Teacher: **Yanjie Li** | Assignment Number: **3**

Course: **Linear Algebra in Control Theory** | Disclosure date: May 27, 2021

Problem 1

Find a polynomial $q \in \mathcal{P}_2(\mathbf{R})$ such that

$$\int_0^1 p(x)(\cos \pi x)\,dx = \int_0^1 p(x)\,q(x)\,dx$$

for every $p \in \mathcal{P}_2(\mathbf{R})$.

Problem 2

Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 12 \\ 6 \\ 18 \end{bmatrix}$$

(a) Use the Gram–Schmidt process to find an orthonormal basis for the column space of A.
(b) Factor A into a product QR, where Q has an orthonormal set of column vectors and R is upper triangular.
(c) Solve the least squares problem $A\mathbf{x} = \mathbf{b}$.

Problem 3

Let U be an m-dimensional subspace of $\mathbb{R}^n$ and let V be a k-dimensional subspace of U, where $0 < k < m$.
(a) Show that any orthonormal basis

$$\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$$

for V can be expanded to form an orthonormal basis $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k, \mathbf{v}_{k+1}, \ldots, \mathbf{v}_m\}$ for U.
(b) Show that if $W = \mathrm{Span}\{\mathbf{v}_{k+1}, \ldots, \mathbf{v}_m\}$, then $U = V \oplus W$.

Problem 4

Suppose $v_1, ..., v_m \in V$. Prove that

$$\{v_1, ..., v_m\}^{\perp} = (\operatorname{span}(v_1, ..., v_m))^{\perp}$$

Problem 5

Suppose U is the subspace of $\mathbf{R}^4$ defined by

$$U = \operatorname{span}((1, 2, 3, -4), (-5, 4, 3, 2)).$$

Find an orthonormal basis of U and an orthonormal basis of $U^{\perp}$.

Pay Attention

a) **Mark your class number, student number and name on the homework**.
b) Try to write your homework on **A4** size paper.
c) Please hand in your homework to your TA before class next Wednesday (June 2). If you really cannot hand in your homework by the time mentioned above, please bring it to office D205a by yourself.

References

[1] Axler, S. (1997). Linear algebra done right. Springer Science Business Media.

[2] Lay, D. C. . Linear algebra and its applications. Academic Press.

[3] Leon, S. J., de Pillis, L., De Pillis, L. G. (2015). Linear algebra with applications (pp. 337-350). Boston: Pearson.