

## FORECASTING HOMEWORK 6

The data sets chaos1 and chaos2 (available on the course website) were generated with  $n=50$  by iterating the "tent map",

$$f(x) = \begin{cases} x/.6 & \text{if } 0 \leq x \leq .6 \\ (1-x)/.4 & \text{if } .6 < x \leq 1 \end{cases}.$$

We used  $x_0=.5$  for chaos1 and  $x_0=.501$  for chaos2.

- 1) Check that  $x_1=f(x_0)$  where  $\{x_t\}$  is the series of observations on chaos1, and  $f$  is the function defined above.
- 2) Plot chaos1 and chaos2, in separate plots. Do the series look random? Are they in fact random? Do the series look stationary?
- 3) Plot the ACF and PACF for chaos1. Based on these, suggest an ARMA model. Would this model provide the best possible forecasts?
- 4) Plot both chaos1 and chaos2 on the same plot. Do the paths look similar? Should they look similar when  $t$  is close to 1? Why? What should happen if chaos1 and chaos2 happen to get very close together at some later time? Use the plot to help justify your answer.
- 5) Plot  $x_2, \dots, x_{50}$  versus  $x_1, \dots, x_{49}$ , where  $\{x_t\}$  is the series of observations on chaos1. Does this reveal the map (in other words, the function  $f$ ) which generated the data? Do you see why this  $f$  is called the tent map? Does this plot help us to see that  $\{x_t\}$  is not an  $AR(1)$  series? How?