way that we may be able to improve the method (supervised or unsupervised) that you believe produces the less accurate representation.

Solution E: *The unsupervised algorithm produced sparser matrices, where some of the entries are nearly 0 in both the transition and observation matrices.*

By looking at the observation matrices and the genre map, we saw that the supervised model produces a distribution over genres that makes intuitive sense to us (e.g. angry -¿ rap, metal, rock; happy -¿ pop, house, rock). The unsupervised model seemed to predict unrelated genres for a given state (for example rock, folk, and classical being the most likely from state 1).



Supervised Model



Unsupervised Model

For improving the unsupervised model, we can gather more data, increase the number of epochs, or try more/fewer hidden states.

Sequence Generation

Hidden Markov Models fall under the umbrella of generative models and therefore can be used to not only predict sequential data, but also to generate it.

Problem F [5 points]: Load the trained HMMs from the files titled `sequence_data0.txt,...,sequence_-data5.txt`. Use the six models to probabilistically generate five sequences of emissions from each model, each of length 20. Show your results.

Solution F:

```
File #0:
Generated Emission
######################################################################
77420454341752715750
72457572640525742077
77646442257647445465
65244754502514721023
62772345127414575542

File #1:
Generated Emission
######################################################################
77420454341752715750
72457572640525742077
77646442257647445465
```

```
65244754502514721023
62772345127414575542

File #2:
Generated Emission
######################################################################
99540565323972926751
72568781750637954077
87836321467659227365
75123966712525932224
73993455259226683524

File #3:
Generated Emission
######################################################################
66330354312651605630
61256660620512622066
36624310326546115244
53012653511322620012
41663243236113433312

File #4:
Generated Emission
######################################################################
66320463212652606630
62465660630514622066
66535310256636115344
53012654502324621122
32662232136214462312

File #5:
Generated Emission
######################################################################
88341483323863806630
63468680640635834166
88836320456648115354
63022865512334821234
62883443238214483314
```

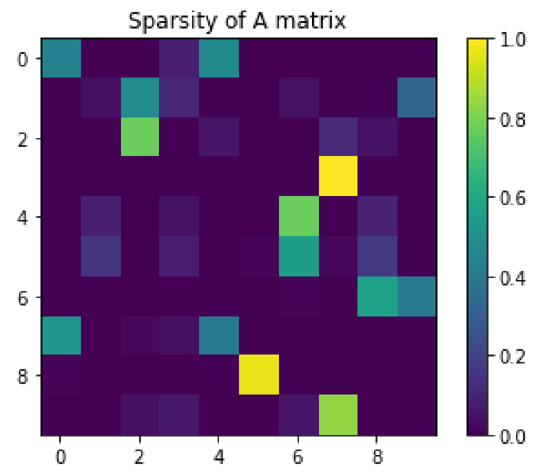
Visualization & Analysis

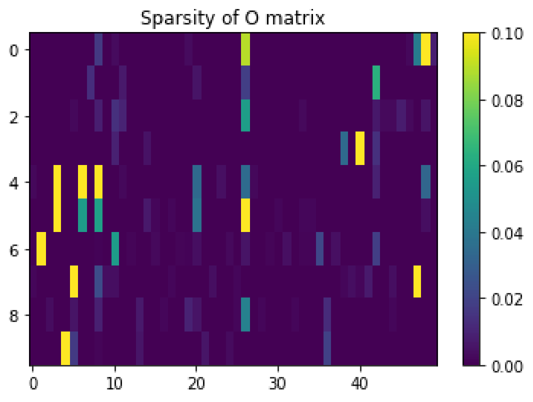
Once you have implemented the above, load and run `2_notebook.ipynb`. In this notebook, you will apply the HMM you have implemented to the Constitution. There is no coding required for this part, only analysis. To run the notebook, however, you will likely need to install the `wordcloud` package. Please refer to the provided installation instructions if you get an error when running `pip install wordcloud`.

Answer the following problems in the context of the visualizations in the notebook.

Problem G [3 points]: What can you say about the sparsity of the trained A and O matrices? How does this sparsity affect the transition and observation behaviour at each state?

Solution G:





They're very sparse. This means that a given state is fairly certain for what observation it will produce and what state it will transition to.

Problem H [5 points]: How do the sample emission sentences from the HMM change as the number of hidden states is increased? What happens in the special case where there is only one hidden state? In general, when the number of hidden states is unknown while training an HMM for a fixed observation set, can we increase the training data likelihood by allowing more hidden states?

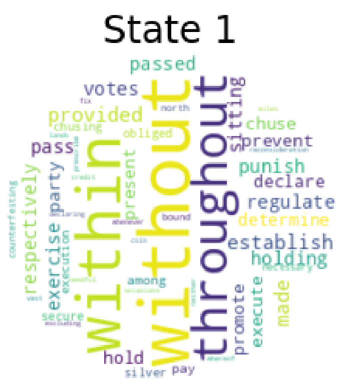
Solution H: *None of the sentences make any sense, but the ones with more hidden states are able to make 2-3 word phrases that sound plausible.*

The 1 state model just randomly selects words according to their frequency in the text.

In general, adding more hidden states will increase the training data likelihood because adding states can capture more complex patterns in the training data.

Problem I [5 points]: Pick a state that you find semantically meaningful, and analyze this state and its wordcloud. What does this state represent? How does this state differ from the other states? Back up your claim with a few key words from the wordcloud.

Solution I:



The biggest words here are "within", "without", and "throughout", which clearly would fall in similar places in a sentence, and suggest that this state represents a relationship of how/when/where things occur. The other states didn't seem to have quite as coherent a meaning, though there were some patterns in them as well.