FORECASTING HOMEWORK 7

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Problem 1.

a) Since $X_t = X_{t-1} + \epsilon_t$ and $\epsilon_t = e_t + \beta e_{t-1}e_{t-2}$
$E(\epsilon_t \epsilon_{t+s}) = E(e_t e_{t+s} + \beta e_{t-1}e_{t-2}e_{t+s} + \beta e_t e_{t+s-1}e_{t+s-2} + \beta^2 e_{t-1}e_{t-2}e_{t+s-1}e_{t+s-2}$
Since e_t are strict white noise. They are independent.
if $s \neq 0$, LHS $= 0$
Therefore, ϵ_t is an uncorrelated white noise, thus not linearly predictable.
The best linear predictor of $X_{n+1} = E(X_n|X_n, X_{n-1}, ...) + 0 = X_n$

b) The best possible predictor of $X_{n+1} = E(X_{n+1}|X_n, X_{n-1}, ...) = E(X_n + \epsilon_{n+1}|X_n, X_{n-1}, ...)$
$= E(X_n|X_n, X_{n-1}, ...) + E(\epsilon_{n+1}|e_n, e_{n-1}, ...) = X_n + \beta e_{t-1}e_{t-2}$

c) X_t is not a martingale, since $E(X_{n+1}|X_n, X_{n-1}, ...) \neq X_n$

Problem 2.

a) $X_t = \alpha X_{t-1} + e_t$
$E(X_{t+1}|X_t, X_{t-1}, ...) = \alpha E(X_t|X_t, X_{t-1}, ...) + E(e_{t+1}|e_t, e_{t-1}, ...) = \alpha X_t$
If $\alpha < 1$, then $\alpha X_t \neq X_t, X_t$ is not a martingale.

b) If $\alpha = 1$,then $\alpha X_t = X_t, X_t$ is a martingale.

Problem 3.

a) $y_1 = \epsilon_1, y_2 = \epsilon_2 + \epsilon_1, \quad ... \quad y_t = \sum_{n=1}^{t} \epsilon_n$
$y_t - \epsilon_t = \sum_{n=1}^{t-1} \epsilon_n = y_{t-1}$
Therefore $y_t = y_{t-1} + \epsilon_t$

b) $E(y_{t+1} - y_t|y_t, y_{t-1}, ...) = E(\epsilon_{t+1}|y_t, y_{t-1}, ...) = E(\epsilon_{t+1}|\epsilon_t, \epsilon_{t-1}, ...)$
Since ϵ_t are martingale differences. $E(\epsilon_{t+1}|\epsilon_t, \epsilon_{t-1}, ...) = 0$
Hence, $E(y_{t+1} - y_t|y_t, y_{t-1}, ...) = 0$ which means, $E(y_{t+1}|y_t, y_{t-1}, ...) = y_t$.
Therefore, y_t is a a martingale.