Problem Session 1

1. *Dependence.* From a well shuffled standard deck of 52 cards, pick two. What is the probability that the *second* card is black? Solve this in two ways (at least).

2. *Independence.* Give an example of three events that are pairwise independent but not independent.

3. *Which box?* Suppose there are three boxes, numbered from one to three, that contains 1 white and 1 black, 2 white and 1 black, 3 white and 1 black marbles, respectively. A box is drawn at random, and a ball is drawn from that box. Your goal is to guess the box that the ball came from.

   a. Which box would you guess if the ball drawn is white? This should be intuitively clear.

   b. What is your chance of winning?

   c. Now repeat the same game, but now I pick the box, and you don't know how I do it. What strategy should you follow?

4. *Factorization.* Show that $\mathrm{P}(A, B|C) = \mathrm{P}(A|C)\,\mathrm{P}(B|A, C)$

5. *Complement of the conditional.* Give an example of three events A, B, C such that A and B are conditionally independent given C, but A and B are not conditionally independent given the complement of C.

6. *Example 4.4 from lecture notes.* An example of two events that are independent but not conditionally independent.

7. *Change of variables.* Let X be uniformly distributed over $[0, 1]$, and $Y = \lambda^{-1}\log(X)$ where λ is positive. Find the distribution of Y.

8. *Sampling with or without replacement.* From a population of size N, sample n individuals one by one, at random. Assume successive draws are independent and each person has the same chance of being chosen. Suppose individuals are either good or bad.

   a. Find the probability of choosing g good and b bad when sampling with replacement.

   b. Find the probability of choosing g good and b bad when sampling without replacement.