

Problem Session 7

1. *CLT and MLE.* You have 100 samples from $\text{binomial}(20, p)$.
 - a. Write the likelihood function for p .
 - b. Write the log-likelihood.
 - c. Compute the maximum likelihood estimator, p .
 - d. Using this find the MLE for the mean, μ_{ML} .
 - e. Is it biased?
 - f. Now suppose you are given $p = 0.65$, approximate $P(\mu_{\text{ML}} > 14)$.
2. *Normal priors.* Suppose the data is realizations of normal distribution with unknown mean μ . Set the prior density to normal with mean m and variance $1/s$.
 - a. Write the prior density.
 - b. Write the density for data given parameter emphasizing the difference between the parameter for the mean and the sample mean.
 - c. Write the posterior density of the parameter given data.
 - d. Recognize the posterior as a normal distribution, what is its mean and variance? How do you interpret the results?
3. *Which method?* A hospital is interested in length of stay for critically ill patients. A sample of such patients contain 30 female patients and 29 for the remaining sexes. Let's say the duration of stay is measured in days. And you would like to look at how the data of females contrasts. You plot the histograms of the duration of stay variable, and see that the distribution for females has a strong right skew, with mean 60 days and the median is 31.5 days. In contrast the remaining data is more symmetric with a mean of 30.8 days and median 29 days.