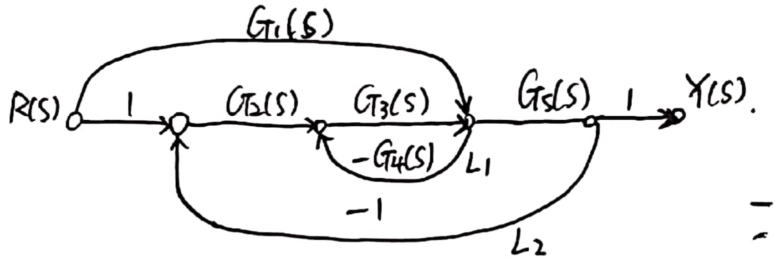
自控A HW-3 190410102 方尧 自动化1班

1.



共两个回路

$L_1 = -G_3G_4$

$L_2 = -G_2G_3G_5$

∴ 无不接触两回路

$$\Delta = 1-[(-G_3G_4)+(-G_2G_3G_5)] = 1+G_3G_4+G_2G_3G_5$$

共两条前向通路.

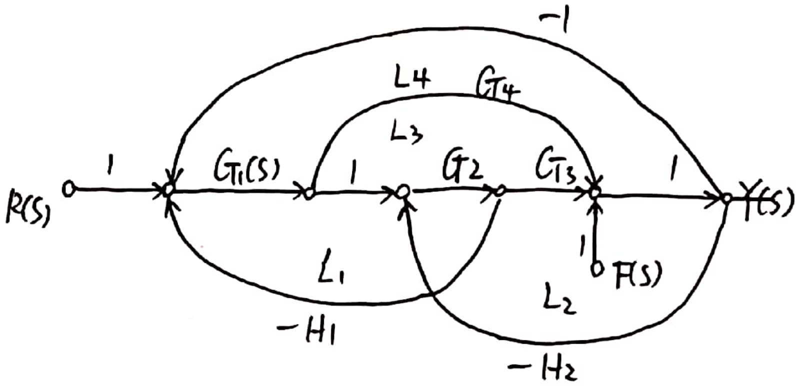
$P_1 = G_1G_5 \quad \Delta_1 = 1$

$P_2 = G_2G_3G_5 \quad \Delta_2 = 1$

$$G(S) = \frac{Y(S)}{R(S)} = P = \frac{1}{\Delta}\sum_{k=1}^{n} P_k\Delta_k = \frac{G_1G_5+G_2G_3G_5}{1+G_3G_4+G_2G_3G_5}$$

$$H(S) = \frac{E(S)}{R(S)} = \frac{R(S)-Y(S)}{R(S)} = 1-G(S) = \frac{1+G_3G_4-G_1G_5}{1+G_3G_4+G_2G_3G_5}$$

2.



共4个回路.

$L_1 = -H_1G_1G_2$.

$L_2 = -H_2G_2G_3$

$L_3 = -G_1G_2G_3$

$L_4 = -G_1G_4$.

无两两不接触回路

$$\Delta = 1+H_1G_1G_2+H_2G_2G_3+G_1G_2G_3+G_1G_4$$

$P_1 = 1 \quad \Delta_1 = 1+G_1G_2H_1$

$$\frac{Y(S)}{F(S)} = \frac{1+G_1G_2H_1}{1+H_1G_1G_2+H_2G_2G_3+G_1G_2G_3+G_1G_4}$$

∴ 则有 $1+G_1G_2H_1=0$ 时，Y(S)不受F(S)影响

3. 选取 $X=[y_1\ \dot{y}_1\ y_2\ \dot{y}_2]$

$$\begin{cases} u-k(y_2-y_1)=m_2\ddot{y}_2 \\ k(y_2-y_1)-b\dot{y}_1=m_1\ddot{y}_1 \end{cases}$$

$$\begin{aligned} \dot{x}_1&=x_2 \\ \dot{x}_2&=-\frac{k}{m_1}x_1+\frac{k}{m_1}x_3-\frac{b}{m_1}x_2 \\ \dot{x}_3&=x_4 \\ \dot{x}_4&=\frac{k}{m_2}x_1-\frac{k}{m_2}x_3+\frac{1}{m_2}u \end{aligned}$$

状态空间表达式 $\dot{x}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & -\frac{b}{m_1} & \frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & 0 \end{bmatrix}x+\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \end{bmatrix}u$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}x$$

4. $G(s)=\frac{Y(s)}{U(s)}=C(sI-A)^{-1}B+D=[-1\ 1\ 0]\begin{bmatrix} s-1 & -1 & 1 \\ -7 & s-3 & 0 \\ 2 & -1 & s-5 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

$$=\frac{3s-30}{s^3-9s^2+14s+33}$$

5. (a) 反变换 $y^{(3)}+7y^{(2)}+14y^{(1)}+8y=8u(t)$

令 $x=[y,y^{(1)},y^{(2)}]^T$

$$\dot{x}=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -14 & -7 \end{bmatrix}x+\begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix}$$

$$y=[1\ 0\ 0]x$$

(b) 同理可得 $G(s)=\frac{s^2+2s+5}{s^3+2s^2+3s+10}$ 的状态空间表达式为：

$$\dot{x}=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -3 & -2 \end{bmatrix}x+\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u$$

$$y=[5\ 2\ 1]x$$