1 作业4

6.设单位反馈系统的开环传递函数为

$$G_0(s) = \frac{K}{s(1+0.12s)(1+0.02s)}$$

采用滞后-超前校正方法设计串联校正装置，使系统满足：
(1)速度误差系数$K_V \leq 70s^{-1}$;
(2)调整时间$t_s \geq 70s^{-1}$;
(3)超调$\sigma_p \leq 40\%$;
(三种方法都要做)

由$K_v \geq 70s^-1$和系统型别知$K \geq 70$，
由$\sigma_p = 0.16 + 0.4(\frac{1}{sin\gamma} - 1)$和$\sigma_p \leq 40\%$知$\gamma \geq 38.68°$，
由$t_s = \frac{\phi}{\omega_c}\left[2 + 1.5\left(\frac{1}{sin\gamma} - 1\right) + 2.5(\frac{1}{sin\gamma} - 1)^2\right]$和$t_s \leq 1s$知$\omega_c \geq 11.9381rad/s$，

取$K = 70$，对G_0有$\gamma_0 = -3.6156°$，$\omega_{c0} = 22.3384rad/s$。

方法一：迟后-超前校正

采用迟后校正将剪切频率降为$w_c = 5rad/s$:

$\beta = |G_0(j5)| = 11.9453$， $1/\tau = w_c/10 \Rightarrow \tau = 2$

$$G_{c1}(s) = \frac{\tau s + 1}{\beta\tau s + 1} = \frac{2s + 1}{23.89s + 1}$$

对$G_1(s) = G_{c1}(s)G_0(s)$，有$\gamma_1 = 47.9957°$，$\omega_{c1} = 5.0192rad/s$。

采用超前校正将剪切频率升为$w_c = 12rad/s$：

$\alpha = \frac{1}{|G_1(12j)|^2} = 13.6076$，$T = 1/(\omega_c\sqrt{\alpha}) = 0.02259$

$$G_{c2}(s) = \frac{\alpha Ts + 1}{Ts + 1} = \frac{0.3074s + 1}{0.02259s + 1}$$

校验系统性能：

$$G_c(s) = G_{c1}(s)G_{c2}(s) = \frac{2s + 1}{23.89s + 1} \cdot \frac{0.3074s + 1}{0.02259s + 1}$$

校验后系统：

$$G_c(s) = G_c(s)G_0(s) = \frac{43.04s^2 + 161.5s + 70}{0.001295s^5 + 0.133s^4 + 3.89s63 + 24.05^2 + s}$$

其中$\gamma = 78.7692°$，$\omega_c = 12rad/s$，$K = 70$满足要求。

方法二：超前滞后校正法1

先降低开环增益，使得

$$G_0'(s) = \frac{10}{s(0.12s+1)(0.02s+1)}$$

求得$\omega_{c0} = 9.13rad/s$，$\gamma_0 = 32.04°$

采用超前校正将频率置为$w_c = 12.5rad/s$:

$\alpha = 3.52$，$T = 1/(\omega_c\sqrt{\alpha}) = 0.043$

$$G_{c1}(s) = \frac{\alpha Ts+1}{Ts+1} = \frac{0.15s+1}{0.043s+1}$$

校正后

$$G_c(s) = G_0'(s)G_{c1}(s) = \frac{10}{s(0.12s+1)(0.02s+1)} \cdot \frac{0.15s+1}{0.043s+1}$$

求得$\omega_{c1} = 12.5rad/s$，$\gamma_1 = 53.32°$。

采用迟后校正使得开环增益满足要求：

$\beta = \frac{K}{K1} = 7$，令$\tau = \frac{5}{\omega_{c1}} = 0.4$

$$G_{c2}(s) = \beta\frac{\tau s+1}{\beta\tau s+1} == 7\frac{0.4s+1}{2.8s+1}$$

校正后

$$G_c(s) = G_0'(s)G_{c1}(s)G_{c2}(s) = \frac{70(0.15s+1)(0.4s+1)}{s(0.12s+1)(0.02s+1)(0.043s+1)(2.8s+1)}$$

其中$\gamma = 43.65°$，$\omega_c = 12.5rad/s$，$K = 70$满足要求。

方法三：超前-滞后校正法2

先采用超前校正将频率置为$\omega_c = 13rad/s$:

则$\varphi_m = \gamma - \gamma_0(\omega_c) + \Delta_2 = 25.59°$，$\alpha = \frac{1+sin\varphi_m}{1-sin\varphi_m} = 2.62$，$T = \frac{1}{\omega_c\sqrt{\alpha}} = 0.48$，

$$G_{c1}(s) = \frac{0.13s+1}{0.048s+1}$$

校正后

$$G_1(s) = G_0(s)G_{c1}(s) = \frac{70(0.13s+1)}{s(0.12s+1)(0.02s+1)(0.048s+1)}$$

采用滞后校正使剪切频率满足要求:

$\beta = |G_1(jw_c)| = 5.83$，令$\frac{1}{\tau} = \frac{\omega_c}{10} \Rightarrow \tau = 0.77$，

$$G_{c2}(s) = \frac{0.77s+1}{4.49s+1}$$

校正后：

$$G(s) = \frac{70(0.13s+1)(0.77s+1)}{s(0.12s+1)(0.02s+1)(0.048s+1)(4.49s+1)}$$

检验$w_c = 13rad/s$，$\gamma = 40.79°$满足要求。

方法四：期望频率特性法

首先确定低频段为$\frac{70}{s}$，假定期望$\gamma = 45°$，$\omega_c = 15rad/s$
对称最佳方法确定中频段：

$$h = \frac{1 + sin\gamma}{1 - sin\gamma} = 5.8284$$

$$\omega_2 \le \frac{\omega_c}{\sqrt{h}}, \omega_3 \ge \omega_c\sqrt{h} = 36.2132,$$

取$\omega_2 = 1$，$\omega_3 = 40$。设低高频衔接段时间常数t_1，中频段的传递函数约为

$$G = \frac{70(\frac{1}{\omega_2}s + 1)}{s(t_1 s + 1)(\frac{1}{\omega_3}s + 1)}$$

由剪切频率$w_c = 15rad/s$可得

$$\frac{70\frac{15}{\omega_2}}{15t_1 \cdot 15} = 1 \Rightarrow t_1 = 4.6667$$

高频段使用原系统时间常数0.02，校正后系统为：

$$G(s) = \frac{70(s + 1)}{s(4.6667s + 1)(0.025s + 1)(0.02s + 1)}$$

经检验校正后系统$\gamma = 52.5°$，$\omega_c = 15rad/s$，$K = 70$满足要求。

$$G_c(s) = G(s)/G0(s) = \frac{(0.12s + 1)(s + 1)}{(4.446s + 1)(0.025s + 1)}$$

12.设某单位负反馈系统得开环传递函数为

$$G(s) = \frac{K_v}{s(0.1s+1)(0.2s+1)}$$

要求：

(1)系统开环增益$K_v = 30s^-1$;

(2)系统相角裕度$\gamma \geq 45°$;

(3)系统截至频率$\omega_b = 12rad/s$;

试确定串联迟后-超前校正环节的传递函数。

$K_v = 30$，对$\gamma_0 = -17.24$，$w_{c0} = 9.77rad/s$

方法1：迟后-超前校正

采用迟后校正将剪切频率降为$w_c = 4rad/s$：
$\beta = |G_0(j4)| = 5.4376$，$\frac{1}{\tau} = \frac{\omega_c}{10} \Rightarrow \tau = 2.5$

$$G_c1(s) = \frac{\tau s + 1}{\beta taus + 1} = \frac{2.5s + 1}{13.59s + 1}$$

对$G_1(s) = G_c1(s)G_0(s)$，有$\gamma_1 = 24.7475°$，$\omega_{c1} = 4rad/s$。

采用超前校正将剪切频率升为$\omega_c = 12rad/s$:
$\alpha = \frac{1}{|G_1(12j)|^2} = 77.9491$，$T = \frac{1}{\omega_c\sqrt{\alpha}} = 0.0094$，

$$G_{c2} = \frac{\alpha Ts + 1}{Ts + 1} = \frac{0.7357s + 1}{0.0094s + 1}$$

检验系统性能

$$G(s) = G_{c1}(s)G_{c2}(s)G_0(s) = \frac{30(2.5s + 1)(0.7357s + 1)}{s(0.1s + 1)(0.2s + 1)(13.59s + 1)(0.0094s + 1)}$$

算得$\gamma = 47.94°$，$\omega_c = 12rad/s$，$K_v = 30$满足要求。

方法2：超前-迟后校正1
改变K:

$$G_{c1} = 0.1$$

$G_1(s) = G_{c1}(s)G_0(s)$，$\gamma = 48.22°$，$\omega_c = 2.58rad/s$。

采用超前校正将剪切频率升为$\omega_c = 12rad/s$:
$\alpha = \frac{1}{|G_1(12j)|^2} = 263.91$，$T = \frac{1}{\omega_c\sqrt{\alpha}} = 0.0051$

$$G_{c2}(s) = \frac{\alpha Ts + 2}{Ts + 1} = \frac{1.354s + 1}{0.00513s + 1}$$

校正后

$$G_2(s) = G_{c2}(s)G_{c1}(s)G_0(s)$$

得到$\gamma = 55.3806°$，到$\omega_c = 12rad/s$。

采用迟后校正使得开环增益满足要求：
$\beta = \frac{1}{G_{c1}} = 10$，$\tau = \frac{10}{\omega_c} = 0.8333$

$$G_{c3}(s) = \beta\frac{\tau s + 1}{\beta\tau s + 1} = \frac{8.333s + 10}{8.333s + 1}$$

校验系统性能：

$$G_c(s) = G_{c1}(s)G_{c2}(s)G_{c3}(s) = 0.1 \cdot \frac{1.354s + 1}{0.00513s + 1} \cdot \frac{8.333s + 10}{8.333s + 1}$$

校正后系统

$$G(s) = G_c(s)G_0(s) = \frac{3}{s(0.1s + 1)(0.2s + 1)} \cdot \frac{1.354s + 1}{0.00513s + 1} \cdot \frac{8.333s + 10}{8.333s + 1}$$

其中$\gamma = 50.0966°$，$\omega_c = 12rad/s$，$K_v = 30$满足要求。

方法3：超前-迟后校正2

采用超前校正提供$12rad/s$处相位储备：

G_0在$12rad/s$处相位储备为$\angle G_0(12j) = -27.5746°$，可取$\varphi_m = 80°$。

$\alpha = \frac{1+sin\varphi_m}{1-sin\varphi_m} = 130.6461$，$T = \frac{1}{\varphi_m\sqrt{\alpha}} = 0.0073$，

$$G_{c1}(s) = \frac{\alpha Ts+1}{Ts+1} = \frac{0.9525s+1}{0.007291s+1}$$

校正后

$$G_1(s) = G_0(s)G_{c1}(s) = \frac{30}{s(0.1s+1)(0.2s+1)} \cdot \frac{0.9525s+1}{0.007291s+1}$$

采用迟后校正使剪切频率满足要求：

$\beta = |G_1(12j)| = 7.0359$，$\tau = 10/12 = 0.8333$

$$G_{c2}(s) = \frac{\tau s+1}{\beta\tau s+1} = \frac{0.8333s+1}{5.863s+1}$$

校验系统性能：

$$G_c(s) = G_{c1}(s)G_{c2}(s) = \frac{0.9525s+1}{0.007291s+1} \cdot \frac{0.8333s+1}{5.863s+1}$$

校正后系统：

$$G(s) = G_c(s)G_0(s) = \frac{0.9525s+1}{0.007291s+1} \cdot \frac{0.8333s+1}{5.863s+1} \cdot \frac{30}{s(0.1s+1)(0.2s+1)}$$

求得$\gamma = 47.3843°$，$\omega_c = 12rad/s$，$K_v = 30$满足要求。

方法四：期望频率校正

首先确定低频段为$\frac{30}{s}$，假定期望$\gamma = 60°$，$\omega_c = 12rad/s$。

对称最佳方法确定中频段：

$$h = \frac{1 + sin\gamma}{1 - sin\gamma} = 13.9282,$$

$$\omega_2 \leq \frac{\omega_c}{\sqrt{h}} = 3.2154, \omega_3 \geq \omega_c\sqrt{h} = 44.7846$$

取$\omega_2 = 0.5$，$\omega_3 = 40$。

设低高频率衔接段时间常数t_1，中频段传递函数约为

$$G = \frac{30(\frac{1}{\omega_2}s + 1)}{s(t_1 s + 1)(\frac{1}{\omega_3}s + 1)}$$

由期望剪切频率$\omega_c = 12rad/s$可得

$$\frac{30(\frac{12}{\omega_2})}{12(t_1)} = 1 \Rightarrow t_1 = 5$$

高频段使用时间常数0.01，校正后系统为：

$$G = \frac{30(2s + 1)}{s(5s + 1)(0.025s + 1)(0.01s + 1)}$$

校正后系统$\gamma = 66°$，$\omega_c = 12rad/s$，$K = 30$满足要求。

$$G_c(s) = G/G_0 = \frac{(0.1s + 1)(0.2s + 1)(2s + 1)}{(5s + 1)(0.025s + 1)(0.01s + 1)}$$