自控A HW-5 自动化1班 19041002 方尧

1.(a) $\Phi(s)=\dfrac{L(s)}{1+L(s)}=\dfrac{k}{s(s+\sqrt{2k})+k}=\dfrac{k}{s^2+\sqrt{2k}\,s+k}$

$w_n=\sqrt{k}$，$\xi=\dfrac{\sqrt{2}}{2}\in(0,1)$

$\sigma\%=\exp\left(-\dfrac{\xi\pi}{\sqrt{1-\xi^2}}\right)=4.32\%$ $T_s=\dfrac{4}{\xi w_n}=\dfrac{4\sqrt{2}}{\sqrt{k}}$

(b) $T_s=\dfrac{4\sqrt{2}}{\sqrt{k}}<1 \Rightarrow k>32$

2 可得 $\xi\in(0,1)$

$T_p=\dfrac{\pi}{w_d}=\dfrac{\pi}{w_n\sqrt{1-\xi^2}}=3$，$\sigma\%=e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}=0.254$

得 $w_n=1.426$ $\xi=0.4$..

$\dfrac{k}{T}=w_n^2$ $\dfrac{1}{T}=2\xi w_n$ 得 $k=1.428$ $T=1.094$

3 设主反馈极性 n_1（$n_1=0$或1或-1） 内反馈 n_2（$n_2=0$或1或-1）

$\Phi(s)=\dfrac{k_1k_2}{s^2+n_2k_2s+n_1k_1k_2}$

$w_n=\sqrt{n_1k_1k_2}$ $\xi=\dfrac{n_2k_2}{2\sqrt{n_1k_1k_2}}$ $(n_1\neq 1)$

(b)图 无阻尼 $\xi=0$ 则 $n_2=0$，$n_1=1$ 即主反馈为负反馈，内反馈无反馈

(c)图 负阻尼 $\xi<0$ 则 $n_2=-1$，$n_1=1$ 即主反馈为负反馈，内反馈为正反馈

(d)图 有 $c(t)=\dfrac{k_1k_2}{2}t^2$ 则 $n_1=n_2=0$ 即主反馈，内反馈都无反馈

(e)图. 欠阻尼 $\xi\in(0,1)$ 则 $n_1=n_2=1$ 即主反馈内反馈都为负反馈

4.(1) 零点 $z_1=-2.5$，极点 $p_1=0$，$p_2=-0.5$

(2) 闭环传递函数 $\Phi=\dfrac{G(s)}{1+G(s)}=\dfrac{as+1}{s^2+(a+b)s+1}=\dfrac{0.4s+1}{s^2+0.9s+1}$

零点 $z_1=-2.5$，极点 $p_{1,2}=\dfrac{-9\pm\sqrt{319}i}{20}$

(3) $w_n^2=1$ $2\xi w_n=0.9$ 得 $\xi=0.45$，$w_n=1$

(4) $\Phi(s)=\dfrac{0.4s+1}{s^2+0.9s+1}$ $\quad y_z(t)=1-\dfrac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\dfrac{l}{z}\sin(\omega_d t+\phi+\psi)$

$\varphi=\arctan\dfrac{\sqrt{1-\zeta^2}}{\zeta}=1.10403\ rad$, $\psi=\arctan\left(\dfrac{\omega_n\sqrt{1-\zeta^2}}{z-\zeta\omega_n}\right)=0.41083\ rad$

上升时间 $T_r=\dfrac{\pi-\varphi-\psi}{\omega_d}=1.822s$ $\quad T_p=\dfrac{\pi-\psi}{\omega_d}=3.0578s$

超调量 $\sigma_z=\dfrac{l}{z}e^{-\zeta\omega_n T_p}e^{\zeta\omega_n\frac{\psi}{\omega_d}}=22.59\%$

调整时间 $T_s=\dfrac{1}{\zeta\omega_n}\left[-\ln\delta-\frac{1}{2}\ln(1-\zeta^2)-\ln\frac{l}{z}\right]=9.1927s$ $\quad(\delta=2\%)$

(5) $\Phi(s)=\dfrac{1}{s^2+0.5s+1}$ $\quad \omega_n=1,\ \zeta=0.25$ $\quad \varphi=\arctan\dfrac{\sqrt{1-\zeta^2}}{\zeta}=1.3181\ rad$

$\sigma\%=e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}=44.43\%$ $\quad T_p=\dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}=3.2446s$

$T_r=\dfrac{\pi-\varphi}{\omega_d\sqrt{1-\zeta^2}}=1.883s$ $\quad T_s=\dfrac{1}{\zeta\omega_n}\left[-\ln\delta-\frac{1}{2}\ln(1-\zeta^2)\right]=15.777s$ (2%)

5. (1) $\Phi(s)=\dfrac{K_2}{s^2+K_1K_2s+K_2}$, $\omega_n=\sqrt{K_2}$, $\zeta=\dfrac{K_1}{2}\sqrt{K_2}$

$\sigma\%=\exp\left(-\dfrac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)=16\%$, $T_p=\dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}=2s$

得 $K_1=0.554$, $K_2=3.307$

(2) $Y(s)=\dfrac{1}{s^2}\Phi(s)=\dfrac{K_2}{s^2(s^2+K_1K_2s+K_2)}$

$$e_{ss}=\lim_{t\to\infty}e(t)=\lim_{s\to 0}sE(s)=\lim_{s\to 0}s\left[R(s)-Y(s)\right]$$

$$=\lim_{s\to 0}\frac{s+K_1K_2}{s^2+K_1K_2s+K_2}=K_1=0.5$$

故 $K_1=0.5$

(4) $Y(s)=\frac{1}{s}\Phi(s)=\frac{1}{s}\cdot\frac{0.4s+1}{s^2+0.9s+1}=\frac{s^2+0.9s+1-0.5s}{s(s^2+0.9s+1)}$

$=\frac{1}{s}-\frac{0.5}{s^2+0.9s+1}$

错误

$=\frac{1}{s}-\frac{\frac{\sqrt{319}}{20}}{(s+0.45)^2+\frac{319}{400}}\cdot\frac{10}{319}\sqrt{319}$

则 $y(t)=1-\frac{10}{\sqrt{319}}e^{-0.45t}\sin\frac{\sqrt{319}}{20}t$.

稳态 $y(t)|_{t\to\infty}=1$ $\quad y'(t)=\frac{10}{\sqrt{319}}e^{-0.45t}(0.45\sin\frac{\sqrt{319}}{20}t-\frac{\sqrt{319}}{20}\cos\frac{\sqrt{319}}{20}t)$

$\arctan\theta=\frac{\sqrt{319}}{0.45\times20}$ $\quad\theta=1.9845\text{rad}$ $\quad y'(t)=\frac{10}{\sqrt{319}}e^{-0.45t}\sin(\frac{\sqrt{319}}{20}t-\theta)$

$T_p=\frac{\theta+\pi}{\frac{\sqrt{319}}{20}}=5.74\,s$, $\quad 6\%=y(t)|_{t=T_p}-1=3.873\%$

$T_r=0$ $\qquad T_s=\frac{1}{0.45}\ln\frac{10\times50}{\sqrt{319}}=7.40s$ $\quad(6=2\%)$

(5) $a=0$ $\quad Y(s)=\frac{1}{s}\cdot\frac{1}{s^2+0.5s+1}$ 得 $y(t)=1-e^{-0.5t}(\cos\frac{\sqrt{15}}{4}t+\frac{1}{\sqrt{15}}\sin\frac{\sqrt{15}}{4}t)$

$y'(t)=e^{-0.5t}(\frac{17}{60}\sqrt{15}\sin\frac{\sqrt{15}}{4}t+\frac{1}{4}\cos\frac{\sqrt{15}}{4}t)$ $\quad\theta=\arctan\frac{\frac{1}{4}}{\frac{17}{60}\sqrt{15}}=0.224$

$T_p=\frac{\pi-\theta}{\frac{\sqrt{15}}{4}}=3.013s$ $\qquad 6\%=y(t)|_{t=T_p}-1=20.34\%$

$T_r=1.883s$ $\qquad T_s=2\ln\frac{4\times50}{\sqrt{15}}=7.89s$.

5 (1) $\Phi(s)=\frac{k_2}{s^2+k_1k_2s+k_2}$. 则 $w_n=\sqrt{k_2}$, $\zeta=\frac{k_1}{2}\sqrt{k_2}$.

$6\%=\exp(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}})=16\%$, $T_p=\frac{\pi}{w_n\sqrt{1-\zeta^2}}=2s$ 得 $k_1=0.554$ $\quad k_2=3.3070$

(2) $Y(s)=\frac{1}{s^2}\Phi(s)=\frac{k_2}{s^2(s^2+k_1k_2s+k_2)}$

$e_{ss}=\lim_{t\to\infty}e(t)=\lim_{s\to0}sE(s)=\lim_{s\to0}s[R(s)-Y(s)]$

$=\lim_{s\to0}\frac{s+k_1k_2}{s^2+k_1k_2s+k_2}=k_1=0.5$

故 $k_1=0.5$