自动控制理论 HW-17　自动化1班　1904010102　方尧

1. $|\Phi(j\omega)| = \frac{2}{2} = 1$，$\angle\Phi(j\omega) = -45^\circ$　($j\omega = j1$ 时)

$\Phi(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2}$　$\Phi(j\omega)\big|_{\omega=1} = 1\angle -45^\circ = \frac{\sqrt2}{2} - \frac{\sqrt2}{2}j$

$\Phi(j\omega)\big|_{\omega=1} = \frac{1}{1 - \frac{1}{\omega_n^2} + 2\frac{\zeta}{\omega_n}j} = \frac{\sqrt2}{2} - \frac{\sqrt2}{2}j$

故 $\begin{cases} 1 - \frac{1}{\omega_n^2} = \frac{\sqrt2}{2} \\ 2\frac{\zeta}{\omega_n} = \frac{\sqrt2}{2} \end{cases}$　联立解得 $\begin{cases} \omega_n = \sqrt{\frac{\sqrt2}{\sqrt2 - 1}} \approx 1.848 \\ \zeta = \frac{1}{4}\sqrt{\frac{2\sqrt2}{\sqrt2 - 1}} \approx 0.6533 \end{cases}$

2. (1) $\omega_1 = 0.125$，$\omega_2 = 0.5$，渐近线为：斜率为0，过(1, 20lg2) 约为(1, 6dB)

截止频率 $\omega_c = 0.25$，(图像于坐标纸绘制)

(2) 转折频率分别为0.1, 1，渐近线为：斜率为-40，过(1, 20lg200) 约为(1, 46)

截止频率 $\omega_c = 2.1$，图像绘于坐标纸

(3) 转折频率依次为0.1, 1, 2，渐近线为：斜率为-20，过(1, 20lg8) 约为(1, 18.1)

截止频率 $\omega_c = 5.43$，图像绘于坐标纸

(4) 转折频率依次为0.1, 1, 20，低频渐近线为：斜率-20，过(1, 20lg10) 即(1, 20)

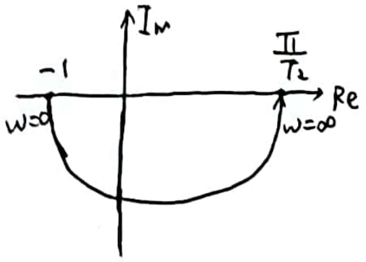
截止频率为 $\omega_c = 1$，图像绘于坐标纸

3. 该环节频率响应为 $G(j\omega) = \frac{1 + j\omega T_1}{-1 + j\omega T_2}$

幅频 $|G(j\omega)| = \frac{\sqrt{1 + (\omega T_1)^2}}{\sqrt{1 + (\omega T_2)^2}}$，相频 $\angle G(j\omega) = \arctan\omega T_1 + (-180^\circ + \arctan\omega T_2)$

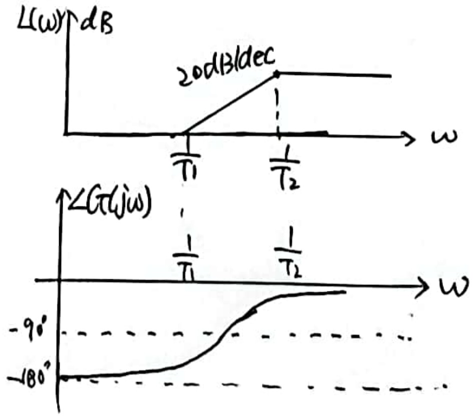
$\omega = 0$，$G(j\omega) = 1\angle -180^\circ$；$\omega = \infty$，$G(j\omega) = \frac{T_1}{T_2}\angle 0^\circ$

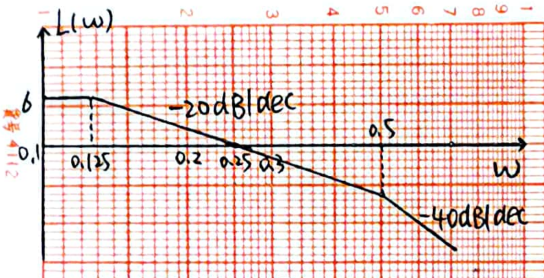
奈奎斯特图和伯德图如下：



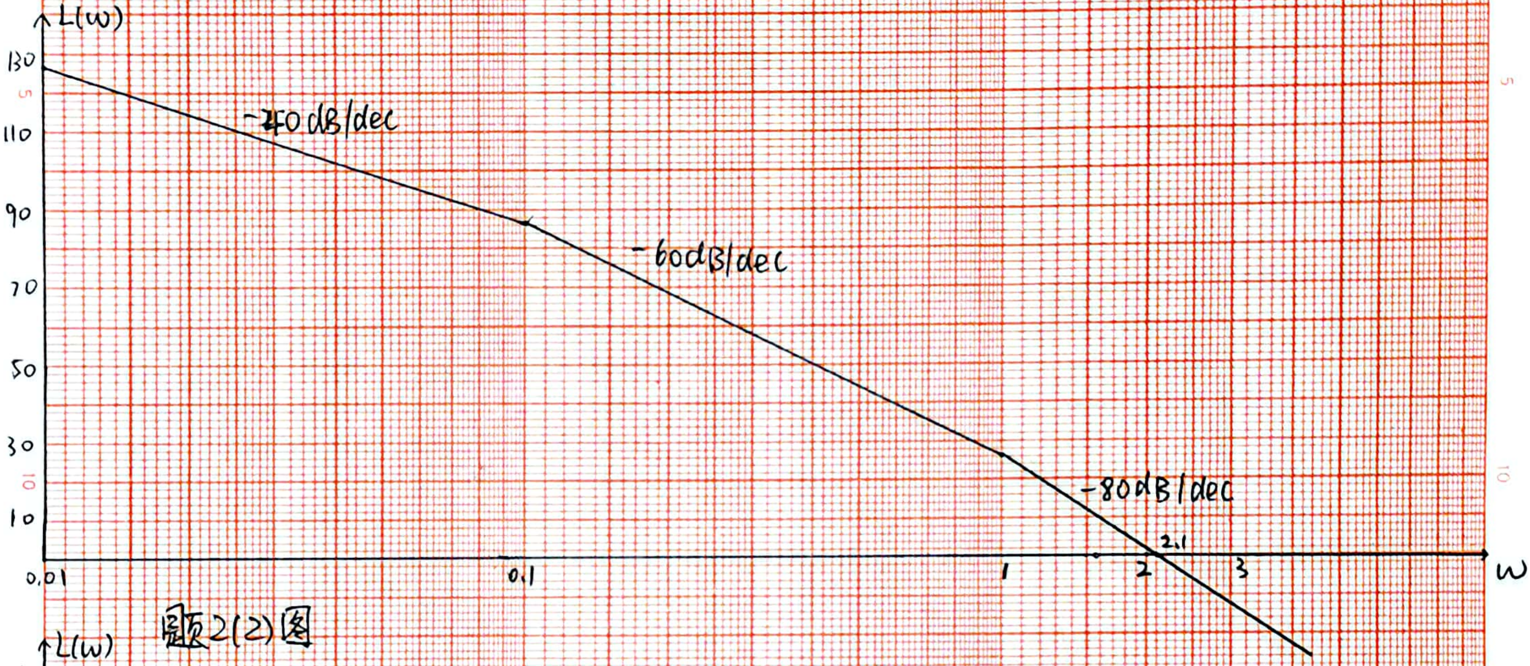
伯德图：转折频率 $\omega_1 = \frac{1}{T_1}$　$\omega_2 = \frac{1}{T_2}$

基准线：过(1, 0)，K=0，斜率为0.

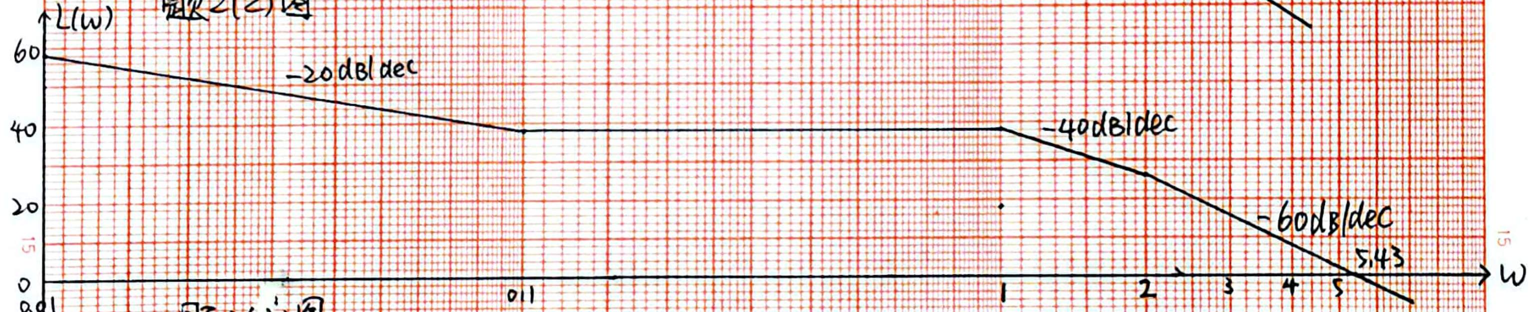




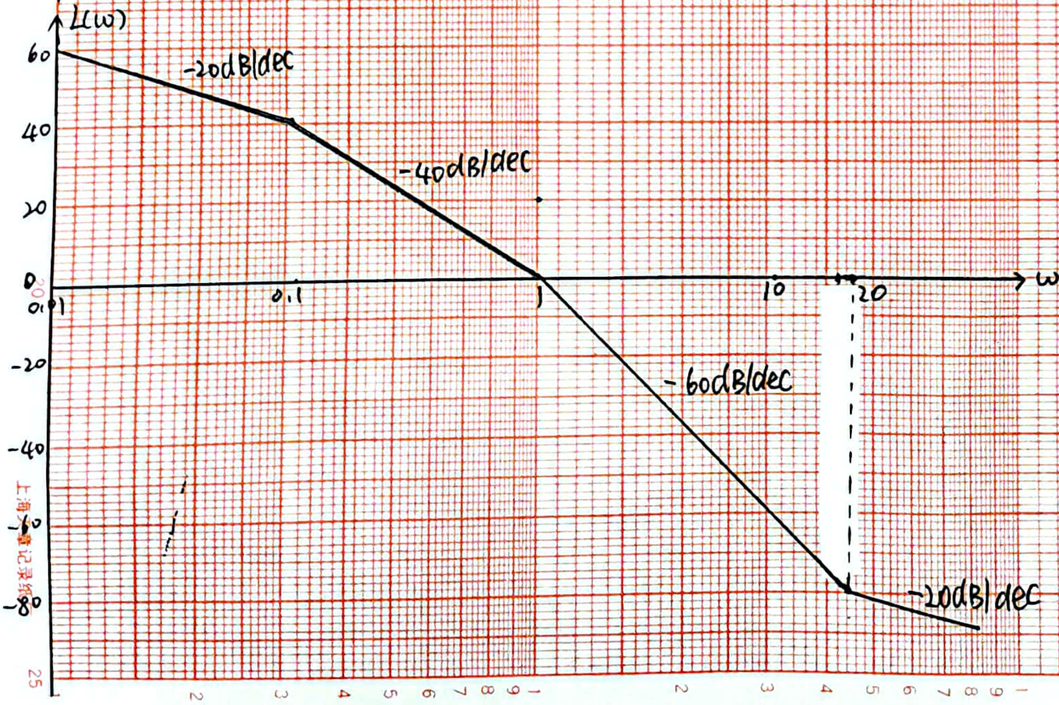
题2(1)图



题2(2)图



题2(3)图



4. $G(j\omega)H(j\omega)=\dfrac{K\cdot e^{-0.1j\omega}}{j\omega(0.1j\omega+1)(j\omega+1)}$

剪切频率 $\omega_c=5rad/s$，即 $|G(j\omega)H(j\omega)|_{\omega=5}=\left|\dfrac{K\cdot e^{-0.5j}}{j5(0.5j+1)(j5+1)}\right|=1$，得 $K=\dfrac{5\sqrt{130}}{2}\approx 28.5$

5. 求拉氏变换 $Y(s)=\dfrac{1}{s}-1.8\dfrac{1}{s+4}+0.8\dfrac{1}{s+9}=\dfrac{36}{s(s+4)(s+9)}$

$$G(s)=\frac{Y(s)}{R(s)}=\frac{\frac{36}{s(s+4)(s+9)}}{\frac{1}{s}}=\frac{36}{(s+4)(s+9)}$$

故该系统频率响应为 $G(j\omega)=\dfrac{36}{(j\omega+4)(j\omega+9)}=\dfrac{36}{\sqrt{\omega^2+4^2}\cdot\sqrt{\omega^2+9^2}}\angle\left(-\arctan\frac{\omega}{4}-\arctan\frac{\omega}{9}\right)$

6. $\omega<\omega_1$ 时，斜率为0，故 $\nu=0$，过 $(1,20\lg K)$ 即 $(1,40)$ 故 $K=100$

转折频率 ω_1，ω_2 各有一惯性环节

$$G(s)=\frac{K}{(\frac{1}{\omega_1}s+1)(\frac{1}{\omega_2}s+1)}，即 G(s)=\frac{100}{(\frac{s}{\omega_1}+1)(\frac{s}{\omega_2}+1)}$$

7. 可知 $\omega<\omega_2$ 时 基准线过 $(1,20\lg K)$，斜率为 $20dB/dec$

故 $\dfrac{20\lg K}{20}=\lg 1-\lg\omega_1$　得 $K=\dfrac{1}{\omega_1}$

转折频率 ω_2，ω_3，各有一个一阶惯性环节

$$G(s)=\frac{K}{s^{-1}(\frac{s}{\omega_2}+1)(\frac{s}{\omega_3}+1)}\quad 即 G(s)=\frac{\frac{s}{\omega_1}}{(\frac{s}{\omega_2}+1)(\frac{s}{\omega_3}+1)}$$

8. $20\lg K=20$ 得 $K=10$

设 $G(s)=\dfrac{10(s+1)}{s(\frac{s^2}{\omega_n^2}+2\frac{\zeta}{\omega_n}s+1)}$

$\omega_n=2.5$，$|G(j2.5)|=\left|\dfrac{10(j2.5+1)}{j2.5(-1+2\frac{\zeta}{2.5}\cdot j2.5+1)}\right|=10^{\frac{28}{20}}$

得 $\zeta=0.214$

故 $G(s)=\dfrac{10(s+1)}{s(0.16s^2+0.17s+1)}$

9. 设 $G(s)=\dfrac{k(\tau^2s^2+2\xi_1\tau s+1)}{(T^2s^2+2\xi_2Ts+1)(T_1s+1)}$

知 $\frac{1}{\tau}=3.16$，$\frac{1}{T}=31.6$，$\frac{1}{T_1}=400$， $20\lg k=-20$ 得 $k=0.1$

$20\lg|G(j3.16)|=20\lg\left|\dfrac{0.1\cdot 2\xi_1}{1}\right|=-28$ 得 $\xi_1=0.2$.

$20\lg|G(j31.6)|=20\lg\left|\dfrac{0.1}{2\xi_2}\right|=14$ 得 $\xi_2=1$

故 $G(s)=\dfrac{0.1\left(\frac{s^2}{3.16^2}+2\times\frac{0.2}{3.16}s+1\right)}{\left(\frac{s^2}{31.6^2}+2\times\frac{1}{31.6}s+1\right)\left(\frac{s}{400}+1\right)}$

10 $\omega=100$，$20\lg G(j\omega)=0$，~~斜率为~~$-20dB/dec$，故 $\omega=1$ 时过 $(1,40)$

即过 $(1,20\lg k)$ 即 $20\lg k=40$ 得 $k=100$

设 $G(s)=\dfrac{100}{s\left(\frac{s^2}{\omega_n^2}+\frac{2\xi}{\omega_n}s+1\right)}$

$$\begin{cases}\omega_n\sqrt{1-2\xi^2}=45.3 \\ 20\lg\dfrac{1}{2\xi\sqrt{1-\xi^2}}=4.85 \\ 0<\xi<0.707\end{cases} \quad 可得 \begin{cases}\xi=0.3 \\ \omega_n=50\end{cases}$$

得 $G(s)=\dfrac{100}{s\left(\frac{s^2}{2500}+2\times\frac{0.3}{50}s+1\right)}$