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信号分析与处理 HW-8 自动化1924班 1904101002 方老

2-9 (1) 先求 $h(t)$，$\lambda+3=0 \Rightarrow \lambda=-3$

设 $h(t)=A_1e^{-3t}u(t)+a\delta(t)$ 代入方程

$$(A_1e^{-3t}+3a)\delta(t)+a\frac{d\delta(t)}{dt}=2\frac{d\delta(t)}{dt} \Rightarrow \begin{cases} A_1=-6 \\ a=2 \end{cases}$$

故 $h(t)=2\delta(t)-6e^{-3t}u(t)$

$$g(t)=\int_0^t h(\tau)d\tau=2e^{-3t}u(t)$$

(2) 特征方程 $\lambda^2+\lambda+1=0$. $\lambda_1=\frac{-1+\sqrt{3}j}{2}$，$\lambda_2=\frac{-1-\sqrt{3}j}{2}$.

设 $h(t)=(A_1e^{\lambda_1t}+A_2e^{\lambda_2t})u(t)$，$e(t)=\delta(t)$

$$\frac{d}{dt}h(t)=[A_1\lambda_1u(t)+A_1\delta(t)]e^{\lambda_1t}+[A_2\lambda_2u(t)+A_2\delta(t)]e^{\lambda_2t}$$

$$\frac{d^2}{dt^2}h(t)=[A_1\lambda_1^2u(t)+2A_1\lambda_1\delta(t)+A_1\delta'(t)]e^{\lambda_1t}+[A_2\lambda_2^2u(t)+2A_2\lambda_2\delta(t)+A_2\delta'(t)]e^{\lambda_2t}$$

代入 $\frac{d^2}{dt^2}h(t)+\frac{d}{dt}h(t)+h(t)=\delta'(t)+\delta(t)$，对应系数相等.

$$\begin{cases} A_1+A_2+2A_1\lambda_1+2A_2\lambda_2=1 \\ A_1+A_2=1 \end{cases} \Rightarrow \begin{cases} A_1=\frac{\sqrt{3}-j}{2\sqrt{3}} \\ A_2=\frac{\sqrt{3}+j}{2\sqrt{3}} \end{cases}$$

$$h(t)=[A_1e^{\lambda_1t}+A_1e^{\lambda_2t}]u(t)=\frac{\sqrt{3}-j}{2\sqrt{3}}e^{-\frac{1}{2}t}(\cos\frac{\sqrt{3}}{2}t+j\sin\frac{\sqrt{3}}{2}t)$$

$$+\frac{\sqrt{3}+j}{2\sqrt{3}}e^{-\frac{1}{2}t}(\cos\frac{\sqrt{3}}{2}t-j\sin\frac{\sqrt{3}}{2}t)=e^{-\frac{1}{2}t}(\cos\frac{\sqrt{3}}{2}t+\frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t)$$

$$=\frac{2}{\sqrt{3}}e^{-\frac{1}{2}t}(\frac{\sqrt{3}}{2}\cos\frac{\sqrt{3}}{2}t+\frac{1}{2}\sin\frac{\sqrt{3}}{2}t)=\frac{2}{\sqrt{3}}e^{-\frac{1}{2}t}\cos(\frac{\sqrt{3}}{2}t-\frac{\pi}{6})u(t)$$

即 $h(t)=\frac{2}{\sqrt{3}}e^{-\frac{1}{2}t}\cos(\frac{\sqrt{3}}{2}t-\frac{\pi}{6})u(t)$

$$g(t)=\int_{-\infty}^t h(\tau)d\tau=\frac{2}{\sqrt{3}}\int_0^t e^{-\frac{1}{2}\tau}\cos(\frac{\sqrt{3}}{2}\tau-\frac{\pi}{6})d\tau=\frac{-4}{\sqrt{3}}e^{-\frac{1}{2}\tau}\cos(\frac{\sqrt{3}}{2}\tau-\frac{\pi}{6})\Big|_0^t$$

$$+\frac{4}{\sqrt{3}}\int_0^t-\frac{\sqrt{3}}{2}e^{-\frac{1}{2}\tau}\sin(\frac{\sqrt{3}}{2}\tau-\frac{\pi}{6})d\tau=-\frac{4}{\sqrt{3}}[e^{-\frac{t}{2}}\cos(\frac{\sqrt{3}}{2}t-\frac{\pi}{6})-\frac{\sqrt{3}}{2}]$$

$$+4e^{-\frac{1}{2}\tau}\sin(\frac{\sqrt{3}}{2}\tau-\frac{\pi}{6})\Big|_0^t-2\sqrt{3}\int_0^t e^{-\frac{1}{2}\tau}\cos(\frac{\sqrt{3}}{2}\tau-\frac{\pi}{6})d\tau$$

整理得 $g(t)=\frac{1}{4}[-\frac{4}{\sqrt{3}}e^{-\frac{t}{2}}\cos(\frac{\sqrt{3}}{2}t-\frac{\pi}{6})+4e^{-\frac{t}{2}}\sin(\frac{\sqrt{3}}{2}t-\frac{\pi}{6})+4]$

$$=[1+\frac{2}{\sqrt{3}}e^{-\frac{1}{2}t}\sin(\frac{\sqrt{3}}{2}t-\frac{\pi}{3})]u(t)$$

(3) $\lambda+2=0$, $\lambda=-2$. 记 $h(t)=A_1e^{-2t}u(t)+a_1\delta(t)+a_2\delta'(t)$

$h'(t)=A_1e^{-2t}(-2u(t)+\delta(t))+a_1\delta'(t)+a_2\delta''(t)$

代入 $r(t)=h(t)$, $e(t)=\delta(t)$ 得

$$\begin{cases}2a_1+A_1=3\\2a_2+a_1=3\\a_2=1\end{cases}\Rightarrow\begin{cases}A_1=1\\a_1=1\\a_2=1.\end{cases}$$ 故 $h(t)=e^{-2t}u(t)+\delta(t)+\delta'(t)$

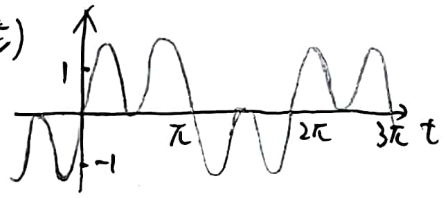
$g(t)=\int_{-\infty}^{t}h(\tau)d\tau=-\frac{1}{2}(e^{-2t}-1)u(t)+u(t)+\delta(t)=(\frac{3}{2}-\frac{1}{2}e^{-2t})u(t)+\delta(t)$

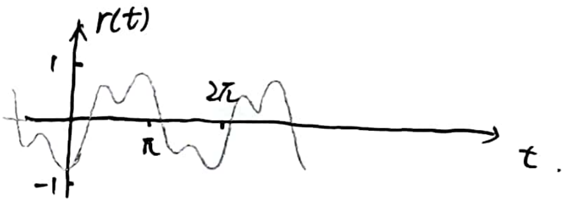
5-2 幅频特性 $|H(j\omega)|=\frac{1}{\sqrt{1+\omega^2}}$. $\varphi(\omega)=-\arctan\omega$

$H_1=|H(j\omega)|=\frac{1}{\sqrt{2}}$, $\varphi_1=-\arctan 1=-\frac{\pi}{4}$

$H_2=\frac{1}{\sqrt{10}}$, $\varphi_2=-\arctan 3$

故 $r(t)=H_1\sin(t+\varphi_1)+H_2\sin(3t+\varphi_2)=\frac{1}{\sqrt{2}}\sin(t-\frac{\pi}{4})+\frac{1}{\sqrt{10}}\sin(3t-\arctan 3)$





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5-9 $R(j\omega)=H(j\omega)E(j\omega)=\tau Sa(\frac{\omega\tau}{2})[u(\omega+\frac{2\pi}{\tau})-u(\omega-\frac{2\pi}{\tau})]$

$r(t)=F^{-1}[R(j\omega)]=\frac{1}{2\pi}\int_{-\infty}^{\infty}R(j\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\int_{-\frac{2\pi}{\tau}}^{\frac{2\pi}{\tau}}\tau Sa(\frac{\omega\tau}{2})e^{j\omega t}d\omega$

令 $x=\frac{\omega\tau}{2}$. $r(t)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\tau Sa(x)e^{j\frac{2x}{\tau}t}\cdot\frac{2}{\tau}\cdot dx=\frac{1}{\pi}\int_{-\pi}^{\pi}Sa(x)[\cos\frac{2x}{\tau}t+j\sin\frac{2x}{\tau}t]dx$

$=\frac{1}{\pi}\int_{-\pi}^{\pi}\frac{\sin x}{x}\cos\frac{2x}{\tau}t\,dx=\frac{1}{\pi}\int_{-\pi}^{\pi}\frac{\sin(x-\frac{2t}{\tau}x)+\sin(x+\frac{2t}{\tau}x)}{2x}dx$

$=\frac{1}{2\pi}\int_{-(1+\frac{2t}{\tau})\pi}^{(1+\frac{2t}{\tau})\pi}Sa(x_1)dx_1+\frac{1}{2\pi}\int_{-(1-\frac{2t}{\tau})\pi}^{(1+\frac{2t}{\tau})\pi}Sa(x_2)dx_2.$

$=\frac{1}{\pi}Si[(1+\frac{2t}{\tau})\pi]+\frac{1}{\pi}Si[(1-\frac{2t}{\tau}\pi)]$

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8-24 (1) 两边求Z变换.

$Y(z)+3z^{-1}Y(z)=X(z)$ （因果系统，初始条件为0）

$Y(z)=\frac{1}{1+3z^{-1}}X(z)$

$x(n)=\delta(n)$时，$X(z)=1$ 故 $H(z)=Y(z)=\frac{1}{1+3z^{-1}}$

$h(n)=\mathcal{Z}^{-1}[H(z)]=(-3)^n u(n)$

(2) 当 $x(n)=(n+n^2)u(n)$

$X(z)=\frac{z}{(z-1)^2}+\frac{z(z+1)}{(z-1)^3}=\frac{2z^2}{(z-1)^3}$

$Y(z)=\frac{X(z)}{1+3z^{-1}}=\frac{2z^3}{(z+3)(z-1)^3}=-\frac{9}{32}\frac{z}{z+3}+\frac{1}{2}\frac{z}{(z-1)^3}+\frac{7}{8}\frac{z}{(z-1)^2}+\frac{9}{32}\frac{z}{(z-1)}$

Z反变换得：

$y(z)=-\frac{9}{32}(-3)^n u(n)+\frac{1}{2}\cdot\frac{n(n-1)}{2!}u(n)+\frac{7}{8}nu(n)+\frac{9}{32}u(n)$

$=\frac{1}{32}[-9\cdot(-3)^n+8n^2+20n+9]u(n)$