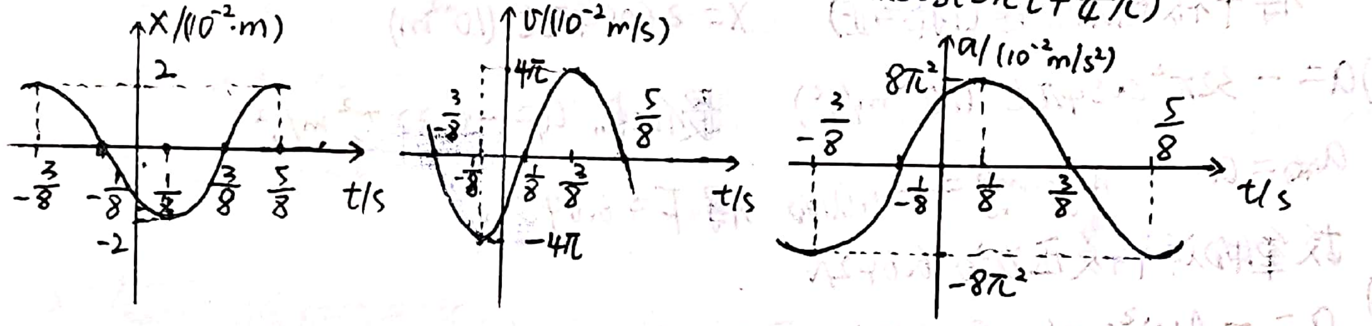
190410102 方尧

第九章

9-1 (B); 9-2 (D); 9-3 (B); 9-4 (C); 9-5 (C); 9-6 (D)

9-7 解: $\omega = \frac{2\pi}{T} = 2\pi\ rad/s$ 故运动方程为 $x = 2.0\times10^{-2}\times\cos(2\pi t+\frac{3}{4}\pi)$ (SI)

$v = -4\pi\times10^{-2}\times\sin(2\pi t+\frac{3}{4}\pi)$ $a = -8\pi^2\times10^{-2}\times\cos(2\pi t+\frac{3}{4}\pi)$



9-8 解: (1) 振幅为 0.1 m; 频率 $f = \frac{\omega}{2\pi} = 10\ Hz$; 角频率 $\omega = 20\pi\ rad/s$

周期 $T = \frac{2\pi}{\omega} = 0.1s$; 初相 $\varphi = \frac{\pi}{4}$

(2) $x|_{t=2s} = 0.071\ m$ $v = -2\pi\sin(20\pi t+\frac{\pi}{4})$ 得 $v|_{t=2s} = -\sqrt{2}\pi\ m/s$
方向沿x轴负向.

$a = -40\pi^2\cos(20\pi t+\frac{\pi}{4})$ 得 $a|_{t=2s} = -20\sqrt{2}\pi^2\ m/s^2$ 方向沿x轴负向

9-14 解: $\omega = \frac{2\pi}{T} = 4\pi\ rad/s$ 由旋转矢量图易知 $\varphi = 0, \frac{\pi}{2}, \frac{\pi}{3}, \frac{4}{3}\pi$

(1) $x = 2\cos 4\pi t\ (10^{-2}m)$; (2) $x = 2\cos(4\pi t+\frac{\pi}{2})\ (10^{-2}m)$;

(3) $x = 2\cos(4\pi t+\frac{\pi}{3})\ (10^{-2}m)$; (4) $x = 2\cos(4\pi t+\frac{4}{3}\pi)\ (10^{-2}m)$;

9-15 解: 有 $mg = k\cdot\Delta L$ 得 $\omega^2 = \frac{k}{m} = \frac{g}{\Delta L}$ 故 $\omega = 10\ rad/s$

(1) 由旋转矢量图 $\varphi = \pi$, $A = 8\times10^{-2}m$ 故 $x = 8\cos(10t+\pi)\ (10^{-2}m)$

(2) 有 $v = -A\omega\sin(\omega t+\varphi)$

此时 $x = A\cos(\omega t+\varphi) = 0$ $v = -0.6m/s$ 得 $A = 0.06m$ $\varphi = \frac{\pi}{2}$

故 $x = 0.06\cos(10t+\frac{\pi}{2})\ (m)$

9-16 解: 设 $x = A\cos(\omega t+\varphi)$, $A = 0.1$ $x(0) = 0.05$, $v(0) > 0$

$v = -A\omega\sin(\omega t+\varphi)$ 得 $\varphi = -\frac{\pi}{3}$ 又 $x(4) = 0$ $\frac{T}{2} = \frac{\pi}{\omega} > 4$ 得 $\omega = \frac{5}{24}\pi$

(1) 运动方程为 $x = 0.1\cos(\frac{5}{24}\pi t - \frac{\pi}{3})\ (m)$ (2) P点相位为0.

(3) 令 $\omega t+\varphi = 0$ 得 $t = 1.6s$

9-17 解 (1) $t_1 = \frac{T}{4}$ 即 $\frac{1}{4}$ (2) 令 $x = A\cos(\omega t+\varphi) = \frac{A}{2}$ 得 $\omega t+\varphi = \frac{\pi}{3} + 2k\pi$

令 $x = A\cos(\omega t+\varphi) = 0$ $\omega t+\varphi = 2k\pi + \frac{\pi}{2}$ 故 $\omega\Delta t = \frac{\pi}{6}$ $\Delta t = \frac{\pi}{6\omega} = \frac{T}{12}$

即 $\frac{1}{12}$ (3) $\frac{1}{4} - \frac{1}{12} = \frac{1}{6}$ 即为周期 $\frac{1}{6}$

9-18 得平板运动方程(向下为正) $x = 2\cos 4\pi t\ (10^{-2}m)$

(1) $a = -32\pi^2 \cos 4\pi t\ (10^{-2} m/s^2)$ 最低点 $a_1 = -0.32\pi^2 m/s^2$

$a_{物} = a$ $mg - F = ma_{物}$ 得 $F = 12.96N$

故重物对平板压力为 6.642N

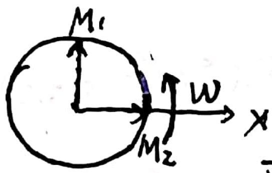
(2) $a = -A\omega^2 \cos t$ 知在最高点最易脱落 $a_{高} = A\omega^2$.

当无压力即最高点加速度大于 g 时重物会脱离平板

即 $A\omega^2 > g$ $\omega = \frac{2\pi}{0.5} = 4\pi\ rad/s$ 得 $A > 0.0621\ m$

(3) $A\omega^2 > g$ $A = 0.02m$, $\omega = 2\pi f$ 得 $f > 3.52\ Hz$

9-20 解:



第一个质点记为M_1, 质点2记为M_2

知质点2较质点1落后 $\frac{\pi}{2}$ 相位

故质点2运动方程 $x_2 = A\cos(\omega t+\varphi - \frac{\pi}{2})$

相位差为 $\frac{\pi}{2}$

9-21 设 $x = A\cos(\omega t+\varphi)$ $v = -A\omega\sin(\omega t+\varphi)$

由题 $A=2$, $A\omega=3$, $v(0) = 1.5\ cm\cdot s^{-1}$ $a(0) > 0$ 则 $\omega = 1.5$, $\varphi = -\frac{5}{6}\pi$

故 $x = 2\cos(\frac{3}{2}t - \frac{5}{6}\pi)$.

(1) 振动周期 $T = \frac{2\pi}{\omega} = \frac{4}{3}\pi\ s = 4.19\ s$

(2) $a_{max} = v_{max}\cdot\omega = 3\times1.5 = 4.5 cm/s^2$

(3) 运动方程为 $x = 2\cos(\frac{3}{2}t - \frac{5}{6}\pi)$ (cm)

9-28 解 (1) 最大加速度 $a_{max} = A\omega^2 = 4\ m/s^2$ 得 $\omega = 20\ rad/s$

周期 $T = \frac{2\pi}{\omega} = \frac{\pi}{10}\ s = 0.314\ s$

(2) 知 $k = m\omega^2 = 40\ N/m$ 总能量 $E_总 = \frac{1}{2}kA^2 = 2\times10^{-3}\ J$

平衡位置 $E_k = E_总 = 2\times10^{-3}\ J$

(3) $v = -\omega A\sin(\omega t+\varphi)$ $E_k = \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\varphi) = \frac{1}{2}E_总$

得 $\omega t+\varphi = \frac{\pi}{4}+\frac{k\pi}{2}$ 此时 $x = \pm\frac{\sqrt{2}}{2}A$ 即在 $\pm 7.071\times10^{-3}\ m$ 处动能势能相等

(4) 由 $E_p = \frac{1}{2}kx^2$ $\because x = \frac{A}{2}$ 时 $E_p = \frac{1}{4}E_总$、$E_k = \frac{3}{4}E_总$

9-30 解 (1) 振动角频率 $\omega = 8\pi\ rad/s$，周期 $T = \frac{2\pi}{\omega} = 0.25\ s$

振幅 $A = 0.5\ cm$，初相 $\varphi = \frac{\pi}{3}$

(2) 有 $k = m\omega^2 = \frac{16}{25}\pi^2\ N/m$ $E = \frac{1}{2}kA^2 = 7.90\times10^{-5}\ J$

(3) $E_p = \frac{1}{2}kx^2 = \frac{k}{8}\cos^2(8\pi t+\frac{\pi}{3})\times10^{-4}$

$\overline{E_p} = \frac{1}{T}\int_0^T E_p\,dt = \frac{1}{16}k\times10^{-4}\ J = \frac{E}{2} = 3.95\times10^{-5}\ J$

$E_p+E_k = E$ 故 $\overline{E_k} = E-\overline{E_p} = 3.95\times10^{-5}\ J$