HW-6 19040102 自动化1班 方充

6-3

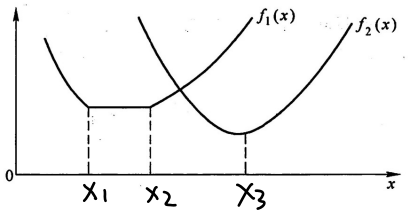


图 6-10

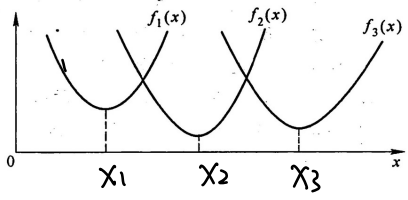
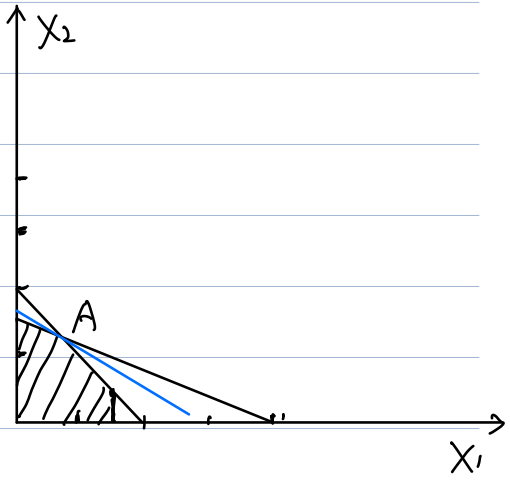


图 6-11

图 6-10 有效解集合 $[x_2, x_3]$　　图 6-10 有效解集合 $[x_1, x_3]$

6-6 $\min h(f(x)) = -\frac{7}{2}x_1 - \frac{9}{2}x_2$

$$\text{s.t.} \begin{cases} 3x_1 + 8x_2 \le 12 \\ x_1 + x_2 \le 2 \\ 0 \le x_1 \le 1.5,\ x_2 \ge 0 \end{cases}$$



$A(\frac{4}{5}, \frac{6}{5})$ 为最优解

$f_1(x) = -10.4$, $f_2(x) = -6$

6-7 设 A, B, C 咨询时间 x_1, x_2, x_3

收入 $f(x) = 20x_1 + 22x_2 + 18x_3$　　对应偏差变量 d_1^-　d_1^+ (500元)

耗费路上时间为 $0.1x_1 + 0.2x_2 + 0.2x_3$　对应偏差变量 d_2^-　d_2^+ (15小时)

建立的模型为

$$\min [p_1 d_1^- + p_2 d_2^+]$$

$$\text{s.t.} \begin{cases} 1.1x_1 + 1.2x_2 + 0.2x_3 \le 40 \\ 20x_1 + 22x_2 + 18x_3 + d_1^- - d_1^+ = 500 \\ 0.1x_1 + 0.2x_2 + 0.2x_3 + d_2^- - d_2^+ = 15 \\ 0 \le x_1 \le 80,\ 0 \le x_2 \le 60,\ 0 \le x_3 \le 20 \end{cases}$$

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6-8 设 A_i 到 B_j 供应 x_{ij}

模型建立为

$$\min F = P_1 d_5^- + P_2(d_6^- + d_7^- + d_8^-) + P_3 d_9^+ + P_4 d_{10}^- + P_5(d_{11}^+ + d_{12}^+) + P_6(d_{13}^- + d_{13}^+)$$

$$\text{s.t.}\begin{cases} x_{11}+x_{12}+x_{13}+d_1^- = 3000 \\ x_{21}+x_{22}+x_{23}+d_2^- = 4000 \\ x_{11}+x_{21}+d_3^- - d_3^+ = 2000 \\ x_{12}+x_{22}+d_4^- - d_4^+ = 1500 \\ x_{13}+x_{23}+d_5^- - d_5^+ = 5000 \\ x_{11}+x_{21}+d_6^- - d_6^+ = 1500 \\ x_{12}+x_{22}+d_7^- - d_7^+ = 1125 \\ x_{13}+x_{23}+d_8^- - d_8^+ = 3750 \\ 10x_{11}+4x_{12}+12x_{13}+8x_{21}+10x_{22}+3x_{23}-d_9^+ = 0 \\ x_{21}+d_9^- - d_{10}^+ = 1000 \\ x_{13} - d_{11}^+ = 0 \\ x_{22} - d_{12}^+ = 0 \\ x_{12}+x_{22}-0.75(x_{11}+x_{21})+d_{13}^- - d_{13}^+ = 0 \end{cases}$$

d_i^-, d_i^+, x_{ij} 均 ≥ 0

6-9 (2)

① 序列法：

P1级目标规划 $\min Z = P_1 d_1^-$

$$\text{s.t.}\ x_1 + x_2 + d_1^- - d_1^+ = 80$$

$x_{1,2} \geq 0,\ d_i^-, d_i^+ \geq 0$

最优解 $d_1^- = 0$

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P_2 级目标规划 $\min z = d_{11}^+$

$$s.t. \quad x_1 + x_2 - d_1^+ = 80$$
$$d_1^+ + d_{11}^- - d_{11}^+ = 11$$

		0	0	0	0	1
		x_1	x_2	d_1^+	d_{11}^-	d_{11}^+
0 x_1	80	1	1	-1	0	0
0 d_{11}^-	10	0	0	1	1	-1

$\min z = 0$ 故 $d_{11}^+ = 0$

P_3 级目标决见划

$$\min z_3 = 5d_2^- + 3d_3^-$$

$$\begin{cases} x_1 + x_2 - d_1^+ = 80 \\ x_1 + d_2^- = 70 \\ x_2 + d_3^- = 45 \\ d_1^+ + d_{11}^- = 10 \\ x_i \geq 0 \quad d_i^- \, d_i^+ \geq 0 \end{cases}$$

		0	0	0	5	3	0
		x_1	x_2	d_1^+	d_2^-	d_3^-	d_{11}^-
0 x_2	80	1	1	-1	0	0	0
5 d_2^-	70	1	0	0	1	0	0
⇐3 d_3^-	-35	-1	0	1	0	1	0
0 d_{11}^-	10	0	0	1	0	0	1
		-2	0	-3	0	0	0

		0	0	0	5	3	0
		x_1	x_2	d_1^+	d_2^-	d_3^-	d_{11}^-
0 x_2	45	0	1	0	0	1	0
5 d_2^-	35	0	0	1	1	1	0
0 x_1	35	1	0	-1	0	-1	0
⇐0 d_{11}^-	10	0	0	1	0	0	1
	-175	0	0	-5	0	-2	0

		0	0	0	5	3	0
		x_1	x_2	d_1^+	d_2^-	d_3^-	d_{11}^-
x_2	45	0	1	0	0	1	0
⇐d_2^-	25	0	0	0	1	1	-1
x_1	45	1	0	0	0	-1	1
d_1^+	10	0	0	1	0	0	1
	-125	0	0	0	0	-2	5

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		x_1	x_2	d_1^+	d_2^-	d_3^-	d_1^-
x_2	20	0	1	0	-1	0	1
d_3^-	25	0	0	0	1	1	-1
x_1	75	1	0	0	1	0	0
d_1^+	10	0	0	1	0	0	1
	-75	0	0	0	0	0	3

故得到最优解

$d_{11}^- = 0$, $d_1^+ = 10$

$5d_2^- + 3d_3^- = 75$

P_4 级 $d_1^+ = 10$

$$\begin{cases} x_1 + x_2 = 90 \\ x_1 + d_2^- = 70 \\ x_2 + d_3^- = 45 \\ 5d_2^- + 3d_3^- = 75 \end{cases} \Rightarrow \begin{cases} x_1 = 70 \\ x_2 = 20 \\ d_2^- = 0 \\ d_3^- = 25 \end{cases}$$

即最优解为 $x_1 = 70$, $x_2 = 20$, $d_1^- = d_2^- = d_{11}^- = d_{11}^+ = 0$, $d_1^+ = 10$, $d_3^- = 25$

$$Z^* = (0, 0, 75, 10)$$

② 单纯形法

C_j			0	0	P_1	$5P_3$	$3P_3$	0	P_4	0	0	P_2
C_B	X_B	b	x_1	x_2	d_1^-	d_2^-	d_3^-	d_{11}^-	d_1^+	d_2^+	d_3^+	d_{11}^+
P_1	d_1^-	80	1	1	1	0	0	0	-1	0	0	0
$5P_3$	d_2^-	70	1	0	0	1	0	0	0	0	0	0
$3P_3$	d_3^-	45	0	1	0	0	1	0	0	0	0	0
0	d_{11}^-	10	0	0	0	0	0	1	1	0	0	-1
	P_1	-80	-1	-1	0	0	0	0	1	0	0	0
	P_2	0	0	0	0	0	0	0	0	0	0	1
	P_3	-485	-5	-3	0	0	0	0	0	0	0	0
	P_4	0	0	0	0	0	0	0	1	0	0	0

表1

P_1 行有 <0 项，x_1 进，d_2^- 出

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C_j		0	0	P_1	$5P_3$	$3P_3$	0	P_4	0	0	P_2
C_B	X_B	b	x_1 ⇓	x_2	d_1^-	d_2^-	d_3^-	d_{11}^-	d_1^+	d_2^+	d_3^+	d_{11}^+
	d_1^-	10	0	1	1	-1	0	0	-1	0	0	0
⇐	x_1	70	1	0	0	1	0	0	0	0	0	0
	d_3^-	45	0	1	0	0	1	0	0	0	0	0
	d_{11}^-	10	0	0	0	0	0	1	1	0	0	-1
	P_1	-10	0	-1	0	1	0	0	1	0	0	0
	P_2	0	0	0	0	0	0	0	0	0	0	1
	P_3	-135	0	-3	0	5	0	0	0	0	0	0
	P_4	0	0	0	0	0	0	0	1	0	0	0

表2

x_2 进基，d_1^- 离基

C_j		0	0	P_1	$5P_3$	$3P_3$	0	P_4	0	0	P_2
C_B	X_B	b	x_1	x_2 ⇓	d_1^-	d_2^-	d_3^-	d_{11}^-	d_1^+	d_2^+	d_3^+	d_{11}^+
⇐	x_2	10	0	1	1	-1	0	0	-1	0	0	0
	x_1	70	1	0	0	1	0	0	0	0	0	0
	d_3^-	35	0	0	-1	1	1	0	1	0	0	0
	d_{11}^-	10	0	0	0	0	0	1	1	0	0	-1
	P_1	0	0	0	0	0	0	0	0	0	0	0
	P_2	0	0	0	0	0	0	0	0	0	0	1
	P_3	-105	0	0	3	2	0	0	-3	0	0	0
	P_4	0	0	0	0	0	0	0	1	0	0	0

表3

P_1、P_2 级满足，P_3 行有负项，d_1^+ 进基，d_{11}^- 离基

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C_j			0	0	P_1	$5P_3$	$3P_3$	0	P_4	0	0	P_2
C_B	X_B	b	x_1	x_2	d_1^-	d_2^-	d_3^-	d_{11}^-	d_1^+	d_2^+	d_3^+	d_{11}^+
	x_2	20	0	1	1	-1	0	1	0	0	0	-1
	x_1	70	1	0	0	1	0	0	0	0	0	0
	d_3^-	25	0	0	-1	1	1	-1	0	0	0	1
←	d_1^+	10	0	0	0	0	0	1	1	0	0	-1
	P_1	0	0	0	0	0	0	0	0	0	0	0
	P_2	0	0	0	0	0	0	0	0	0	0	1
	P_3	-75	0	0	3	2	0	3	0	0	0	-3
	P_4	-10	0	0	0	0	0	-1	0	0	0	1

表4

d_{11}^+ 检验数 $P_2 - 3P_3 > 0$ P_3 级已为最优

d_{11}^- 检验数 $3P_3 - P_4 > 0$ P_4 级已为最优

故当前基本可行解对 P_1, P_2, P_3 级目标都达最优

$$\begin{cases} d_1^- = 0 \\ d_{11}^+ = 0 \\ 5d_2^- + 3d_3^- = 75 \\ d_1^+ = 10 \\ x_1 + x_2 + d_1^- - d_1^+ = 80 \\ x_1 + d_2^- = 70 \\ x_2 + d_3^- = 45 \\ d_1^+ + d_{11}^- - d_{11}^+ = 10 \end{cases}$$

联立可解得 $d_{11}^- = d_{11}^+ = 0$

$d_1^- = 0$, $d_1^+ = 10$, $x_1 = 70$, $x_2 = 20$

$d_2^- = d_2^+ = 0$, $d_3^- = 25$, $d_3^+ = 0$

即最优解为 $x_1 = 70$, $x_2 = 20$, $d_1^- = d_2^- = d_{11}^- = d_{11}^+ = 0$, $d_1^+ = 10$, $d_3^- = 25$

$X^* = (0, 0, 75, 10)$