

HW-2 190410102 自动化1班 方尧

2-1(1) $\min S = 5x_1 + 3x_2$

$$\begin{cases} -2x_1 + x_2 - 4x_3 \geq -4 \\ -x_1 - x_2 - 2x_3 \geq -5 \\ 2x_1 - x_2 + x_3 \geq 1 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

(D) $\max z = -4\lambda_1 - 5\lambda_2 + \lambda_3$

$$\Rightarrow \begin{cases} -2\lambda_1 - \lambda_2 + 2\lambda_3 \leq 5 \\ \lambda_1 - \lambda_2 - \lambda_3 \leq 3 \\ -4\lambda_1 - 2\lambda_2 + \lambda_3 \leq 0 \\ \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0 \end{cases}$$

(2) (D) $\min z = 10\lambda_1 + 10\lambda_2$

$$\begin{cases} \lambda_1 + 2\lambda_2 \geq 4 \\ 2\lambda_1 + 3\lambda_2 \geq 7 \\ \lambda_1 + 3\lambda_2 \geq 2 \\ \lambda_1 \geq 0, \lambda_2 \geq 0 \end{cases}$$

(3) (D) $\max z = 3\lambda_1 + 5\lambda_2$

$$\begin{cases} \lambda_1 + 2\lambda_2 \leq 2 \\ 2\lambda_1 + \lambda_2 \leq 1 \\ 2\lambda_1 + 3\lambda_2 = 4 \\ \lambda_1 \geq 0, \lambda_2 \geq 0 \end{cases}$$

2-2(1)

$$\min S = x_1 + 2x_2 + 3x_3$$

$$\text{S.t.} \begin{cases} -x_1 + x_2 - x_3 + x_4 = -4 \\ x_1 + x_2 + 2x_3 + x_5 = 8 \\ -x_2 + x_3 + x_6 = -2 \\ x_i \geq 0, i=1, \dots, 6 \end{cases}$$

		x_1	x_2	x_3	x_4	x_5	x_6
	0	1	2	3	0	0	0
$\leftarrow x_4$	-4	-1	1	-1	1	0	0
x_5	8	1	1	2	0	1	0
x_6	-2	0	-1	1	0	0	1

$\varepsilon = \min\{\frac{1}{1}, \frac{3}{1}\} = 1$

		x_1	x_2	x_3	x_4	x_5	x_6
	-4	0	3	2	1	0	0
x_1	4	1	-1	1	-1	0	0
x_5	4	0	2	1	1	1	0
$\leftarrow x_6$	-2	0	-1	1	0	0	1

		x_1	x_2	x_3	x_4	x_5	x_6
	-10	0	0	5	1	0	3
x_1	6	1	0	0	-1	0	-1
x_5	0	0	0	3	1	1	2
x_2	2	0	1	-1	0	0	-1

最优解 $x^* = (6, 2, 0)^T$, $S = 10$

2-2(2)

$$\min S = 3x_1 + 2x_2 + x_3 + 4x_4$$

$$\text{s.t.} \begin{cases} -2x_1 - 4x_2 - 5x_3 - x_4 + x_5 = 0 \\ -3x_1 + x_2 - 7x_3 + 2x_4 + x_6 = -2 \\ -5x_1 - 2x_2 - x_3 - 6x_4 + x_7 = -15 \\ x_j \geq 0, j=1 \dots 7 \end{cases}$$

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	0	3	2	1	4	0	0	0
x_5	0	-2	-4	-5	-1	1	0	0
x_6	-2	-3	1	-7	2	0	1	0
x_7	-15	-5	-2	-1	-6	0	0	1

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	-9	0	$\frac{4}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	0	0	$\frac{3}{5}$
x_5	6	0	$-\frac{14}{5}$	$-\frac{23}{5}$	$\frac{7}{5}$	1	0	$-\frac{2}{5}$
x_6	7	0	$\frac{11}{5}$	$\frac{32}{5}$	2	0	1	0
x_1	3	1	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{6}{5}$	0	0	$-\frac{1}{5}$

最优解 $X = (3, 0, 0)^T$, $S = 9$

2-3 先求解最优解

$\min S = 5X_1 - 5X_2 - 13X_3$			X_1	X_2	X_3	X_4	X_5
s.t. $\begin{cases} -X_1 + X_2 + 3X_3 + X_4 = 20 \\ 12X_1 + 4X_2 + 10X_3 + X_5 = 90 \\ X_j \geq 0, j=1, \dots, 5 \end{cases}$		0	5	-5	-13	0	0
		X_4 20	-1	1	3	1	0
		X_5 90	12	4	10	0	1

		X_1	X_2	X_3	X_4	X_5
	100	0	0	2	5	0
X_2	20	-1	1	3	1	0
X_5	10	16	0	-2	-4	1

最优解 $X = (0, 20, 0)^T$
 $S = -100$

(1) 右端常数20变为30

$$B^{-1}b = \begin{pmatrix} 1 & 0 \\ -4 & 1 \end{pmatrix} \cdot \begin{pmatrix} 30 \\ 90 \end{pmatrix} = \begin{pmatrix} 30 \\ -30 \end{pmatrix} \neq 0 \quad \text{最优基改变}$$

		X_1	X_2	X_3	X_4	X_5
	150	0	0	2	5	0
X_2	30	-1	1	3	1	0
X_5	-30	16	0	-2	-4	1

 \Rightarrow

		X_1	X_2	X_3	X_4	X_5
	120	16	0	0	1	1
X_2	-15	23	1	0	-5	$\frac{3}{2}$
X_3	15	-8	0	1	2	$-\frac{1}{2}$

 \Rightarrow

		X_1	X_2	X_3	X_4	X_5
	117	$\frac{103}{5}$	$\frac{1}{5}$	0	0	$\frac{13}{10}$
X_4	3	$\frac{23}{5}$	$-\frac{1}{5}$	0	1	$-\frac{3}{10}$
X_3	9	$\frac{6}{5}$	$\frac{2}{5}$	1	0	$\frac{1}{10}$

最优解 $X = (0, 0, 9)^T$
 $S = -117$

(2) 右端项由90变为70

$$B^{-1}b = \begin{pmatrix} 1 & 0 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} 20 \\ 70 \end{pmatrix} = \begin{pmatrix} 20 \\ -10 \end{pmatrix} \neq 0 \quad \text{最优基改变}$$

		x_1	x_2	x_3	x_4	x_5			x_1	x_2	x_3	x_4	x_5
	100	0	0	2	5	0	\Rightarrow	90	16	0	0	1	1
x_2	20	-1	1	3	1	0		x_2	5	23	1	0	$-\frac{3}{2}$
x_5	-10	16	0	-2	-4	1		x_3	5	-8	0	1	$-\frac{1}{2}$

最优解 $x = (0, 5, 5)^T$, $S = -90$

(3) x_3 系数由-13变为-8

为非基变量 $\Delta C_3 = 5$

$$y'_{03} = \Delta C_3 + y_{03} = 5 + 2 = 7 > 0, \text{故不发生改变}$$

(4) x_1 由 $(-1, 12)^T$ 变为 $(0, 5)^T$, 非基列

$$y'_{01} = C_1 - C_B B^{-1} \bar{P}_j = 5 - (-5, 0) \begin{pmatrix} 1 & 0 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 5 \end{pmatrix} = 5 > 0$$

故不发生改变

(5) 增加一约束条件 $2x_1 + 3x_2 + 5x_3 \leq 50$

$x = (0, 20, 0)^T$ 代入不满足, 故最优基改变

		x_1	x_2	x_3	x_4	x_5	x_6
	100	0	0	2	5	0	0
x_2	20	-1	1	3	1	0	0
x_5	10	16	0	-2	-4	1	0
x_6	50	2	3	5	0	0	1

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↓

		x_1	x_2	x_3	x_4	x_5	x_6
	100	0	0	2	5	0	0
x_2	20	-1	1	3	1	0	0
x_5	10	16	0	-2	-4	1	0
x_6	10	5	0	-4	-3	0	1

↓

		x_1	x_2	x_3	x_4	x_5	x_6
	95	$\frac{5}{2}$	0	0	$\frac{7}{2}$	0	$\frac{1}{2}$
x_2	$\frac{25}{2}$	$\frac{11}{4}$	1	0	$-\frac{5}{4}$	0	$\frac{3}{4}$
x_5	10	16	0	-2	-4	1	0
x_3	$\frac{5}{2}$	$-\frac{5}{4}$	0	1	$\frac{3}{4}$	0	$-\frac{1}{4}$

故最优解为 $x = (0, \frac{25}{2}, \frac{5}{2})^T$

$$S = -95$$

(16) 增加一变量 x_4 , $C_4 = -3$, $a_{14} = 2$, $a_{24} = 6$

$$P_{0.4} = C_4 - C_B B^{-1} P_4$$

$$= -3 - (-5, 0) \begin{pmatrix} 1 & 0 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 6 \end{pmatrix} = 7 > 0$$

故不发生改变

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2-5(1)

销地		B ₁	B ₂	B ₃	B ₄	产量	
产地	A ₁	3 $0 \geq 0$	5 $0 \geq 0$	9 $0 \geq 0$	1 3^*	3	$u_1=0$
	A ₂	4 $0 < 0$	2 1^*	3 5^*	8 1^*	7	$u_2=7$
	A ₃	2 2^*	7 0	6 0	4 2^*	4	$u_3=3$
销量		2	1	5	6		

$v_1=-1 \quad v_2=-5 \quad v_3=-4 \quad v_4=1$

$\theta = \min\{1, 2\} = 1$, x_{21} 进基, x_{24} 出基

销地		B ₁	B ₂	B ₃	B ₄	产量	
产地	A ₁	3 ≥ 0	5 ≥ 0	9 ≥ 0	1 3^*	3	$u_1=0$
	A ₂	4 1^*	2 1^*	3 5^*	8 20	7	$u_2=5$
	A ₃	2 1^*	7 20	6 ≥ 0	4 3^*	4	$u_3=3$
销量		2	1	5	6		

$v_1=-1 \quad v_2=-3 \quad v_3=-2 \quad v_4=1$

故最优解 $X = (0, 0, 0, 3, 1, 1, 5, 0, 1, 0, 0, 3)^T$, $S = 38$

2-5(3)

销地		B ₁	B ₂	B ₃	B ₄	B ₅	产量	
产地	A ₁	10 $0 \geq 0$	20 0^*	5 4^*	9 6^*	10 $0 \geq 0$	12	$u_1=0$
	A ₂	2 $0 \geq 0$	10 4^*	8 $0 \geq 0$	30 $0 \geq 0$	6 $0 \geq 0$	4	$u_2=70$
	A ₃	1 3^*	20 1^*	7 $0 \geq 0$	10 $0 \geq 0$	4 3^*	8	$u_3=0$
销量		3	5	4	6	3		

$v_1=1 \quad v_2=20 \quad v_3=5 \quad v_4=9 \quad v_5=4$

故已为最优解

最优解为 $X = (0, 0, 4, 6, 0, 0, 4, 0, 0, 0, 3, 1, 0, 0, 3)^T$

$S = 149$