

1-1 设生产A、B各 x 千克, y 千克; 产值为 W

建立模型如下:

$$\begin{aligned} \max W &= 7x + 12y \\ \begin{cases} 9x + 4y &\leq 360 \\ 4x + 5y &\leq 200 \\ 3x + 10y &\leq 300 \end{cases} \end{aligned}$$

解得 $x = 20 \text{ kg}$, $y = 24 \text{ kg}$, 产值最高为 428 万元

1-3 设甲、乙、丙种植水稻、大豆、玉米分别为 x_1, x_2, x_3 ;

y_1, y_2, y_3 ; z_1, z_2, z_3 . 总产量设为 W

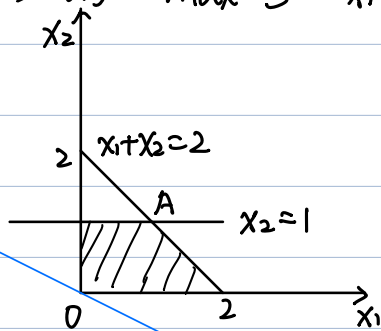
建立模型如下:

$$\begin{aligned} \max W &= 700x_1 + 300x_2 + 900x_3 + 600y_1 + 350y_2 + 800y_3 \\ &\quad + 600z_1 + 250z_2 + 700z_3 \\ \begin{cases} x_1 + x_2 + x_3 &= 200 \\ y_1 + y_2 + y_3 &= 400 \\ z_1 + z_2 + z_3 &= 600 \\ 700x_1 + 600y_1 + 600z_1 &\geq 13 \times 10^4 \\ 300x_2 + 350y_2 + 250z_2 &\geq 4 \times 10^4 \\ 900x_3 + 800y_3 + 700z_3 &\geq 25 \times 10^4 \end{cases} \end{aligned}$$

解得 $x_1 = x_2 = y_1 = z_1 = 0$, $x_3 = 200$, $y_2 = 114.29$, $y_3 = 285.71$, $z_1 = 216.67$, $z_3 = 383.33$

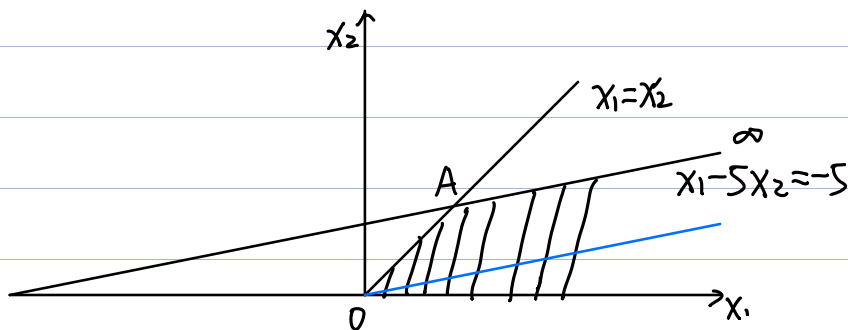
总产量最高为 846904.8 kg

1-5 (1) $\max S = x_1 + 2x_2$



可知, 在A点可得最大值, 此时 $x_1 = x_2 = 1$, 最大值 $S = 3$, $X = (1, 1)^T$

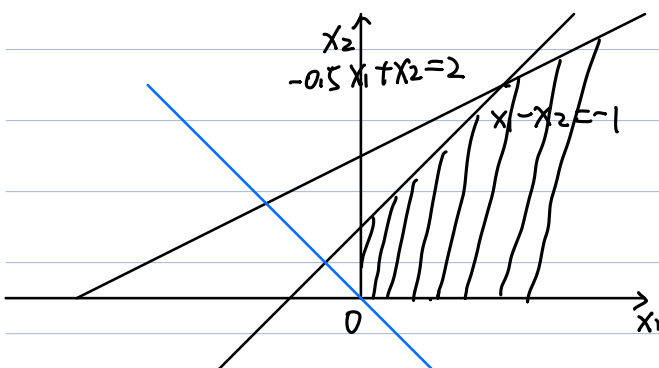
1-5 (2) $\min S = 2x_1 - 10x_2$



可知, 在A点 (直线 $x_1 - 5x_2 = -5$ 上) 取到最小值, $A(\frac{5}{4}, \frac{5}{4})$

不妨代入 $(0, 1)$, $\min S = -10$, $X = \{(x_1, x_2) \mid x_1 - 5x_2 = -5, x_1 \geq \frac{5}{4}\}$

1-5 (6) $\max S = 2x_1 + 2x_2$



由图可知, S 最大可取至无穷大, 无解。

1-6(1)

$$\min -S = -x_1 - x_2 - x_3$$

$$\begin{cases} -x_1 - 2x_3 + x_4 = 5 \\ 2x_1 - 3x_2 + x_3 + x_5 = 3 \\ 2x_1 - 5x_2 + 6x_3 + x_6 = 5 \end{cases} \quad x_i \geq 0$$

			x_1	x_2	x_3	x_4	x_5	x_6
	-13		-1	-1	-1	0	0	0
0	x_4	5	-1	0	-2	1	0	0
0	x_5	3	2	-3	1	0	1	0
0	x_6	5	2	-5	6	0	0	1

由于 x_2 列 $y_{02} < 0$, $y_{i2} \leq 0$ 故无有限最优解

1-6(2)

			x_1	x_2	x_3	x_4	x_5	x_6
	15		8	0	3	0	3	0
-3	x_2	5	3	1	2	0	1	0
0	x_6	2	1	0	1	0	2	1
0	x_4	6	1	0	2	1	2	0

由于非基变量检验数 $y_{0q} > 0$, 故已为最优解

$$\min S = -15 \quad \text{此时 } X = (0 \ 5 \ 0 \ 6 \ 0 \ 2)^T$$

1-6(3)

表一			x_1	x_2	x_3	x_4	x_5	x_6
	-13		2	0	-6	0	0	-7
	x_4	9	1	0	0	1	0	6
	x_2	2	3	1	-4	0	0	2
	x_5	6	1	0	3	0	1	2

表二			x_1	x_2	x_3	x_4	x_5	x_6
		-1	4	0	0	0	2	-3
	x_4	9	1	0	0	1	0	6
	x_2	10	$\frac{13}{3}$	1	0	0	$\frac{4}{3}$	$\frac{14}{3}$
	x_3	2	$\frac{1}{3}$	0	1	0	$\frac{1}{3}$	$\frac{2}{3}$

表三			x_1	x_2	x_3	x_4	x_5	x_6
		$\frac{7}{2}$	$\frac{9}{2}$	0	0	$\frac{1}{2}$	2	0
	x_6	$\frac{3}{2}$	$\frac{1}{8}$	0	0	$\frac{1}{8}$	0	1
	x_2	3	$\frac{32}{9}$	1	0	$-\frac{7}{9}$	$\frac{4}{3}$	0
	x_3	1	$\frac{2}{9}$	0	1	$-\frac{1}{9}$	$\frac{1}{3}$	0

此时, 取得最优解 $x = (0, 3, 1, 0, 0, \frac{3}{2})^T$, $\min S = -\frac{7}{2}$

1-7(1) 阶段一

构造辅助(LP)

$$\min S = y_1 + y_2$$

$$\begin{cases} x_1 + 2x_2 + 3x_3 + y_1 = 6 \\ 4x_1 + 5x_2 - 6x_3 + y_2 = 6 \\ x_i \geq 0 \quad i=1 \dots 6 \end{cases}$$

$$4x_1 + 5x_2 - 6x_3 + y_2 = 6$$

$$x_i \geq 0 \quad i=1 \dots 6$$

表一			x_1	x_2	x_3	y_1	y_2
		-12	-5	-7	3	0	0
	y_1	6	1	2	3	1	0
	y_2	6	4	5	-6	0	1

表二			x_1	x_2	x_3	y_1	y_2
		$-\frac{9}{2}$	0	$-\frac{3}{4}$	$-\frac{9}{2}$	0	$\frac{5}{4}$
	y_1	$\frac{9}{2}$	0	$\frac{3}{4}$	$\frac{9}{2}$	1	$-\frac{1}{4}$
	x_1	$\frac{3}{2}$	1	$\frac{5}{4}$	$-\frac{3}{2}$	0	$\frac{1}{4}$

表二			x_1	x_2	x_3	y_1	y_2
		-3.6	0.6	0	-5.4	0	1.4
	y_1	3.6	-0.6	0	5.4	1	-0.4
	x_2	1.2	0.8	1	-1.2	0	0.2

		x_1	x_2	x_3	y_1	y_2
	0	0	0	0	1	1
x_3	$\frac{2}{3}$	$-\frac{1}{9}$	0	1	$\frac{5}{27}$	$-\frac{2}{27}$
x_2	2	$\frac{2}{3}$	1	0	$\frac{2}{9}$	$\frac{1}{9}$

第二阶段

		x_1	x_2	x_3
	$\frac{4}{3}$	$\frac{16}{9}$	0	0
x_3	$\frac{2}{3}$	$-\frac{1}{9}$	0	1
x_2	2	$\frac{2}{3}$	1	0

知 $X = (0, 2, \frac{2}{3})^T$ 时为最优解
 $\min S = -\frac{4}{3}$

1-7 (3)

$$\min S = 4x_1 + 5x_2 + 0x_3$$

$$\begin{cases} x_1 + x_2 + x_3 = 5 \\ -6x_1 + 10x_2 + 5x_3 + x_4 = 20 \\ 5x_1 - 3x_2 + x_3 - x_5 = 15 \end{cases}$$

辅助(LP) \Rightarrow

$$\min S = y_1 + y_2 + y_3$$

$$\begin{cases} x_1 + x_2 + x_3 + y_1 = 5 \\ -6x_1 + 10x_2 + 5x_3 + x_4 + y_2 = 20 \\ 5x_1 - 3x_2 + x_3 - x_5 + y_3 = 15 \end{cases}$$

		x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3
	-40	0	-8	-7	-1	1	0	0	0
y_1	5	1	1	1	0	0	1	0	0
y_2	20	-6	10	5	1	0	0	1	0
y_3	15	5	-3	1	0	-1	0	0	1
		x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3
	-24	-4.8	0	-3	-0.2	1	0	0.8	0
y_1	3	1.6	0	0.5	-0.1	0	1	-0.1	0
x_2	2	-0.6	1	0.5	0.1	0	0	0.1	0
y_3	21	3.2	0	2.5	0.3	-1	0	0.3	1

		x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3
	-15	0	0	-1.5	-0.5	1	3	0.5	0
x_1	$\frac{15}{8}$	1	0	$\frac{5}{16}$	$-\frac{1}{16}$	0	$\frac{5}{8}$	$-\frac{1}{16}$	0
x_2	3.125	0	1	0.6875	$\frac{1}{16}$	0	0.375	$\frac{1}{16}$	0
y_3	15	0	0	1.5	0.5	-1	-2	0.5	1

		x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3
	-8.18	0	2.18	0	-0.364	1	3.82	0.182	0
x_1	0.45	1	-0.45	0	-0.091	0	0.45	-0.091	0
x_3	4.54	0	1.45	1	-0.091	0	0.54	0.091	0
y_3	8.18	0	-2.18	0	0.364	-1	-3.82	0.0364	1

		x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3
	0	0	0	0	0	0	1	1	1
x_1	2.5	1	-1	0	0	-0.25	-0.25	0	0.25
x_3	2.5	0	2	1	0	-0.25	1.25	0	-0.25
x_4	21.5	0	-6	0	1	-2.75	-7.75	1	2.75

第3阶段 $\min S = 4x_1 + 5x_2 + 6x_3$

		x_1	x_2	x_3	x_4	x_5
	-25	0	-3	0	0	-0.5
x_1	2.5	1	-1	0	0	-0.25
x_3	2.5	0	2	1	0	-0.25
x_4	21.5	0	-6	0	1	-2.75

		x_1	x_2	x_3	x_4	x_5
	-21.25	0	0	1.5	0	-0.125
x_1	3.75	1	0	0.5	0	-0.125
x_2	1.25	0	1	0.5	0	0.125
x_4	30	0	0	3	1.0	-2

		x_1	x_2	x_3	x_4	x_5
	-20	0	1	2	0	0
x_1	5	1	1	1	0	0
x_5	10	0	8	4	0	1
x_4	50	0	16	11	1	0

知 $y_{0i} > 0 \quad i=1 \dots 5$, 故最优解为 $\min S = 20$

对应 $x^* = (5, 0, 0, 50, 10)^T$

即 $\min S = 20, x = (5, 0, 0)^T$