自控A-HW-2　190410102　方尧　自动化1班

1. 证明:

$\forall s>\max(\alpha,\beta)$, $s>\alpha$, $s>\beta$, 则 $L^{-1}(F_1(s))=f_1(t)$, $L^{-1}(F_2(s))=f_2(t)$

$L^{-1}(aF_1(s)+bF_2(s))=aL^{-1}(F_1(s))+bL^{-1}(F_2(s))=af_1(t)+bf_2(t)$

故 $L(af_1(t)+bf_2(t))=aF_1(s)+bF_2(s)$, $\forall s>\max(\alpha,\beta)$

2 (1) $F(s)=\int_0^\infty e^{at}e^{-st}dt=\int_0^\infty e^{-(s-a)t}dt=\frac{1}{s-a}$ $(s>a)$

(2) $F(s)=\int_0^\infty t^n e^{-st}dt=\int_0^\infty -\frac{1}{s}t^n de^{-st}=-\frac{1}{s}t^n\cdot e^{-st}\Big|_0^\infty+\frac{1}{s}\int_0^\infty e^{-st}dt^n$

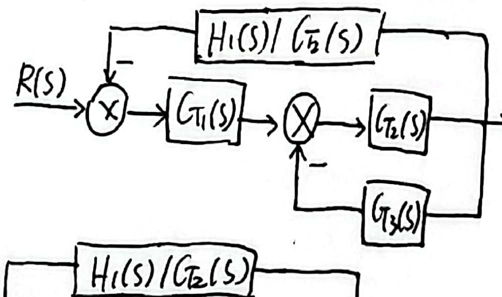
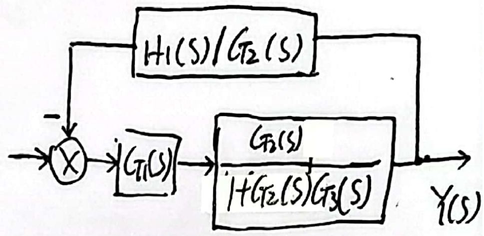
$=\frac{n}{s}\int_0^\infty e^{-st}t^{n-1}dt\ \underset{\text{不断分部积分}}{=}\ \frac{n!}{s^n}\int_0^\infty e^{-st}dt=\frac{n!}{s^{n+1}}$, $(s>0)$

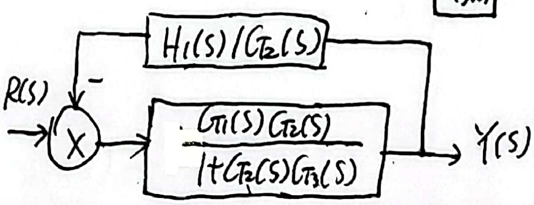
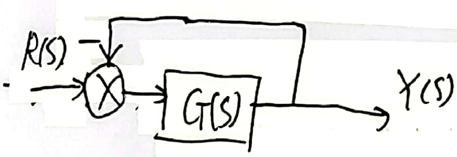
3. (1) $F(s)=\frac{2s+2}{s^2+2s+5}=\frac{2s+2}{[s-(2i-1)][s-(-2i-1)]}=\frac{1}{s-(2i-1)}+\frac{1}{s-(-2i-1)}$

$=e^{(2i-1)t}+e^{(-2i-1)t}=2e^{-t}\cos 2t$

(2) $F(s)=\frac{2}{(s-1)^3}=\mathrm{Res}[F(s)\cdot e^{st},1]=\frac{1}{2!}\cdot\lim_{s\to1}\frac{d^2}{ds^2}[F(s)e^{st}\cdot(s-1)^3]$

$=e^t+t^2e^t$

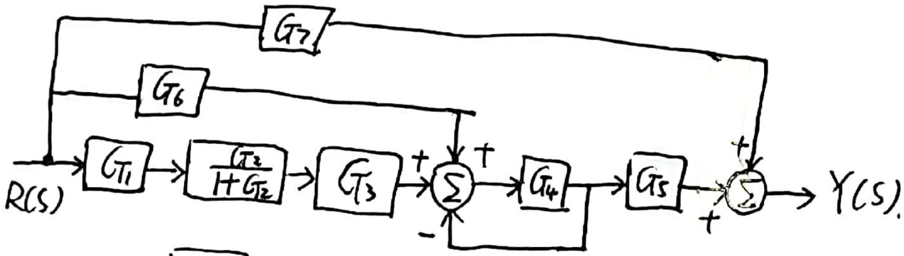
4. 原图⇒  ⇒ 

⇒  ⇒ 

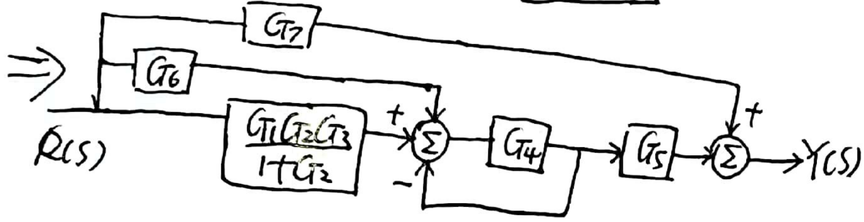
有 $\frac{G(s)}{1+G(s)}=\frac{\frac{G_1(s)G_2(s)}{1+G_2(s)G_3(s)}}{1+H_1(s)/G_2(s)\cdot\frac{G_1(s)G_2(s)}{1+G_2(s)G_3(s)}}$

得 $G(s)=\frac{G_1(s)G_2(s)}{1+G_2(s)G_3(s)+H_1(s)G_1(s)-G_1(s)G_2(s)}$

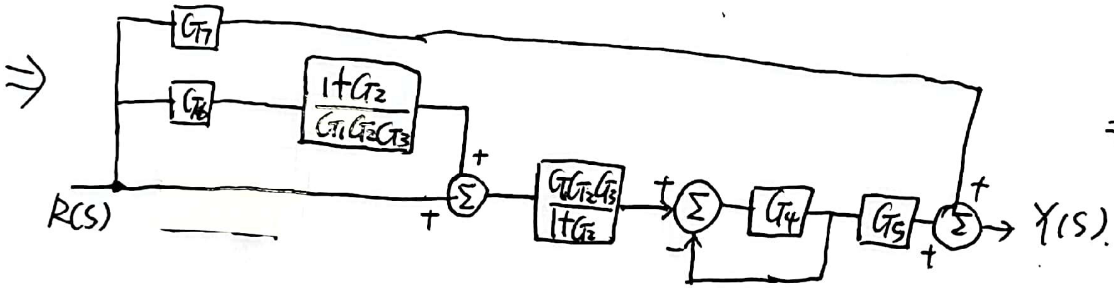
5.



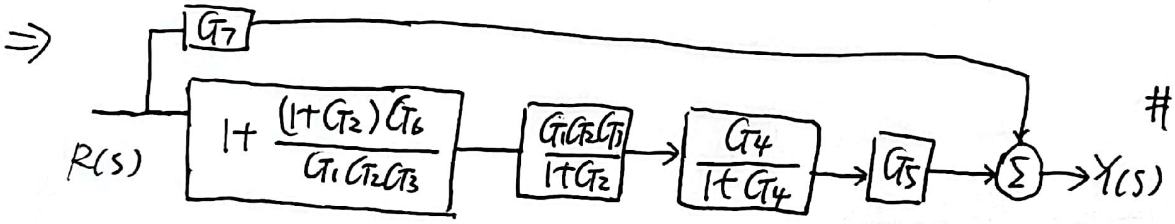
#消去反馈回路.



#合并串联支路

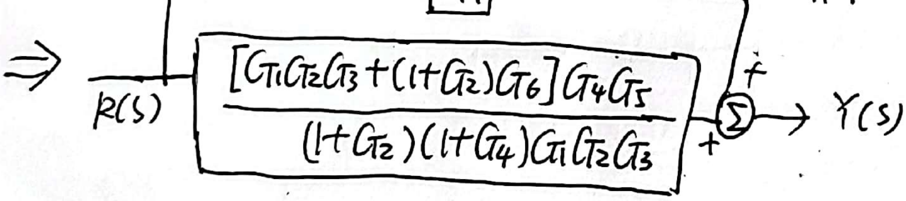


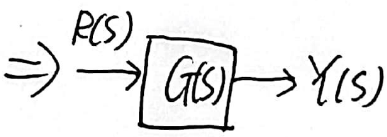
#相加点前移



合并相加
消去反馈回路.

#串联合并



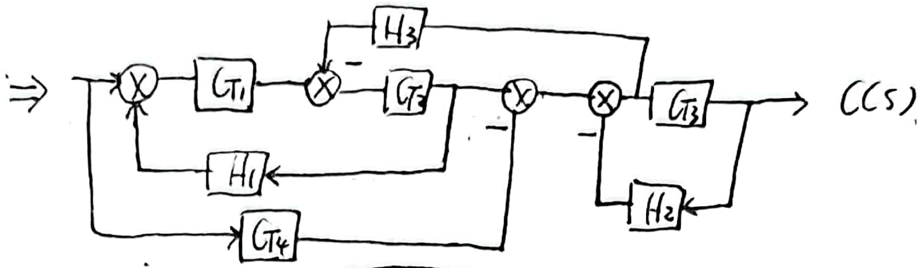


#合并相加

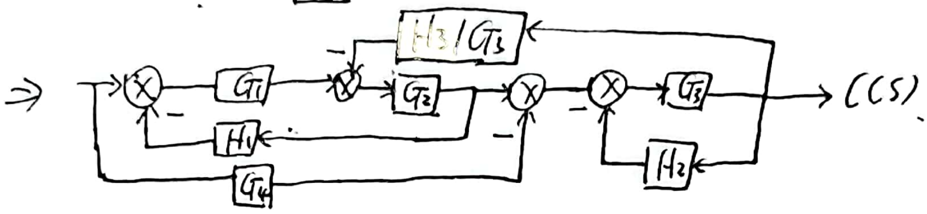
$$\Rightarrow G(S)=\frac{[G_1G_2G_3+(1+G_2)G_6]G_4G_5}{(1+G_2)(1+G_4)G_1G_2G_3}+G_7$$

6.

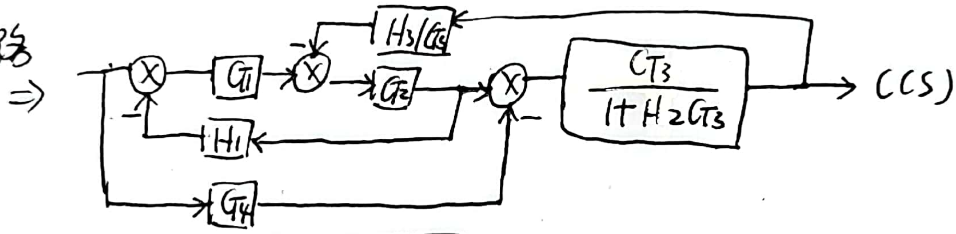
① 相加点后移



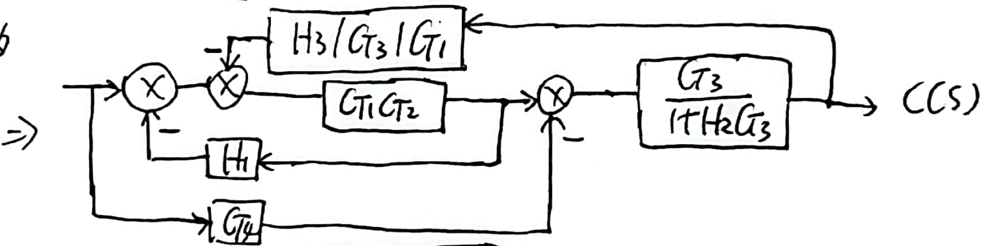
② 分支点后移



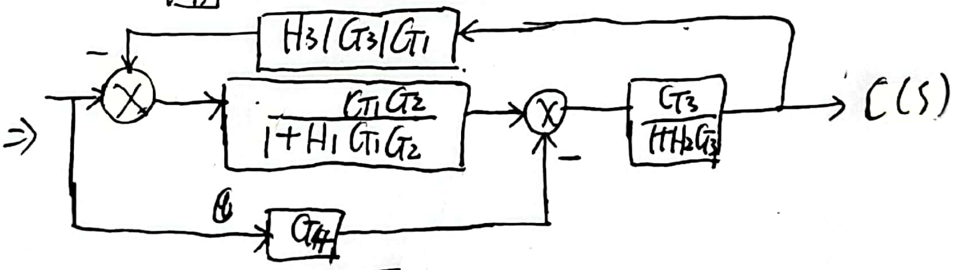
③ 消去反馈支路



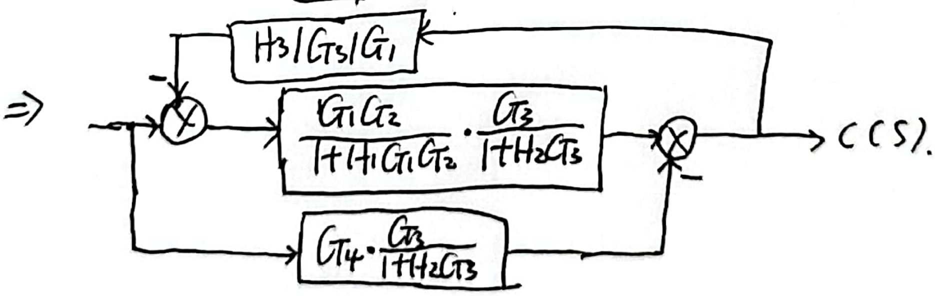
④ 相加点前移 并合并串联支路



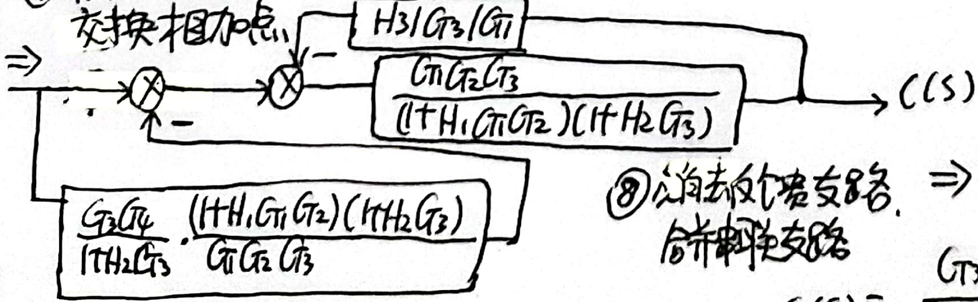
⑤ 交换相加点 并消去反馈回路



⑥ 相加点后移，合并串联项



⑦ 相加点前移 交换相加点



⑧ 消去反馈支路 合并串联支路

$$G(S)=\frac{G_3(G_1G_2-H_1G_1G_2G_4-G_4)}{(1+H_1G_1G_2)(1+H_2G_3)+G_2H_3}$$