

3-1 解： 知 $\vec{F} = \vec{OA} = (1, 0, 1)$ $\vec{F} = (-5\sqrt{2} \text{ kN}, 5\sqrt{2} \text{ kN}, 0)$

$$\vec{M}_O(\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ -5\sqrt{2} & 5\sqrt{2} & 0 \end{vmatrix} = -5\sqrt{2} \text{ kN} \cdot \text{m} \cdot \vec{i} - 5\sqrt{2} \text{ kN} \cdot \text{m} \cdot \vec{j} + 5\sqrt{2} \text{ kN} \cdot \text{m} \cdot \vec{k}$$

故 $M_x(\vec{F}) = -7070 \text{ N} \cdot \text{m}$, $M_y(\vec{F}) = -7070 \text{ N} \cdot \text{m}$, $M_z(\vec{F}) = 7070 \text{ N} \cdot \text{m}$

3-2 解： $\vec{r} = \vec{OC} = (\frac{1}{2}r, \frac{\sqrt{3}}{2}r, h)$, $\vec{F}_z = -F \cdot \sin 60^\circ \vec{k} = -\frac{\sqrt{3}}{2}F \cdot \vec{k}$

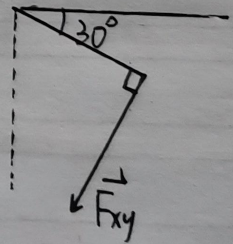
$$|\vec{F}_{xy}| = F \cdot \cos 60^\circ = \frac{1}{2}F, \quad \vec{F}_x = \frac{1}{2}F \cdot \sin 60^\circ \cdot \vec{i} = \frac{\sqrt{3}}{4}F \vec{i}$$

$$\vec{F}_y = -\frac{1}{2}F \cdot \cos 60^\circ \vec{j} = -\frac{1}{4}F \vec{j}$$

故 $\vec{M}_O(\vec{F}) = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2}r & \frac{\sqrt{3}}{2}r & h \\ \frac{\sqrt{3}}{4}F & -\frac{1}{4}F & -\frac{\sqrt{3}}{2}F \end{vmatrix}$

$$= (\frac{1}{4}hF - \frac{3}{4}rF) \vec{i} + \frac{\sqrt{3}}{4}F(r+h) \vec{j} - \frac{1}{2}rF \vec{k}$$

故力 \vec{F} 对 x 轴之矩为 $\frac{1}{4}hF - \frac{3}{4}rF$, 对 y 轴之矩为 $\frac{\sqrt{3}}{4}F(r+h)$
对 z 轴之矩为 $-\frac{1}{2}rF$



3-3 解: 记 A, B, C 三轮上的力偶分别为 \vec{M}_A , \vec{M}_B , \vec{M}_C

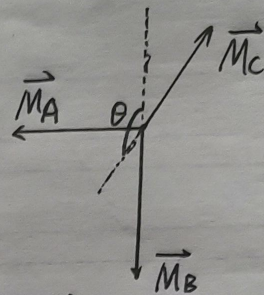
$$|\vec{M}_A| = 10\text{N} \times 2 \times 150 \times 10^{-3}\text{m} = 3\text{N}\cdot\text{m}$$

$$|\vec{M}_B| = 20\text{N} \times 2 \times 100 \times 10^{-3}\text{m} = 4\text{N}\cdot\text{m}$$

知 $\sum \vec{M}_i = 0$ 故 $|\vec{M}_C| = \sqrt{M_A^2 + M_B^2} = 5\text{N}\cdot\text{m}$

$$\frac{|\vec{M}_A|}{|\vec{M}_B|} = \frac{3}{4} \quad \arctan \frac{3}{4} = 37^\circ, \quad |\vec{M}_C| = F \times 2 \times 50 \times 10^{-3}\text{m}$$

得 $F = 50\text{N}$, $\theta = 143^\circ$



3-4 解: $\sum \vec{M}_z = 0$ 即 $M_z - F_t \cdot OB = 0$

得 $F_t = 2000\text{N}$, 故 $F_a = 640\text{N}$, $F_r = 340\text{N}$

$$\sum \vec{M}_y(\vec{F}_i) = 0, \quad F_t \times 4\text{m} - F_{ax} \times 3\text{m} = 0, \quad \text{故 } F_{ax} = 2666.4\text{N}$$

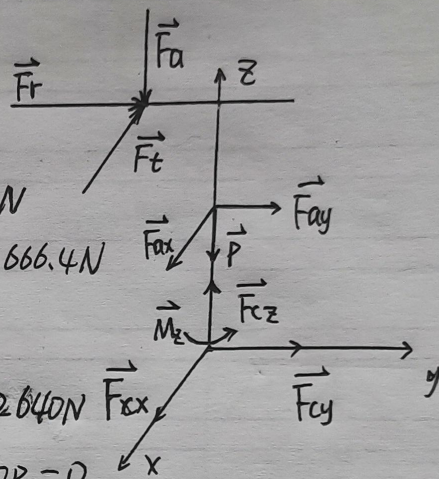
$$\sum \vec{M}_x(\vec{F}_i) = 0 \quad F_t \times 1\text{m} + F_{cx} \times 3\text{m} = 0$$

故 $F_{cx} = -666.7\text{N}$, $\sum \vec{F}_z = 0$, $F_{cz} = F_a + P = 12640\text{N}$

$$\sum \vec{M}_x(\vec{F}_i) = 0 \quad F_r \times 4\text{m} + F_{ay} \times 3\text{m} - F_a \times OB = 0$$

故 $F_{ay} = -325.3\text{N}$, $\sum F_y = 0$, $F_r + F_{ay} + F_{cy} = 0$ 得 $F_{cy} = -14.7\text{N}$

故 $\vec{F}_a = 2666.4\text{N} \cdot \vec{i} - 325.3\text{N} \cdot \vec{j}$; $\vec{F}_c = -666.7\text{N} \cdot \vec{i} - 14.7\text{N} \cdot \vec{j} + 12640\text{N} \cdot \vec{k}$



3-5 解: $\sum \vec{M}_y(\vec{F}_i) = 0$, $M - F \cdot \frac{d}{2} \cos 20^\circ = 0$

故 $F = \frac{2M}{d \cos 20^\circ}$

$$\sum \vec{M}_z(\vec{F}_i) = 0, \quad -F_{bx} \times (22\text{cm} + 12.2\text{cm}) + F \cos \alpha \times 22\text{cm} = 0$$

得 $F_{bx} = 7659.8\text{N}$

$$\sum F_x = 0 \quad F_{ax} + F_{bx} - F \cdot \cos \alpha = 0$$

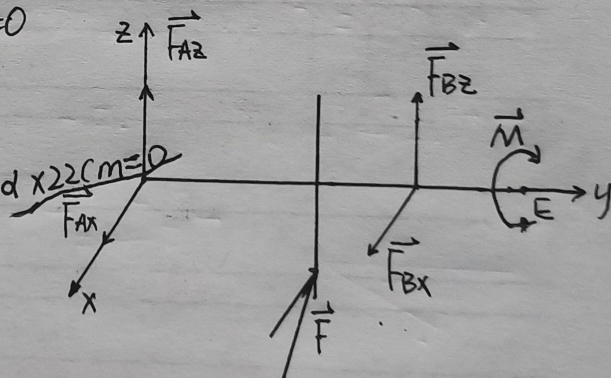
得 $F_{ax} = 4247.7\text{N}$

$$\sum \vec{M}_x(\vec{F}_i) = 0 \quad F_{bz} \times (22\text{cm} + 12.2\text{cm}) + F \sin \alpha \times 22\text{cm} = 0$$

得 $F_{bz} = -2787.9\text{N}$

$$\sum F_z = 0 \quad F_{az} + F_{bz} + F \cdot \sin \alpha = 0 \quad \text{得 } F_{az} = -1546.08\text{N}$$

故 $\vec{F}_A = 4247.7\text{N} \cdot \vec{i} - 1746.1\text{N} \cdot \vec{k}$, $\vec{F}_B = 7659.8\text{N} \cdot \vec{i} - 2787.9\text{N} \cdot \vec{k}$



3-6 解: 设 $\triangle OBE$, OB 边高 h .

由 $\triangle OEB$ 等腰三角形, 故 $E(\frac{L}{2}, h)$

对 x 轴用合力矩定理, $\rho(L^2 - \frac{1}{2}Lh) \cdot h = \rho L^2 \cdot \frac{L}{2} + \rho(-\frac{1}{2}L \cdot h) \cdot \frac{h}{3}$

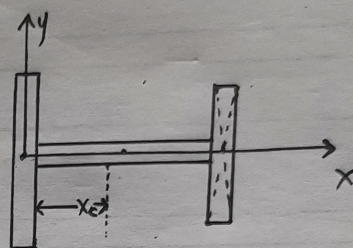
得 $h = \frac{3-\sqrt{3}}{2} L$ 故 E 为 $(\frac{L}{2}, \frac{3-\sqrt{3}}{2} L)$ 即 $(\frac{L}{2}, 0.634L)$

3-7 解: 把该工字钢分为三部分, 如图.

由对称性 $y_c = 0$.

对 y 轴应用合力矩定理.

$$(10+x_c)(200 \times 20 + 200 \times 20 + 150 \times 20) = (10+100) \times 200 \times 20 + (10+200+10) \times 150 \times 20$$



得 $x_c = 90$, 故该截面几何中心在中轴线上距左侧钢翼右侧边缘 90mm 处

3-8 解: 将组合体旋转放置重力方向如图所示

建立直角坐标系, 由对称性 $x_c = 0, y_c = 0$

对下方半球体, $\frac{2}{3}\pi r^3 \cdot z_c' = \int_{-r}^0 \pi(r^2 - z^2) \cdot dz \cdot z$

得 $z_c' = -\frac{3}{8}r$

对整体 $0 = \pi r^2 \cdot h \cdot \frac{h}{2} + \frac{2}{3}\pi r^3 \cdot (-\frac{3}{8}r)$

得 $h = \frac{\sqrt{2}}{2}r = 0.707r$

故高度为 $0.707r$

