HW-5 19040102 自动化1班 方亮

5-1 证明：$\nabla f(x)=\begin{bmatrix}-1\\0\end{bmatrix}$，$\nabla g_1(x)=\begin{bmatrix}-2x_1\\-2x_2\end{bmatrix}$，$\nabla g_2(x)=\begin{bmatrix}-3(x_1-1)^2\\1\end{bmatrix}$

$\nabla f(x^*)=\begin{bmatrix}-1\\0\end{bmatrix}$，$\nabla g_1(x^*)=\begin{bmatrix}-2\\0\end{bmatrix}$，$\nabla g_2(x^*)=\begin{bmatrix}0\\1\end{bmatrix}$

$\begin{bmatrix}-1\\0\end{bmatrix}-\lambda_1\begin{bmatrix}-2\\0\end{bmatrix}-\lambda_2\begin{bmatrix}0\\1\end{bmatrix}=0$ 得 $\lambda_1=\frac{1}{2}$，$\lambda_2=0$

故 x^* 是 K-T 点

$\nabla f(\bar{x})=\begin{bmatrix}-1\\0\end{bmatrix}$，$\nabla g_1(\bar{x})=\begin{bmatrix}0\\2\end{bmatrix}$，$\nabla g_2(\bar{x})=\begin{bmatrix}-3\\1\end{bmatrix}$

$\begin{bmatrix}-1\\0\end{bmatrix}-\lambda_1\begin{bmatrix}0\\2\end{bmatrix}-\lambda_2\begin{bmatrix}-3\\1\end{bmatrix}=0$ $\lambda_1=-\frac{1}{6}<0$ $\lambda_2=\frac{1}{3}$

故不满足 K-T 条件，$\bar{x}$ 不是 K-T 点

5-2 (1) $\nabla f(x)=\begin{bmatrix}2(x_1-2)\\2(x_2-1)\end{bmatrix}$ $\nabla g(x)=\begin{bmatrix}-2x_1\\-2x_2\end{bmatrix}$

K-T 条件 $\begin{cases}\nabla f(x)-\lambda\nabla g(x)=0\\ \lambda g(x)=0\end{cases}$ 即 $\begin{cases}2(x_1-2)+2\lambda x_1=0\\ 2(x_2-1)+2\lambda x_2=0\\ \lambda(1-x_1^2-x_2^2)=0\end{cases}$

$\lambda=0$，$x_1=2$，$x_2=1$ 不满足条件

$g(x)=0$ 即 $1-x_1^2-x_2^2=0$ 联立得

$\begin{cases}x_1=\frac{2\sqrt{5}}{5}\\ x_2=\frac{\sqrt{5}}{5}\end{cases}$ $\begin{cases}x_1=-\frac{2}{5}\sqrt{5}\\ x_2=-\frac{\sqrt{5}}{5}\end{cases}$ 得最优解为 $x=[\frac{2}{5}\sqrt{5},\frac{\sqrt{5}}{5}]^T$

5-3 (2) 设 $\varphi(x)=x_1^2+2x_2^2+M\cdot[\min\{0,x_1+x_2-1\}]^2$

∵ 外点法，∴ $x_1+x_2-1\le 0$

∴ $\varphi(x)=x_1^2+2x_2^2+M(x_1+x_2-1)^2$

$\nabla\varphi(x)=\begin{bmatrix}2x_1+2M(x_1+x_2-1)\\4x_2+2M(x_1+x_2-1)\end{bmatrix}=0$ 得 $x=\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}\frac{2M}{3M+2}\\ \frac{M}{3M+2}\end{bmatrix}$

$x^*=\lim\limits_{M\to\infty}x=\begin{bmatrix}\frac{2}{3}\\ \frac{1}{3}\end{bmatrix}$ 即 $x^*=[\frac{2}{3},\frac{1}{3}]^T$ $S^*=\frac{2}{3}$

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5-4(2) $\varphi(x)=f(x)-\sum r_i \ln g(x_i)=5x_1+4x_2^2-r_1\ln(x_1-1)-r_2\ln(x_2)$

$$\nabla\varphi(x)=\begin{bmatrix}5-\frac{r_1}{x_1-1}\\ 8x_2-\frac{r_2}{x_2}\end{bmatrix}=0 \quad 得\begin{cases}x_1=1+\frac{r_1}{5}\\ x_2=\sqrt{\frac{r_2}{8}}\end{cases}$$

$$x^*=\lim_{r\to 0}\begin{bmatrix}1+\frac{r_1}{5}\\ \sqrt{\frac{r_2}{8}}\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix} \quad 即 x^*=[1,0]^T \quad S^*=5$$

5-5(1) $\varphi(x,\mu^{(k)},c)=f(x)+\sum \mu_j h_j(x)+\frac{c}{2}\sum h_j^2(x)$，取 $c=2$

$$\varphi(x,\mu^{(k)},c)=x_1^2+2x_2^2+2x_3^2+\mu^{(k)}(x_1+x_2+x_3-4)+(x_1+x_2+x_3-4)^2$$

$$0=\frac{\partial\varphi}{\partial x_1}=2x_1+\mu^{(k)}+2(x_1+x_2+x_3-4)$$

$$0=\frac{\partial\varphi}{\partial x_2}=4x_2+\mu^{(k)}+2(x_1+x_2+x_3-4) \quad 得 x^{(k)}=\begin{pmatrix}\frac{8-\mu}{6}\\ \frac{8-\mu}{12}\\ \frac{8-\mu}{12}\end{pmatrix}$$

$$0=\frac{\partial\varphi}{\partial x_3}=4x_3+\mu^{(k)}+2(x_1+x_2+x_3-4)$$

$$\mu^{(k+1)}=\mu^{(k)}+c\,h(x^{(k)})$$
$$=\frac{\mu^{(k)}}{3}-\frac{8}{3}$$

$\mu^{(1)}=1 \quad \mu^{(15)}=\mu^{(16)}=\cdots=\mu^{(20)}=-4 \quad \mu^*=-4$

$$x^*=x^{(k)}\big|_{\mu^{(k)}=\mu^*}=\begin{pmatrix}2\\1\\1\end{pmatrix}$$