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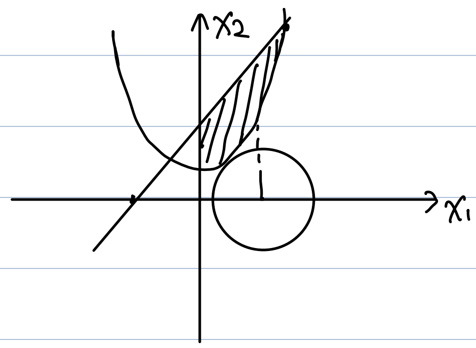
4-2 $\max S = 3.6x_1 - 0.4x_1^2 + 1.6x_2 - 0.2x_2^2$

$$\text{s.t.} \begin{cases} x_1 + 0.5x_2 \le 5 \\ x_1 \ge 0,\ x_2 \ge 0 \end{cases}$$

4-4(2) $\min S = x_1^2 + x_2^2 - 4x_1 + 4 = (x_1-2)^2 + x_2^2$ 即至(2,0)距离平方



由图知 从(2,0)为圆心圆与抛物线相切时,取到最小值

$$\begin{cases} (x_1-2)^2 + x_2^2 = a \\ x_2 = x_1^2 + 1 \end{cases}$$

即 $2x_1' \cdot \dfrac{x_1'^2+1}{x_1'-2} = -1$ 得 $x_1' = 0.5536$

即 $X = (0.5536, 1.31)^T$, $\min = 3.81$

4-5(1) $\nabla f(x) = (2x_1 - 4x_3,\ 4x_2,\ 6x_3 - 4x_1)^T$

$$\nabla^2 f(x) = \begin{pmatrix} 2 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 6 \end{pmatrix}$$

4-5(2) $\nabla f(x) = (3x_2^2 + x_2 e^{x_1x_2},\ 6x_1x_2 + x_1 e^{x_1x_2})^T$

$$\nabla^2 f(x) = \begin{pmatrix} x_2^2 e^{x_1x_2}, & 6x_2 + e^{x_1x_2} + x_1x_2 e^{x_1x_2} \\ 6x_2 + e^{x_1x_2} + x_1x_2 e^{x_1x_2}, & 6x_1 + x_1^2 e^{x_1x_2} \end{pmatrix}$$

4-8 证明:

$\nabla f(x_1,x_2) = (8x_1 - 2x_1x_2,\ 2x_2 - x_1^2)^T$, $\nabla^2 f(x_1,x_2) = \begin{pmatrix} 8-2x_2, & -2x_1 \\ -2x_1, & 2 \end{pmatrix}$

$\because \nabla f(0,0) = (0,0)^T$, $\nabla^2 f(0,0) = \begin{pmatrix} 8 & 0 \\ 0 & 2 \end{pmatrix}$, $A>0$, $AC-B^2>0$

$\therefore X^* = (0,0)^T$ 是 $f(x_1,x_2)$ 的严格局部极小值点

$\because \nabla f(X_1) = \nabla f(X_2) = (0,0)^T$, $\nabla^2 f(X_1) = \begin{pmatrix} 0 & 4\sqrt{2} \\ 4\sqrt{2} & 2 \end{pmatrix}$, $\nabla^2 f(X_2) = \begin{pmatrix} 0 & -4\sqrt{2} \\ -4\sqrt{2} & 2 \end{pmatrix}$

$AC-B^2<0$, 故 $f(x_1,x_2)$ 在 X_1, X_2 未取到极值

$\therefore X_1, X_2$ 是鞍点, 不是极值点

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4-10 (1) $\nabla f(x) = -3(4-x)^2$, $\nabla^2 f(x) = 6(4-x)$

$x \le 4$, $\nabla^2 f(x)$ 半正定, 故 $f(x)$ 为凸函数

4-10 (2) $\nabla f(x) = (2x_1+2x_2, 2x_1+6x_2)^T$, $\nabla^2 f(x) = \begin{pmatrix} 2 & 2 \\ 2 & 6 \end{pmatrix}$

$\nabla^2 f(x)$ 正定, 故 $f(x)$ 是严格凸函数

4-10 (3) $\nabla f(x) = (x_2, x_1)^T$, $\nabla^2 f(x) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ 不定

故 $f(x)$ 既不是凸函数也不是凹函数

4-13

n	a	b	λ_1	λ_2	$F(\lambda_1)$	$F(\lambda_2)$	$b-a$
0	0	10	3.82	6.18	-6.3	3.1	10
1	0	6.18	2.36	3.82	-6.6	-6.3	6.18
2	0	3.82	1.46	2.36	-4.6	-6.6	3.82
3	1.46	3.82	2.36	2.92	-6.6	-7	2.36
4	2.36	3.82	2.92	3.26	-7	-6.93	1.46
5	2.36	3.26	2.7	2.92	-6.91	-7	0.9
6	2.7	3.26	2.92	3.05	-6.9936	-6.9975	0.56
7	2.92	3.26	3.05	3.13	-6.9975	-6.9831	0.34
8	2.92	3.13					0.21

$0.21 < 10 \times 3\% = 0.3$ $[a,b] = [2.92, 3.13]$

求得 $f(x) = x^2 - 6x + 2$ 极小点为 $x = 3.025$

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4-14 $f(x)=x_1^2+2x_2^2$ $Q=\begin{bmatrix}2&0\\0&4\end{bmatrix}$

$g(x)=\nabla f(x)=[2x_1,\ 4x_2]^T$

$p^{(1)}=-\nabla f(x^0)=[-8,-16]^T$

$\lambda_1=-\dfrac{g^{(1)T}p(k)}{p(k)^T Q\, p(k)}=\dfrac{5}{18}$

$x_1=x_0+p^{(1)}\lambda_1=[\frac{16}{9},\ -\frac{4}{9}]^T$

$g^{(2)}=[\frac{32}{9},-\frac{16}{9}]^T$, $p^{(2)}=[-\frac{32}{9},\ \frac{16}{9}]^T$, $\lambda_2=-\dfrac{g^{(2)T}p(2)}{p(2)^T Q\, p(2)}=\dfrac{5}{12}$

$x_2=x_1+p^{(2)}\lambda_2=[\frac{8}{27},\ \frac{8}{27}]^T$

$g^{(3)}=[\frac{16}{27},\frac{32}{27}]^T$, $p^{(3)}=[-\frac{16}{27},-\frac{32}{27}]^T$, $\lambda_3=-\dfrac{g^{(3)T}p(3)}{p(3)^T Q\, p(3)}=\dfrac{5}{18}$

$x_3=x_2+p^{(3)}\lambda_3=[\frac{32}{243},\ -\frac{8}{243}]^T=[0.132,\ -0.03]^T$

4-15(1) $f(x)=\frac{1}{2}x^TQx+b^Tx+C$

$g(x)=\nabla f(x)=[2x_1-2,\ 8x_2,\ 18x_3+18]^T$

$$G(x)=\nabla^2 f(x)=\begin{bmatrix}2&0&0\\0&8&0\\0&0&18\end{bmatrix}$$

取 $x^{(0)}=(1,1,1)^T$

$g(x^{(0)})=(0,8,36)^T$, $G(x^{(0)})=\begin{bmatrix}2&0&0\\0&8&0\\0&0&18\end{bmatrix}$

$x_1-x^{(0)}=-G(x^{(0)})^{-1}g(x^{(0)})=\begin{bmatrix}0\\-1\\-2\end{bmatrix}$

即 $x^{(1)}=(1,0,-1)^T$ $g(x^{(1)})=0$，即极值点为 $(1,0,-1)^T$

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4-16 $x^{(0)} = (2, 2)^T$

$$g(x) = \begin{bmatrix} 2x_1 - x_2 + 2 \\ -x_1 + 2x_2 - 4 \end{bmatrix} \qquad Q = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$p^{(0)} = -g^{(0)} = \begin{bmatrix} -4 \\ 2 \end{bmatrix} \qquad \lambda_0 = \frac{g^{(0)T} g^{(0)}}{p^{(0)T} Q p^{(0)}} = \frac{5}{14}$$

$$x^{(1)} = x^{(0)} + \lambda_0 p^{(0)} = \begin{bmatrix} \frac{4}{7} \\ \frac{19}{7} \end{bmatrix} \qquad g^{(1)} = \begin{bmatrix} \frac{3}{7} \\ \frac{6}{7} \end{bmatrix}$$

$$\beta_0 = \frac{g^{(1)T} Q p^{(0)}}{p^{(0)T} Q p^{(0)}} = \frac{9}{196} \qquad p^{(1)} = -g^{(1)} + \beta_0 p^{(0)} = \begin{bmatrix} -\frac{30}{49} \\ -\frac{75}{98} \end{bmatrix}$$

$$\lambda_1 = \frac{g^{(1)T} g^{(1)}}{p^{(1)T} Q p^{(1)}} = \frac{14}{15}$$

$$x^{(2)} = x^{(1)} + \lambda_1 p^{(1)} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \qquad g^{(2)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ 迭代终止

最优解 $x^* = x^{(2)} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ $\min(x_1^2 - x_1x_2 + x_2^2 + 2x_1 - 4x_2) = -4$