FINA3351B Assignment 3
Due by the end of May 7, 2025 (Wednesday)

Q1. In the Black-Scholes-Merton model, the European call and put option prices are given by

$$c = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$$
$$p = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$$

where

$$d_1 = \frac{\ln(S/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

The Greeks of the option also have analytical solutions.

Gamma represents how delta changes with respect to stock prices. It is given by

$$\Gamma_{\text{call}} = \Gamma_{\text{put}} = \frac{N'(d_1)e^{-qT}}{S\sigma\sqrt{T}}$$

where $N'(d_1)$ is the probability density function of standard normal distribution.

Vega represents the sensitivity of option price to stock volatility. It is given by

$$\kappa_{\text{call}} = \kappa_{\text{put}} = Se^{-qT}N'(d_1)\sqrt{T}$$

Rho represents the sensitivity of option price to risk-free rate.

$$\rho_{\text{call}} = KTe^{-rT}N(d_2)$$
$$\rho_{\text{put}} = -KTe^{-rT}N(-d_2)$$

Theta represents the sensitivity of the option price to a negative change in time-to-maturity. It is given by

$$\Theta_{\text{call}} = -\frac{\partial C}{\partial T} = -\frac{\sigma}{2\sqrt{T}}e^{-qT}SN'(d_1) + qe^{-qT}SN(d_1) - rKe^{-rT}N(d_2)$$
$$\Theta_{\text{put}} = -\frac{\partial P}{\partial T} = -\frac{\sigma}{2\sqrt{T}}e^{-qT}SN'(d_1) - qe^{-qT}SN(-d_1) + rKe^{-rT}N(-d_2)$$

(1) In Module `Q1`, write VBA function procedures `BSGamma`, `BSVega`, `BSRho`, `BSTheta` that return the Gamma, Vega, Rho and Theta of European options, respectively, based on the Black-Scholes-Merton model. You determine the arguments of functions.

(2) In worksheet “Q1”, calculate the Greek letters using the VBA functions you wrote.

Q2. In CRR binomial tree with n periods, denote time to maturity as T. Then the total returns of up move and down move are given by

$$u = e^{\sigma\sqrt{\Delta t}}, \qquad d = \frac{1}{u}$$

where $\Delta t = \frac{T}{n}$. The risk-neutral probability of up move is $p = \frac{e^{(r-q)\Delta t} - d}{u-d}$.

At maturity date (at step n), in state i (defined as the total number of down moves from step 0 to n), the stock price is given by

$$S_n(i) = S_0 u^{(n-i)} d^i, \qquad \text{for } i = 0, \dots, n$$

The probability that state i occurs is

$$\binom{n}{i} p^{n-i} (1-p)^i$$

where $\binom{n}{i} = \frac{n!}{i!(n-i)!}$ is binomial coefficient. Excel function `Combin(n,i)` calculates the value for the binomial coefficients.

Based on the risk-neutral valuation method, the current price of a European option is the discounted value of expected payoff at maturity:

$$\begin{cases} \text{European call price} = e^{-rT} \displaystyle\sum_{i=0}^{n} \binom{n}{i} p^{n-i} (1-p)^i \max\left(S_0 u^{(n-i)} d^i - K, 0\right) \\ \text{European put price} = e^{-rT} \displaystyle\sum_{i=0}^{n} \binom{n}{i} p^{n-i} (1-p)^i \max\left(K - S_0 u^{(n-i)} d^i, 0\right) \end{cases}$$

(1) In VBA Module `Q2`, complete VBA function procedure `Binomial_euro_new` and use above equations to calculate European call and put option prices.

(2) In worksheet "Q2", the European option inputs values are given. Use the VBA function `Binomial_euro_new` to calculate the option price in cell B13.