DS-GA 1003: Homework 5
Trees and Boosting

Due on Monday, April 4, 2016

Professor David Ronsenberg

See complete code at: *git@github.com:cryanzpj/1003.git*

Yuhao Zhao
Yz3085

2 Decision Trees

- **2.1 Trees on the Banana Dataset.**

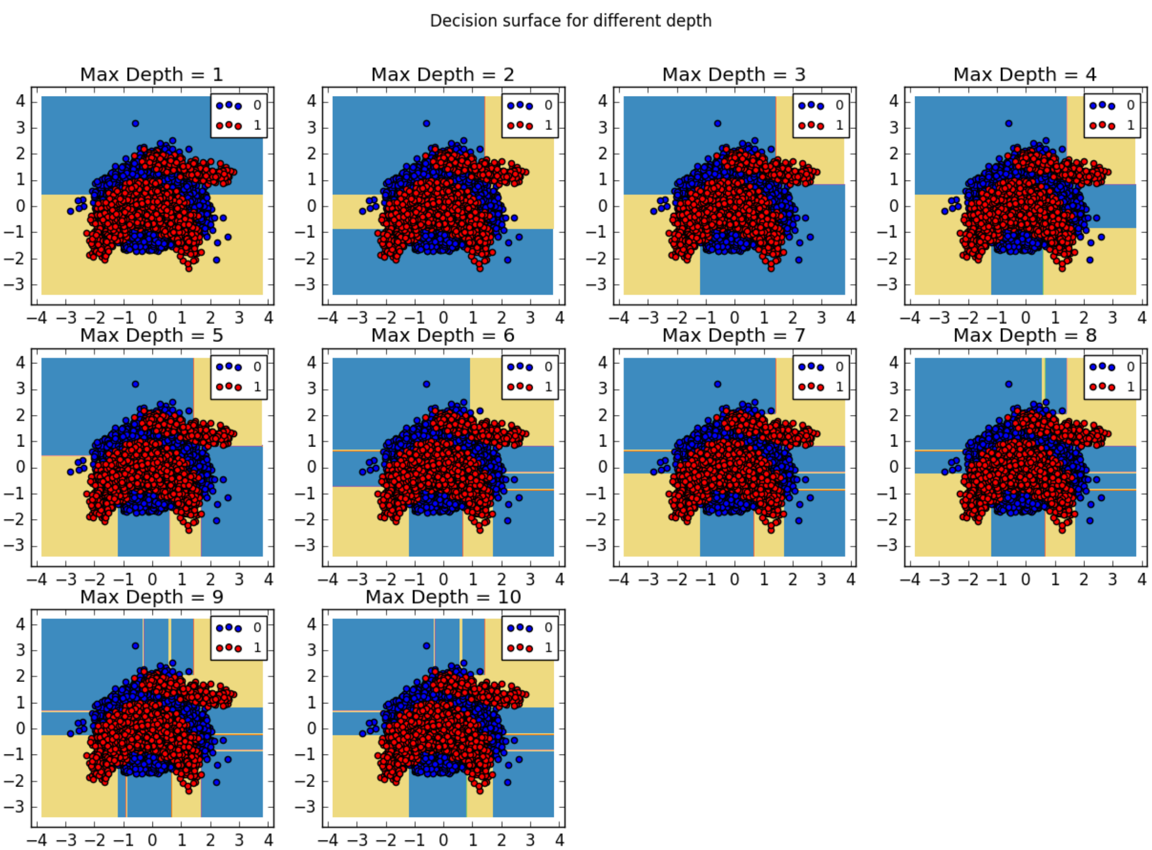
– **2.1.1.**

```
import numpy as np
import scipy as sp
import matplotlib.pyplot as plt
from sklearn.tree import DecisionTreeClassifier

file_train = open('data/banana_train.csv')
file_test = open('data/banana_test.csv')

train = np.array(map(lambda x: x[:2] + [x[-1].strip()],
                         [i.split(',') for i in file_train]), dtype='float')
test = np.array(map(lambda x: x[:2] + [x[-1].strip()],
                         [i.split(',') for i in file_test]), dtype='float')

y_train = np.array([0 if i == -1 else 1 for i in train[:, 0]])
y_test = np.array([0 if i == -1 else 1 for i in test[:, 0]])
X_train = train[:, 1:]
X_test = test[:, 1:]
```

– **2.1.2.**

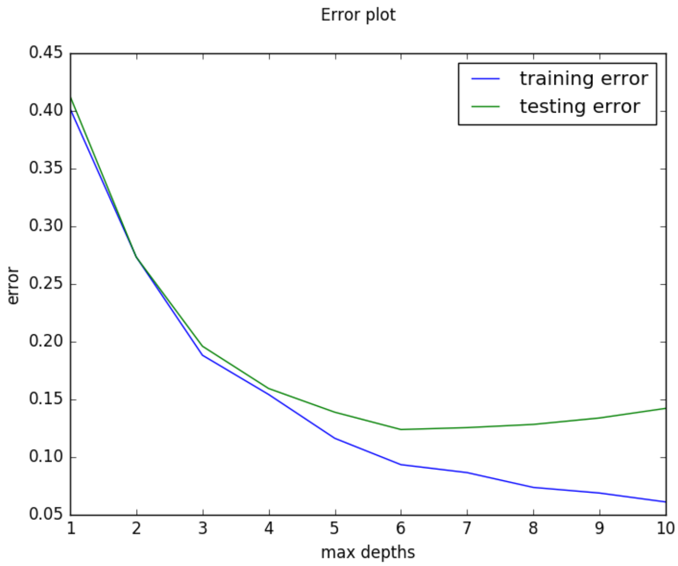
```
n_classes = 2
plot_colors = "bry"
plot_step = 0.02

error = np.zeros((2, 10))

for i in xrange(1, 11):
        idx = np.arange(X_train.shape[0])
        np.random.seed(1)
        np.random.shuffle(idx)
        X = X_train[idx]
        y = y_train[idx]

        mean = X.mean(axis=0)
        std = X.std(axis=0)
        X = (X - mean) / std

        clf = DecisionTreeClassifier(max_depth=i).fit(X, y)
        plt.subplot(2, 5, i)
        x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
        y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
        xx, yy = np.meshgrid(np.arange(x_min, x_max, plot_step),
        np.arange(y_min, y_max, plot_step))

        Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
```

```
        Z = Z.reshape(xx.shape)

        training_error = np.sum(np.equal(clf.predict(X_train),
                                        1 - y_train)) / float(y_train.shape[0])
        testing_error = np.sum(np.equal(clf.predict(X_test),
                                        1 - y_test)) / float(y_test.shape[0])
        error[:, i - 1] = np.array([training_error, testing_error])

        cs = plt.contourf(xx, yy, Z, cmap=plt.cm.Paired)

        plt.axis("tight")

        for i, color in zip(range(n_classes), plot_colors):
                idx = np.where(y == i)
                plt.scatter(X[idx, 0], X[idx, 1], c=color, label=str(i),
                                cmap=plt.cm.Paired)

        plt.axis("tight")
        plt.legend(fontsize=10)
plt.suptitle('Decision surface for different depth')
plt.show()
```



From the plot we can see that, the decision surface has very little change after max depth = 6.

– **2.1.3.**



From the plot we see that the model is clearly over-fitting for large max depths. For max depth greater than 6 the testing error is increasing.

– **2.1.4.**

```
min_error = 1
ite = 0
for a in xrange(1, 21):
    for b in xrange(1, 21):
        for c in xrange(1, 21):
            idx = np.arange(X_train.shape[0])
            np.random.seed(1)
            np.random.shuffle(idx)
            X = X_train[idx]
            y = y_train[idx]

            mean = X.mean(axis=0)
            std = X.std(axis=0)
            X = (X - mean) / std
            #normalize testing data
            X_test_temp = (X_test -mean)/std

            clf = DecisionTreeClassifier(max_depth=a, min_samples_leaf=b,
                                         min_samples_split=c).fit(X, y)

            testing_error = np.sum(np.equal(clf.predict(X_test_temp), 1 - y_test)) /
                                         float(y_test.shape[0])
            if testing_error < min_error:
                min_error = testing_error
                par = [a, b, c]
            ite += 1
```

```
>>> min_error
>>> 0.1111111111111
>>> par
>>> [10, 11, 1]
```

I searched max depth, min samples leaf, min samples split from 1 to 20, the best testing error is 0.1111111111111, and the corresponding parameters are 10,11 and 1

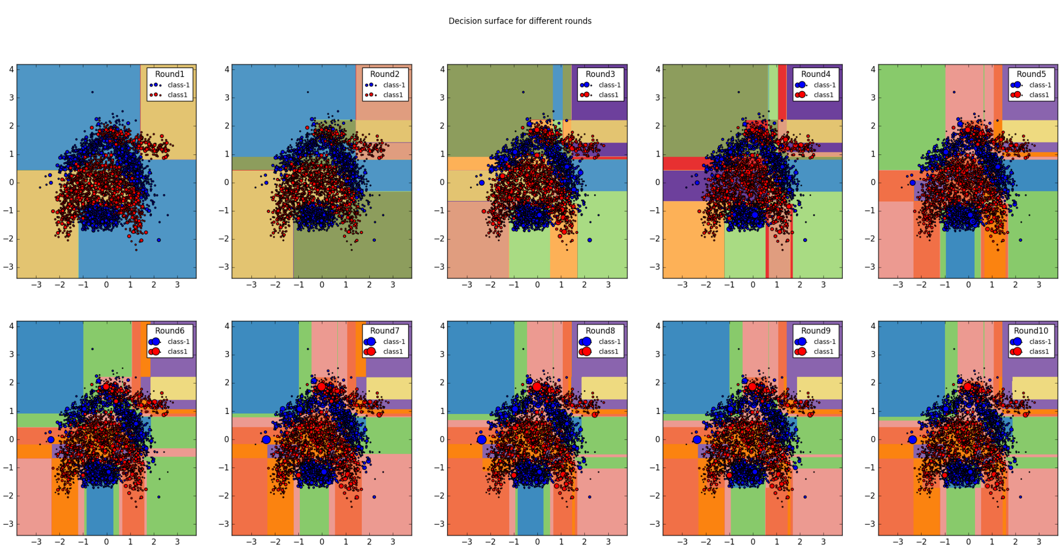
3 Ada Boost

- ## 3.1 Implementation.

  - **3.1.1.**

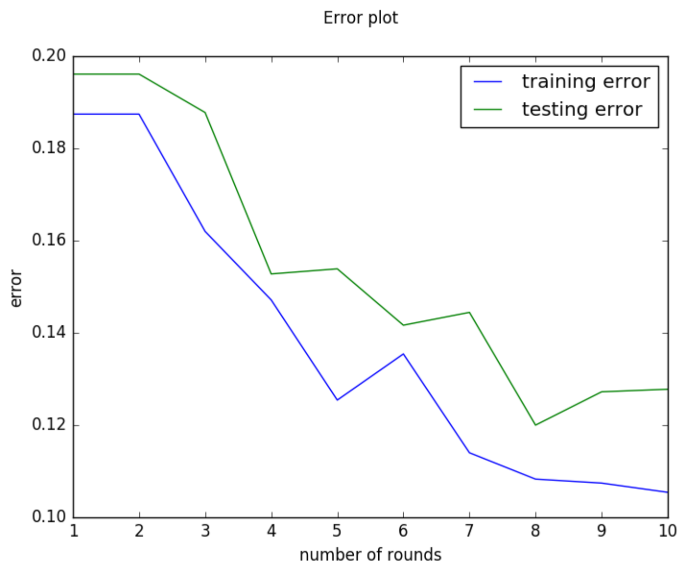
```
def AdaBoost(X, y,n_round=5,test_x = None,test_y= None,visual = False):
    '''
    :param nd-array X: Training data
    :param 1d-array y: Training labels
    :param int n_round: Number of rounds default = 5
    :param nd-array test_x: Testing data default None
    :param 1d-array test_y: Testing labels default None
    :param Boolean visual: True for visualize default None
    :return: trainining error/ testing error of the Final model
    '''

    n_instance = X.shape[0]
    w = np.ones(n_instance) / n_instance
    models = np.zeros(n_round,dtype = object)
    error = np.zeros(n_round)
    alphas = np.zeros(n_round)

    mean = X.mean(axis=0)
    std = X.std(axis=0)
    X = (X - mean) / std


    for n in xrange(1, n_round + 1):
        W = np.sum(w)
        clf = DecisionTreeClassifier(max_depth=3).fit(X,y,sample_weight=w)
        models[n-1] = clf
        error = np.sum((1-np.equal(clf.predict(X), y)) * w)/W
        alphas[n-1] = np.log((1-error)/error)
        w = np.exp((1- np.equal(clf.predict(X),y))*alphas[n-1])*w

        #visualizer
        if visual:
            plot_colors = 'br'
            plt.subplot(2, 5, n)
            x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
            y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
```

```
            xx, yy = np.meshgrid(np.arange(x_min, x_max, plot_step),
                                 np.arange(y_min, y_max, plot_step))

            Z = np.sum(map(lambda i:alphas[i] * models[i].predict(np.c_[xx.ravel(),
                                   yy.ravel()]),list(xrange(n))),0)
            Z = Z.reshape(xx.shape)
            cs = plt.contourf(xx, yy, Z, cmap=plt.cm.Paired)
            plt.axis("tight")

            for i, color in zip([-1,1], plot_colors):
                idx = np.where(y == i)
                plt.scatter(X[idx, 0], X[idx, 1],s=10000*(w/W) , c=color,
                                        label="class" + str(i), cmap=plt.cm.Paired)
                plt.legend(fontsize=10,title = 'Round' + str(n))
    plt.suptitle('Decision surface for different rounds')
    plt.show()

    # record rounds errors
    G_n = np.array(map(lambda i: alphas[i] * models[i].predict(X), list(xrange(n))))
    train_error = 1 - np.sum(np.equal(np.sign(np.sum(G_n,0)),y))/float(n_instance)

    if  (test_x != None) and  (test_y != None) :
        G_n_test = np.array(map(lambda i: alphas[i] * models[i].predict(test_x),
                                 list(xrange(n))))
        test_error = 1 - np.sum(np.equal(np.sign(np.sum(G_n_test,0)),test_y))/
                                float(test_y.shape[0])
        return [train_error,test_error]

    else:
        return train_error
>>> AdaBoost(X_train,y_train,10,visual = True)
```



From the plot we can see that some points near the decision surface are getting larger weights with further boosting.

– **3.1.3.**



From the plot we see that the both of the training and testing error are decreasing when adding rounds. The over-fitting is not clear yet, so we may try to add more rounds to see how the AdaBoost works on this data set.

4 Gradient Boosting Machines

- **4.1.**

For square loss: $l(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2$
From the Gradient Boosting Machines,

$$\begin{aligned}
(g_m)_i &= \frac{\partial}{\partial f(x_i)} \sum_{i=1}^{n} \frac{1}{2}(y_i - f(x_i))^2|_{f(x_i)=f_{m-1}(x_i)} && (1)\\
&= \frac{\partial}{\partial f(x_i)} \frac{1}{2}(y_i - f(x_i))^2|_{f(x_i)=f_{m-1}(x_i)} && (2)\\
&= -(y_i - f(x_i))|_{f(x_i)=f_{m-1}(x_i)} && (3)\\
&= -(y_i - f_{m-1}(x_i)) && (4)
\end{aligned}$$

Then

$$\begin{aligned}
h_m &= \underset{h \in \mathcal{F}}{\operatorname{argmin}} \sum_{i=1}^{n} ((-g_m)_i - h(x_i))^2 && (5)\\
&= \underset{h \in \mathcal{F}}{\operatorname{argmin}} \sum_{i=1}^{n} ((y_i - f_{m-1}(x_i)) - h(x_i))^2 && (6)
\end{aligned}$$

- **4.2.**

For logistic loss: $l(m) = ln(1+e^{-m})$
From the Gradient Boosting Machines,

$$\begin{aligned}
(g_m)_i &= \frac{\partial}{\partial f(x_i)} \sum_{i=1}^{n} ln(1+e^{-y_i f(x_i)})|_{f(x_i)=f_{m-1}(x_i)} && (7)\\
&= \frac{\partial}{\partial f(x_i)} ln(1+e^{-y_i f(x_i)})|_{f(x_i)=f_{m-1}(x_i)} && (8)\\
&= \frac{-y_i e^{-y_i f(x_i)}}{1+e^{-y_i f(x_i)}}|_{f(x_i)=f_{m-1}(x_i)} && (9)\\
&= \frac{-y_i e^{-y_i f_{m-1}(x_i)}}{1+e^{-y_i f_{m-1}(x_i)}} && (10)
\end{aligned}$$

Then

$$\begin{aligned}
h_m &= \underset{h\in\mathcal{F}}{\operatorname{argmin}} \sum_{i=1}^{n} ((-g_m)_i - h(x_i))^2 && (11)\\
&= \underset{h\in\mathcal{F}}{\operatorname{argmin}} \sum_{i=1}^{n} (\frac{y_i e^{-y_i f_{m-1}(x_i)}}{1+e^{-y_i f_{m-1}(x_i)}} - h(x_i))^2 && (12)
\end{aligned}$$

5 From Margins to Conditional Probabilities

• 5.1.

Since $y \in \{1,-1\}$

$$\begin{aligned}
E_y[l(yf(x))|x] &= P(y=-1|x)l(-f(x)) + P(y=1|x)l(f(x)) && (13)\\
&= (1-P(y=1|x))l(-f(x)) + P(y=1|x)l(f(x)) && (14)\\
&= (1-\pi(x))l(-f(x)) + \pi(x)l(f(x)) && (15)
\end{aligned}$$

• 5.2.

For exponential loss $l(y, f(x)) = e^{-yf(x)}$:

$$\begin{aligned}
f^* &= \underset{f}{\operatorname{argmin}}\ E_y[l(yf(x))|x] && (16)\\
&= \underset{f}{\operatorname{argmin}}\ (1-\pi(x))e^{f(x)} + \pi(x)e^{-f(x)} && (17)
\end{aligned}$$

If we take partial derivative of the target function w.r.t f:

$$\frac{\partial}{\partial f}(1-\pi(x))e^{f(x)} + \pi(x)e^{-f(x)} = (1-\pi(x))e^{f(x)} - \pi(x)e^{-f(x)} = 0 \quad (18)$$

We have:

$$\begin{aligned}
e^{2f(x)} &= \frac{\pi(x)}{1-\pi(x)} && (19)\\
f^*(x) = f(x) &= \frac{1}{2} ln(\frac{\pi(x)}{1-\pi(x)}) && (20)
\end{aligned}$$

On the contrary, if we are given f^*, we can solve $\pi(x)$ from eqn(20):

$$\frac{1}{2}ln(\frac{\pi(x)}{1-\pi(x)}) = \frac{1}{2}ln(\frac{1}{1-\pi(x)} - 1) = f^*(x) \tag{21}$$

$$\frac{1}{1-\pi(x)} = e^{2f^*(x)} + 1 \tag{22}$$

$$\pi(x) = \frac{e^{2f^*(x)}}{e^{2f^*(x)}+1} = \frac{1}{1+e^{-2f^*(x)}} \tag{23}$$

$$\tag{24}$$

- **5.3.**

For the logistic loss function $l(y, f(x)) = ln(1 + e^{-yf(x)})$:

$$f^* = \underset{f}{\text{argmin}}\ E_y[l(yf(x))|x] \tag{25}$$

$$= \underset{f}{\text{argmin}}\ (1-\pi(x))ln(1+e^{f(x)}) + \pi(x)ln(1+e^{-f(x)}) \tag{26}$$

If we take partial derivative of the target function w.r.t f:

$$(1-\pi(x))\frac{e^{f(x)}}{1+e^{f(x)}} + \pi(x)\frac{-e^{-f(x)}}{1+e^{-f(x)}}) = 0 \tag{27}$$

We have:

$$\frac{(1-\pi(x))e^{f(x)}}{1+e^{f(x)}} - \frac{\pi(x)}{e^{f(x)}+1} = 0 \tag{28}$$

Since $e^{f(x)} + 1 > 0$:

$$\pi(x) = (1-\pi(x))e^{f(x)} \tag{29}$$

$$f^*(x) = f(x) = ln(\frac{\pi(x)}{1-\pi(x)}) \tag{30}$$

If we are given f^*, we can solve $\pi(x)$ from eqn(30):

$$f^*(x) = ln(\frac{1}{1-\pi(x)} - 1) \tag{31}$$

$$\pi(x) = 1 - \frac{1}{e^{f^*(x)}+1} \tag{32}$$

$$= \frac{e^{f^*(x)}}{e^{f^*(x)}+1} \tag{33}$$

$$= \frac{1}{1+e^{-f^*(x)}} \tag{34}$$

- **5.4.**

For the hinge loss $l(y, f(x)) = max(0, 1 - yf(x))$:

$$f^* = \underset{f}{\text{argmin}}\ E_y[l(yf(x))|x] \tag{35}$$

$$= \underset{f}{\text{argmin}}\ (1-\pi(x))max(0, 1+f(x)) + \pi(x)max(0, 1-f(x)) \tag{36}$$

$$= \underset{f}{\text{argmin}}\ F \tag{37}$$

i) if $f \leq -1$:

$$F = \pi(x)(1 - f(x)) \tag{38}$$

$$\frac{\partial F}{\partial f} = -\pi(x) < 0 \tag{39}$$

The function F is decreasing, therefore to minimize F we choose $f^*(x)$ = -1
ii) if $f \geq 1$:

$$F = (1 - \pi(x))(1 + f(x)) \tag{40}$$

$$\frac{\partial F}{\partial f} = 1 - \pi(x) > 0 \tag{41}$$

The function F is increasing, therefore to minimize F we choose $f^*(x)s$ 1
iii) if $-1 \leq f \leq 1$:

$$F = (1 - \pi(x))(1 + f(x)) + \pi(x)(1 - f(x)) \tag{42}$$

$$\frac{\partial F}{\partial f} = 1 - 2\pi(x) \tag{43}$$

For $\pi(x) \leq \frac{1}{2}$, $\frac{\partial F}{\partial f} \geq 0$, the function is increasing, then we choose $f^*(x) = -1$
For $\pi(x) \geq \frac{1}{2}$, $\frac{\partial F}{\partial f} \leq 0$, the function is decreasing, then we choose $f^*(x) = 1$
From above it's equivalent to $f^*(x) = sign(\pi(x) - \frac{1}{2})$