Teacher: **Yanjie Li** Assignment Number: **5**

Course: **Linear Algebra in Control Theory** Disclosure date: June 12, 2021

Problem 1

Define $T \in \mathcal{L}(\mathbf{F}^3)$ by

$$T(z_1, z_2, z_3) = (2z_2, 0, 5z_3)$$

Find all eigenvalues and eigenvectors of T.

Problem 2

Define $T : \mathcal{P}(\mathbf{R}) \to \mathcal{P}(\mathbf{R})$ by $Tp = p'$. Find all eigenvalues and eigenvectors of T.

Problem 3

Suppose $T \in \mathcal{L}(V)$. Suppose $S \in \mathcal{L}(V)$ is invertible. (a) Prove that T and $S^{-1}TS$ have the same eigenvalues. (b) What is the relationship between the eigenvectors of T and the eigenvectors of $S^{-1}TS$?

Problem 4

Find all eigenvalues and eigenvectors of the backward shift operator $T \in \mathcal{L}(\mathbf{F}^\infty)$ defined by

$$T(z_1, z_2, z_3, \ldots) = (z_2, z_3, \ldots).$$

Problem 5

If A is a matrix with $m \times n$ dimension, please show that A^TA and AA^T have the same nonzero eigenvalues.

Pay Attention

References

[1] Axler, S. (1997). Linear algebra done right. Springer Science Business Media.

[2] Lay, D. C. . Linear algebra and its applications. Academic Press.

[3] Leon, S. J., de Pillis, L., De Pillis, L. G. (2015). Linear algebra with applications (pp. 337-350). Boston: Pearson.