# Homework 2

April 14, 2021

1. For each subspace in (a)-(d), (1) find a basis, and (2) state the dimension.

   (a)
$$\left\{ \begin{bmatrix} s-2t \\ s+t \\ 3t \end{bmatrix} : s, t \text{ in } \mathbb{R} \right\}$$

   (b)
$$\left\{ \begin{bmatrix} 2c \\ a-b \\ b-3c \\ a+2b \end{bmatrix} : a, b, c \text{ in } \mathbb{R} \right\}$$

   (c)
$$\left\{ \begin{bmatrix} a-4b-2c \\ 2a+5b-4c \\ -a+2c \\ -3a+7b+6c \end{bmatrix} : a, b, c \text{ in } \mathbb{R} \right\}$$

   (d)
$$\{(a, b, c, d) : a - 3b + c = 0\}$$

2. Let
$$\mathbf{x}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

   (a) Show that $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ are linearly dependent.

   (b) Show that $\mathbf{x}_1$ and $\mathbf{x}_2$ are linearly independent.

   (c) What is the dimension of Span$(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$?

   (d) Give a geometric description of Span$(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$.

3. Let
$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ a \end{bmatrix}$$

If dim(span$(x_1, x_2, x_3)) = 2$, compute $a$.

4. $V$ is a nonzero finite-dimensional vector spaces, and the vectors listed belong to $V$. Mark each statement True or False. Justify each answer (Prove it if True or give an anti-example if False).

    a. If there exists a set $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ that spans $V$, then dim $V \leq p$.
    b. If there exists a linearly independent set $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ in $V$, then then dim $V \geq p$.
    c. If dim $V = p$, then there exists a spanning set of $p + 1$ vectors in $V$.
    d. If there exists a linearly dependent set $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ in $V$, then dim $V \leq p$.
    e. If every set of $p$ elements in $V$ fails to span $V$, then dim $V > p$.
    f. If $p \geq 2$ and dim $V = p$, then every set of $p - 1$ nonzero vectors is linearly independent.

5. Without computing $A$, find bases for its four fundamental subspaces:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 9 & 8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$