

Problem set 1

Q1. A monopoly insurance company provides accident insurance to two types of customers: low risk customers, for whom the probability of an accident is 0.25, and high risk customers, for whom the probability of an accident is 0.5. There is an equal number of both types of customers. Without insurance, each customer's wealth is 16 if there is no accident, but 0 if there is an accident. Customers' utility of wealth is: $u(w) = \sqrt{w}$. The insurance company cannot identify the type of a customer when the customer applies for an insurance contract. Suppose the insurance company offers the following two contracts. The first contract offers a payout of 8 in case there is an accident, and requires customers to pay a premium of 7. The second contract offers a payout of 16 in case there is an accident, but requires customers to pay a premium of 10.

(a) Determine for low risk customers and then also for high risk customers which, if any, of these contracts they will buy.

(b) Does the insurance company manage to screen its customers with these contracts?

(c) Calculate the insurance company's expected profit if it offers these contracts.

Q2. The driver of a car can exert effort to avoid an accident ($e = 1$) or not exert any effort ($e = 0$). If $e = 1$, the probability of an accident is $1/2$. If $e = 0$, the probability of an accident is 1. The driver's wealth without accident is: $w = 100$. In case of an accident, the repair of the car costs 64. So, if there is an accident, the driver has $w = 100 - 64 = 36$ left. The driver's utility of wealth is \sqrt{w} , that is, the driver is risk averse. The cost of effort, $C(e)$, are 0 if effort is $e = 0$, and 1 if effort is $e = 1$. The driver's utility function is: $u(w, e) = \sqrt{w} - C(e)$.

(a) Will the driver choose to exert effort? Compare the expected utility of the driver when exerting effort ($e = 1$) with the expected utility when exerting no effort ($e = 0$).

(b) Now suppose there is a risk neutral insurance company. This insurance company acts like a principal with the driver being the agent. Suppose the insurance company cannot monitor the driver's behavior. The insurance company considers three contracts, labeled A, B and C. Each contract specifies the price p and the amount of money the driver gets in case of an accident, d . Given p and d , the final wealth of the driver in case of no accident is

$w_0 = 100 - p$ and the final wealth in case of an accident is $w_A = 36 - p + d$. The contracts are:

Contract A: price $p = 36$ and payment $d = 64$

Contract B: price $p = 19$ and payment $d = 47$

Contract C: price $p = 19$ and payment $d = 32$

For each of the contracts, calculate the final wealth, w_0 and w_A , in the two outcomes, and list them in a table of the following type:

Contract A: $w_0 = ?$ and $w_A = ?$

Contract B: $w_0 = ?$ and $w_A = ?$

Contract C: $w_0 = ?$ and $w_A = ?$

(c) Which of these contracts offers full insurance ($w_0 = w_A$) to the driver?

(d) For each of these contracts, determine which of the two effort levels, $e = 0$ or $e = 1$, would be expected utility maximizing for the driver if he accepted that contract. Assume that the driver, if both effort levels yield the same expected utility, chooses $e = 1$.

(e) Which of these contracts are such that the driver would accept the contract rather than staying uninsured? (For each contract, compare the expected utility from being uninsured with the expected utility when having the contract and choosing the optimal effort level. Assume that the driver accepts a contract if indifferent between insuring and not insuring.)

(f) Which of the three contracts gives the insurance company the highest expected profits? What are the expected profits of the insurance company if it offers this contract?