Homework 8 DS-GA 1002

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Problem 1.

a) The condition mean minimize the mean square error.
Therefore, $g_{MSE}(Y) = E(X|Y)$.
for $y \in [-3, -1], x = -1$ with probability 1,$g_{MSE}(Y) = 1$
for $y \in [1, 3], x = 1$ with probability 1,$g_{MSE}(Y) = -1$
for $y \in [-1, 1], x = 1$ with p = 0.5 and x =-1 with p=0.5,$g_{MSE}(Y) = 0.5 - 0.5 = 0$

b) $P(X \neq g_{MSE}) = P(X = 1)P(g_{MSE} \neq 1|X = 1) + P(X = 0)P(g_{MSE} \neq 0|X = 0)$
$= P(y \in [-1, 1]) \times 1 = P(x = 1)P(y \in [-1, 1]|x = 1) + P(x = -1)P(y \in [-1, 1]|x = -1) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

c) MAP estimation minimize the probability of error
$g_{error} = g_{MAP}(y) = \underset{x \in \{1,-1\}}{\text{Argmin}}\ P_{X|Y}(X|Y) = \underset{x \in \{1,-1\}}{\text{Argmin}}\ \frac{f_{Y|X}(y|X=x)P(X=x)}{f_y(y)}$
$Y|X \sim Uniform(X - 2, X + 2)$
Therefore, $g_{MAP}(y) = 1$ if $\frac{1}{4} \times I(y \in [-1, 3]) \times \frac{1}{2} > \frac{1}{4} \times I(y \in [-3, 1]) \times \frac{1}{2}$
This means, $g_{MAP}(y) = 1$ if $y \in [1, 3]$ and $g_{MAP}(y) = 0$ if $y \in [-3, -1]$
For $y \in [-1, 1]$ we choose arbitrarily.

$P(x \neq g_{MAP}(y)) = P(X = 1)P(g_{MAP} \neq 1|X = 1) + P(X = 0)P(g_{MAP} \neq 0|X = 0)$
$= \frac{1}{2} \times P(y \in [-1, 1]|X = 1) \times \frac{1}{2} + \frac{1}{2} \times P(y \in [-1, 1]|X = 0) \times \frac{1}{2}$
$= \frac{1}{2}\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{2}\frac{1}{2} = \frac{1}{4}$

d)$MSE(g_{error}) = E((x - g_{MAP}(y))^2) = E(x^2) + E(g^2_{MPA}) - 2E(xg_{MAP})$
$= 1 + 1$ -$2E(xE(g_{MAP}(y|x))) = 2 - 2(\frac{1}{2}(1 \times \frac{1}{2}) + \frac{1}{2}(-1 \times -\frac{1}{2})) = 1$

Problem 2.

a) $g_{MAP}(w) = \underset{R \in \{1,0\}}{\text{Argmin}}\ P_{R|W}(R|W) = \underset{R \in \{1,0\}}{\text{Argmin}}\ \frac{P_{W|R}(W|R)P(R)}{P(W)}$
$g_{MAP}(w) = 1$ if $\frac{P_{W|R=1}(W|R=1)P(R=1)}{P(W)} > \frac{P_{W|R=0}(W|R=0)P(R=0)}{P(W)}$ and $g_{MAP}(w) = 0$ otherwise
for $w = 1, 0.7 \times 0.2 < 0.3 \times 0.8, g_{MAP}(w) = 0$

for $w = 0, 0.3 \times 0.2 < 0.7 \times 0.8, g_{MAP}(w) = 0$

Therefore, $g_{MAP} = 0$ whatever w is. This means the prediction given the forecast is not rain whatever the forecast is.

b) It's more reasonable to assume H and W are independent given R. H and W are not mutually independent, because rain or not should be dependent on the humidity, the weather forecast should be dependent on humidity.

c) $H|R = 1 \sim uinf(0.5, 0.7), H|R = 0 \sim unif(0.1, 0.6)$

$g_{MAP}(w) = \underset{R\in\{1,0\}}{\text{Argmin}}\ P_{R|W,H}(R|W,H) = \underset{R\in\{1,0\}}{\text{Argmin}}\ \frac{f_{W,H|R}\times P_R(R)}{f_{W,H}} = \underset{R\in\{1,0\}}{\text{Argmin}}\ \frac{P_{W|R}\times f_{H|R}\times P_R(R)}{f_{W,H}}$

by conditional Independence.

$g_{MAP}(w) = 1$ if $0.2P_{W|R=1} \times \frac{1}{0.2} I(H \in [0.5, 0.7]) > 0.8 f_{W|R=0} \times \frac{1}{0.5} I(H \in [0.1, 0.6])$ and $g_{MAP}(w) = 0$ otherwise

H= 0.65,W=0, $\quad 0.2 \times 0.3 \times 5 > 0.8 \times 0.7 \times 2$

$g_{MAP} = 1$

d) H=0.55, w = 1, $\quad 0.2 \times 0.7 \times 5 > 0.8 \times 0.3 \times 2$

$g_{MAP} = 1$

e)$P(R \neq g_{MAP}) = P(R = 0)P(g_{MAP} = 1|R = 0) + P(R = 1)P(g_{MAP} = 0|R = 1)$

The case that the estimation could be wrong is w=1,H$\in [0.1, 0.5]$ and w =0, H$\in [0.5, 0.6]$

$P(R \neq g_{MAP}) = 0.8 \times P(w = 1|R = 0) \times P(H \in [0.1, 0.5]|R = 0)) + 0.2 \times P(w = 0|R = 0 \times P(H \in [0.6, 0.7]|R = 1)) = 0.8 \times 0.3 \times 0.2 + 0.2 \times 0.3 \times 0.5 = 0.048 + 0.03 = 0.078$

Problem 3.

a)$g_{MAP}(S, C) = \underset{H\in\{1,0\}}{\text{Argmin}}\ P_{H|S,C}(H|S, C) = \underset{H\in\{1,0\}}{\text{Argmin}}\ \frac{P_{S,C|H}\times P_H(H)}{P_{S,C}} = \underset{H\in\{1,0\}}{\text{Argmin}}\ \frac{P_{S|H}\times P_{C|H}\times P_H(H)}{P_{S,C}}$

by conditional independence

$g_{MAP} = 1$ if $\frac{P_{S|H=1}\times P_{C|H=1}\times P_H(H=1)}{P_{S,C}} > \frac{P_{S|H=0}\times P_{C|H=0}\times P_H(H=0)}{P_{S,C}}$

$g_{MAP} = 0$ otherwise.

b)

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P_S_H0 >>> array([ 0.42735043,  0.57264957])
P_S_H1 >>> array([ 0.14851485,  0.85148515])
P_C_H0 >>> array([ 0.09401709,  0.24786325,  0.4017094 ,  0.25641026])
P_C_H1 >>> array([ 0.03960396,  0.06930693,  0.16831683,  0.72277228])

Probability of error 0.18
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c)$g_{MAP}(S,C,X) = \underset{H\in\{1,0\}}{\text{Argmin}}\ P_{H|S,C}(H|S,C,X) = \underset{H\in\{1,0\}}{\text{Argmin}}\ \frac{P_{S,C,X|H}\times P_H(H)}{P_{S,C,X}} =$
$\underset{H\in\{1,0\}}{\text{Argmin}}\ \frac{P_{S|H}\times P_{C|H}\times f(X|H)\times P_H(H)}{P_{S,C}}$ by conditional independence

$g_{MAP} = 1$ if $\frac{P_{S|H=1}\times P_{C|H=1}\times f(X|H=1)\times P_H(H=1)}{P_{S,C}} > \frac{P_{S|H=0}\times P_{C|H=0}\times f(X|H=0)\times P_H(H=0)}{P_{S,C}}$
$g_{MAP} = 0$ otherwise

d)

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mean_X_H >>> array([ 245.57264957,  254.02970297])
std_X_H >>> array([ 46.52070879,  45.01890812])
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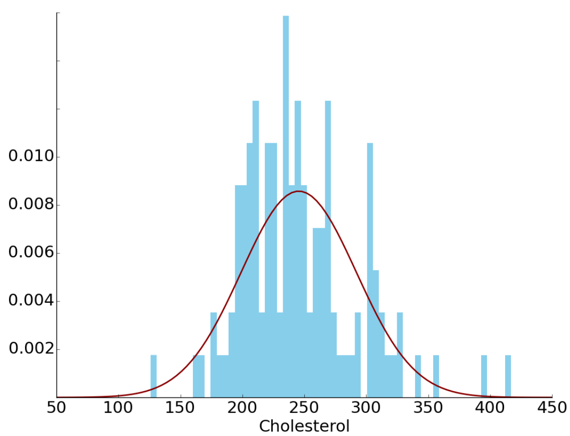
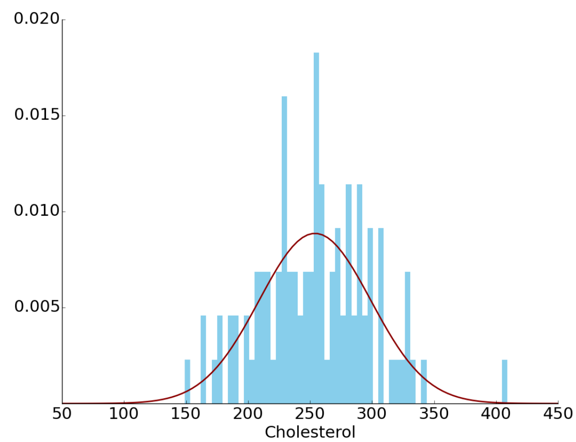


Cholesterol with H=0



Cholesterol with H=1

e)Incorporating with the cholesterol data, we have a new probability of error 0.14.

We can't trust this result, because the Cholesterol data doesn't seems to be perfectly normally distributed from the histograms. What's more, if we conduct a pairwise t-test to cholesterol level with H=0 and H=1, we have a p-value 0.177. This means we don't have enough evidence to conclude that people with different H have difference in cholesterol. Therefore it's not reasonable to include the cholesterol into the Bayesian model.

f)