Simulate Stock Prices – Factor Model

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FINA3351 – Spreadsheet Modelling in Finance

Roadmap for Today

1. Simulate Stock Prices Using Single-Index Model: Monte Carlo
2. Simulate Stock Prices Using Single-Index Model: Bootstrapping

Monte Carlo Simulation of Stock Prices: Single-Index Model

Motivation

- Now we simulate stock returns based on factor models.
- Let's take single-index model as an example.
- The stock i's excess return follows

$$R_{i,t}^e = \alpha + \beta R_{M,t}^e + e_{i,t}$$

  - $R_{m,t}^e$ is the market factor (i.e., market returns minus the risk-free rate).
  - $e_{i,t} \sim N(0, \sigma_e^2)$ is idiosyncratic shock, i.i.d. normally distributed
  - Two random variables in this model:
    - pricing factor $R_{M,t}^e$
    - idiosyncratic shock $e_{i,t}$
- We will start with simulating systematic risk factor $R_{M,t}^e$.

Simulating the Market Factor -1

- We *cannot* deny that the market factor may be affected by some macroeconomic conditions, such as business cycles, interest rates, economic prospects, tax policies, etc.
- You can write

$$R_{M,t}^{e} = a + b_1 F_{1,t} + \cdots + b_K F_{K,t} + e_{M,t}$$

where *a* is the constant, b_j is the sensitivity of the market factor to the j-th macroeconomic factor $F_{j,t}$.

- Then if you can simulate the stochastic processes of $F_{1,t}, \cdots, F_{K,t}$, you can simulate $R_{M,t}^{e}$.
- Here, we illustrate the simplest approach. That is, we assume that the market factor is normally distributed

$$R_{M,t}^{e} = a + e_{M,t} \sim N(a, \sigma_M^2)$$

  - *a* is the market risk premium, $E\left(R_{M,t}^{e}\right)$
  - σ_M^2 is the variance of market index

Simulating the Market Factor -2

- Go to Excel file "Lec8_ StockSimulation-Factor.xlsm".
- See tab "MCM_Excel".
- We simulate Café de Coral (0341.HK) stock prices.
- The market factor is measured by Hang Seng Index (HSI) excess returns.
- **Sample**: Daily data from 31/10/2023 to 31/10/2024.
- Risk-free rate is overnight HIBOR rate announced on HK Monetary Authority website. See the tab "T7.3.1".
- Note: HIBOR rate is updated monthly, we choose one-month lagged rate from Sep. 2023 to Sep. 2024. Then we transform annual interest rate (in percentage) to daily data.

Simulating Firm-Specific Shock $e_{i,t}$

- Recall that, based on single-index model, the stock i's excess return satisfies

$$R_{i,t}^{e} = \alpha + \beta R_{M,t}^{e} + e_{i,t}$$

- We first use the mean and standard deviation of HSI excess returns to simulate future market excess returns $\{R_{M,t}^{e,\mathrm{sim}}\}_{t=1,2,\cdots,T}$.
- Then we simulate idiosyncratic shock $\{e_{i,t}\}_{t=1,2,\cdots,T}$.
  - From the regression, we estimate α, β and σ_e (residual standard deviation) using `LINEST` function.
  - Given that $e_{i,t} \sim N(0, \sigma_e^2)$, we can simulate idiosyncratic shocks $\{e_{i,t}^{\mathrm{sim}}\}_{t=1,2,\cdots,T}$.

Simulating CAF's Excess Return and Price

- We calculate simulated CAF's excess returns $\{R_{i,t}^{e,\text{sim}}\}_{t=1,2,\cdots,T}$, given simulated market factor $\{R_{M,t}^{e,\text{sim}}\}_{t=1,2,\cdots,T}$ and simulated idiosyncratic shocks $\{e_{i,t}^{\text{sim}}\}_{t=1,2,\cdots,T}$.

$$R_{i,t}^{e,\text{sim}} = \hat{\alpha} + \hat{\beta} R_{M,t}^{e,\text{sim}} + e_{i,t}^{\text{sim}}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are coefficient estimation from linear regression.

- Given that the adjusted closing price on 2024-10-30 is $8.164, we can calculate the simulated price for the next 22 days.

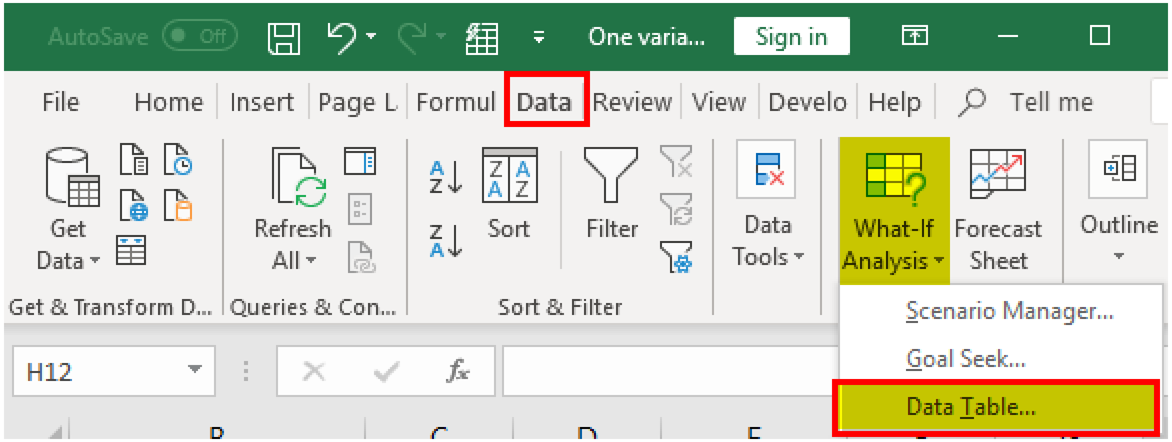
$$R_{i,t}^{e} = \ln\left(\frac{S_t}{S_{t-1}}\right) - R_f$$

$$S_t = S_{t-1} \exp\left(R_{M,t}^{e} + R_f\right)$$

- Note: you need to "add back" the risk-free rate.
- Stock price equation is based on the definition of excess return, not a specific model.

Simulating CAF’s Excess Return and Price

- Now we have the time series of one simulation; what if we want to have multiple simulations?
- We can use Data Table function to generate 20 simulated CAF scenarios in 22 days since 30/10/2024.
- **Data Table** in Excel allows users to test how the value of one variable or two variables affects the result in the created formula.
- Go to Data → What-If Analysis → Data Table



- Go to tab “DataTable” for details

Simulating CAF's excess return using VBA

- Now we write a VBA function to simulate stock price.
- The function "`simfactor`" in Module MCM returns a simulated excess return of a stock based on factor model, with up to four factors (number of factors can be easily modified).
- Refer to Excel tab "Simfactor".
- The factor model is

$$R_{i,t}^{e} = \alpha + \beta_1 F_{1,t} + \beta_2 F_{2,t} + \beta_3 F_{3,t} + \beta_4 F_{4,t} + e_{i,t}$$

where

  - $F_{1,t} \sim N(\mu_1, \sigma_1^2), F_{2,t} \sim N(\mu_2, \sigma_2^2), F_{3,t} \sim N(\mu_3, \sigma_3^2), F_{4,t} \sim N(\mu_4, \sigma_4^2)$
  - $e_{i,t} \sim N(0, \sigma_e^2)$
  - Four factors are independent with each other and are independent with firm-specific shock $e_{i,t}$.
  - To exclude a factor from the model, you can simply set beta coefficient to be 0.

Simulating CAF’s excess return using VBA

- Conducting simulations in Excel is slow.
- Now we write a VBA sub procedure `Factor_mcm` in Module `MCM`.
- The parameters in the single-index model are listed in Sheet “MCM_VBA”.
- Sub procedure reads the parameter values into VBA, then simulate stock prices from day 1 to day 22. Then repeat the simulation for 10000 times.
- Simulated stock prices are written into the worksheet.

Bootstrap Simulation of Stock Prices: Single-Index Model

Bootstrap

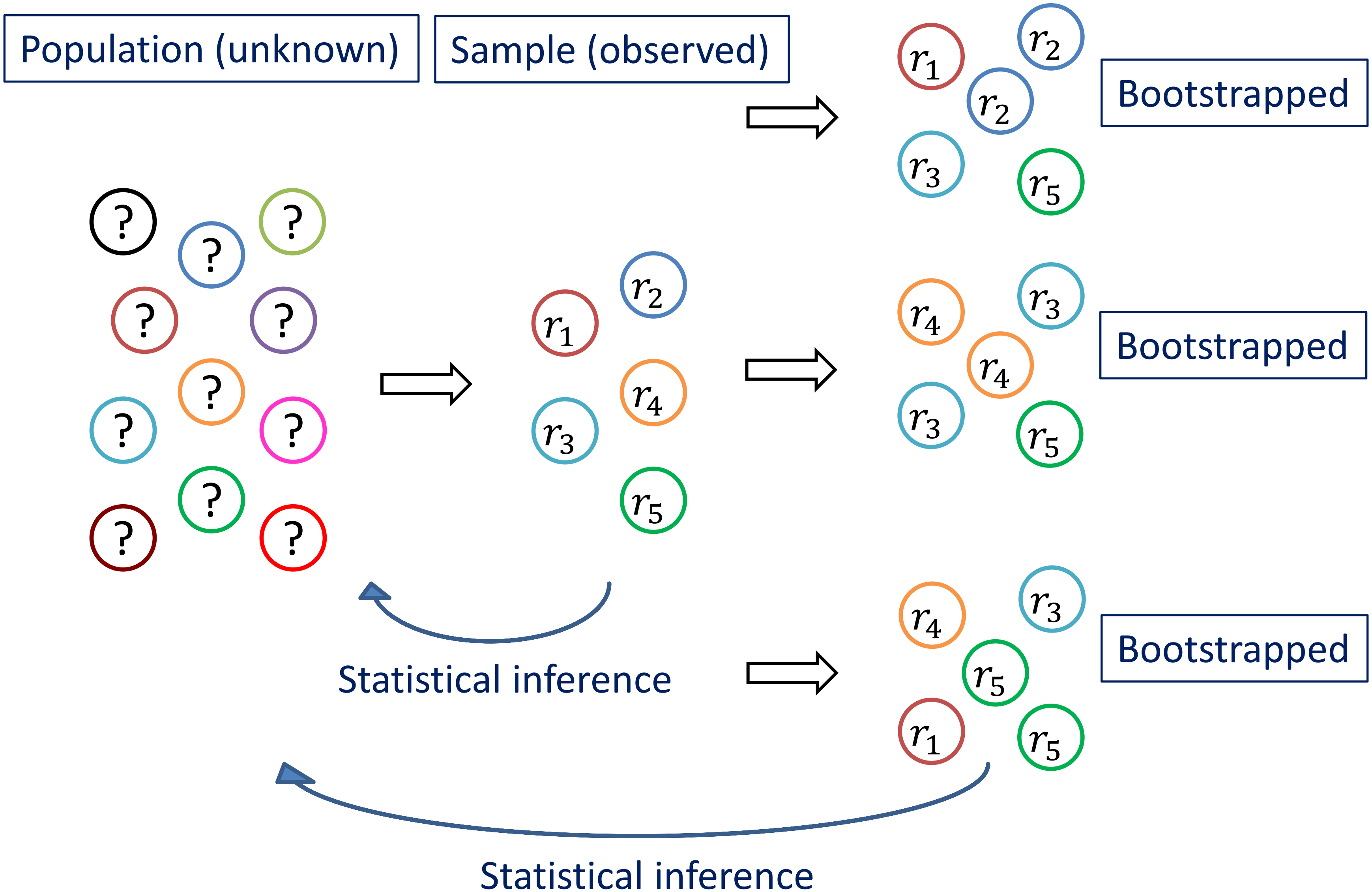
- The idea of bootstrapping is to sample randomly, with replacement, from observed historical observations under the assumption that future observations will be drawn from the same distribution.
- It is a non-parametric simulation method.
- The bootstrap is to estimate the distribution of an estimator or test statistic by resampling one's data.
- The bootstrap provides a way to substitute computation for mathematical analysis if calculating the asymptotic distribution of an estimator or statistic is difficult.
- You must be wondering: Why is this method called "bootstrap"?

=> Come from the term "pulling yourself up by your own bootstraps".

Bootstrap

- For example, assuming normal stock returns has trouble accounting for events like 2008 financial crisis.
- The advantage of bootstrapping is that, since it is not based on a particular assumed distribution (e.g., normal distribution), it is consistent with any distribution of returns, if observations can be assumed to be from an independent and identically distributed population.
- It has better "small-sample property" (especially high-order properties such as skewness).
- A disadvantage of simple bootstrapping is that the autocorrelation property in the data might be lost when the data are reshuffled.

Bootstrap



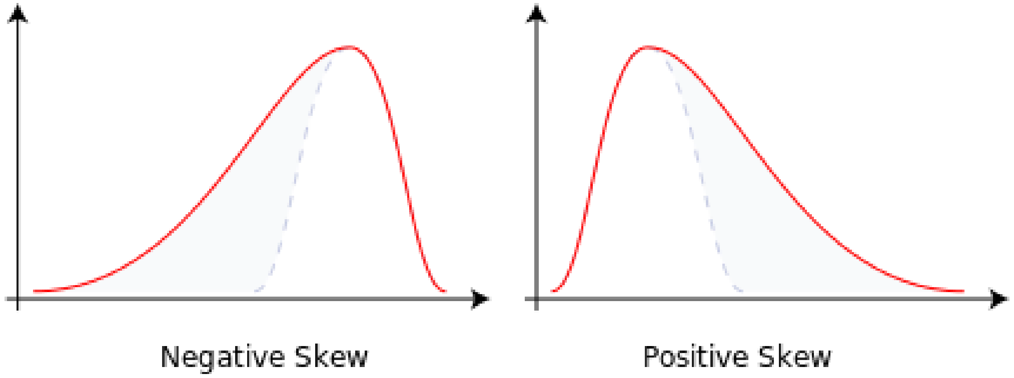
Population (unknown)
Sample (observed)
Bootstrapped
Bootstrapped
Bootstrapped
?
r_1
r_2
r_3
r_4
r_5
Statistical inference
Statistical inference

Simple Bootstrapping

- The most basic approach can be observed in sheet “Bootstrap_Excel”.
- Basically, we simulate the “position” of observations in the sample, rather than simulating data based on any distribution function.
- Steps:
  1. Let N denote the total number of observations in the sample. We index the observations from 1 to N. It is the “position” of each data.
  2. Generate random integer number u between 1 and N.
  3. Find the observation at the position u. This is the bootstrapped data.

Simple Bootstrapping

- As you can see, bootstrapped HSI excess returns have similar mean and standard deviation.
- More importantly, bootstrapped HSI series captures the “skewness” of stock returns, which cannot be achieved by our previous simulation method.
- Skewness is an important property in stock returns.



Simple Bootstrapping using VBA

- The function procedure `bootseeds` in Module `Bootstrap` is used to bootstrap data position using simple bootstrapping method.

Factor Model Simulation - Bootstrap

- Excel sheet "Bootstrap_Excel" tab continues the example of single-factor model simulation of CAF stock prices.

$$R_{i,t}^{e} = \alpha + \beta R_{M,t}^{e} + e_{i,t}$$

- Now we simulate HSI excess return ($R_{m,t}^{e}$) and firm-specific shock ($e_{i,t}$) with bootstrap method:
  - HSI excess returns $R_{M,t}^{e}$ are bootstrapped from sample observations.
  - Shocks $e_{i,t}$ are bootstrapped from residuals of OLS regression.
  - $\alpha,\ \beta$ are still estimated from regression.
- In the example, we can simulate the next 22 days CAF returns and prices – time-series simulation.

Simulating CAF's excess return using Excel

- We can then use Data Table function to do cross-sectional simulation of month-end CAF prices for 100 times.
- **Note**: We don't impose assumptions on the distribution of $R_{m,t}^{e}$ and $e_{i,t}$, but require them to be independent with each other, and has no autocorrelation.

Simulating CAF’s excess return using VBA

- Conducting simulations in Excel is slow.
- Now we write a VBA sub procedure `Factor_bootstrap` in Module `Bootstrap.`
- The parameters in the single-index model are listed in Sheet “Bootstrap_VBA”.
- Sub procedure reads the parameter values into VBA, then simulate stock prices from day 1 to day 22. Then repeat the simulation for 10000 times.
- Simulated stock prices are written into the worksheet.