Homework 1

Due Monday, September 21

1. *Stranded.* You are stranded by the road, so you decide to call some of your friends. You know they are not very trustworthy– in fact, each one will only come with probability 0.1– so you decide to call five of them and ask them to come get you without telling them that you called the others.

   a. If each of your friends decides to go get you independently from the others, what is the probability that three of your friends come?

   b. You realize that the independence assumption does not make sense (your friends tend to be in similar moods). Lower bound the probability that you are left stranded.

2. *Army camp.* In an army camp, there are 10 soldiers. For each soldier there is a probability of 0.2 that they have a certain disease (independently from each other). The doctor wants to determine whether any of them have it. Since the test is very expensive, he pools together blood samples from all the soldiers. If any of the soldiers has the disease the test will be positive with probability 0.9. If none of them have it, then the test will be positive with probability 0.1.

   a. One of the soldiers is called Joe. What is the probability that Joe has the disease if the test is positive?

   b. If the fridge of the camp is broken, which happens with probability 0.4, then the test always reads positive. Is the event *test is positive* independent of the event *Joe has the disease* given the event *fridge is broken*? Make assumptions if you need to and explain why they are reasonable.

   c. What is the probability that the fridge is broken if the test is positive?

3. *Old car.* Dani and Felix have a 2000 Ford Taurus. Every time they drive it, the car breaks down with probability 1/4 independently from the other times they drive it.

   a. The car luckily has not broken down for k drives; let us call this event E. Compare the probability that the car breaks down in the k'th drive *for the first time* and the probability that it breaks down in the $(k+k')$th drive *for the first time* given E. What does this imply about the distribution of the waiting time until the car breaks?

   b. What is the probability that the car breaks down n times in the first k drives?

   c. What is the probability that the nth time the car breaks down is in the kth drive?

   d. Another possible model is that the car breaks down with probability $1-2^{-k}$ in the kth drive (independently from the event that it break down during any other drive). Answer part (a) again under this new model. Which model do you think is more realistic?

4. *Radioactive decay.* You have access to the readings of a device that indicates whether a radioactive particle has decayed. However you do not get a continuous reading, you get a reading every hour.

   a. A reasonable model for the time the particle takes to decay is that it is an exponential random variable with parameter λ. What is the pmf of the number of hours it takes to get a reading indicating that the particle has decayed?

b. What is the pdf of the error between your reading and the true time of decay?

5. *Generating random variables.* You are building an embedded system which runs a program that needs to generate a continuous random variable with cdf F_X. However your only source of randomness is a hardware random generator that is able to generate a continuous random variable with cdf F_Y. You ask a friend what to do.

   a. Your friend suggests computing the cdf of $W = F_Y(Y)$. Assume that F_Y is invertible.

   b. Now your friend suggests computing the cdf of $F_X^{-1}(W)$. Why is your friend telling you to do all this?

   c. You realize that F_Y is not invertible and you get very upset. Your friend tells you it doesn't matter. Is it true or is she just being nice? (Hint: If your calculus is a bit rusty, assume that for a fixed constant w all the values of y such that $F_Y(y) = w$ belong to a closed interval $[a(w), b(w)]$.)