Statistical and Mathematical Methods DS-GA 1002 November 11, 2015

Homework 7 Solutions

1. (10 points) *Short questions.*

   a. No. In fact, it can still be quite clear that it doesn't hold, but we cannot rule it out up to the significance level that we want to.

   b. No. In a frequentist setting the null hypothesis is either true or not deterministically.

   c. A test with large size and large power. We don't mind having false positives, but we want to be able to observe effects.

   d. To make sure we need the test to have small size.

   e. We might lose power to the point where with high probability we will not detect anything at all.

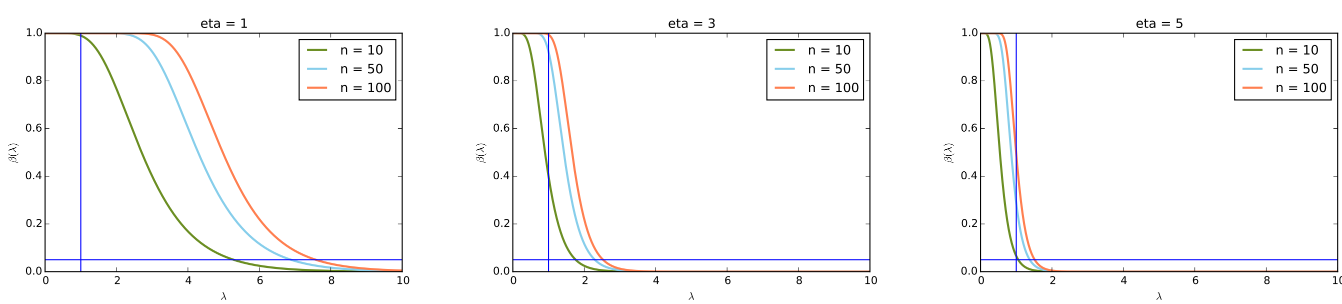
2. (10 points) *Cars*

   a. Let τ be the time a car breaks down for the first time. $\tau \sim exp(\lambda)$ so that the expected time a car breaks is $1/\lambda$ measured in years. The company wants have this time to be more than a year so we set the alternative hypothesis as $\lambda \leq 1$. Therefore the null hypothesis is $\lambda > 1$. It seems reasonable to reject the null hypothesis if the observation of the maximum time of break down is even higher than a threshold.

   b. Power function is the probability of rejecting the null hypotheses when the alternative hypotheses is true. For $\lambda \leq 1$:

$$
\begin{aligned}
\mathrm{P}(T(\tau) \geq \eta) &= 1 - \mathrm{P}(max(\tau_i) < \eta) && (1)\\
&= 1 - \prod_{i=1}^{n} \mathrm{P}(\tau_i < \eta) && (2)\\
&= 1 - \prod_{i=1}^{n} (1 - e^{-\lambda\eta}) = 1 - (1 - e^{-\lambda\eta})^n && (3)
\end{aligned}
$$

   c. For three different eta and n the plots show that higher threshold gives a sharper change in power function when λ passes from the alternative hypothesis to null hypothesis. However increasing the number of samples goes against this effect. For instance if you set a 0.005 significance level for the null hypothesis its apparent that for fixed number of samples eta should be rather large. Since increasing number of samples implies higher chances of observing more extreme values to reach the same size one needs to use a larger threshold.



3. (10 points) *Sign test.*

a. There is no difference in the number of people who have larger left ear vs larger right ear.

b. $L_1, \ldots, L_n$ be the counting random variable for the ones who have larger left ear.

$$\mathrm{P}(T(L_1, \ldots, L_n) \geq \eta) = \sum_{k \geq \eta} \binom{n}{k} \frac{1}{2^n} \tag{4}$$

c. 7 out of 10 has larger left ear. The probability that a statistic exceeding this value under the null hypothesis is

$$p = \mathrm{P}(T(L) \geq 7) = \frac{1}{2^{10}}(\binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}) \tag{5}$$

$$= 11/64 \tag{6}$$

Not small enough to reject the null hypothesis.

4. (10 points) *Permutation test for the median.*

a. The p values should be around 0.02.

b. The histogram of cholesterol levels in men has several very high values that make the sample mean large with respect to that of the women. The median does not take into account the values of these points, so the difference is not that extreme. The p value that we obtain, which is significantly larger than the p value for the difference of the sample means reflects this.