HW-5 19040102 自动化1班 方苑

例4.1 $G_0(S)=\frac{4}{S(S+2)}$ 要求 $\sigma\% \le 20\%$，$t_s \le 2S$

根据 $\sigma\% = e^{-\pi\xi/\sqrt{1-\xi^2}} \le 20\%$ 得 $\xi \ge 0.456$ 取 $\xi = 0.5$

$t_s \approx \frac{3.5}{\xi w_n} < 2S \Rightarrow w_n > 3.5$ 取 $w_n = 5$

$S_{1,2} = -\xi w_n \pm j w_n\sqrt{1-\xi^2} = -2.5 \pm 2.5\sqrt{3}j$，$\theta = 60^\circ$

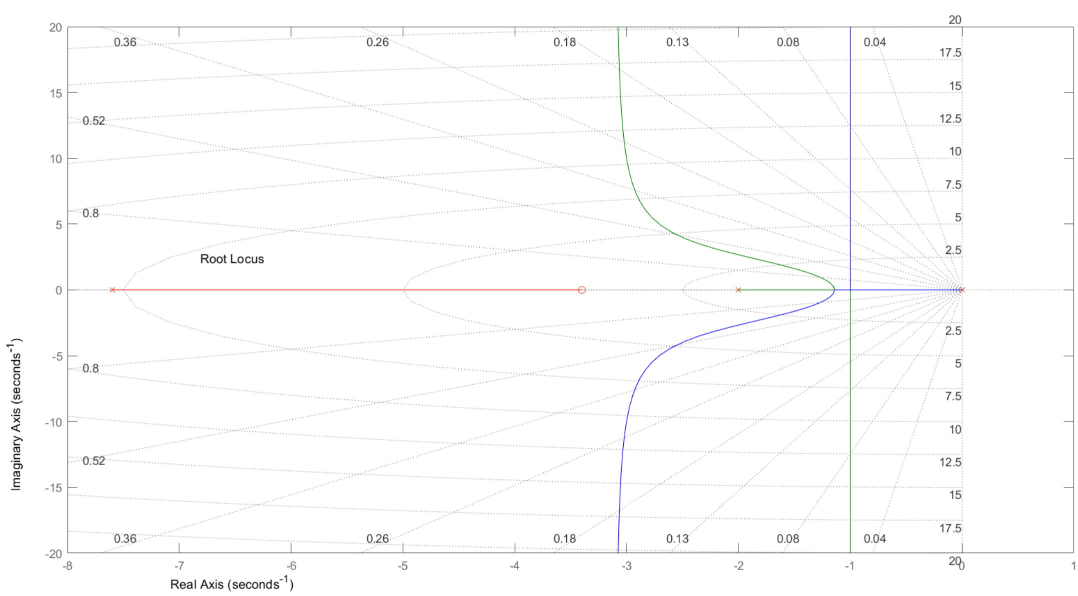
$\angle G_0(S_1) = -\angle S_1 - \angle(S_1+2) = 143.41^\circ$

$\phi = (2l+1)180^\circ - \angle G_0(S_1) = 36.6^\circ$

$$\begin{cases} P_c = -|S_1| \dfrac{\cos\frac{1}{2}(\phi-\theta)}{\cos\frac{1}{2}(\phi+\theta)} = -7.36 \\ z_c = -|S_1| \dfrac{\cos\frac{1}{2}(\phi+\theta)}{\cos\frac{1}{2}(\phi-\theta)} = -3.4 \end{cases}$$

幅值条件 $K_c \dfrac{|G_0(S_1)||S_1-z_c|}{|S_1-P_c|} = 1$ 得 $K_c = 8.02$

得校正网络 $G_c(S) = 8.02\dfrac{S+3.4}{S+7.6}$



校正前后的根轨迹

HW-5 19040102 自动化1班 方尧

例4.3

$\sigma_p \le 25\%$, $t_s \le 0.75$ $K_v \ge 12s^{-1}$ $\Rightarrow$ $\xi = 0.4$, $\omega_n = 14.3 rad/s$

取 $\xi = 0.4$, $\omega_n = 15 rad/s$ $S_{1,2} = -6 \pm 3\sqrt{21}j$, $|S_1| = 15$

$K = 12 \times 5 \times 20 = 1200$, $M = |S_1||S_1+5||S_1+20| = 4057$

$\phi = (2l+1)180^\circ - \angle G_0(S_1) = 180^\circ - 107.78^\circ = 72.22^\circ$

$$\frac{1}{\tan\eta} = \frac{4057}{1200}\frac{1}{\sin 72.22^\circ} - \frac{1}{\tan 72.22^\circ} = 3.23$$

得 $\eta = 17.2^\circ$, $\theta = \arccos\xi = 66.42^\circ$, $\delta = 180^\circ - \eta - \theta = 96.38^\circ$

$$|z_c| = \omega_n \frac{\sin\eta}{\sin\delta} = 4.25$$

$$|p_c| = \omega_n \frac{\sin(\eta+\phi)}{\sin(\delta-\phi)} = 34.94$$

$$G_c(S) = \frac{34.94}{4.25}\frac{S+4.25}{S+34.94} = 8.22\frac{S+4.25}{S+33.78}$$

例4.4

$\sigma_p < 20\%$, $t_s \le 2.6s$ $\Rightarrow$ $\xi = 0.5$, $\omega_n = 2.5 rad/s$, $S_{1,2} = -1.25 \pm 1.25\sqrt{3}j$

开环增益 $K_v = \frac{800K_v}{4 \times 10 \times 20} = K_v = 12$

$\angle G_0(S_1) = -\angle(S_1) - \angle(S_1+4) - \angle(S_1+10) - \angle(S_1+20) = -178.81^\circ \approx -180^\circ$

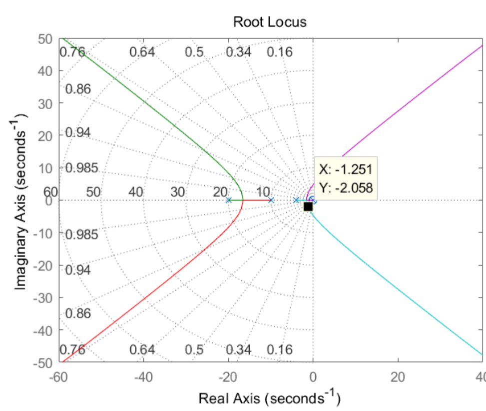
故幅角条件基本满足

S_1 对应 $K_0 = \frac{|S_1||S_1+4||S_1+10||S_1+20|}{800} \approx 1.86$

$\beta = \frac{K}{K_0} = \frac{12}{1.86} = 6.45$

选取 $p_c = 0.02$, $z_c = p_c \cdot \beta = 0.129$

校正系统 $G_c(S) = \frac{S+0.129}{S+0.02}$



HW-5 19040102 自动化1班 方无

例4.5 $\sigma_p \le 20\%$ $t_s \le 10s\ (\Delta = 0.05)$ $\Rightarrow \xi \ge 0.45,\ \xi\omega_n \ge 0.3$

在原系统根轨迹上选取 $s_{1,2} = -0.3328 \pm 0.5792j$

对应 $\xi = 0.4982$, $\xi\omega_n = 0.3328$ 满足要求

设 s_1 对应开环增益为 K_0

$$K_0 = \frac{|s_1||s_1+1||s_1+2|}{2} = 0.521$$

开环增益 $K = 5$

$$\beta = \frac{K}{K_0} = 9.6$$

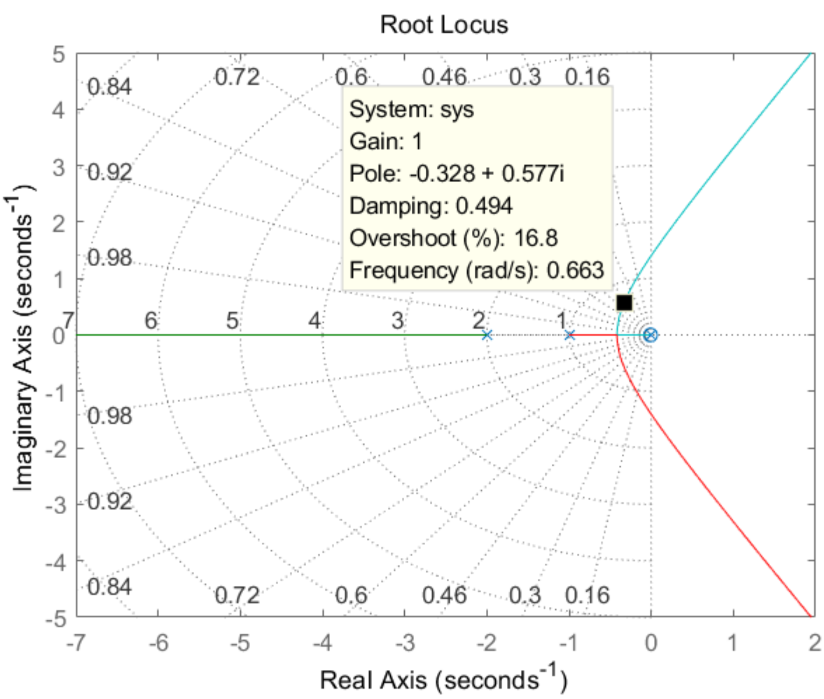
取 $p_c = -0.001$, $z_c = \beta p_c = -0.0096$

原系统开环增益 $K' = 1$, $K_c = \frac{5}{K'} = 5$

校正环节 $G_c(s) = \frac{5(104.2s+1)}{1000s+1}$

校正后 $K=5$ 对应 $s_{1,2} = -0.328 + 0.577j$

$\xi = 0.494$, $\xi\omega_n = 0.328$ $\sigma_p = 16.8\%$



HW-5 19040102 自动化1班 方尧

例4.6

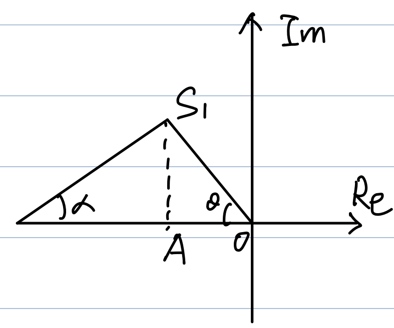
$K = K_v = \frac{1}{e_{ss}} \geq 50$ 取 $K=50$

$\sigma\% = e^{-\pi\xi/\sqrt{1-\xi^2}} \leq 20\% \Rightarrow \xi \geq 0.45$，取 $\xi=0.5$

$t_s = 3/(\xi\omega_n) \leq 1.5$ 得 $\xi\omega_n \geq 2$

考虑选取 $z_c=-1, p_c=-10$，$G_{c1}(s) = 10\frac{s+1}{s+10} = \frac{s+1}{0.1s+1}$

超前校正后 $G_1(s) = \frac{K}{s(0.1s+1)^2}$



$2\alpha + 120° = 180°$ 得 $\alpha = 30°$

$\sqrt{3}x = \frac{1}{\sqrt{3}}(10-x)$ 得 $x = 2.5$

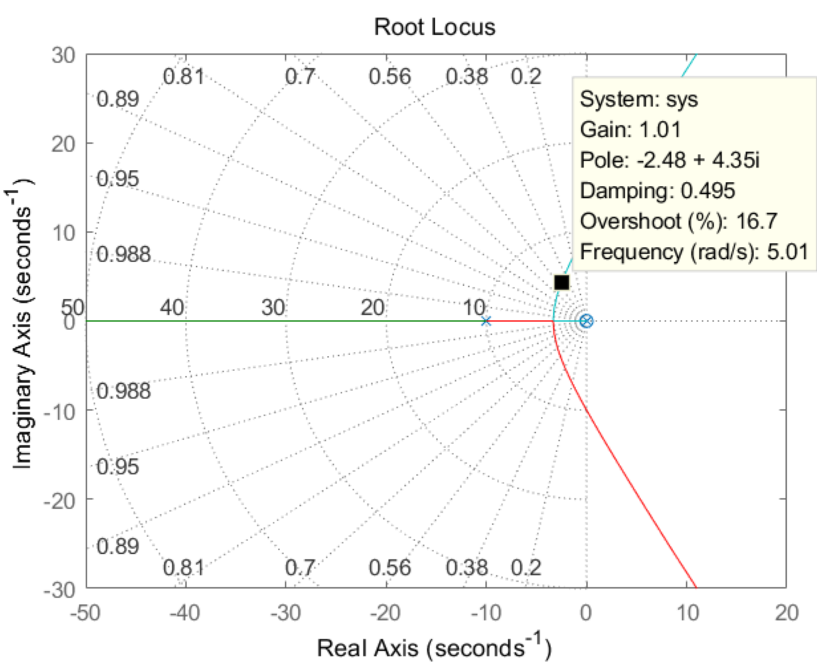
即主导极点 $s_1 = -2.5+4.33i$

由幅值条件，$\frac{100K_0}{|s_1||s_1+10|^2} = 1$ 得 $K_0 = 3.75$

$\beta = \frac{K}{K_0} = \frac{50}{3.75} = 13.33$

迟后校正 $G_{c2}(s) = \frac{s+0.01}{s+\frac{0.01}{13.33}} = 13.33\frac{100s+1}{1330s+1}$

校正环节 $G_c(s) = \frac{s+1}{0.1s+1}\cdot 13.33\frac{100s+1}{1330s+1}$

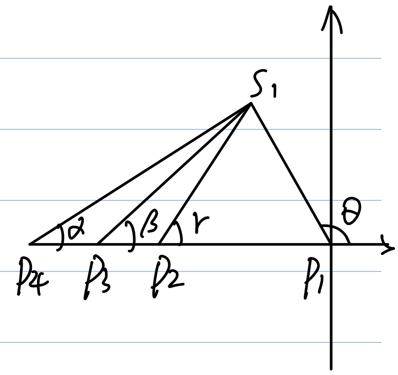


HW-5 19040102 自动化1班 方亮

例 4.7

考虑将小闭环极点移至负实轴 $p_4=-20,\ p_1=0$

小闭环 $G_1(s)=\dfrac{k_2}{0.2s^2+(0.4+k_2\tau)s+k_2+1}$



取 $p_3=-10$

$\theta=150^\circ,\ \alpha=\arctan\dfrac{\sqrt{3}}{17}=5.83^\circ$

$\beta=\arctan\dfrac{\sqrt{3}}{7}\approx 13.89^\circ$

$\alpha+\beta+\gamma+\theta=180^\circ$ 得 $\gamma=10.28^\circ$

根据几何关系 得 $p_2=-12.55$

G_1 特征方程应满足

$0.2s^2+(0.4+k_2\tau)s+k_2+1$

$=0.2(s-p_2)(s-p_3)=0.2(s^2+22.55s+125.5)$

解得 $k_2=24.1,\ \tau=0.1705$

幅值条件 $k=\dfrac{|0.05s_1+1||s_1||s_1-p_2||s_1-p_3|}{|p_2||p_3|}=1.65$

小闭环 $\dfrac{k_2}{1+k_2}=0.96$

$k_1=\dfrac{k}{0.96}=1.72$

即 $k_1=1.72,\ k_2=24.1,\ \tau=0.1705$

HW-5 19040102 自动化1班 方克

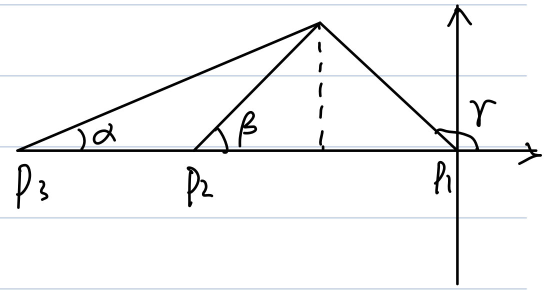
例4.8

两内循环 $G_1(S)=\dfrac{20}{0.5S^2+(10\alpha+1)S+20\beta}$, $P_1=0$

取 $P_2=-5$, $\gamma=135°$, $\beta=33.69°$

$\alpha+\beta+\gamma=180°$ 得 $\alpha=11.31°$

得 $P_3=-12$



G_1 特征方程 $0.5S^2+(10\alpha+1)S+20\beta$

$=0.5(S+12)(S+5)$

得 $\beta=1.5$, $\alpha=0.75$

幅值条件 $K_1\dfrac{20}{0.5|S_1+12||S_1+5|}\cdot\dfrac{1}{|S_1|}=1$

得 $K_1=2.6$

最终得到结果 $\alpha=0.75$, $\beta=1.5$, $K_1=2.6$