基本概念练习题

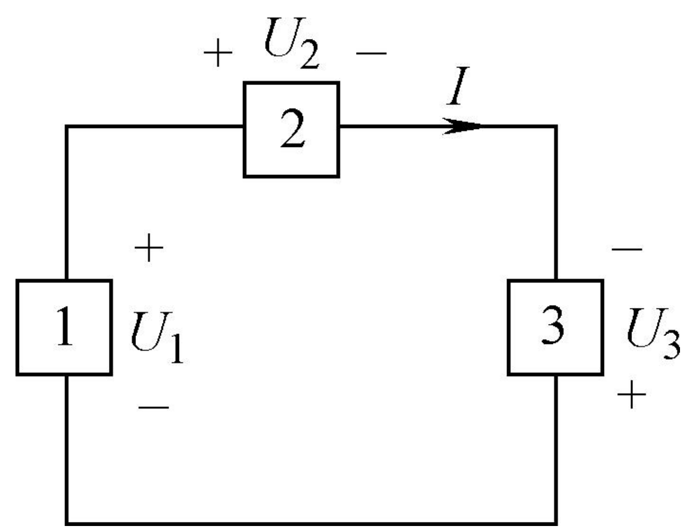
【例1】如图所示的电路中，已知 $I=2\text{A}$，$U_1=10\text{V}$，$U_2=6\text{V}$，$U_3=-4\text{V}$，试问哪些元件是电源？哪些元件是负载？

第一种方法：用实际方向判断电源与负载。

$U_1 \rightarrow$ 电源，发出功率

$U_2 \rightarrow$ 负载，吸收功率

$U_3 \rightarrow$ 负载，吸收功率



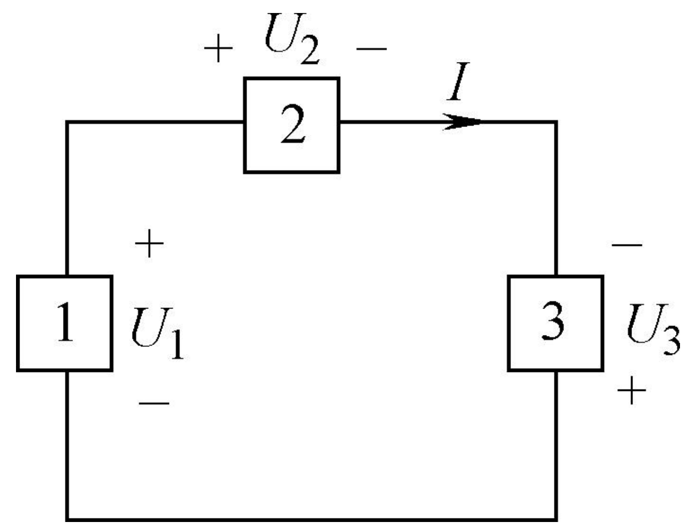
第二种方法：计算功率判断电源与负载。

功率与参考方向之间的关系

- 关联参考方向
  - 吸收的功率 $p = ui$
    - $p > 0$ 实际吸收
    - $p < 0$ 实际发出
  - 发出的功率 $p = -ui$
    - $p > 0$ 实际发出
    - $p < 0$ 实际吸收

- 非关联参考方向
  - 发出的功率 $p = ui$
    - $p > 0$ 实际发出
    - $p < 0$ 实际吸收
  - 吸收的功率 $p = -ui$
    - $p > 0$ 实际吸收
    - $p < 0$ 实际发出

【例1】如图所示的电路中，已知 $I=2\text{A}$，$U_1=10\text{V}$，$U_2=6\text{V}$，$U_3=-4\text{V}$，试问哪些元件是电源？哪些元件是负载？



第二种方法：计算功率P。

U_1发出功率为

$P=U_1I=10\times2=20\text{W}$（实际发出）

U_1吸收功率为

$P=-U_1I=-10\times2=-20\text{W}$（实际发出）

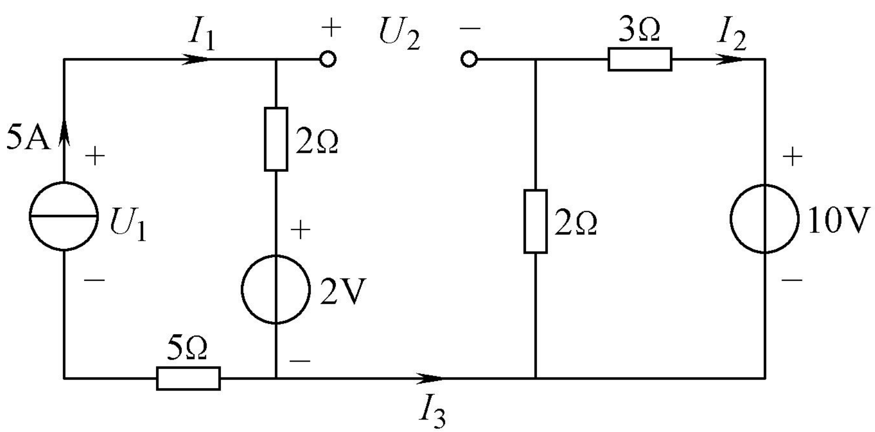
U_2发出功率为$P=-U_2I=-6\times2=-12\text{W}$（实际吸收）

U_2吸收功率为$P=U_2I=6\times2=12\text{W}$（实际吸收）

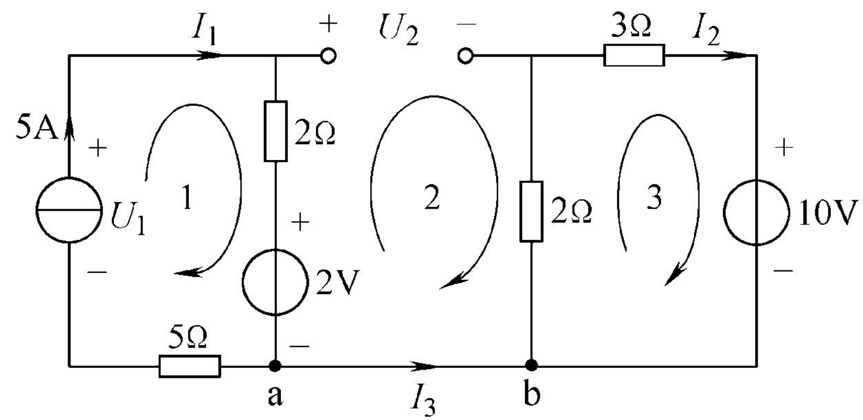
U_3发出功率为$P=U_3I=-4\times2=-8\text{W}$（实际吸收）

U_3吸收功率为$P=-U_3I=4\times2=8\text{W}$（实际吸收）

【例2】 电路如图a所示，求电流 I_1、I_2、I_3 和电压 U_1、U_2。



a)



b)

【解】 设三个回路的绕行参考方向如图b所示。

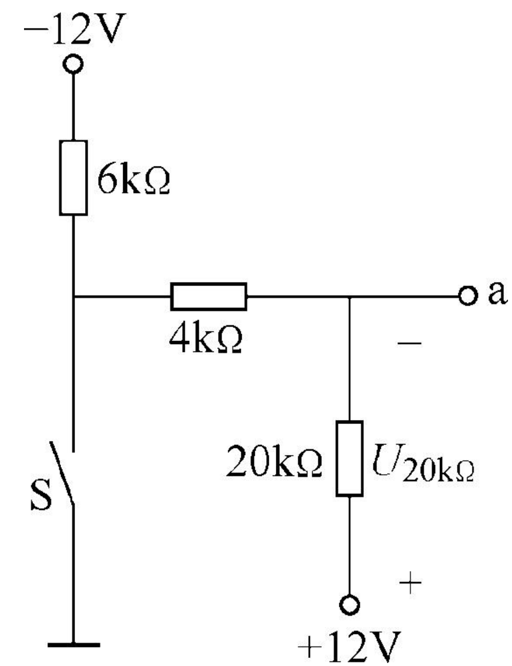
$$I_1 = 5\text{A} \qquad I_2 = -\frac{10}{2+3}\text{A} = -2\text{A} \qquad I_3 = 0$$

$$U_1 = 2I_1 + 2 + 5I_1 = 7I_1 + 2 = (7\times5+2)\text{V} = 37\text{V}$$

$$U_2 = 2I_1 + 2 + 2I_2 = [2\times5+2+2\times(-2)]\text{V} = 8\text{V}$$

【例3】求图示电路中开关S打开和闭合时a点的电位值。

【解】开关S断开，三个电阻串联。电路两端点电压为



$$U=(12-(-12))\text{V}=24\text{V}$$

20kΩ 电阻两端的电压为

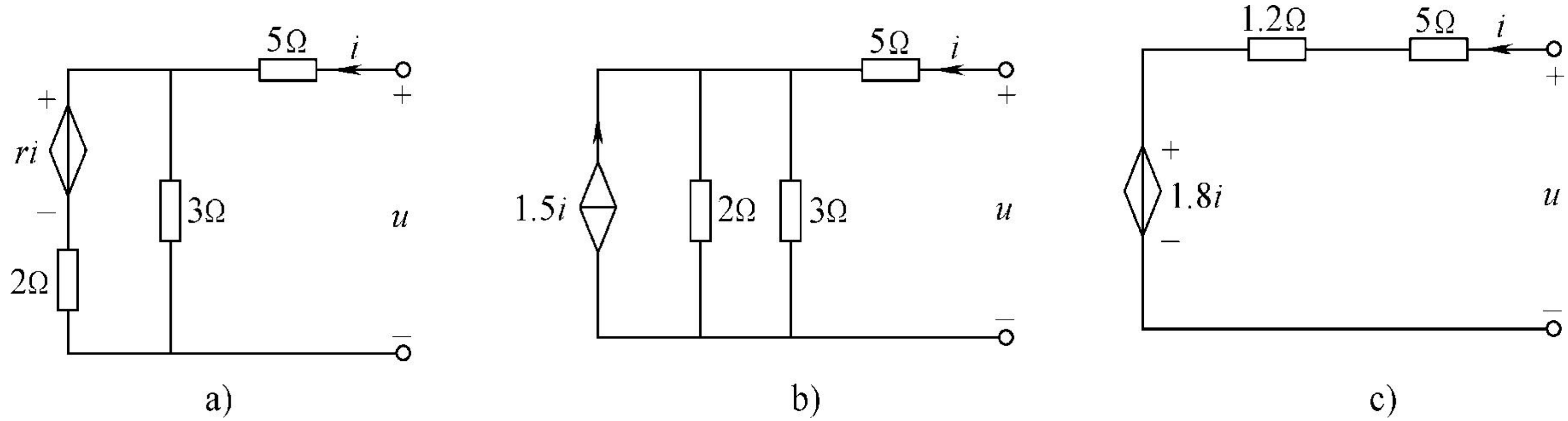
$$U_{20\text{k}\Omega}=20\times\frac{24}{6+4+20}\text{V}=16\text{V}$$

$$V_{\text{a}}=12-U_{20\text{k}\Omega}=(12-16)\text{V}=-4\text{V}$$

开关S闭合后，有

$$V_{\text{a}}=\frac{12}{4+20}\times 4\text{V}=2\text{V}$$

【例4】在图a所示的电路中，已知转移电阻系数 $r=3\Omega$ 求一端口网络的等效电阻。

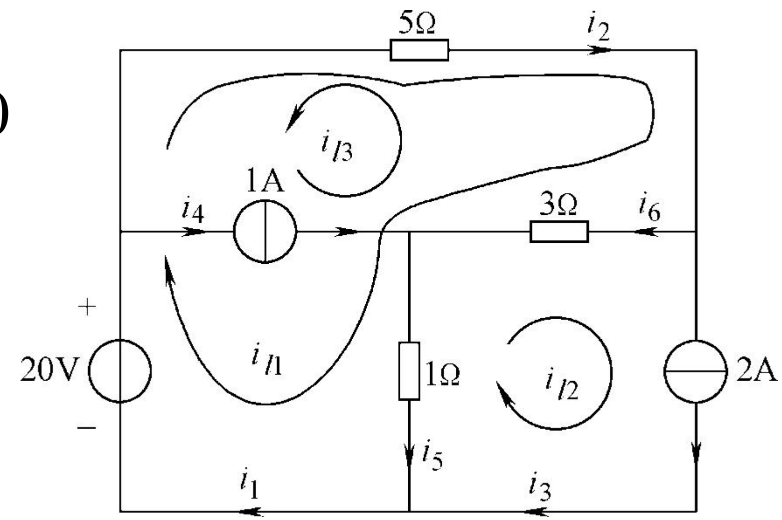


【解】首先将图a中的串联支路等效变换为图b中的并联支路，依次化简如图c所示。

$$u=(5+1.2+1.8)i=8i \quad \Rightarrow \quad R=\frac{u}{i}=8\Omega$$

【例5】 用回路电流法求各支路电流。

【解】选定三个回路电流 i_{l1} 、i_{l2} 和 i_{l3} 的参考方向如图所示。列写回路电流方程



$$(5+3+1)i_{l1}-(1+3)i_{l2}-(5+3)i_{l3}=20$$

$$i_{l2}=2A$$

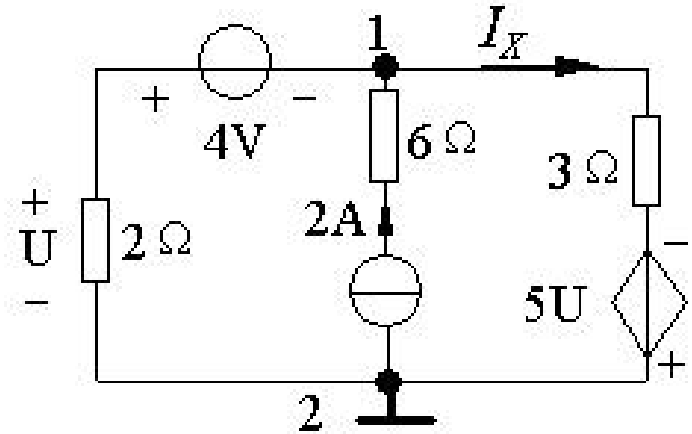
$$i_{l3}=1A \qquad i_{l1}=4A$$

各支路电流分别为

$$i_1=i_{l1}=4A \qquad i_2=i_{l1}-i_{l3}=3A \qquad i_3=i_{l2}=2A$$

$$i_4=i_{l3}=1A \qquad i_5=i_{l1}-i_{l2}=2A \qquad i_6=i_{l1}-i_{l2}-i_{l3}=1A$$

【例6】 已知电路如图所示，用节点电压法求 I_X 。

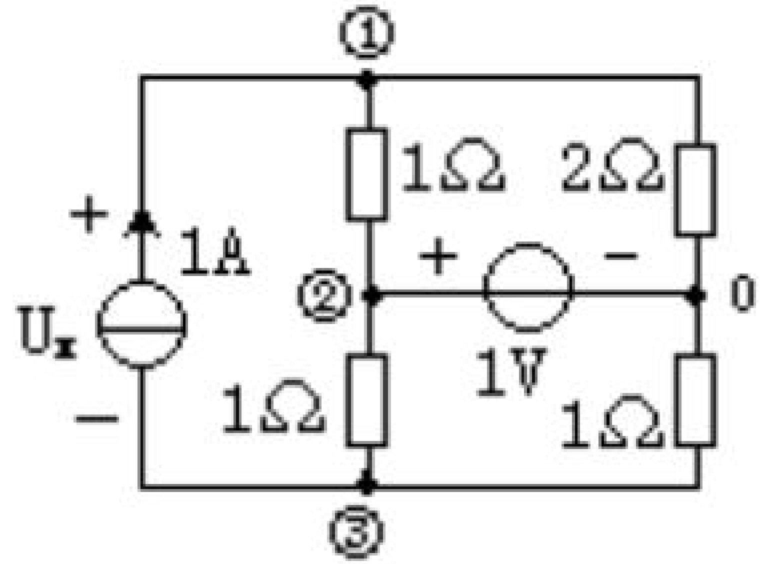


解：

$$\begin{cases} U_1 = \dfrac{2-4/2-5U/3}{1/2+1/3} \\ U = 4+U_1 \end{cases} \qquad \begin{cases} U_1 = -\dfrac{8}{3}\text{V} \\ U = \dfrac{4}{3}\text{V} \end{cases}$$

$$I_X = \frac{U_1+5U}{3} = \frac{4}{3}\text{A}$$

【例7】 试用结点法求电压Ux



【解】 $u_{n2} = 1$

$$(\frac{1}{1}+\frac{1}{2})u_{n1} - \frac{1}{1}u_{n2} = 1$$

$$-\frac{1}{1}u_{n2} + (\frac{1}{1}+\frac{1}{1})u_{n3} = -1$$

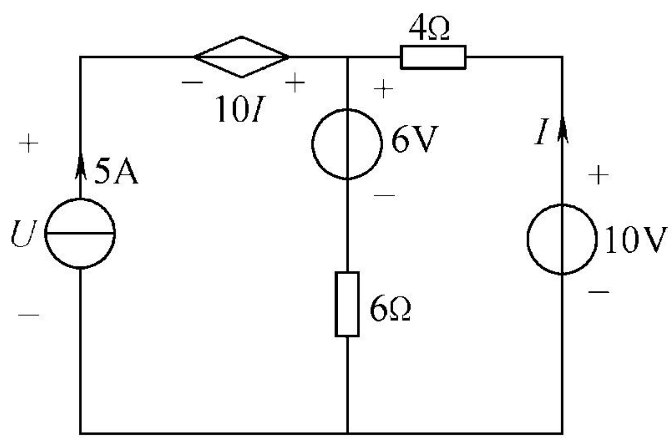
$$u_{n1} = 1.33\text{V}$$

$$-1 + 2u_{n3} = -1$$

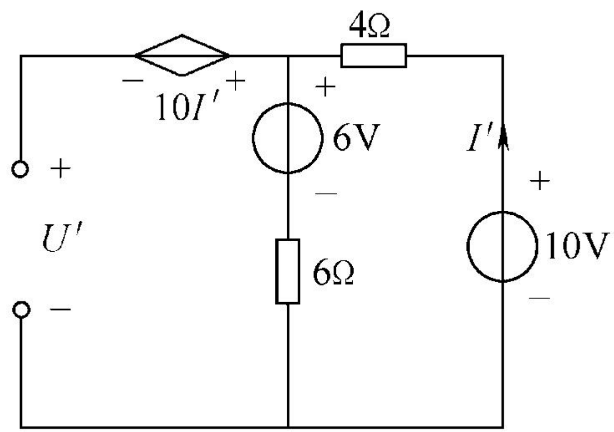
$$u_{n3} = 0$$

$$u_{X} = u_{n1} - u_{n3} = 1.33\text{V}$$

【例8】电路如图a所示，试用叠加定理求U和I 。



a)

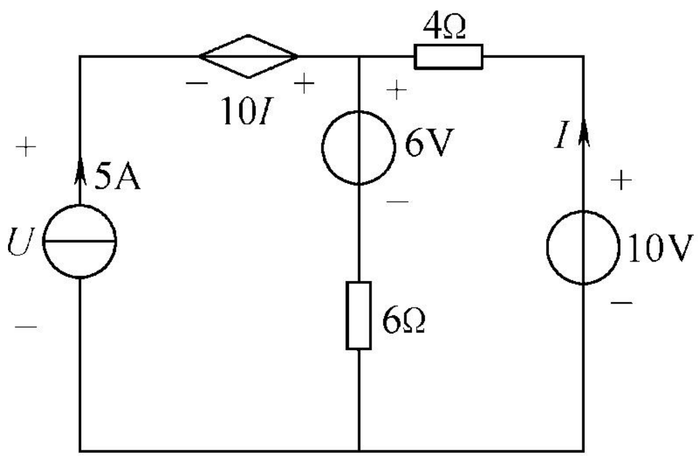


b)

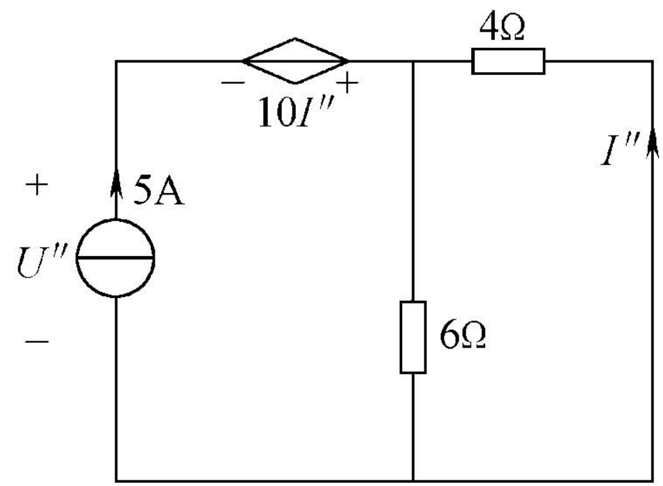
【解】 (1)将电源分成组，即当6V电压源和10V电压源共同作用时，5A电流源用开路代替，电路如图b所示。根据KVL和欧姆定律，得

$$I' = \frac{10-6}{6+4}\text{A} = 0.4\text{A} \qquad U' = -10I' - 4I' + 10 = 4.4\text{V}$$

（2）5A电流源单独作用时的电路如图c所示，根据分流公式得



a)



c)

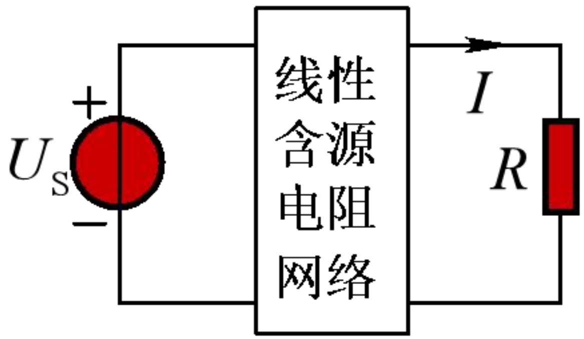
$$I'' = -\frac{6}{4+6} \times 5\text{A} = -3\text{A}$$

$$U'' = -10I'' - 4I'' = 42\text{V}$$

（3）利用叠加定理得 $I = I' + I'' = (0.4-3)\text{A} = -2.6\text{A}$

$$U = U' + U'' = 46.4\text{V}$$

【例9】已知电路如图所示，当 $U_S = 10\text{V}$ 时 $I = 6\text{A}$



$U_S = 15\text{V}$ 时 $I = 7\text{A}$

$I = 10\text{A}$ 时 $U_S = ?$

【解】 根据齐性定理和叠加定理

$$I = I' + I'' = I' + kU_S$$

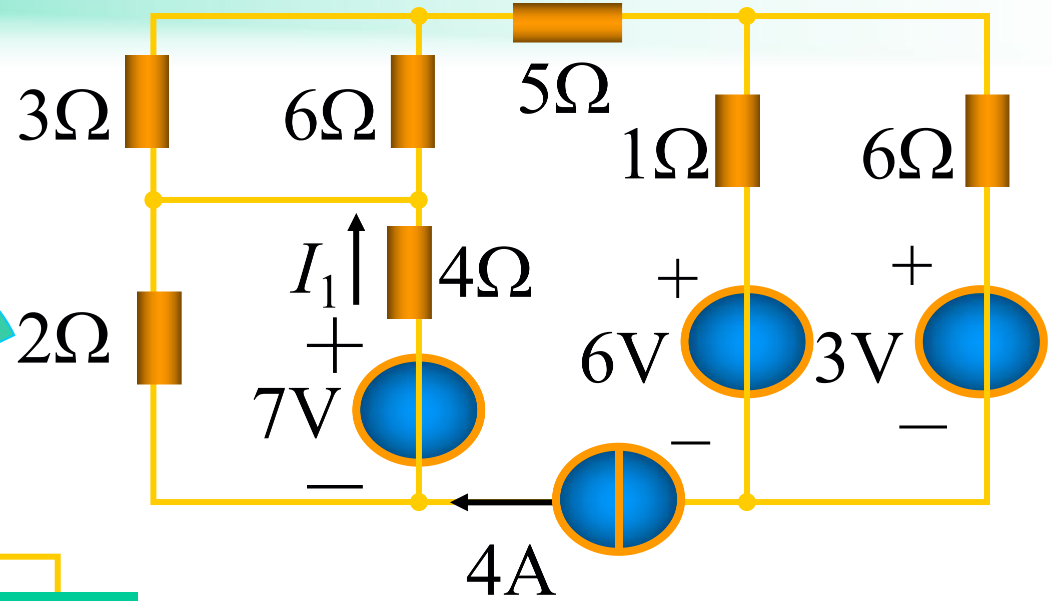
$$\begin{cases} 6\text{A} = I' + k \times 10\text{V} \\ 7\text{A} = I' + k \times 15\text{V} \end{cases} \Rightarrow \begin{cases} k = 0.2\text{S} \\ I' = 4\text{A} \end{cases}$$

$$I = 4\text{A} + 0.2\text{S} \times U_S$$

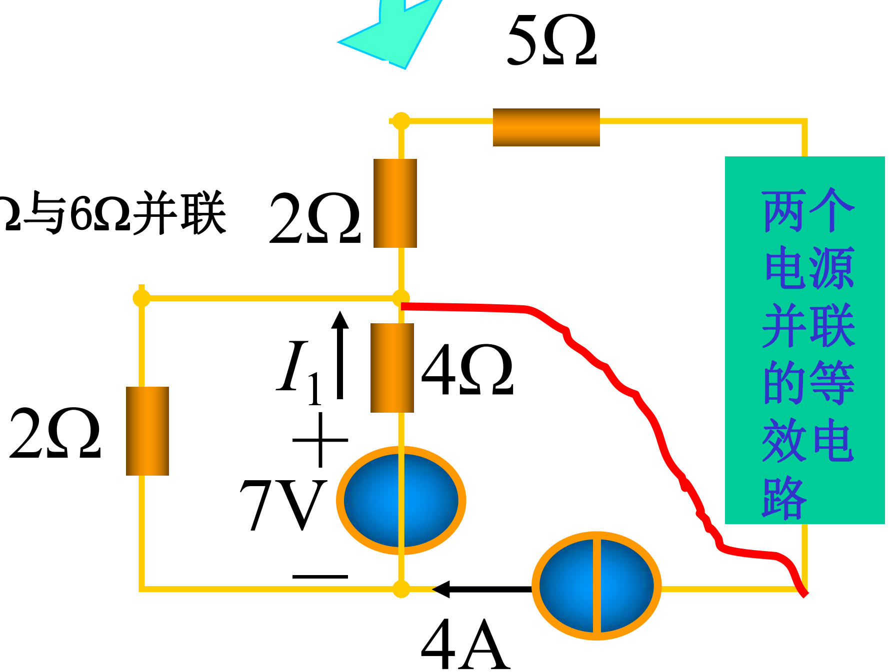
当 $I = 10\text{A}$ 得 $U_S = 30\text{V}$

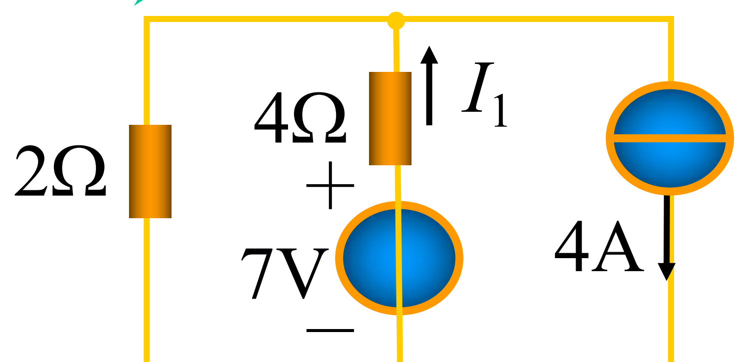
【例10】求电流I_1

【解】用替代定理：



3Ω与6Ω并联

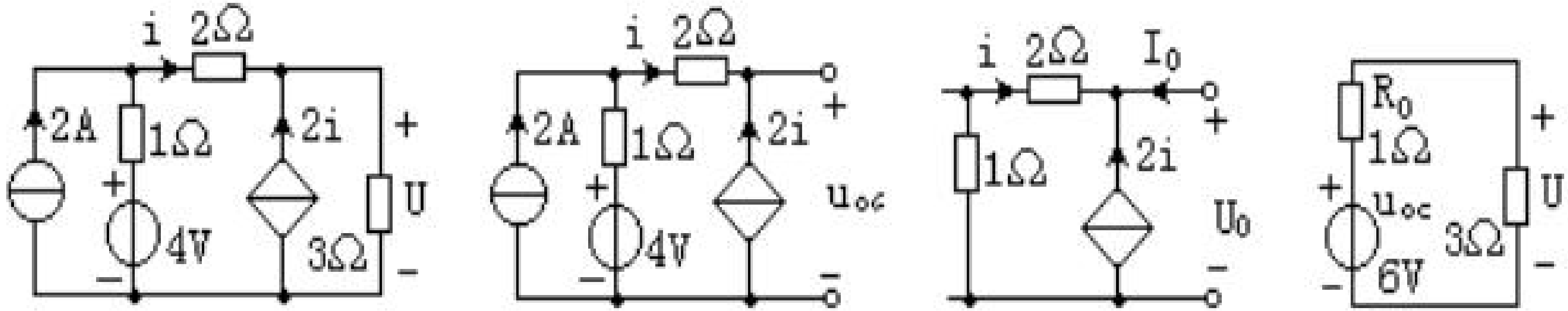




应用叠加原理

$$I_1=\frac{7}{6}+\frac{2\times4}{2+4}=\frac{15}{6}=2.5\text{A}$$

【例11】电路如图(a)所示，试应用戴维宁定理求电压U。



(a) (b) (c) (d)

解 求开路电压u_{oc}：将图 (a)中3Ω电阻断开，则得图 (b)电路。

$$\because i=-2i \quad \therefore \quad i=0 \quad \text{则有} \quad u_{oc}=4+2\times 1=6\text{V}$$

用加压求流法求等效电阻R_0：令网络内所有独立电源为零值，电路如图 (c)所示。

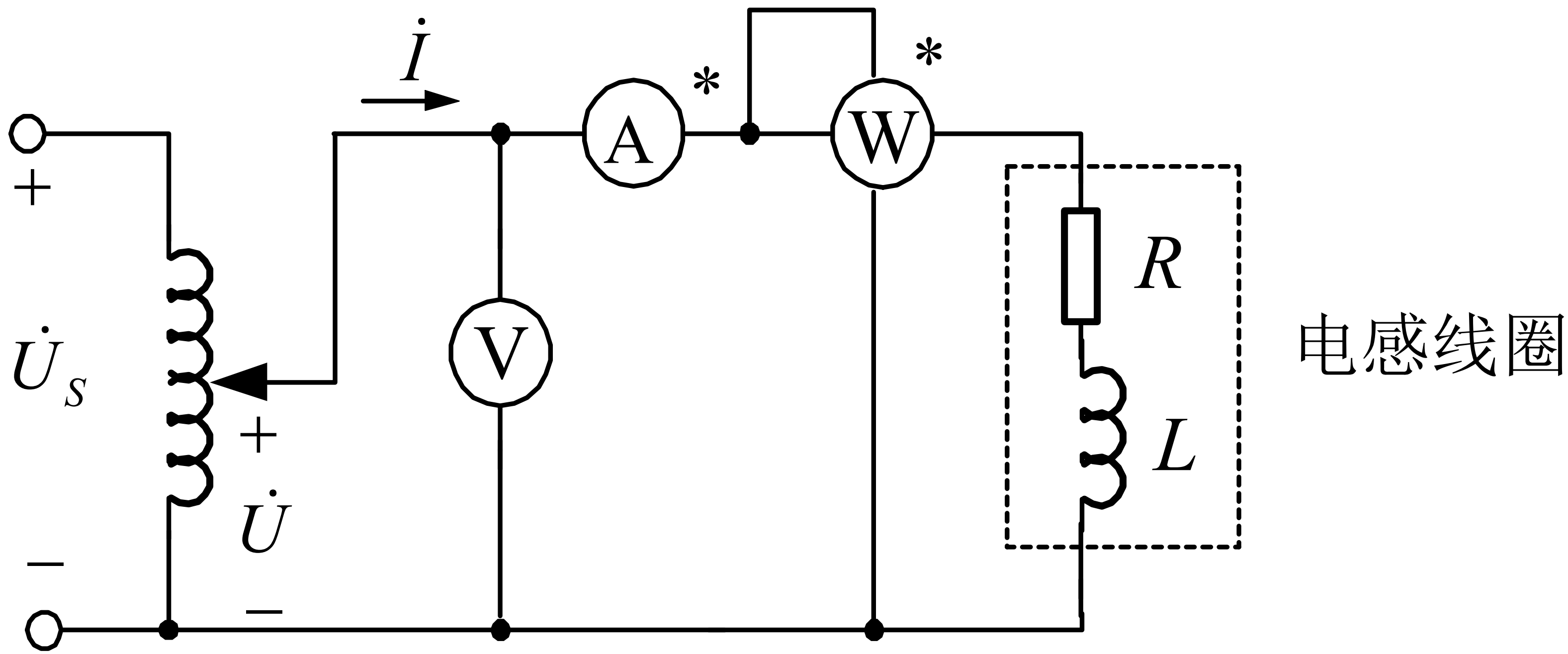
$$\because U_0=-i\times(1+2) \quad \therefore \quad i=-\frac{1}{3}U_0 \quad \Rightarrow \quad R_0=\frac{U_0}{I_0}=1\Omega$$

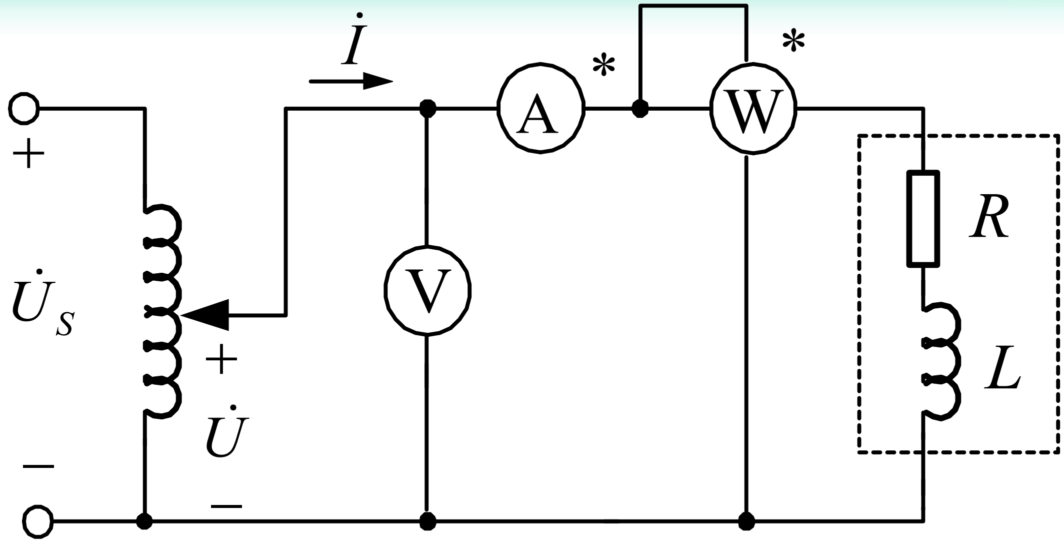
$$I_0=-i-2i=-3i=U_0$$

$$\boxed{U=\frac{3}{R_0+3}\cdot u_{oc}=\frac{3}{1+3}\times 6=4.5\text{V}}$$

【例12】

已知电路如图所示，电压表的读数为**50V**,电流表的读数为**1A**，功率表的读数为**30W**，电源的频率为**50Hz**。试求**R**、**L**的数值、无功功率、视在功率。





电压表=50V

电流表=1A

功率表=30W

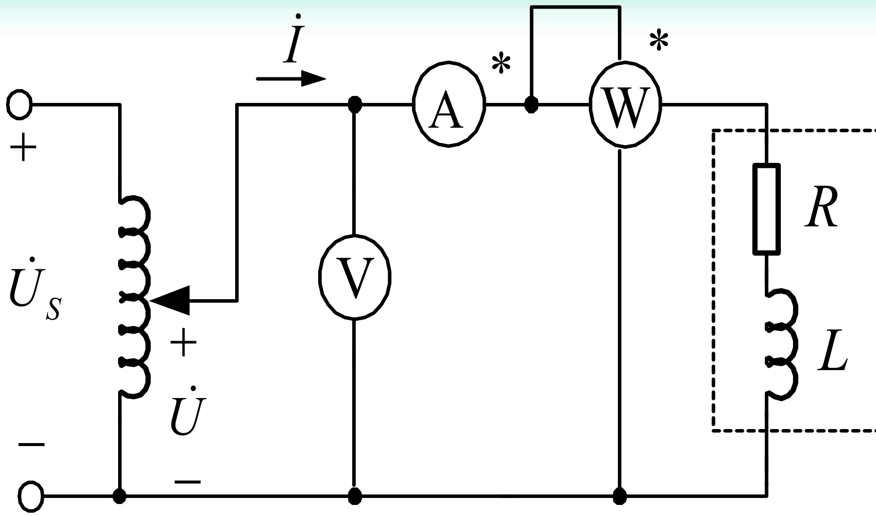
解：1. $\boxed{Z = R + jX_L = |Z|\angle\varphi}$ $\quad\because\ P = UI\cos\varphi$

$$|Z| = \frac{U}{I} = \frac{50}{1} = 50\Omega \qquad \therefore \cos\varphi = \frac{P}{UI} = \frac{30}{50\times1} = 0.6$$

$$Z = R + jX_L = |Z|\angle\varphi = 50\angle53.13^\circ\ \Omega \qquad \varphi = 53.13^\circ$$

$$Z = 50\angle53.13^\circ = (30 + j40)\Omega$$

$$\boxed{R = 30\Omega,\ L = \frac{X_L}{\omega} = \frac{40}{314} = 127\ \text{mH}}$$



电压表＝50V

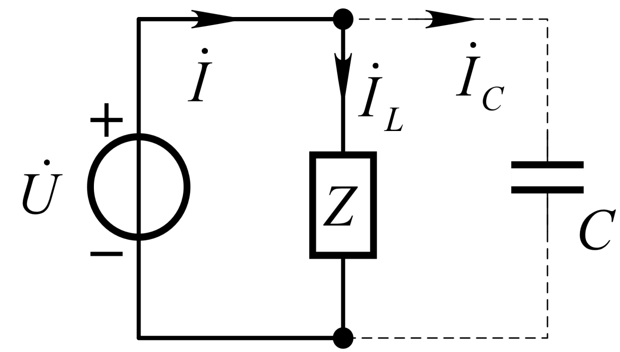
电流表＝1A

功率表＝30W

2. $Q = UI\sin\varphi = 50\times 1\times \sin 53.13^{\circ} = 40$乏

3. $S = UI = \sqrt{P^2 + Q^2} = 50\times 1 = 50\,\mathrm{VA}$

【例13】 在如图所示电路中，已知$U = 220\text{V}$，$f = 50\text{Hz}$，感性负载的功率为 40W，额定电流为0.4A。试求（1）电路的功率因数，电感L和电感上的电压；（2）若要将电路的功率因数提高到0.95，需要并联多大电容？（3）并联电容后电源的总电流为多少？电源提供的无功功率为多少？



【解】（1）因为 $U = 220\text{V}$

$$I_{\text{N}} = 0.4\text{A}$$

$$\cos\varphi = \cos\varphi_{\text{L}} = \frac{P}{UI_{\text{N}}} = \frac{40}{220\times 0.4} = 0.45$$

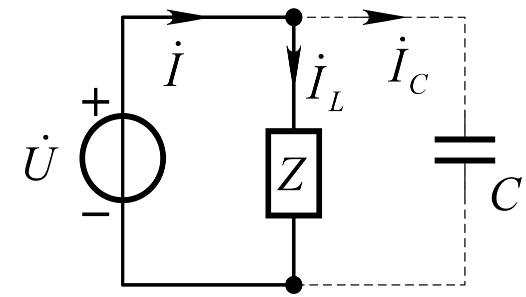
$$|Z| = \frac{U}{I_{\text{N}}} = \frac{220}{0.4} = 550\ \Omega \qquad \varphi_{\text{L}} = \arccos\varphi_{\text{L}} = 63^\circ$$

$$Z_{\text{L}} = |Z|\angle\varphi_{\text{L}} = 550\angle 63^\circ = (250 + \text{j}490)\Omega$$

$$R = 250\,\Omega \qquad X_{\text{L}} = 490\,\Omega \qquad L = \frac{X_{\text{L}}}{\omega} = \frac{490}{314}\text{H} = 1.56\text{H}$$

$$U_{\text{L}} = X_{\text{L}} I_{\text{N}} = 490 \times 0.4\text{V} = 196\,\text{V}$$

（2）并电容后，电路的功率因数为0.95



$$\varphi = \arccos 0.95 = 18.2^\circ$$

由公式有 $C = \dfrac{P}{\omega U^2}(\tan\varphi_{\mathrm{L}} - \tan\varphi)$

$$= \frac{40}{2\pi \times 50 \times 220^2}\left(\tan 63^\circ - \tan 18.2^\circ\right) \approx 4.3\mathrm{uF}$$

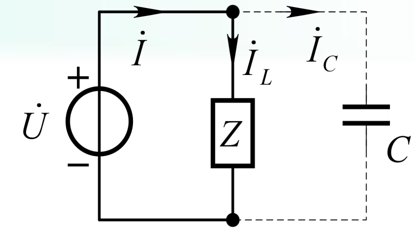
或者：

$$Q_{\mathrm{L}} = I_{\mathrm{L}}^2 X_{\mathrm{L}} = 0.4^2 \times 490 = 78.4\,\mathrm{var}$$

$$Q' = P\tan 18.2^\circ = 13.1\,\mathrm{var}$$

$$Q_{\mathrm{C}} = Q' - Q_L = 13.1 - 78.4 = \text{-}65.2\,\mathrm{var}$$

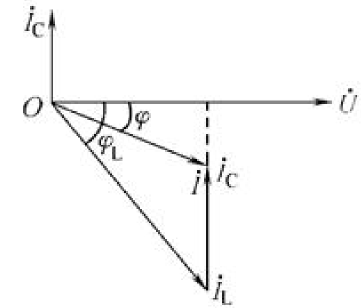
$$C = \frac{Q_{\mathrm{C}}}{\omega U^2} = \frac{65.2}{314 \times 220^2} = 4.3\mu\mathrm{F}$$



（3）并联电容后，电源的总电流为

$$I=\frac{P}{U\cos\varphi}=\frac{40}{220\times 0.95}\,\mathrm{A}=0.191\,\mathrm{A}$$

电源提供的无功功率为



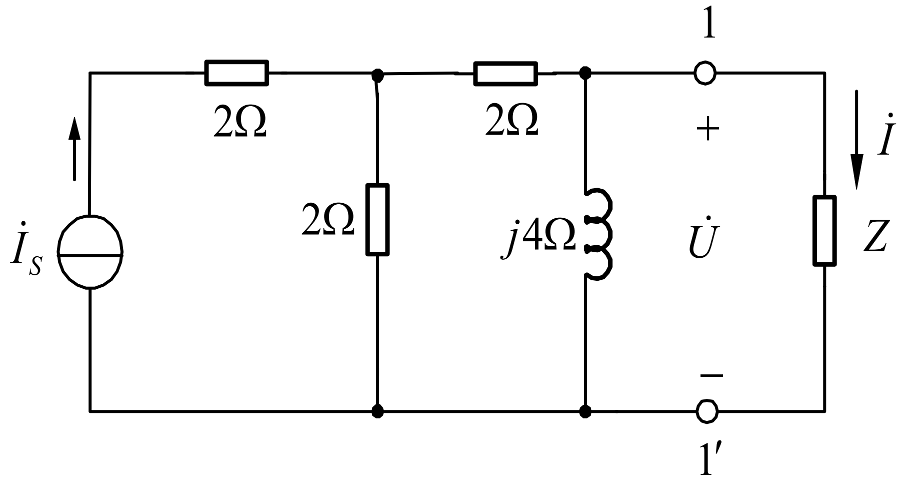
$$Q=P\tan 18.2^{\circ}=13.1\,\mathrm{var}$$

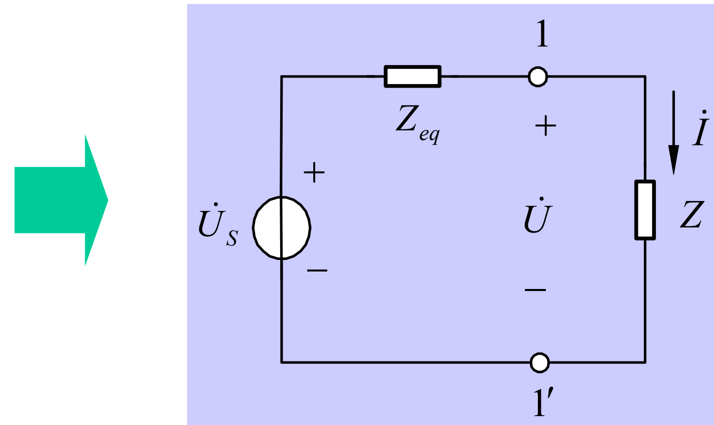
$$Q=Q_{\mathrm{L}}+Q_{\mathrm{C}}=78.4\,\mathrm{var}-65.24\,\mathrm{var}=13.1\,\mathrm{var}$$

$$Q=UI\sin\varphi=220\times 0.191\times\sin 18.2=13.1\,\mathrm{var}$$

【例14】电路如图所示，已知 $\dot{I}_S = 2\angle 0^\circ A$。

求最佳匹配时获得的最大功率。





解： 先求得一端口得戴维南等效电路

$$\boxed{Z = Z^*_{eq} = 2 - j2\,\Omega}$$

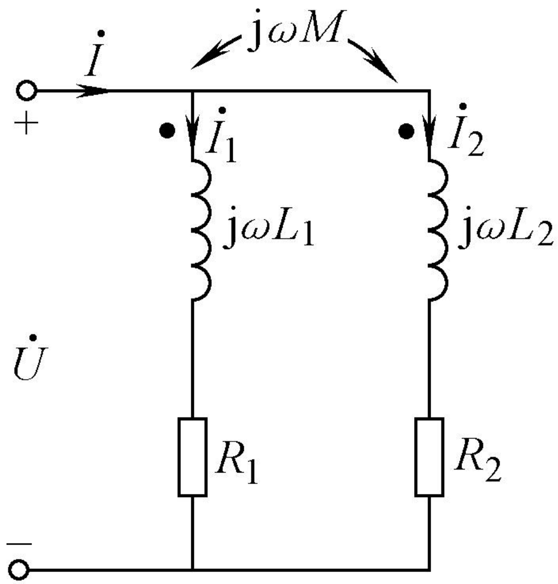
$$\dot{U}_{OC} = \dot{U}_S = \frac{2\times 2\angle 0^\circ}{2+2+\mathrm{j}4}\times \mathrm{j}4 = \frac{\mathrm{j}16}{5.65\angle 45^\circ}$$

$$= 2.83\angle 45^\circ V$$

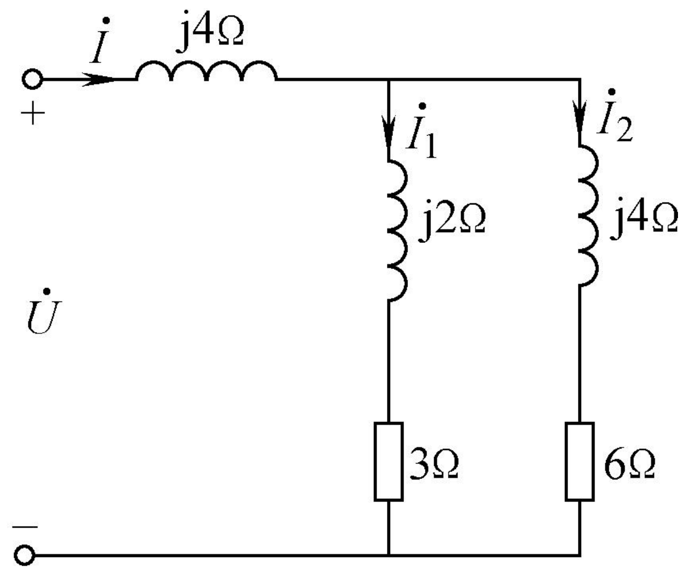
$$\boxed{P_{\max} = \frac{U_{OC}^2}{4R_{eq}} = \frac{2.83^2}{4\times 2} = 1\,\mathrm{W}}$$

$$Z_{eq} = \frac{4\times \mathrm{j}4}{4+\mathrm{j}4} = \frac{\mathrm{j}16}{5.65\angle 45^\circ} = 2.83\angle 45^\circ = 2+\mathrm{j}2\Omega$$

【例15】在图a)电路中，已知 $R_1=3\Omega$, $R_2=6\Omega$, $\omega L_1=6\Omega$, $\omega L_2=8\Omega$, $\omega M=4\Omega$，$\dot{U}=10\angle 0^\circ\text{V}$ 。试用互感消去法求 $\dot{I}_1$, $\dot{I}_2$ 和输入阻抗 Z_i 。



a) 原电路



b) 等效电路

【解】将图a)电路中的互感消去，其等效电路如图 b)所示。

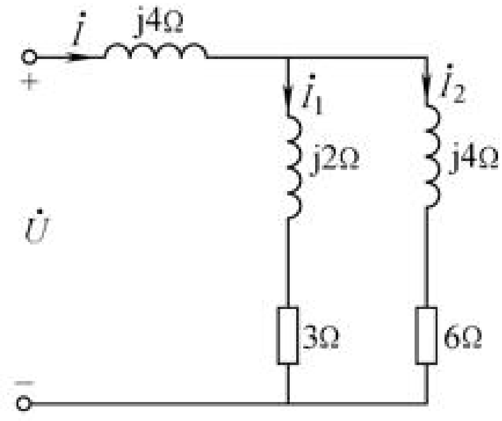
由图b)得

$$Z_1=(3+\mathrm{j}2)\Omega=3.6\angle 33.6^\circ\Omega$$

$$Z_2=(6+\mathrm{j}4)\Omega=7.2\angle 33.6^\circ\Omega$$

$$Z_1//Z_2=\frac{(3+\mathrm{j}2)(6+\mathrm{j}4)}{3+\mathrm{j}2+6+\mathrm{j}4}\Omega=\frac{25.92\angle 67.2^\circ}{9+\mathrm{j}6}\Omega$$

$$=2.4\angle 33.6^\circ\Omega=(2+\mathrm{j}1.32)\Omega$$



b) 等效电路

输入阻抗为 $Z_\mathrm{i}=\mathrm{j}4+Z_1//Z_2=(\mathrm{j}4+2+\mathrm{j}1.32)\Omega=(2+\mathrm{j}5.32)\Omega$

$$\dot{I}=\frac{\dot{U}}{Z_i}=\frac{10\angle 0^\circ}{2+\mathrm{j}5.32}\mathrm{A}=1.76\angle-69.4^\circ\mathrm{A}$$

$$\dot{I}_1=\frac{6+\mathrm{j}4}{3+\mathrm{j}2+6+\mathrm{j}4}\dot{I}=\frac{7.2\angle 33.6^\circ}{10.8\angle 33.6^\circ}\times 1.76\angle-69.4^\circ\mathrm{A}$$

$$=1.16\angle-69.4^\circ\mathrm{A}$$

$$\dot{I}_2=\frac{3+\mathrm{j}2}{3+\mathrm{j}2+6+\mathrm{j}4}\dot{I}=\frac{3.6\angle 33.6^\circ}{10.8\angle 33.6^\circ}\times 1.76\angle-69.4^\circ\mathrm{A}$$

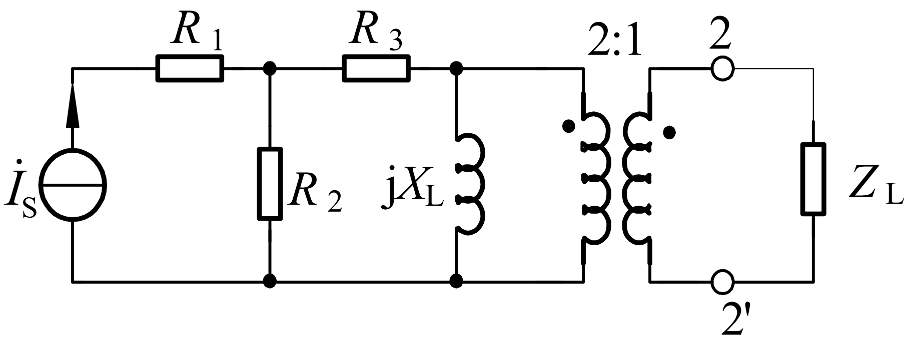
$$=0.58\angle-69.4^\circ\mathrm{A}$$

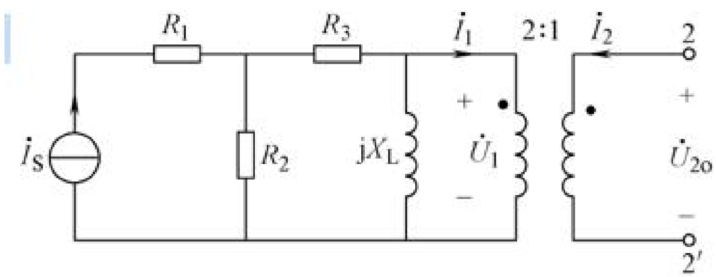
【例16】在图7.31所示的电路中，已知 $\dot{I}_S = 4\angle 0^\circ \text{A}$，

$R_1 = R_2 = R_3 = 2\Omega$，$jX_L = j4\Omega$。试求：

（1）22'端口电路的戴维南等效电路；

（2）最佳匹配时负载Z_L上获得的最大功率。





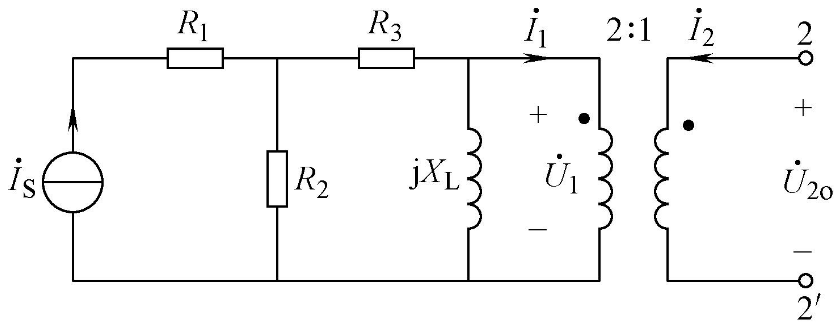
a) 求一端口开路电压

【解】（1）求开路电压的一端口电路如图a）所示。

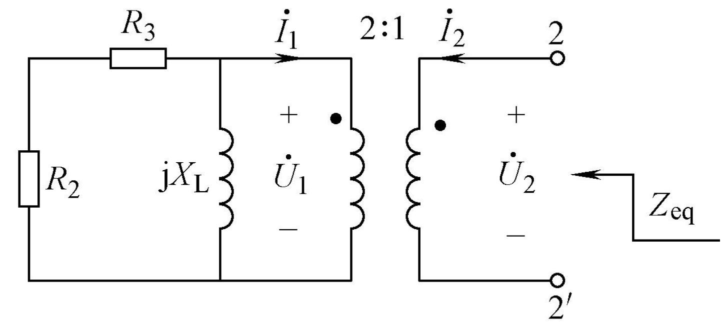
$\dot{U}_{2o} = \dfrac{1}{K}\dot{U}_1 = \dfrac{1}{2}\dot{U}_1$ 由于 $\dot{I}_2 = 0$，所以 $\dot{I}_1 = -\dfrac{1}{K}\dot{I}_2 = 0$，则

$$\dot{U}_1 = jX_L \frac{R_2}{R_2 + R_3 + jX_L}\dot{I}_S = j4 \times \frac{2}{2+2+j4} \times 4\angle 0^\circ = 4\sqrt{2}\angle 45^\circ \text{V}$$

$$\boxed{\dot{U}_{2O} = \frac{1}{2}\dot{U}_1 = \frac{1}{2} \times 4\sqrt{2}\angle 45^\circ \text{V} = 2\sqrt{2}\angle 45^\circ \text{V}}$$



a) 求一端口开路电压



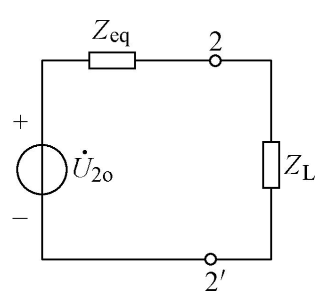
b)求等效阻抗

由图b)，用外加电压法得

$$Z_{eq}=\frac{\dot{U}_2}{\dot{I}_2}=\frac{\frac{\dot{U}_1}{K}}{-K\dot{I}_1}=\frac{1}{K^2}\frac{\dot{U}_1}{(-\dot{I}_1)}=\frac{1}{K^2}\frac{[(R_2+R_3)//\,jX_L](-\dot{I}_1)}{(-\dot{I}_1)}=\frac{1}{4}\times 4//\,j4\Omega=(\frac{1}{2}+\frac{j}{2})\Omega$$

图a)的 $22'$ 端口的戴维宁等效电路如下图所示。

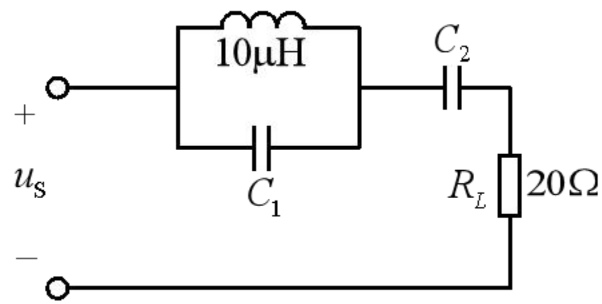
当 $Z_L=Z_{eq}^*=(\frac{1}{2}-j\frac{1}{2})\Omega$ 时，



$$\boxed{P_L=\frac{U_{2O}^2}{4R_{eq}}=\frac{(2\sqrt{2})^2}{4\times\frac{1}{2}}W=4W}$$

【例17】

图示电路，已知 $f_1=100\text{kHz}$时，电流不能通过负载 R_L ，而在频率为 $f_2=50\text{kHz}$ 时流过R_L的电流为最大。求C_1和C_2。



【解】L和C_1 发生并联谐振时，电流不能通过负载。

则有 $\omega_1 C_1=\dfrac{1}{\omega_1 L}$

$$\Rightarrow C_1=\frac{1}{\omega_1^2 L}=\frac{1}{(2\pi f_1)^2\times 10\times 10^{-6}}\approx 0.25\mu\text{F}$$

$L//C_1$和C_2发生串联谐振时，电流最大

$$\frac{1}{\mathrm{j}\omega_2 C_2}+\frac{\mathrm{j}\omega_2 L\cdot\dfrac{1}{\mathrm{j}\omega_2 C_1}}{\mathrm{j}\omega_2 L+\dfrac{1}{\mathrm{j}\omega_2 C_1}}=0 \quad \Rightarrow C_2=\frac{1}{\omega_2^2 L}-C_1\approx 0.76\mu\text{F}$$

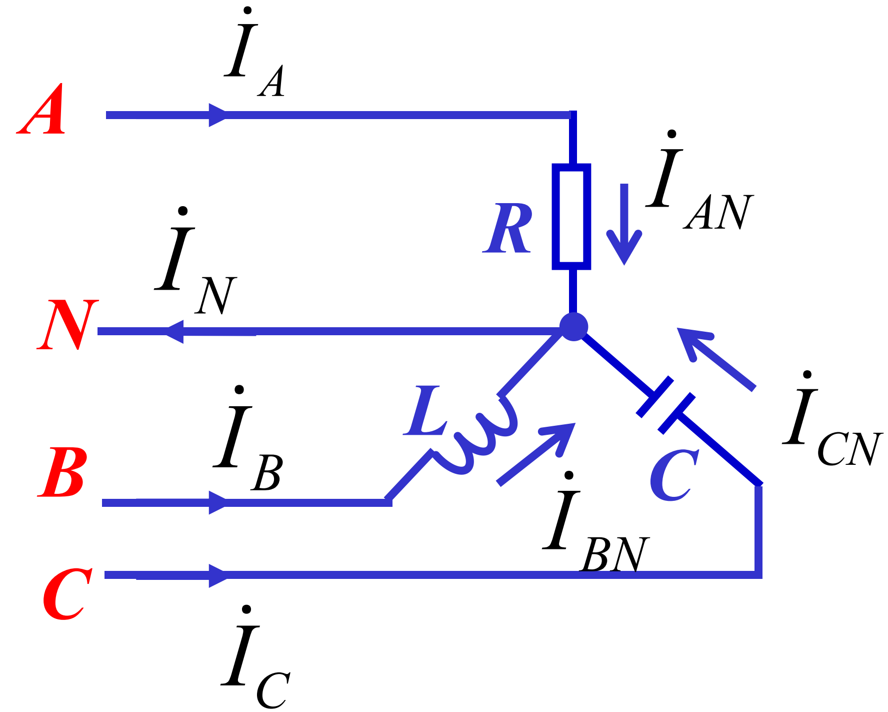
【例18】

已知：$U_l = 380\text{V}$

三相负载

$R = X_L = X_C = 10\Omega$

求：相电流及中线电流

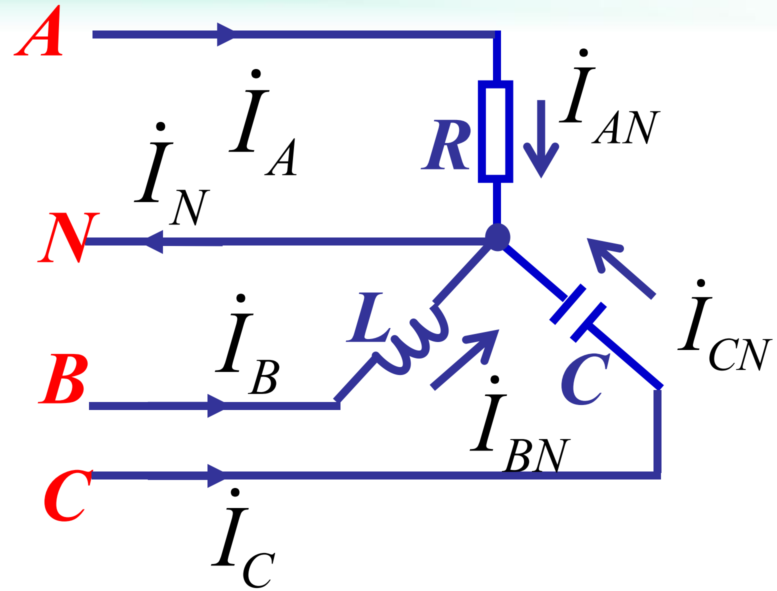


$$\dot{U}_{AN} = 220\angle 0^\circ \text{ V}$$

$$\dot{U}_{BN} = 220\angle -120^\circ \text{ V}$$

$$\dot{U}_{CN} = 220\angle 120^\circ \text{ V}$$

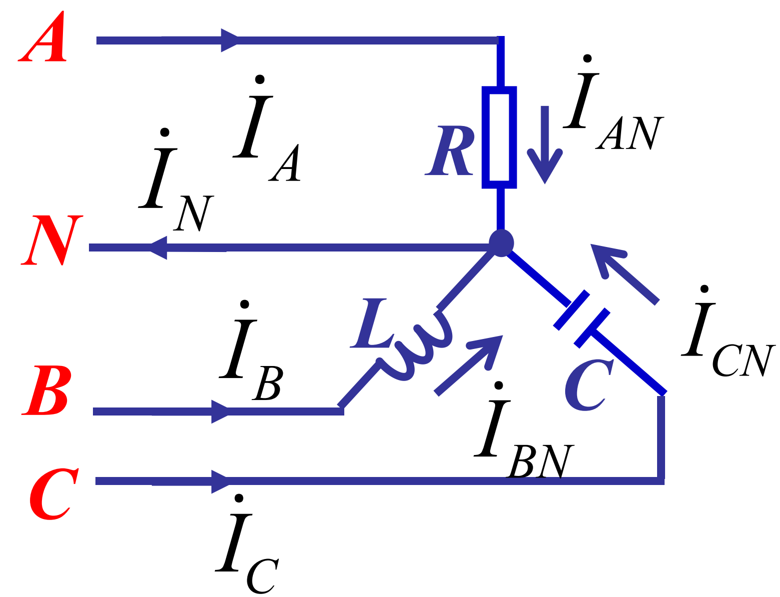
$$\dot{U}_{AN} = 220\angle 0° \text{ V}$$
$$\dot{U}_{BN} = 220\angle -120° \text{ V}$$
$$\dot{U}_{CN} = 220\angle 120° \text{ V}$$



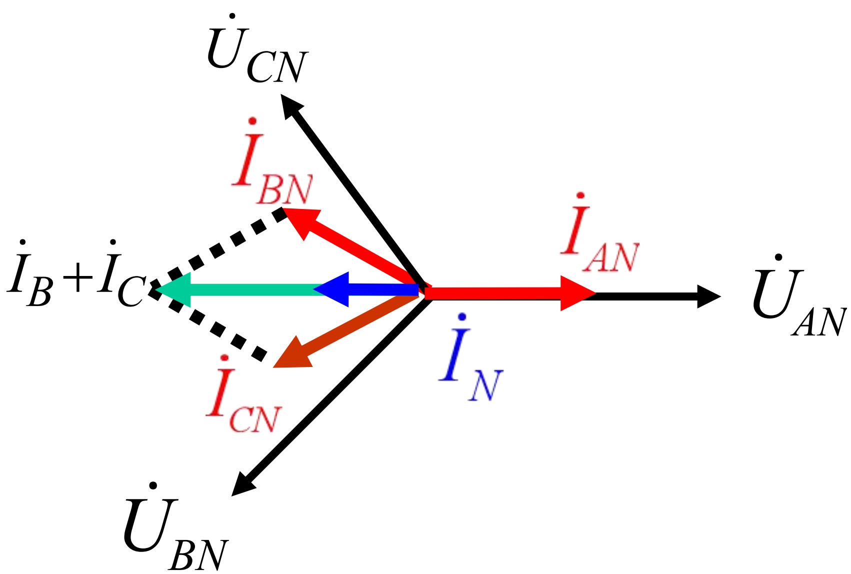
$$\begin{cases} \dot{I}_{AN} = \dfrac{\dot{U}_{AN}}{R} = \dfrac{U_P}{R} = \dfrac{220}{10} = 22\angle 0^{\circ} \text{ A} \\ \dot{I}_{BN} = \dfrac{\dot{U}_{BN}}{j\omega L} = \dfrac{U_P\angle -120^{\circ}}{j\omega L} = \dfrac{220}{10}\angle -210^{\circ} = (22\angle -210^{\circ})\text{A} \\ \dot{I}_{CN} = \dfrac{\dot{U}_{CN}}{-j\dfrac{1}{\omega C}} = \dfrac{U_P\angle 120^{\circ}}{-j\dfrac{1}{\omega C}} = \dfrac{220}{10}\angle 210^{\circ} = （22\angle\text{-}150^{\circ}）\text{ A} \end{cases}$$

中线电流

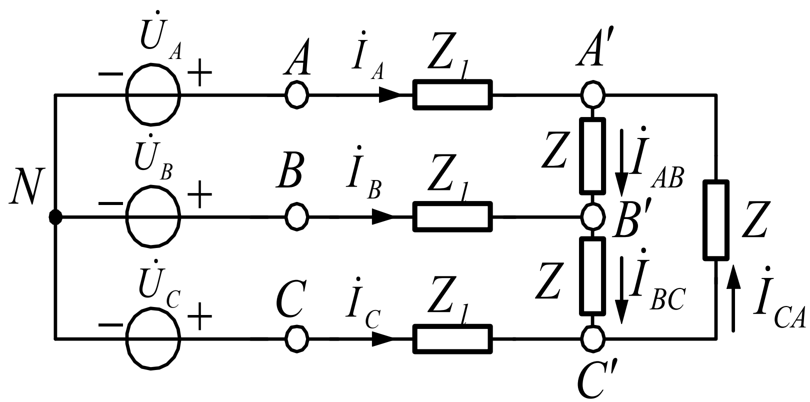
$$\dot{I}_N = \dot{I}_A + \dot{I}_B + \dot{I}_C$$
$$= 22 + 22\angle -210^\circ + 22\angle -150^\circ$$
$$= 22 - 19 - j11 - 19 + j11$$
$$= 22 - 38 = -16\,\text{A}$$

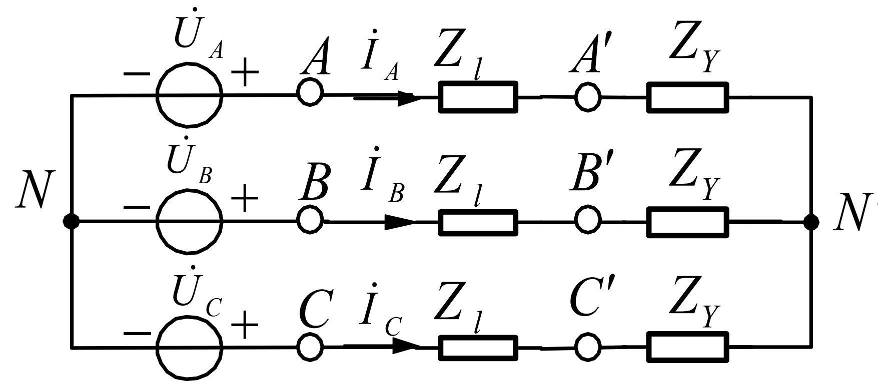


用相量图求中线电流



【例19】已知电路如图所示，$Z=(19.2+j14.4)\Omega$，$U_{AB}=380\text{V}$。$Z_l=(3+j4)\Omega$，试求负载Z的相电压和相电流。



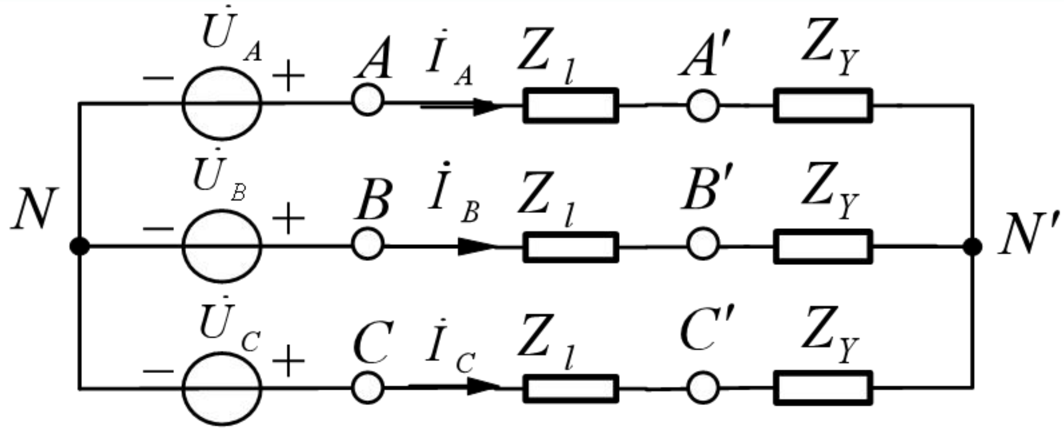


解：将电路变换为对称的Y——Y电路 $\boxed{Z_\Delta=3Z_Y}$

$$Z_Y=\frac{Z_\Delta}{3}=\frac{19.2+j14.4}{3}=(6.4+j4.8)\Omega \quad \dot{U}_A=220\angle 0^\circ\ \text{V}$$

$$\dot{I}_A=\frac{\dot{U}_A}{Z_l+Z_Y}=\frac{220\angle 0^\circ}{3+j4+6.4+j4.8}=17.1\angle -43.2^\circ A$$

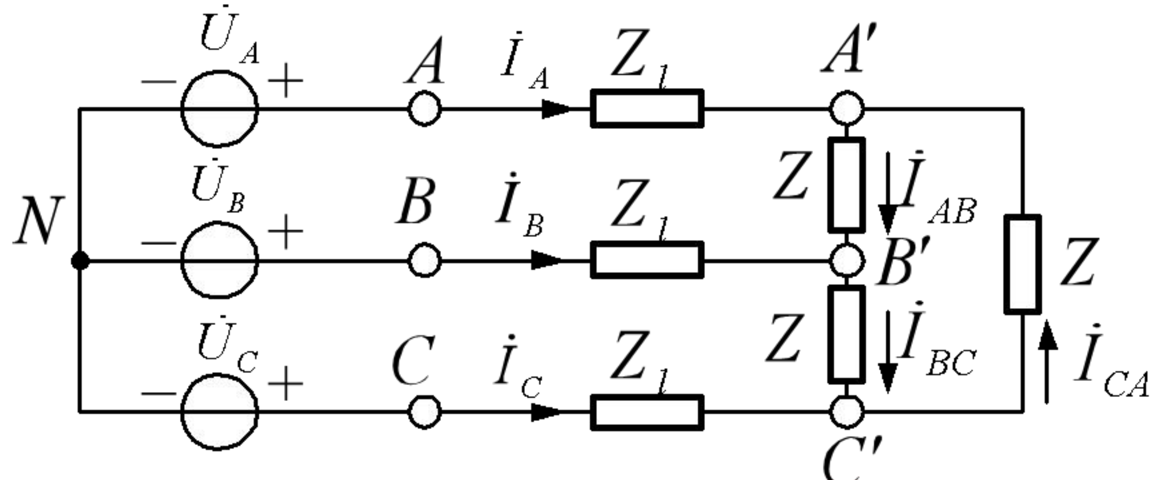
$$\dot{I}_B=17.1\angle -163.2^\circ A \qquad \dot{I}_C=17.1\angle 76.8^\circ A$$



$$\dot{I}_A = 17.1\angle -43.2^\circ\ A$$

$$\dot{I}_B = 17.1\angle -163.2^\circ\ A$$

$$\dot{I}_C = 17.1\angle 76.8^\circ\ A$$



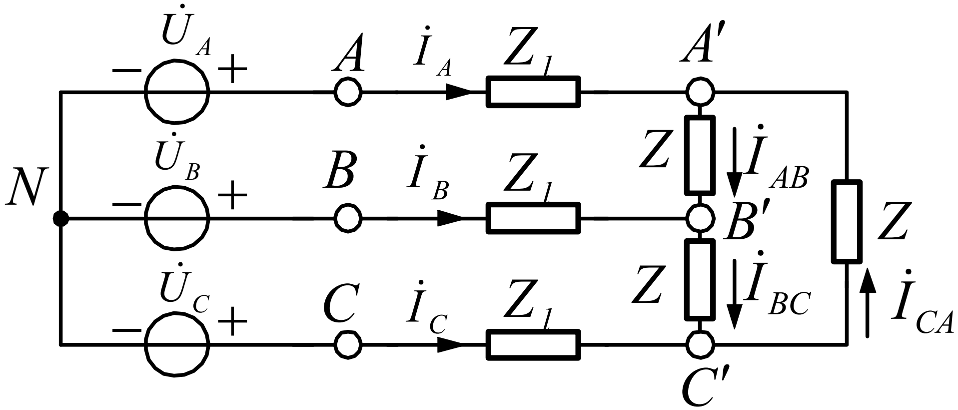
$$\dot{I}_l = \sqrt{3}\dot{I}_p\angle -30^\circ$$

$$Z = (19.2 + j14.4)\Omega$$

$$\dot{I}_{\mathrm{AB}} = \frac{\dot{I}_{\mathrm{A}}}{\sqrt{3}\angle -30^\circ} = \frac{17.1\angle -43.2^\circ}{\sqrt{3}\angle -30^\circ} = 9.9\angle -13.2^\circ\ A$$

$$\dot{I}_{\mathrm{BC}} = 9.9\angle -132.2^\circ\ A$$

$$\dot{I}_{\mathrm{CA}} = 9.9\angle 106.8^\circ\ A$$



$$Z=(19.2+j14.4)\Omega$$

相电流：

$$\dot{I}_{\rm AB}=\frac{\dot{I}_{\rm A}}{\sqrt{3}\angle-30^\circ}=\frac{17.1\angle-43.2^\circ}{\sqrt{3}\angle-30^\circ}=9.9\angle-13.2^\circ\ A$$

$$\dot{I}_{\rm BC}=9.9\angle-132.2^\circ\ A \qquad \dot{I}_{\rm CA}=9.9\angle106.8^\circ\ A$$

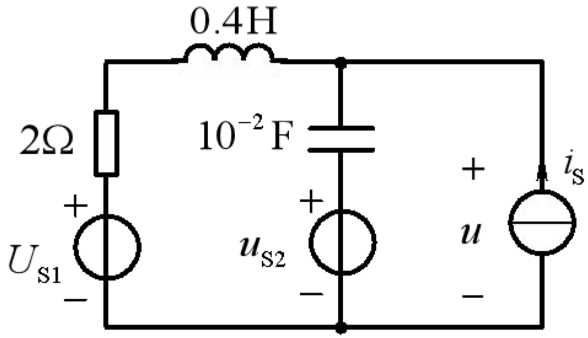
相电压：

$$\dot{U}_{\rm A'B'}=Z\dot{I}_{\rm AB}=(19.2+{\rm j}14.4)\times9.9\angle-13.2^\circ=237.6\angle23.7^\circ\,{\rm V}$$

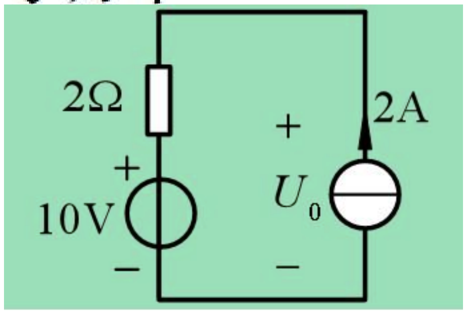
$$\dot{U}_{\rm B'C'}=237.6\angle-96.3^\circ\,{\rm V} \qquad \dot{U}_{\rm C'A'}=237.6\angle143.7^\circ\,{\rm V}$$

【例20】

图示电路$U_{S1}=10\text{V}$,$u_{S2}=20\sqrt{2}\cos\omega_1 t\text{V}$, $i_S=(2+2\sqrt{2}\cos\omega_1 t)\text{A}$ $\omega_1=10\text{rad/s}$ 。(1)求电流源的端电压u及其有效值;(2) 求电流源发出的平均功率。

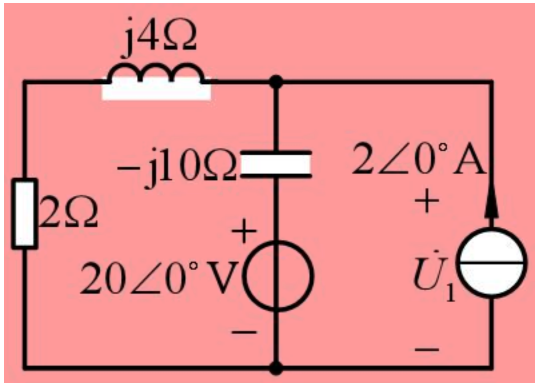


【解】 直流分量作用:



$$U_0=10\text{V}+2\Omega\times 2\text{A}$$
$$=14\text{V}$$

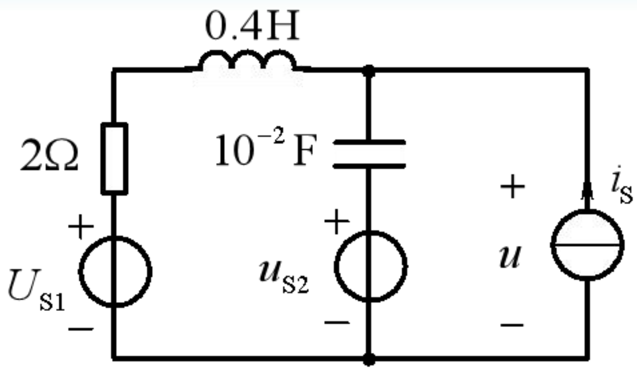
交流分量作用:



$$\dot{U}_1=\frac{\dfrac{20}{-\text{j}10}+2}{\dfrac{1}{2+\text{j}4}+\dfrac{1}{-\text{j}10}}$$

解得

$$\dot{U}_1=20\angle 90^\circ\text{V}$$



$$U_0 = 14\text{V}$$

$$\dot{U}_1 = 20\angle 90^\circ \text{V}$$

电流源的端电压及其有效值分别为

$$u = U_0 + u_1 = [14 + 20\sqrt{2}\cos(\omega_1 t + 90^\circ)]\text{V}$$

$$U = \sqrt{U_0^2 + U_1^2} = \sqrt{(14)^2 + (20)^2}\text{V} = 24.4\text{V}$$

电流源发出的平均功率

$$P = 2U_0 + 2U_1 \cos(90^\circ - 0^\circ)$$

$$= (14 \times 2 + 20 \times 2\cos 90^\circ)\text{W} = 28\text{W}$$

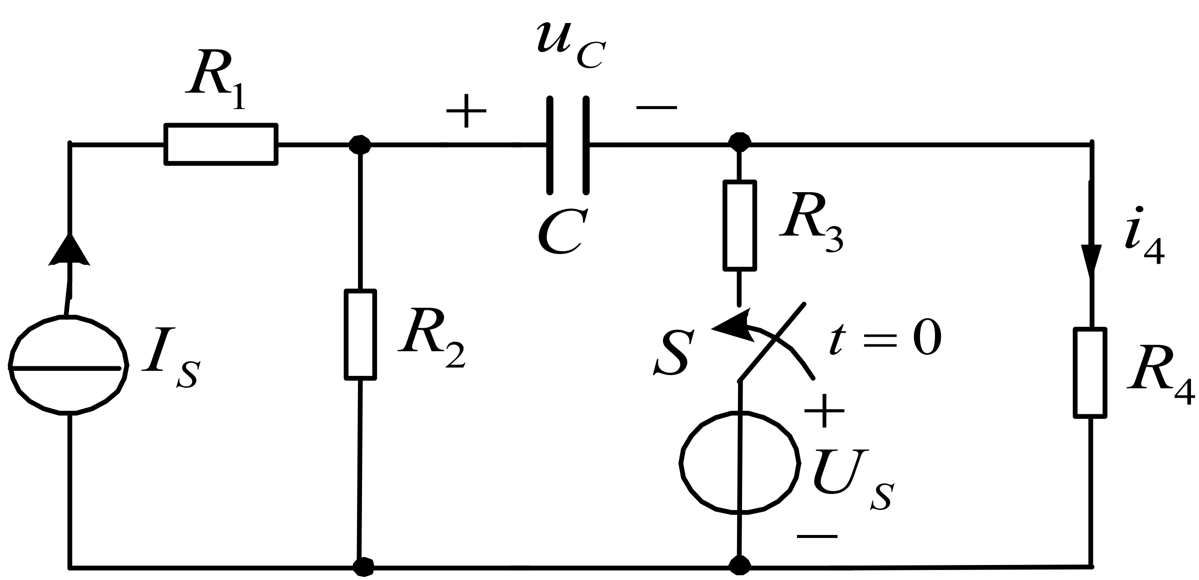
【例21】

图示电路原已稳定，t = 0时将开关S闭合。已知：

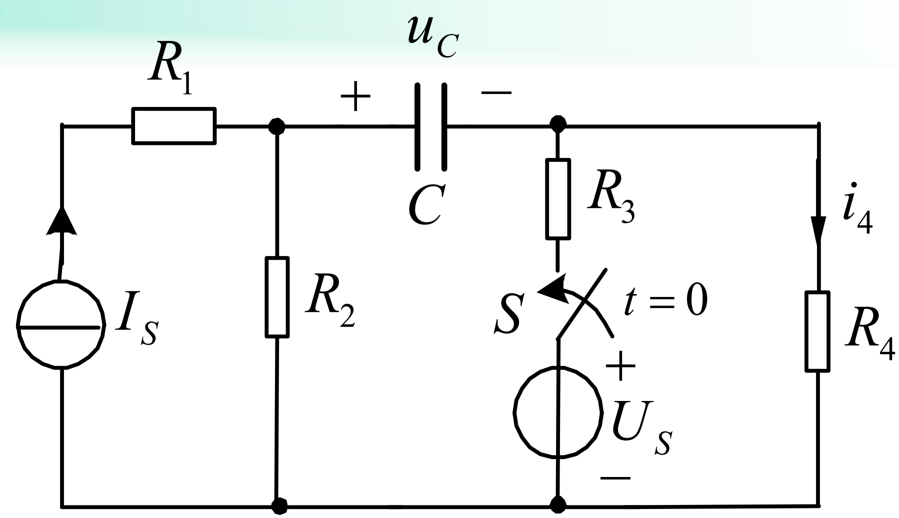
$R_1 = 6\,\mathrm{k\Omega}, R_2 = 3\,\mathrm{k\Omega}, R_3 = 4\,\mathrm{k\Omega}, R_4 = 1\,\mathrm{k\Omega}, C = 1\,\mu\mathrm{F};$

$I_S = 2\,\mathrm{mA}, U_S = 5\,\mathrm{V}$。试求开关S闭合后

$u_C(t)$和$i_4(t)$，并绘出它们的变化曲线。



解：1 求 $u_C(t)$



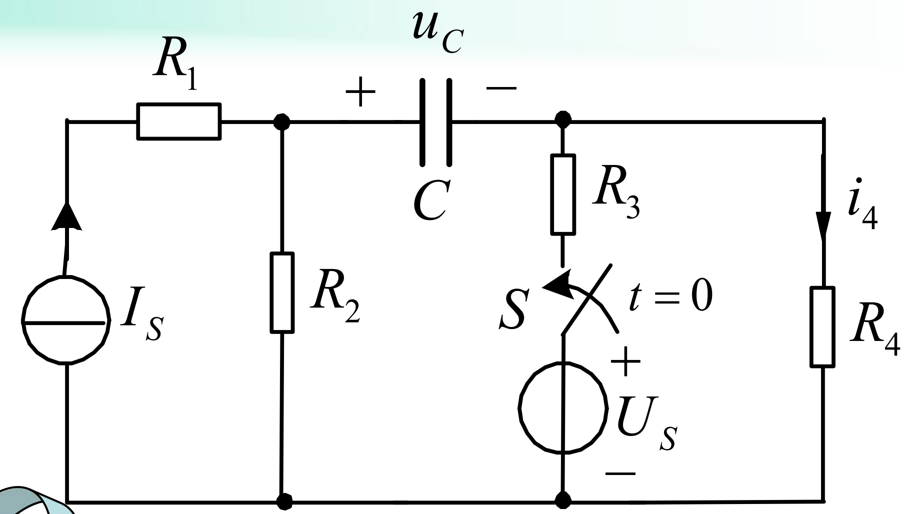
$$u_C(0_+) = u_C(0_-) = I_S R_2$$
$$= 2 \times 3 = 6\,\text{V}$$

$$u_C(\infty) = I_S R_2 - \frac{R_4}{R_3 + R_4} U_S$$
$$= 6 - \frac{1}{4+1} \times 5 = 5\,\text{V}$$

$$\tau = (R_2 + R_3 \,//\, R_4)C = (3 + \frac{4 \times 1}{4+1}) \times 1 \times 10^{-3} = 3.8\,\text{mS}$$

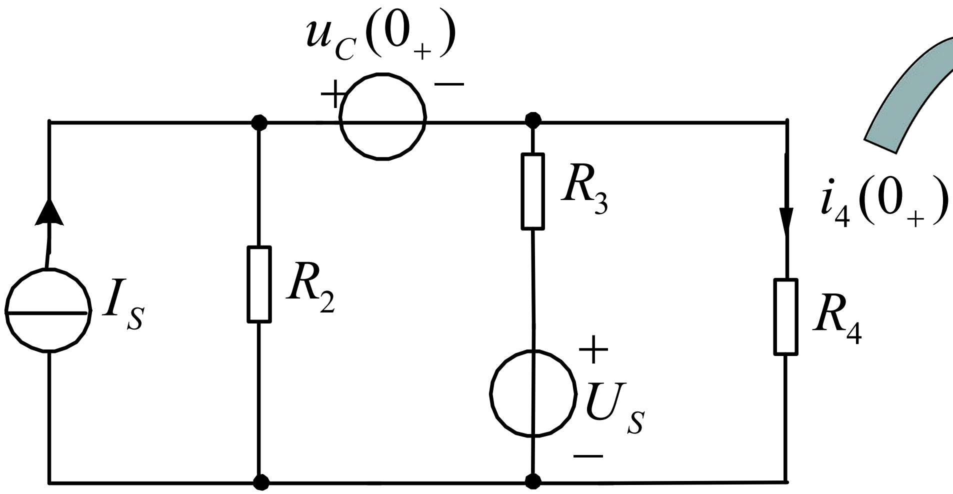
$$\boxed{u_C(t) = 5 + (6-5)e^{-\frac{t}{\tau}} = 5 + e^{-263t}}$$

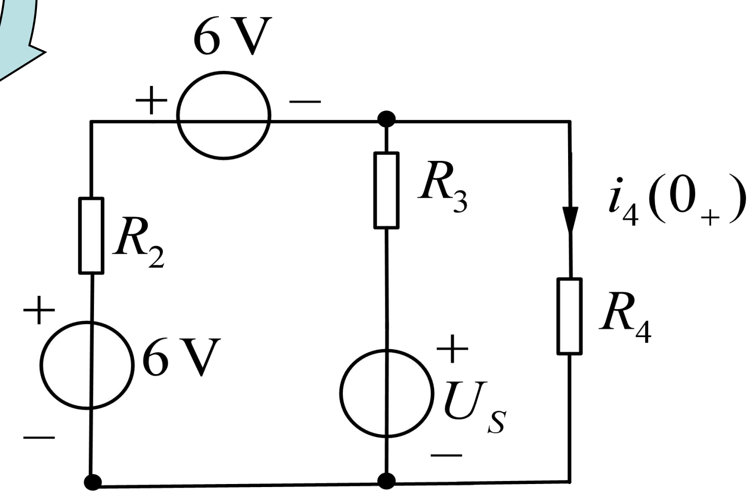
2 求 $i_4(t)$



求$i_4(0_+)$ $\because u_C(0_+)=6\,\text{V}$

$t=0+$



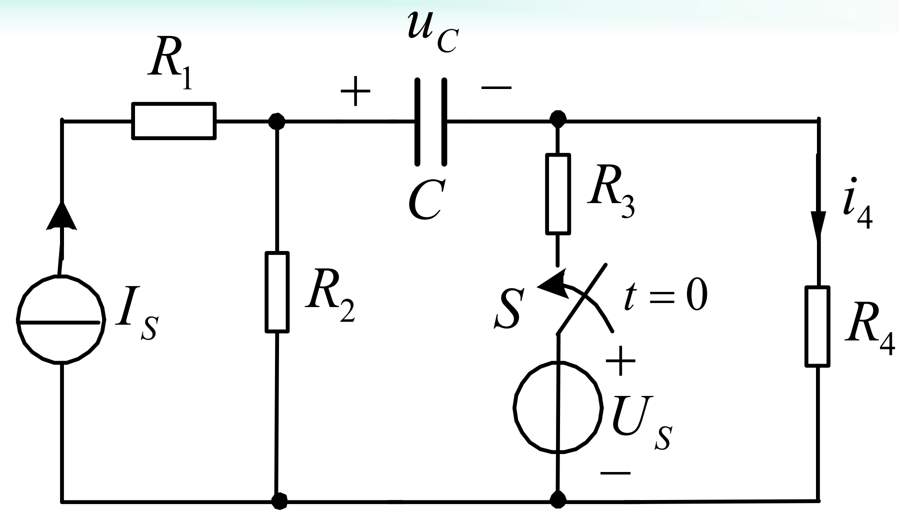


$$I_S R_2 = 2\times 3 = 6\,\text{V}$$

$$\boxed{i_4(0_+) = \frac{U_S}{R_2 /\!/ R_4 + R_3}\frac{R_2}{R_2+R_4} = 0.8\,\text{mA}}$$

求$i_4(\infty)$

$$i_4(0_+) = 0.8\,\text{mA}$$



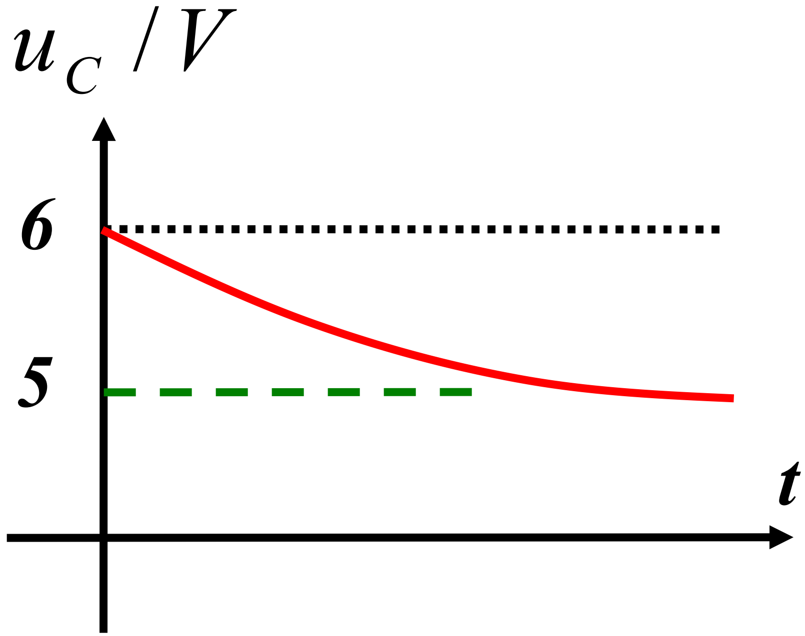
$$i_4(\infty) = \frac{U_S}{R_3 + R_4} = \frac{5}{4+1} = 1\,\text{mA}$$

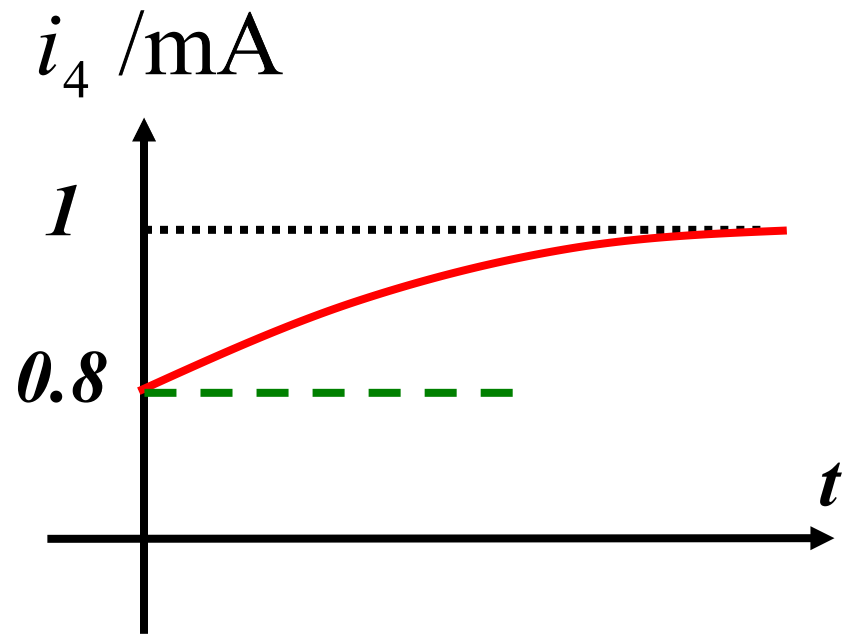
$$\tau = 3.8\,\text{mS}$$

$$\boxed{i_4(t) = 1 + (0.8 - 1)e^{-\frac{t}{\tau}} = 1 - 0.2e^{-263t}\ \text{mA}}$$

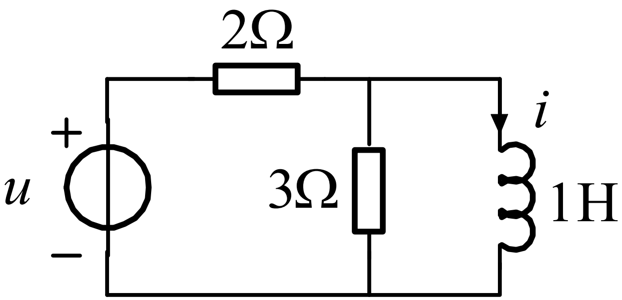
$$u_C(t) = 5 + (6-5)e^{-\frac{t}{\tau}} = 5 + e^{-263t}$$

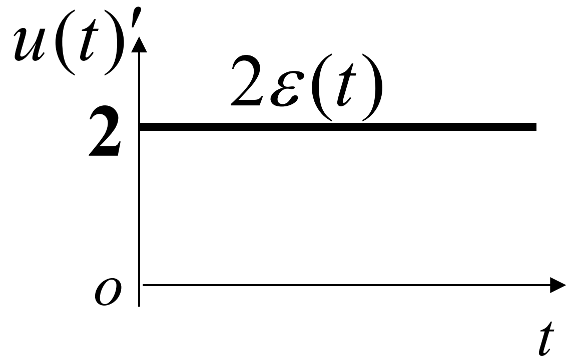
$$i_4(t) = 1 + (0.8-1)e^{-\frac{t}{\tau}} = 1 - 0.2e^{-263t}\ \mathrm{mA}$$

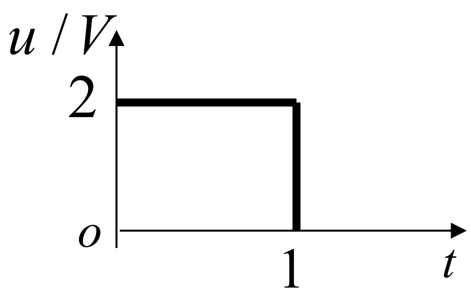


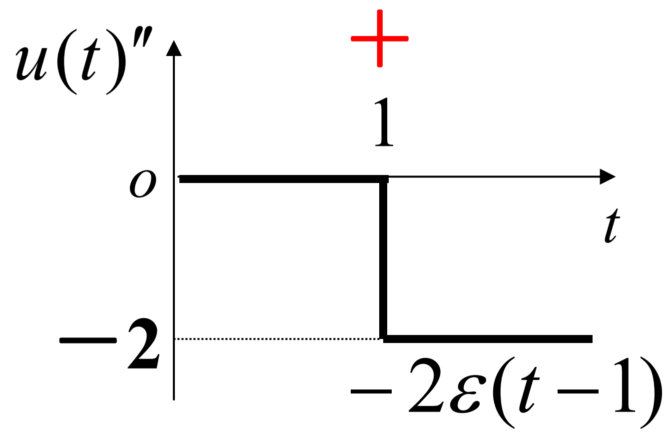


【例22】已知RL电路的输入波形如图所示，求其阶跃响应 $i(t)$。

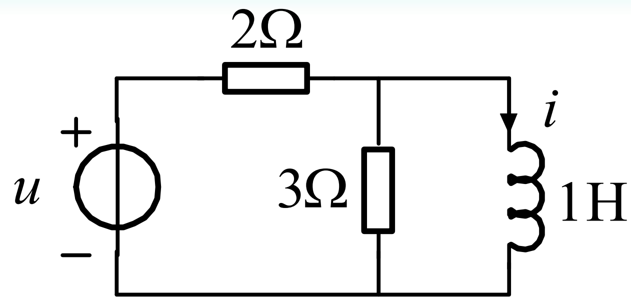






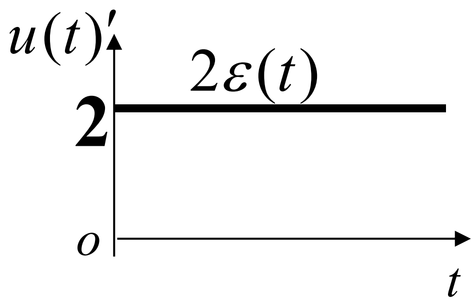


$$u(t)=u(t)'+u(t)''=2\varepsilon(t)-2\varepsilon(t-1)$$



$$u(t)=u(t)'+u(t)''=2\varepsilon(t)-2\varepsilon(t-1)$$

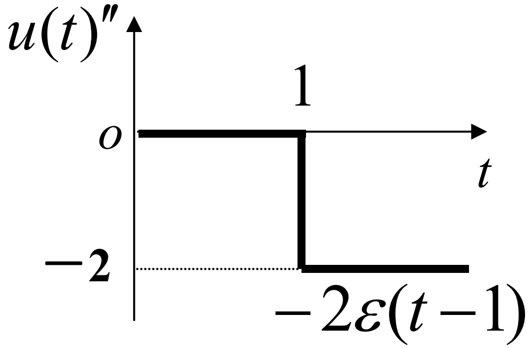
$$i'(t)=i'(\infty)(1-e^{-\frac{t}{\tau}})$$



$$i'(\infty)=\frac{2}{2}=1\mathrm{A}\quad \tau=\frac{L}{R_{eq}}=\frac{1}{2//3}=\frac{5}{6}S$$

其阶跃响应

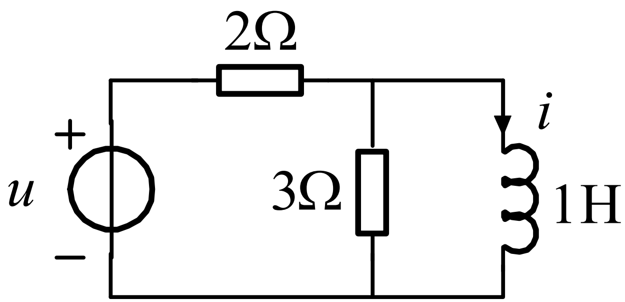
$$i'(t)=(1-e^{-\frac{6}{5}t})\varepsilon(t)\ \mathrm{A}$$

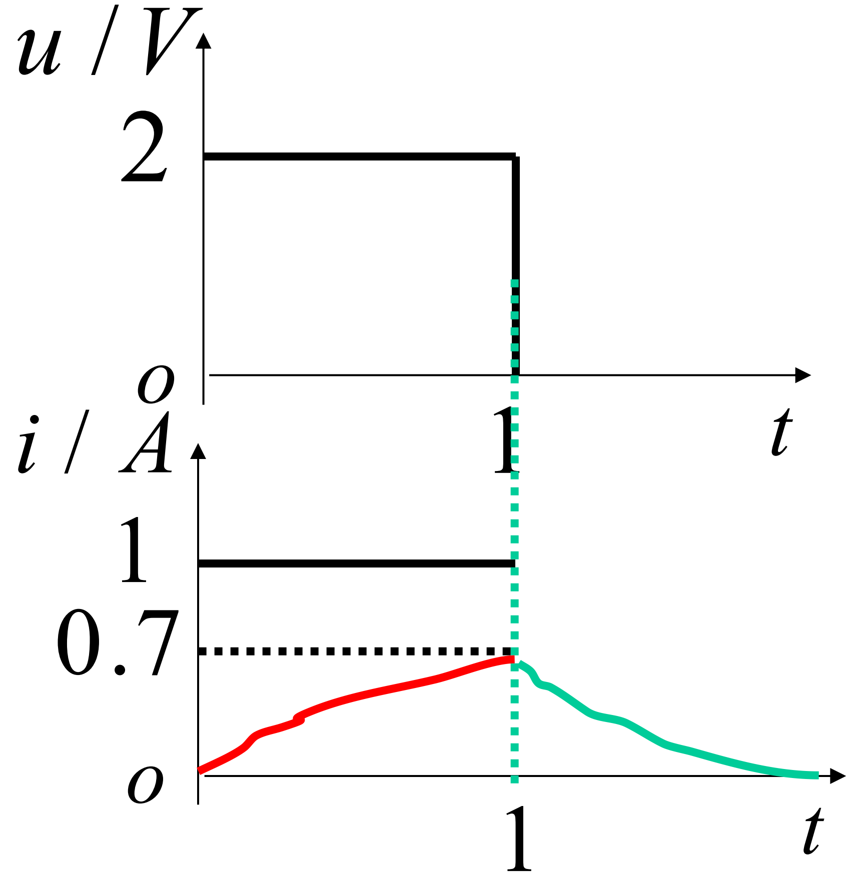


$$i''(t)=-(1-e^{-\frac{6}{5}(t-1)})\varepsilon(t-1)\mathrm{A}$$

$$\boxed{i\ (t)=i'(t)+i''(t)=\left((1-e^{-\frac{6}{5}t})\varepsilon(t)-(1-e^{-\frac{6}{5}(t-1)})\varepsilon(t-1)\right)A}$$

此题也可以用三要素求解：





1、$0 < t < 1$

$$i_L(\infty) = \frac{2}{2} = 1\,\mathrm{A} \quad \tau = 0.83\ s$$

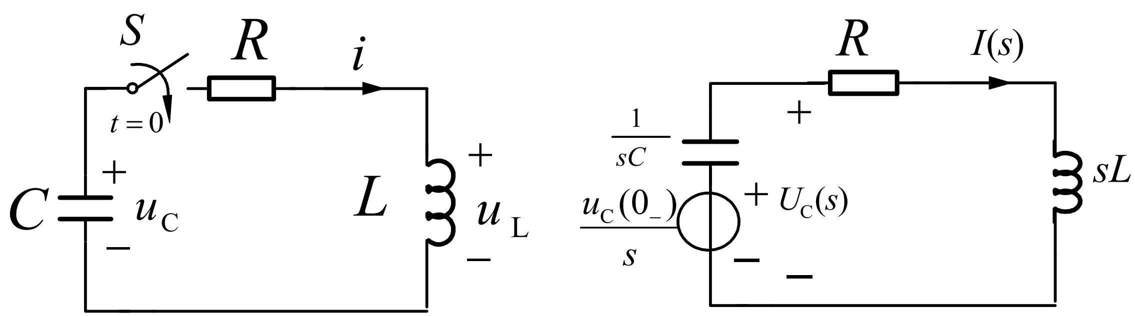
$$i_L(t) = 1 - e^{-1.2t}\ \mathrm{A}$$

2、$t > 1$

$$i_L(1_+) = 1 - e^{-1.2\times 1} = 1 - \frac{1}{e^{1.2}} = 1 - \frac{1}{3.32} = 0.7\ A$$

$$i_L(\infty) = 0 \qquad i_L(t) = 0.7e^{-1.2(t-1)}\ \mathrm{A}$$

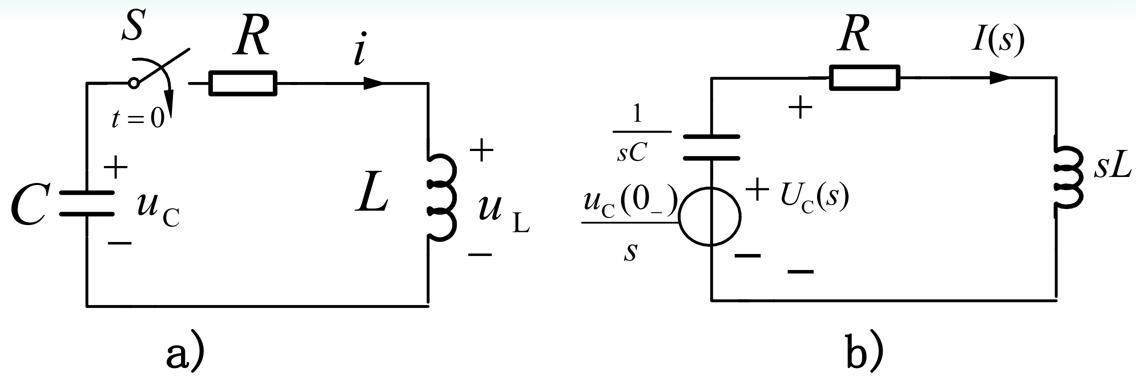
【例23】在图a中，已知 $R=500\Omega$， $L=800\text{mH}$，$C=40\mu\text{F}$，$u_C(0_-)=10\text{V}$，$i_L(0_-)=0$。试求 $t>0$ 后的 $u_C(t)$。



a)　　　　　　　　　　b)

【解】复频域电路如图b所示。由图b得电流、电容电压的象函数为

$$I(s)=\frac{\dfrac{u_C(0_-)}{s}}{R+sL+\dfrac{1}{sC}}=\frac{\dfrac{10}{s}}{500+0.8s+\dfrac{25\times10^3}{s}}=\frac{10}{500s+0.8s^2+25\times10^3}$$



$$I(s)=\frac{\frac{u_C(0_-)}{s}}{R+sL+\frac{1}{sC}}=\frac{\frac{10}{s}}{500+0.8s+\frac{25\times10^3}{s}}=\frac{10}{500s+0.8s^2+25\times10^3}$$

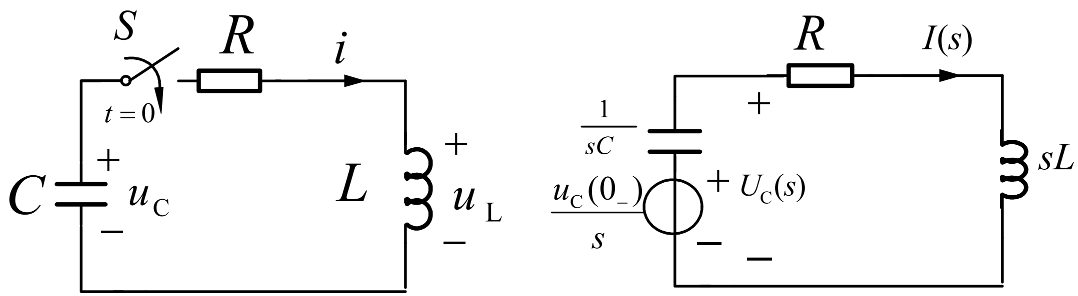
$$U_C(s)=I(s)(R+sL)=\frac{10}{500s+0.8s^2+25\times10^3}\times(500+0.8s)=\frac{5000+8s}{0.8s^2+500s+25\times10^3}$$

令 $0.8s^2+500s+25\times10^3=0$

求出根 $p_1=-54.8$，$p_2=-570.2$。

$U_C(s)$ 可以展开为 $U_C(s)=\frac{K_1}{s-p_1}+\frac{K_2}{s-p_2}=\frac{K_1}{s+54.8}+\frac{K_2}{s+570.2}$

由公式 $\boxed{K_i=\left.\frac{F_1(s)}{F_2'(s)}\right|_{s\to p_i}=\frac{F_1(p_i)}{F_2'(p_i)}}$ 求出待定系数 K_1、K_2，即



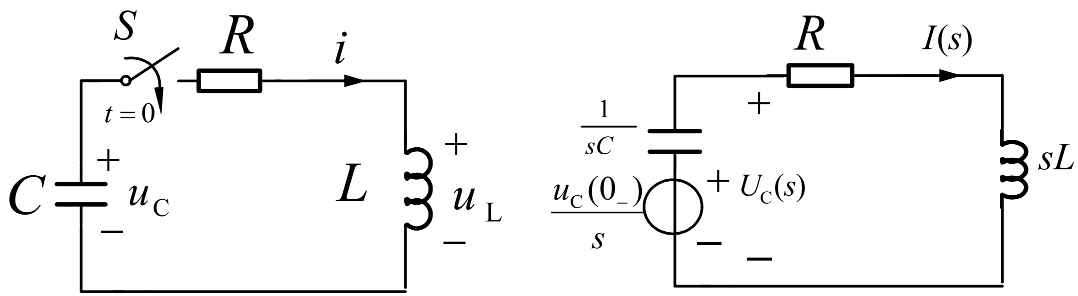
$$U_C(s)=\frac{K_1}{s-p_1}+\frac{K_2}{s-p_2}=\frac{K_1}{s+54.8}+\frac{K_2}{s+570.2}$$

$$K_1=\left.\frac{5000+8s}{(0.8s^2+500s+25\times10^3)'}\right|_{s=-54.8}$$

$$=\left.\frac{5000+8s}{1.6s+500}\right|_{s=-54.8}=\frac{4561.6}{412.32}=11.06\text{V}$$

$$K_2=\left.\frac{5000+8s}{(0.8s^2+500s+25\times10^3)'}\right|_{s=-570.2}$$

$$=\left.\frac{5000+8s}{1.6s+500}\right|_{s=-570.2}=\frac{438.4}{-412.32}=-1.06\text{V}$$



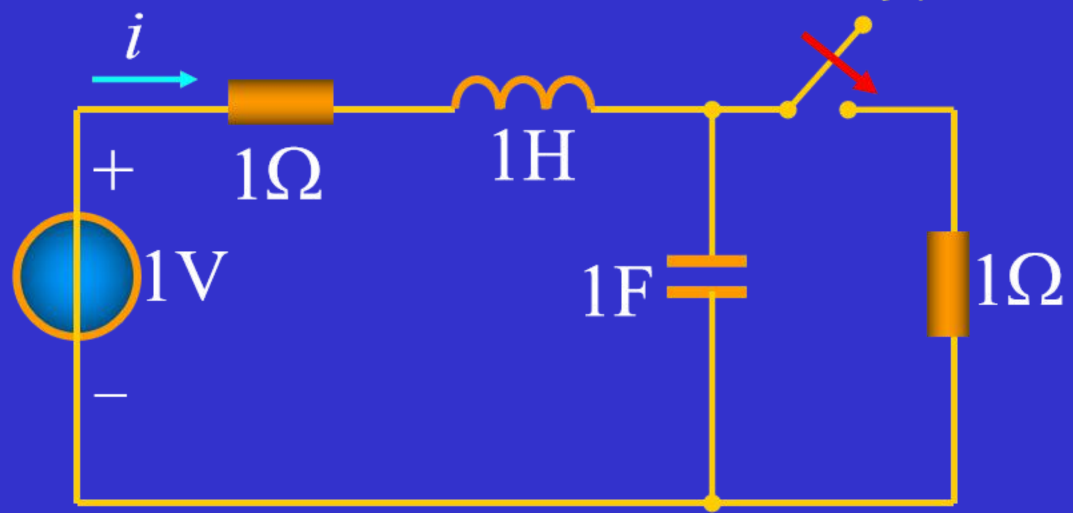
$$U_C(s)=\frac{K_1}{s-p_1}+\frac{K_2}{s-p_2}=\frac{K_1}{s+54.8}+\frac{K_2}{s+570.2}$$

$$K_1=11.06\text{V} \qquad K_2=-1.06\text{V}$$

作拉氏反变换，得

$$u_C(t)=(11.06e^{-54.8t}-1.06e^{-570.2t})\text{V}$$

例24 电路原处于稳态，$t=0$ 时开关闭合，试用运算法求电流 $i(t)$。

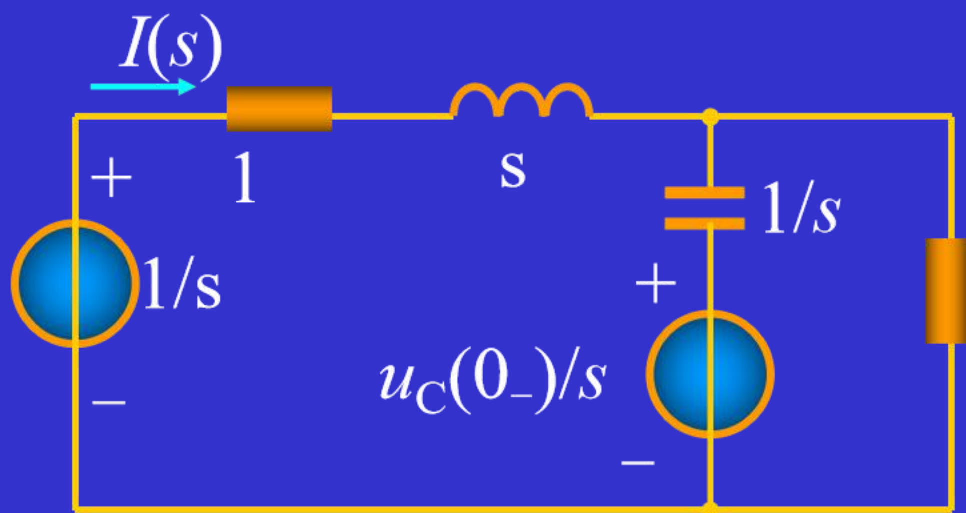


解 (1) 计算初值

$$u_c(0_-)=1\text{V}$$

$$i_L(0_-)=0$$

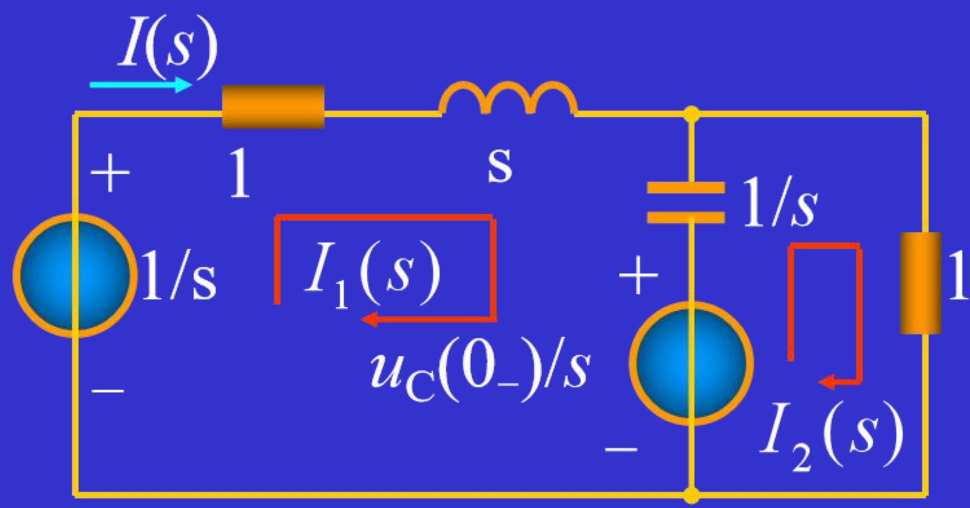
(2) 画运算电路



$$\text{s}L=1s$$

$$\frac{1}{sC}=\frac{1}{\text{s}\times 1}=\frac{1}{\text{s}}$$

（3）应用回路电流法



$$\begin{cases}(1+s+\dfrac{1}{s})I_1(s)-\dfrac{1}{s}I_2(\mathrm{s})=\dfrac{1}{s}-\dfrac{u_C(0\_)}{s}=0\\ -\dfrac{1}{s}I_1(\mathrm{s})+(1+\dfrac{1}{s})I_2(\mathrm{s})=\dfrac{u_C(0\_)}{s}=\dfrac{1}{s}\end{cases}$$

$$I_1(s)=I(s)=\frac{1}{s(s^2+2s+2)}$$

$$I_1(s)=I(s)=\frac{1}{s(s^2+2s+2)}$$

(4) 反变换求原函数

$\mathrm{D}(s)=0$有3个根： $p_1=0$，$p_2=-1+\mathrm{j}$，$p_3=-1-\mathrm{j}$

$$I(s)=\frac{K_1}{s}+\frac{K_2}{s+1-\mathrm{j}}+\frac{K_3}{(\mathrm{s}+1+\mathrm{j})}$$

$$K_1=I(\mathrm{s})s\Big|_{s=0}=\frac{1}{2}$$

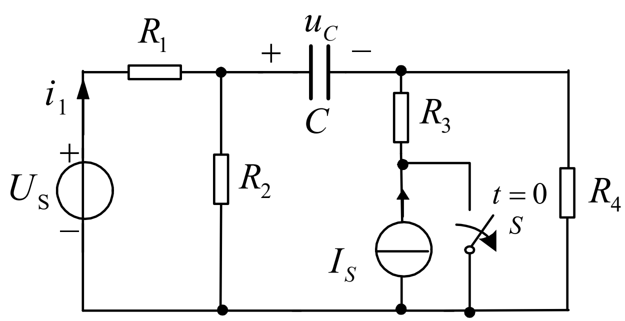
$$K_2=I(s)(s+1-\mathrm{j})\Big|_{s=-1+\mathrm{j}}=-\frac{1}{2(1+\mathrm{j})}$$

$$K_3=I(s)(s+1+\mathrm{j})\Big|_{s=-1-\mathrm{j}}=-\frac{1}{2(1-\mathrm{j})}$$

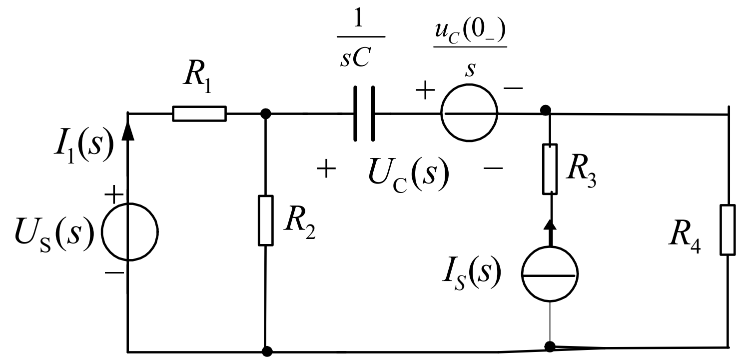
$$I(s)=\frac{1/2}{s}-\frac{1/2(1+\mathrm{j})}{s+1-\mathrm{j}}-\frac{1/2(1-\mathrm{j})}{(s+1+\mathrm{j})}$$

$$\mathrm{L}^{-1}I(s)=i(t)=\frac{1}{2}(1-\mathrm{e}^{-t}\cos t-\mathrm{e}^{-t}\sin t)$$

【例25】在图a中，已知 $U_S = 9\text{V}$, $I_S = 1\text{A}$, $R_1 = 3\Omega$, $R_2 = 6\Omega$, $R_3 = 1\Omega$, $R_4 = 2\Omega$, $C = 10\mu\text{F}$ 。试求开关S打开后的 $u_C(t)$。



a)



b)

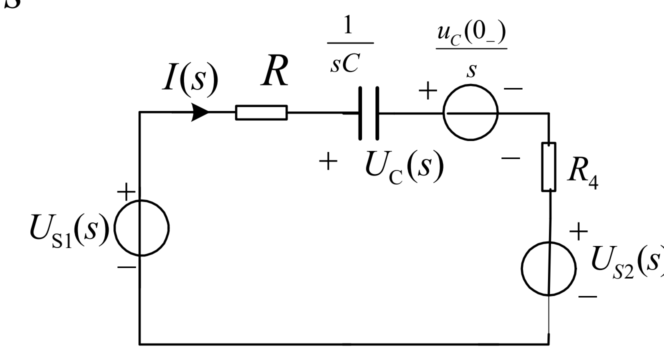
【解】复频域电路如图b所示。

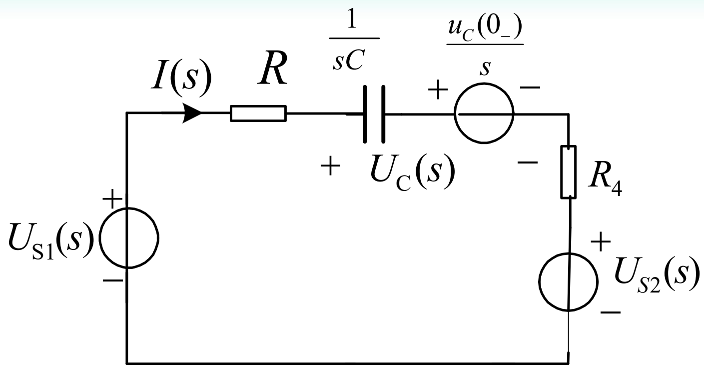
外加激励的象函数和电容电压的原始值为

$$U_S(s) = \frac{U_S}{s} = \frac{9\text{V}}{s} \qquad I_S(s) = \frac{I_S}{s} = \frac{1\text{A}}{s} \qquad u_C(0_-) = \frac{R_2}{R_1 + R_2} U_S = (\frac{6}{3+6} \times 9)\text{V} = 6\text{V}$$

用电源的等效变换方法将图b等效成图C所示的电路，则

c)





$$U_{S1}(s)=\frac{U_S(s)}{R_1}\frac{R_1R_2}{(R_1+R_2)}=\frac{R_2}{R_1+R_2}U_S(s)=\frac{6}{3+6}\times\frac{9\text{V}}{s}=\frac{6\text{V}}{s}$$

$$U_{S2}(s)=I_S(s)R_4=\frac{1\text{A}}{s}\times 2=\frac{2\text{V}}{s}$$

$$U_{S2}(s)=I_S(s)R_4=\frac{1\text{A}}{s}\times 2=\frac{2\text{V}}{s} \qquad R=R_1//R_2=2\Omega$$

$$I(s)=\frac{U_{S1}(s)-\dfrac{u_C(0_-)}{s}-U_{S2}(s)}{R+\dfrac{1}{sC}+R_4}=\frac{\dfrac{6}{s}-\dfrac{6}{s}-\dfrac{2}{s}}{2+\dfrac{10^5}{s}+2}=\frac{-\dfrac{2}{s}}{4+\dfrac{10^5}{s}}=\frac{-2}{4s+10^5}$$

$u_C(t)$ 的象函数为

$$U_C(s)=I(s)\frac{1}{sC}+\frac{u_C(0_-)}{s}=\frac{-2}{4s+10^5}\times\frac{10^5}{s}+\frac{6}{s}=-2\times\frac{\dfrac{1}{4}\times10^5}{s(s+\dfrac{1}{4}\times10^5)}+\frac{6}{s}$$

作拉氏反变换，得

$$\boxed{u_C(t)=-2(1-e^{-\frac{1}{4}\times10^5t})+6=(-2+2e^{-25\times10^3t}+6)\text{V}=(4+2e^{-25\times10^3t})\text{V}}$$

练习结束