DIP HW-4 19041002 方尧 自动化1902班

4.38 proof:

(a) $(f*h)(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)\, h(x-m,y-n)$

$$F[f(x,y)*h(x,y)] = \sum_{x=0}^{M-1}\sum_{y=0}^{M-1}\left[\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)\,h(x-m,y-n)\right]\cdot e^{-j2\pi(ux/M+vy/N)}$$

$$= \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} h(x-m,y-n)\cdot e^{-j2\pi(ux/M+vy/N)}$$

$$= \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)\, e^{-j2\pi(um/M+vn/N)}\, H(u,v)$$

$$= F(u,v)\cdot H(u,v)$$

(b) $$F^{-1}\left[\frac{1}{MN}F(u,v)*H(u,v)\right] = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\left[\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} F(m,n)\,H(u-m,v-n)\cdot\frac{1}{MN}\right]e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

$$= \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} F(m,n)\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\frac{1}{MN}\, H(u-m,v-n)\,e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

$$= \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} F(m,n)\, e^{j2\pi(mx/M+ny/N)}\cdot h(x,y)$$

$$= f(x,y)\cdot h(x,y)$$

即 (a) $(f*h)(x,y) \Longleftrightarrow (F\cdot H)(u,v)$

(b) $(f\cdot h)(x,y) \Longleftrightarrow \frac{1}{MN}[(F*h)(u,v)]$

4.48 $$h(t,z) = F^{-1}[H(u,v)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A e^{-(u^2+v^2)/2\sigma^2} e^{j2\pi(ut+vz)}\,du\,dv$$

$$= A\int_{-\infty}^{\infty} e^{-\frac{u^2}{2\sigma^2}+j2\pi ut}\,du\cdot\int_{-\infty}^{\infty} e^{-\frac{v^2}{2\sigma^2}+j2\pi vz}\,dv$$

$$= Ae^{-2\pi^2\sigma^2(t^2+z^2)}\int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}(u-j2\pi\sigma^2 t)^2}\,du\int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}(v-j2\pi\sigma^2 z)^2}\,dv$$

$$= Ae^{-2\pi^2\sigma^2(t^2+z^2)}\int_{-\infty}^{\infty} e^{-\frac{r^2}{2\sigma^2}}\,dr\int_{-\infty}^{\infty} e^{-\frac{k^2}{2\sigma^2}}\,dk$$

$$= A2\pi\sigma e^{-2\pi^2\sigma^2(t^2+z^2)}$$

即 $H(u,v)=Ae^{-(u^2+v^2)/2\sigma^2}$ 对应时域连续变换为 $h(t,z)=A2\pi\sigma\, e^{-2\pi^2\sigma^2(t^2+z^2)}$

4.49 (a)

$$G(u,v) = H(u,v)F(u,v) = e^{-D^2(u,v)/2D_0^2}F(u,v)$$

Kth Gaussian LPF

$$G_K(u,v) = e^{-KD^2(u,v)/2D_0^2}F(u,v)$$

由低通滤波知最终若K很大，最终仅有$F(0,0)$能通过.

$$H_K(u,v) = e^{-KD^2(u,v)/2D_0^2} = \begin{cases} 1 & \text{if}(u,v)=(0,0) \\ 0 & \text{others} \end{cases}$$

最终图像每一个像素将会等于原图像的平均灰度.

4.56

$$\begin{aligned} H_{HP}(u,v) &= 1 - H_{LP}(u,v) \\ &= 1 - \frac{1}{1+[D(u,v)/D_0]^{2n}} \\ &= \frac{[D(u,v)/D_0]^{2n}}{1+[D(u,v)/D_0]^{2n}} \\ &= \frac{1}{1+[D_0/D(u,v)]^{2n}} \end{aligned}$$

即 $H_{hp}(u,v) = \dfrac{1}{1+[D_0/D(u,v)]^{2n}}$ Butterworth