Problem Session 8

1. *Estimating mean of Gaussian.* A device receives information from a channel. The input received is normally distributed with unknown mean μ and known variance P.

   a. Represent the input as the sum of μ and a normal random variable, what is its mean and variance?

   b. Given a value of the input what is the maximum likelihood estimator for the mean μ?

   c. Suppose μ is distributed normally with mean zero and variance N. Find the posterior density. (Hint: Recall the 2^{nd} problem of Recitation Session 7!)

   d. Calculate the mean square error of the information coming from the first channel given the output. Find the minimizer of this.

   e. In which regime do the Bayesian estimation come close to the maximum likelihood estimation?

2. *Estimating mean of Poisson.* Another signal is assumed to be Poisson(λ).

   a. Given an observation what is the ML estimator?

   b. Suppose the prior distribution for λ is uniform over $[0, \theta]$ for some unknown θ. Given an observation what is the maximum a posteriori estimator?

   c. When does the MAP estimator converge to ML estimator?

   d. Impose a hard constraint on the ML to get the same MAP estimator without using Bayesian methods.

3. *Functions of estimators.* Suppose the task is estimating a parameter $\theta \in [0, 1]$ from an observation Y. Assume the prior density of θ is available for the Bayes estimators, MAP and MMSE, and the conditional density of Y given θ is known. Which ones of the below are true? If true, tell why and give an example in which it is not true.

   a. $2 + 3\hat{\theta}_{MAP} = (\widehat{2 + 3\theta})_{MAP}$

   b. $2 + 3\hat{\theta}_{MMSE} = (\widehat{2 + 3\theta})_{MMSE}$

   c. $2 + 3\hat{\theta}_{ML} = (\widehat{2 + 3\theta})_{ML}$