Teacher: **Yanjie Li** | Assignment Number: **4**
Course: **Linear Algebra in Control Theory** | Disclosure date: June 2, 2021

## Problem 1

In $\mathbf{R}^4$, let

$$U = \text{span}((1,1,0,0),(1,1,1,2)).$$

Find $u \in U$ such that $\|u-(1,2,3,4)\|$ is as small as possible.

## Problem 2

Find $p \in \mathcal{P}_3(\mathbf{R})$ such that $p(0)=0, p'(0)=0$, and

$$\int_0^1 |2+3x-p(x)|^2 dx$$

is as small as possible.

## Problem 3

Suppose $T \in \mathcal{L}(V)$ and $U$ is a subspace of $V$.
(a) Prove that if $U \subset \text{null}\, T$, then $U$ is invariant under $T$.
(b) Prove that if range $T \subset U$, then $U$ is invariant under $T$.

## Problem 4

Suppose $S, T \in \mathcal{L}(V)$ are such that $ST = TS$. Prove that range $S$ is invariant under $T$.

## Problem 5

Suppose $S, T \in \mathcal{L}(V)$ are such that $ST = TS$. Prove that null $S$ is invariant under $T$.

## Pay Attention

a) **Mark your class number, student number and name on the homework**.
b) Try to write your homework on **A4** size paper.
c) Please hand in your homework to your TA before class next Wednesday (June 9). If you really cannot hand in your homework by the time mentioned above, please bring it to office D205a by yourself.

# References

[1] Axler, S. (1997). Linear algebra done right. Springer Science Business Media.

[2] Lay, D. C. . Linear algebra and its applications. Academic Press.

[3] Leon, S. J., de Pillis, L., De Pillis, L. G. (2015). Linear algebra with applications (pp. 337-350). Boston: Pearson.