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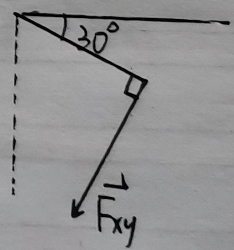
3-1 解：知 $\vec{r}=\vec{OA}=(1,0,1)$　$\vec{F}=(-5\sqrt{2}\,kN,\ 5\sqrt{2}\,kN,\ 0)$

$$\vec{M}_O(\vec{F})=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 0 & 1\\ -5\sqrt{2} & 5\sqrt{2} & 0\end{vmatrix}=-5\sqrt{2}\,kN\cdot m\cdot\vec{i}-5\sqrt{2}\,kN\cdot m\cdot\vec{j}+5\sqrt{2}\,kN\cdot m\,\vec{k}$$

故 $M_x(\vec{F})=-7070\,N\cdot m$、$M_y(\vec{F})=-7070\,N\cdot m$，$M_z(\vec{F})=7070\,N\cdot m$

3-2 解：$\vec{r}=\vec{OC}=(\frac{1}{2}r,\ \frac{\sqrt{3}}{2}r,\ h)$，　$\vec{F_z}=-F\cdot\sin60^\circ\,\vec{k}=-\frac{\sqrt{3}}{2}F\cdot\vec{k}$

$|\vec{F_{xy}}|=F\cdot\cos60^\circ=\frac{1}{2}F$，$\vec{F_x}=\frac{1}{2}F\cdot\sin60^\circ\cdot\vec{i}=\frac{\sqrt{3}}{4}F\vec{i}$



$\vec{F_y}=-\frac{1}{2}F\cdot\cos60^\circ\,\vec{j}=-\frac{1}{4}F\cdot\vec{j}$

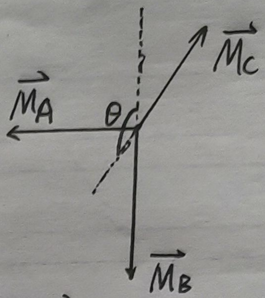
$$故\ \vec{M}_{OA}(\vec{F})=\vec{r}\times\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{1}{2}r & \frac{\sqrt{3}}{2}r & h\\ \frac{\sqrt{3}}{4}F & -\frac{1}{4}F & -\frac{\sqrt{3}}{2}F\end{vmatrix}$$

$$=(\frac{1}{4}hF-\frac{3}{4}rF)\vec{i}+\frac{\sqrt{3}}{4}F(r+h)\vec{j}-\frac{1}{2}rF\,\vec{k}$$

故力F对x轴之矩为 $\frac{1}{4}hF-\frac{3}{4}rF$，对y轴之矩为 $\frac{\sqrt{3}}{4}F(r+h)$

对z轴之矩为 $-\frac{1}{2}rF$

3-3解：记A,B,C三轮上的力偶分别为 $\vec{M}_A$，$\vec{M}_B$，$\vec{M}_C$



$|\vec{M}_A| = 10N\times 2\times 150\times 10^{-3}m = 3\,N\cdot m$

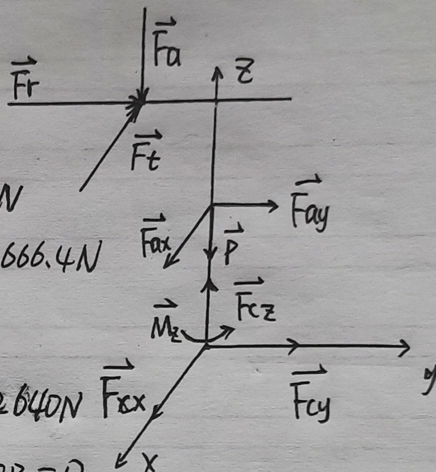
$|\vec{M}_B| = 20N\times 2\times 100\times 10^{-3}m = 4\,N\cdot m$

知 $\sum \vec{M}_i = 0$ 故 $|\vec{M}_C| = \sqrt{M_A^2+M_B^2} = 5\,N\cdot m$

$\frac{|\vec{M}_A|}{|\vec{M}_B|} = \frac{3}{4}$ $\arctan\frac{3}{4} = 37^\circ$，$|\vec{M}_C| = F\times 2\times 50\times 10^{-3}m$

得 $F = 50N$，$\theta = 143^\circ$

3-4解：$\sum \vec{M}_z = 0$ 即 $M_z - F_t\cdot OB = 0$



得 $F_t = 2000N$，故 $F_a = 640N$，$F_r = 340N$

$\sum \vec{M}_y(\vec{F}_i) = 0$，$F_t\times 4m - F_{ax}\times 3m = 0$，故 $F_{ax} = 2666.4N$

$\sum \vec{M}_{ay}(\vec{F}_i) = 0$ $F_t\times 1m + F_{cx}\times 3m = 0$

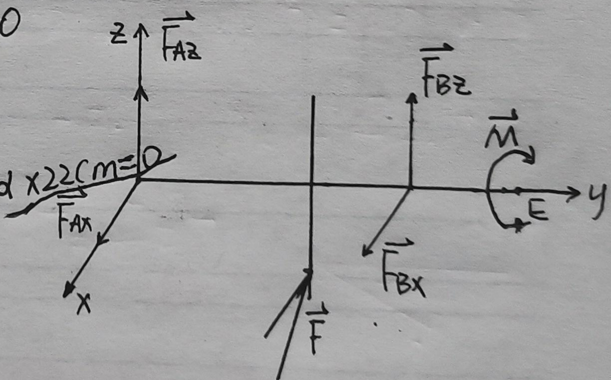
故 $F_{cx} = -666.7N$，$\sum \vec{F}_z = 0$，$F_{cz} = F_a + P = 12640N$

$\sum \vec{M}_x(\vec{F}_i) = 0$ $F_r\times 4m + F_{ay}\times 3m - F_a\times OB = 0$

故 $F_{ay} = -325.3N$ $\sum F_y = 0$，$F_r + F_{ay} + F_{cy} = 0$ 得 $F_{cy} = -14.7N$

故 $\vec{F}_a = 2666.4N\cdot\vec{i} - 325.3N\cdot\vec{j}$；$\vec{F}_c = -666.7N\cdot\vec{i} - 14.7N\cdot\vec{j} + 12640N\cdot\vec{k}$

3-5解：$\sum \vec{M}_y(\vec{F}_i) = 0$，$M - F\cdot\frac{d}{2}\cos 20^\circ = 0$



故 $F = \frac{2M}{d\cos 20^\circ}$

$\sum \vec{M}_z(\vec{F}_i) = 0$，$-F_{Bx}\times(22cm + 12.2cm) + F\cos\alpha\times 22cm = 0$

得 $F_{Bx} = 7659.8N$.

$\sum F_x = 0$ $F_{Ax} + F_{Bx} - F\cdot\cos\alpha = 0$

得 $F_{Ax} = 4247.7N$

$\sum \vec{M}_x(\vec{F}_i) = 0$ $F_{Bz}\times(22cm + 12.2cm) + F\sin\alpha\times 22cm = 0$

得 $F_{Bz} = -2787.9N$.

$\sum F_z = 0$. $F_{Az} + F_{Bz} + F\cdot\sin\alpha = 0$ 得 $F_{Az} = -1546.08N$

故 $\vec{F}_A = 4247.7N\cdot\vec{i} - 1746.1\cdot\vec{k}$，$\vec{F}_B = 7659.8N\cdot\vec{i} - 2787.9N\cdot\vec{k}$

3-6 解：设△OBE，OB边高h.　　由△OEB等腰三角形，故E($\frac{L}{2}$, h)

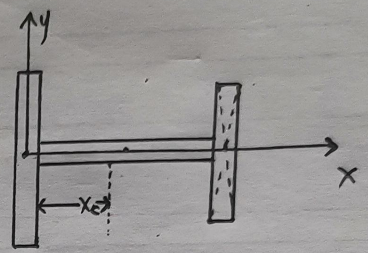
对X轴用合力矩定理，$\rho(L^2-\frac{1}{2}Lh)\cdot h = \rho L^2\cdot\frac{L}{2} + \rho(-\frac{1}{2}L\cdot h)\cdot\frac{h}{3}$

得 $h=\frac{3-\sqrt{3}}{2}L$　故E为($\frac{L}{2}$, $\frac{3-\sqrt{3}}{2}L$)　即($\frac{L}{2}$, 0.634L)

3-7 解：把该工字钢分为三部分，如图。

由对称性 $y_c=0$，

对y轴应用合力矩定理、



$$(10+x_c)(200\times20+200\times20+150\times20) = (10+100)\times200\times20 + (10+200+10)\times150\times20$$

得 $x_c=90$，故该截面几何中心在中轴线上距左侧钢右侧壁90mm处

3-8 解：　将组合体旋转放置重力方向如图所示

建立直角坐标系，由对称性 $x_c=0$，$y_c=0$

对下方半球体，$\frac{2}{3}\pi r^3\cdot z_c' = \int_{-r}^{0}\pi(r^2-z^2)\cdot dz\cdot z$

得 $z_c' = -\frac{3}{8}r$

对整体　$0 = \pi r^2\cdot h\cdot\frac{h}{2} + \frac{2}{3}\pi r^3\cdot(-\frac{3}{8}r)$

得 $h=\frac{\sqrt{2}}{2}r = 0.707r$

故高度为0.707r