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Teacher: **Yanjie Li**

Assignment Number: **1**

Course: **Linear Algebra in Control Theory**

Disclosure date: May 13, 2021

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## Homework 1

Show that the function that takes  $((x_1, x_2), (y_1, y_2)) \in R^2 \times R^2$  to  $|x_1 y_1| + |x_2 y_2|$  is not an inner product on  $R^2$ .

## Homework 2

Suppose  $V$  is a real inner product space, show that:

- a) the inner product  $\langle u + v, u - v \rangle = \|u\|^2 - \|v\|^2$  for every  $u, v \in V$ .
- b) if  $u, v \in V$  have the same norm, then  $u + v$  is orthogonal to  $u - v$ .
- c) use part(b) to show that the diagonals of a rhombus are perpendicular to each other.

## Homework 3

Suppose  $u, v \in V$ , prove that the inner product  $\langle u, v \rangle = 0$  if and only if  $\|u\| \leq \|u + av\|$  for all  $a \in R$ .

## Homework 4

Suppose  $u, v \in V$ , prove that  $\|au + bv\| = \|bu + av\|$  for all  $a, b \in R$  if and only if  $\|u\| = \|v\|$ .

## Homework 5

Suppose  $u, v \in V$ ,  $\|u\| = \|v\| = 1$  and  $\langle u, v \rangle = 1$ , prove that  $u = v$ .

## Homework 6

Find vectors  $u, v \in \mathbb{R}^2$  such that  $u$  is a scalar multiple of  $(1, 3)$ ,  $v$  is orthogonal to  $(1, 3)$ , and  $(1, 2) = u + v$ .

## Homework 7

Prove that  $(x_1 + \cdots + x_n)^2 \leq n(x_1^2 + \cdots + x_n^2)$  for all positive integers  $n$  and all real numbers  $x_1, \dots, x_n$ .

## Homework 8

Suppose  $V$  is a real inner product space, prove that

$$\langle u, v \rangle = \frac{\|u + v\|^2 - \|u - v\|^2}{4}$$

for all  $u, v \in V$ .

## Pay Attention

- a) Mark your class number, name and student number on the homework.
- b) Please hand in your homework to your TA before class next Wednesday (May 19).

## References

- [1] Axler, S. (1997). Linear algebra done right. Springer Science Business Media.
- [2] Lay, D. C. . Linear algebra and its applications. Academic Press.