

$$9-5 \quad X_{p1}(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-jk\Omega_0 n} \quad \Omega_0 = \frac{2\pi}{N}$$

$$X_{p2}(k) = \frac{1}{2N} \sum_{n=0}^{2N-1} x(n) e^{-jk\Omega_0' n} \quad \Omega_0' = \frac{2\pi}{2N}$$

$$X_{p2}(k) = \frac{1}{2N} \sum_{n=0}^{2N-1} x(n) e^{-jk \cdot \frac{\pi}{N} n} = \frac{1}{2N} \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{k}{2}\Omega_0 n} + \frac{1}{2N} \sum_{n=N}^{2N-1} x(n) \cdot e^{-jk\frac{\pi}{N} n}$$

$$= \frac{1}{2} X_{p1}\left(\frac{k}{2}\right) + \frac{1}{2N} \sum_{n=0}^{N-1} x(n) e^{-jk\frac{\pi}{N}(n+N)}$$

$$= \frac{1}{2} X_{p1}\left(\frac{k}{2}\right) + \frac{1}{2N} \sum_{n=0}^{N-1} x(n) \cdot e^{-jk\frac{\pi}{N} n} \cdot (-1)^k$$

$$= \frac{1}{2} [1 + (-1)^k] X_{p1}\left(\frac{k}{2}\right) = \begin{cases} X_{p1}\left(\frac{k}{2}\right) & ; k \text{ 为偶数} \\ 0 & , k \text{ 为奇数} \end{cases}$$

$$9-8 \quad X(k) = W_N^{nk} x(n) = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^3 \\ W_N^0 & W_N^2 & W_N^4 & W_N^6 \\ W_N^0 & W_N^3 & W_N^6 & W_N^9 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}$$

$$W_N = e^{-j\frac{2\pi}{N}} = -j$$

$$\text{得 } X(k) = [5, 2+j, -5, 2-j]^T$$

验证 IDFT[X(k)] = x(n)

$$; \quad x(n) = \frac{1}{4} \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & W_N^0 \\ W_N^0 & W_N^{-1} & W_N^{-2} & W_N^{-3} \\ W_N^0 & W_N^{-2} & W_N^{-4} & W_N^{-6} \\ W_N^0 & W_N^{-3} & W_N^{-6} & W_N^{-9} \end{bmatrix} \begin{bmatrix} 5 \\ 2+j \\ -5 \\ 2-j \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix} \quad \text{得证}$$

9-17

(1) $x(n)$ 与 $x(n)$ 线性卷积

$x(m)$	0.5	1	1	0.5
$x(0-m)$	0.5	1	1	0.5
$x(1-m)$	0.5	1	1	0.5
$x(2-m)$	0.5	1	1	0.5
$x(3-m)$	0.5	1	1	0.5
$x(4-m)$	0.5	1	1	0.5
$x(5-m)$	0.5	1	1	0.5
$x(6-m)$	0.5	1	1	0.5
				$y_1(0) = 0.25$
				$y_1(1) = 1$
				$y_1(2) = 2$
				$y_1(3) = 2.5$
				$y_1(4) = 2$
				$y_1(5) = 1$
				$y_1(6) = 0.25$

(2) $x(n)$ 与 $x(n)$ 4点图卷积

$x(m)$	0.5	1	1	0.5
$x((0-m))_4 R_4(m)$	0.5	0.5	1	1
$x((1-m))_4 R_4(m)$	1	0.5	0.5	1
$x((2-m))_4 R_4(m)$	1	1	0.5	0.5
$x((3-m))_4 R_4(m)$	0.5	1	1	0.5
				$y_2(0) = 2.25$
				$y_2(1) = 2$
				$y_2(2) = 2.25$
				$y_2(3) = 2.5$

(3) $x(n)$ 与 $x(n)$ 10点图卷积

$x(m)$	0.5	1	1	0.5
$x((0-m))_{10} R_{10}(m)$	0.5			
$x((1-m))_{10} R_{10}(m)$	1	0.5		
$x((2-m))_{10} R_{10}(m)$	1	1	0.5	
$x((3-m))_{10} R_{10}(m)$	0.5	1	1	0.5
$x((4-m))_{10} R_{10}(m)$		0.5	1	1
$x((5-m))_{10} R_{10}(m)$			0.5	1
$x((6-m))_{10} R_{10}(m)$				0.5
$x((7-m))_{10} R_{10}(m)$				
$x((8-m))_{10} R_{10}(m)$				
$x((9-m))_{10} R_{10}(m)$				
				$y_3(0) = 0.25$
				$y_3(1) = 1$
				$y_3(2) = 2$
				$y_3(3) = 2.5$
				$y_3(4) = 2$
				$y_3(5) = 1$
				$y_3(6) = 0.25$
				$y_3(7) = 0$
				$y_3(8) = 0$
				$y_3(9) = 0$

(4) 最小长度 $L = M + N - 1 = 4 + 4 - 1 = 7$

