FORECASTING HOMEWORK 8

In these problems, we will consider Rupee, the exchange rate for the Indian Rupee to 1 U.S. Dollar. The data is daily from July 1st, 2002 to April 8th, 2011. (n=2235). We will work with the logs of the exchange rates. We will be fitting ARIMA-ARCH models to the data. To do this we will use R. You can use R on apps.stern.nyu.edu or download it from http://cran.r-project.org/. See the handout on using R (Windows and Mac versions), which has some basic facts about R and also details on transferring files between R and Minitab. Examples of fitting ARCH models in R are given in the handout on Estimation and Automatic Selection of ARCH Models.

1)

Plot the logs of Rupee. Based on this plot, and the ACF and PACF of the logs and differenced logs, does the series appear to be stationary? Can you identify an $ARIMA(p, d, q)$ model from these plots?

2)

Using AIC_C, select an $ARIMA(p, 1, q)$ (without constant) with $0 \le p \le 2$, $0 \le q \le 2$. Save the residuals and fitted values for the model you selected, using Storage $\rightarrow$ Residuals, Fits. The residuals will be stored in RESI1 and the fitted values will be stored in FITS1. (Note that FITS1 starts with one missing value, while at time t it represents $f_{t-1,1}$, the one-step forecast for the log exchange rate at time t made from time $t-1$). Also, get Minitab to compute the (ARIMA) one step ahead forecast and 95% forecast interval.

3)

Plot the residuals, as well as ACF and PACF of both the residuals and the squared residuals. Use these plots to argue that the residuals, although approximately uncorrelated, are not independent; instead, they show evidence of conditional heteroscedasticity.

Save the residuals (RESI1), using File → Other Files → Export Special Text. Be sure to save the file as an ANSI data (.DAT) file. I will use RES.DAT for the output file name. If you will be using R from apps.stern, save the file in your H: directory. If you will be using a version of R that is installed on your computer, save the file in a local directory (see the documents usingRwin or usingRmac for detailed instructions). Next, minimize the Minitab window, and open R.

To read the data set into R use the command >scan() as described in the usingR document. Download and load the tseries package that will be used to fit ARCH models, as described in the usingR document. You can now fit your arch models, as described in class and in the handout on Estimation and Automatic Selection of ARCH models.

4)

Using R on the residuals from the ARIMA model, find the log likelihood values and AIC_C values for $ARCH(q)$ models where q ranges from 0 to 10. You will need to calculate the log likelihood for the $ARCH(0)$ model by hand. See the handout on Estimation and Automatic Selection of ARCH models.

Next, consider a $GARCH(1,1)$ model. If the residuals from Minitab are stored in an R data set x, then the R command is >model=garch(x,c(1,1)). Evaluate AIC_C for the $GARCH(1,1)$ model, using $q=2$ in the formula for AIC_C. If the $GARCH(1,1)$ is preferred by AIC_C, use it as your selected model. Comment on the statistical significance of the parameter values of your selected model, as given by the summary(model) command. Write the complete form of the ARCH or GARCH model you have selected. Hand in the R output for the selected model, that is, the results of both summary(model) and logLik(model), but only for the one model that was selected by AIC_C.

5)

Using the Minitab output from problem 2, and the R output from your selected model in problem 4, construct a 95% one step ahead forecast interval for the log exchange rate, based on your ARIMA-ARCH model. (If you decided to use a *GARCH*(1,1) model, you will need to first get the conditional variances from R. See Problem 6.) Compare this to the interval based on the ARIMA only model from problem 2.

6)

Plot the conditional variances, ht, for your fitted ARCH model from problem 4. (See instructions below). Use this plot to locate bursts of high volatility. Do these highly volatile periods agree with those found from examination of the time series plot of the log exchange rates themselves?

To save the conditional variances and read them into Minitab, proceed as follows. First, re-fit your selected ARCH model and store it in the variable "model". Compute and store the conditional variances with the command >ht=model$fit[,1]^2.

Next, write the ht dataset to a file using the write() command as described in the usingR document. Minimize R and re-enter Minitab. Read htfile.dat into your worksheet, using File → Other Files → Import Special Text. The resulting column (let's call it ht) should have a length of 2235. So, for example, the last value is h_{2235}, the conditional variance for time 2235 (which can be computed based on information available at time 2234).

7)

Make a time series plot which simultaneously shows the log exchange rates, together with the ARIMA-ARCH one-step-ahead 95% forecast intervals based on information available the previous day. (See instructions below). Using the plot, together with the numerical values in your Minitab worksheet, comment on the accuracy and practical usefulness of the forecast intervals. Keep in mind that the performance may be somewhat better here than in an actual forecasting context, since the ARIMA-ARCH parameters are estimated from the entire data set, not just the

observations up to the time at which the forecast is to be constructed.

To compute the forecast intervals in Minitab, proceed as follows. First, get "low" and "high", the lower and upper endpoints of the 95% forecast intervals, using Calc → Calculator → Store result in variable: low , Expression : FITS1-1.96*sqrt(ht) → OK, and similarly for high. For a given t, the interval between low and high represents a one-step-ahead 95% forecast interval for the log exchange rate at time t based on information which was available at time $t-1$.

To plot the intervals along with the log exchange rates in Minitab, use Graph → Time Series Plot → Multiple. In Data View, click the box for "Connect Line" but un-check the box for "Symbols".

8)

Compute the residuals from your ARIMA-ARCH model, that is, $e_t = \varepsilon_t / \sqrt{h_t}$. If the ARIMA-ARCH model is adequate, these residuals should be normally distributed with mean zero and variance 1. To compute these residuals in Minitab, use Calc → Calculator → Store result in variable: archres, Expression: RESI1/sqrt(ht). Make a normal probability plot of archres, using Stat → Basic Statistics → Normality Test. Does the model seem to have adequately described the leptokurtosis ("long-tailedness") in the data?

9)

From the formula for the prediction intervals, it follows that the 95% prediction interval constructed yesterday fails to cover today's log exchange rate whenever today's residual exceeds 1.96 in absolute value. Use Calculator to count up how many failures there were, using sum(abs(archres)>1.96). What percentage of the time did the intervals fail? (Keep in mind that there are not 2235 data values in archres).