Forecasting Homework 4

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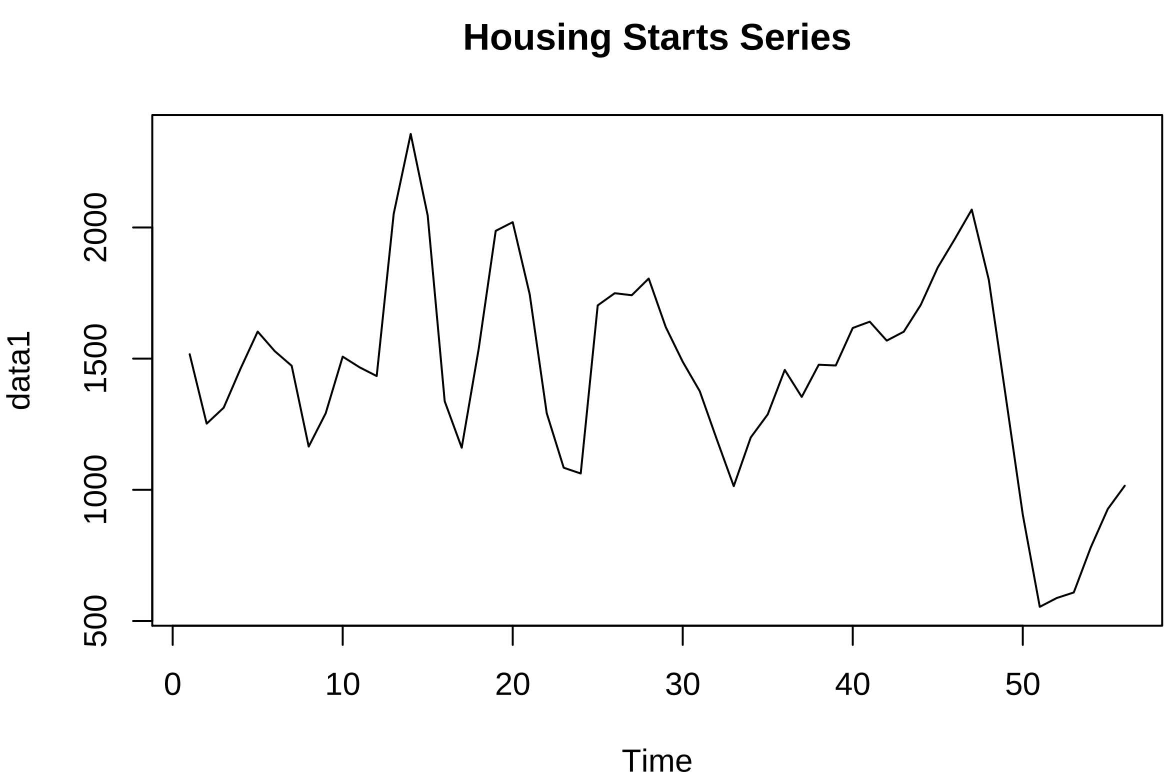
1)

$X_t = X_{t-1} - 5X_{t-2} + \epsilon_t$
we need to solve $Z^2 = Z - 5, Z = \frac{1 \pm \sqrt{-19}}{2} = \frac{1 \pm \sqrt{19}i}{2}$
$|Z| = \sqrt{5} > 1$, Therefore, the process is not stationary

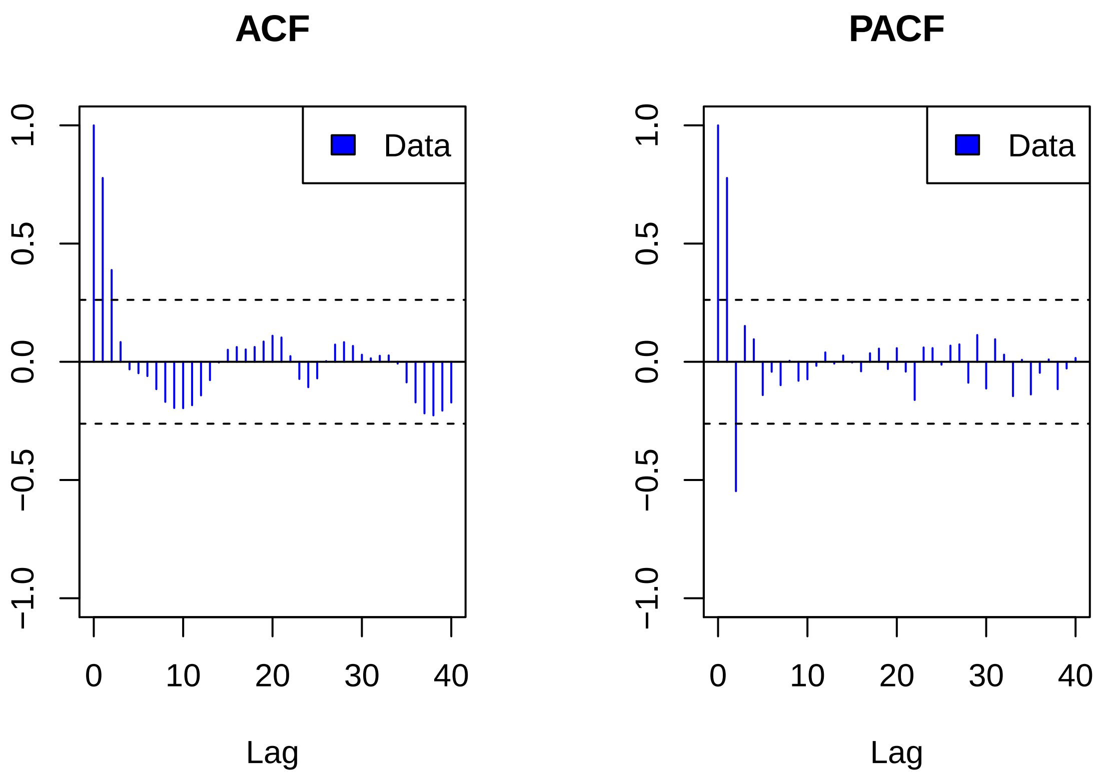
2)

```
library('itsmr')
data1 = read.csv("http://people.stern.nyu.edu/churvich/Forecasting/Data/HousingStarts.CSV"
                 ,header = T)[,2]
data2 = log(read.csv("http://people.stern.nyu.edu/churvich/Forecasting/Data/GDP.CSV"
                 ,header = T)[,2])
data3 =  diff(data2,1)
data4 = diff(log(read.csv('http://people.stern.nyu.edu/churvich/Forecasting/Data/CPI.CSV'
                 ,header = T)[,2]),1)
```

```
ts.plot(data1,main = 'Housing Starts Series')
```

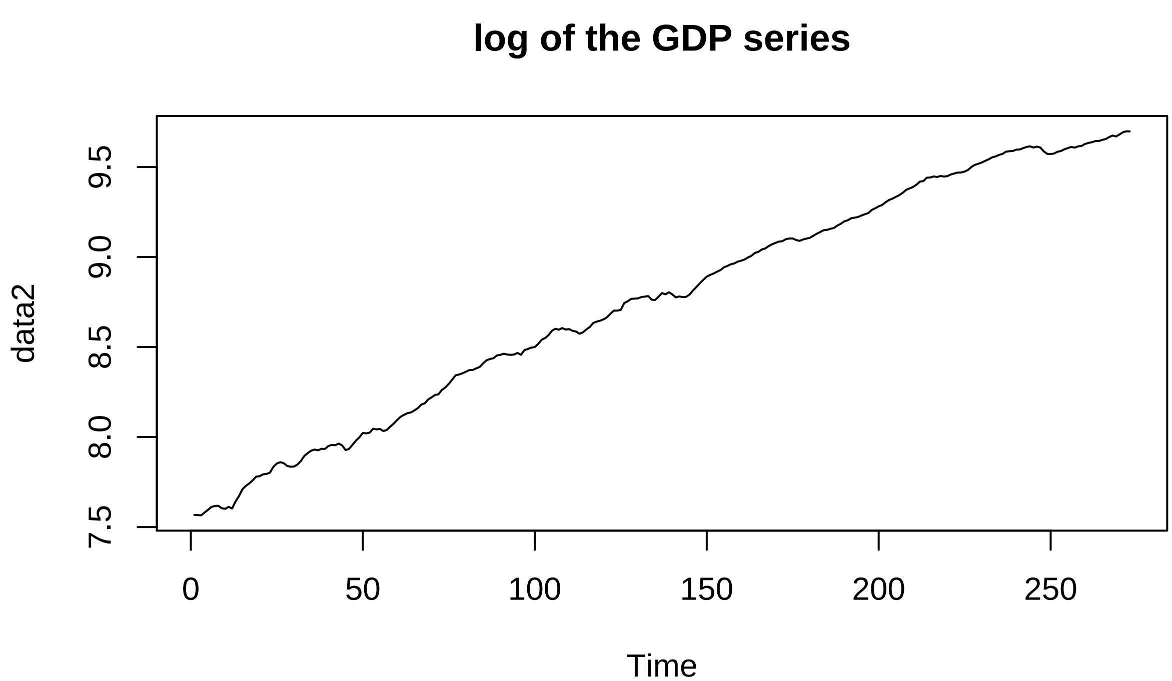


```r
plota(data1)
```

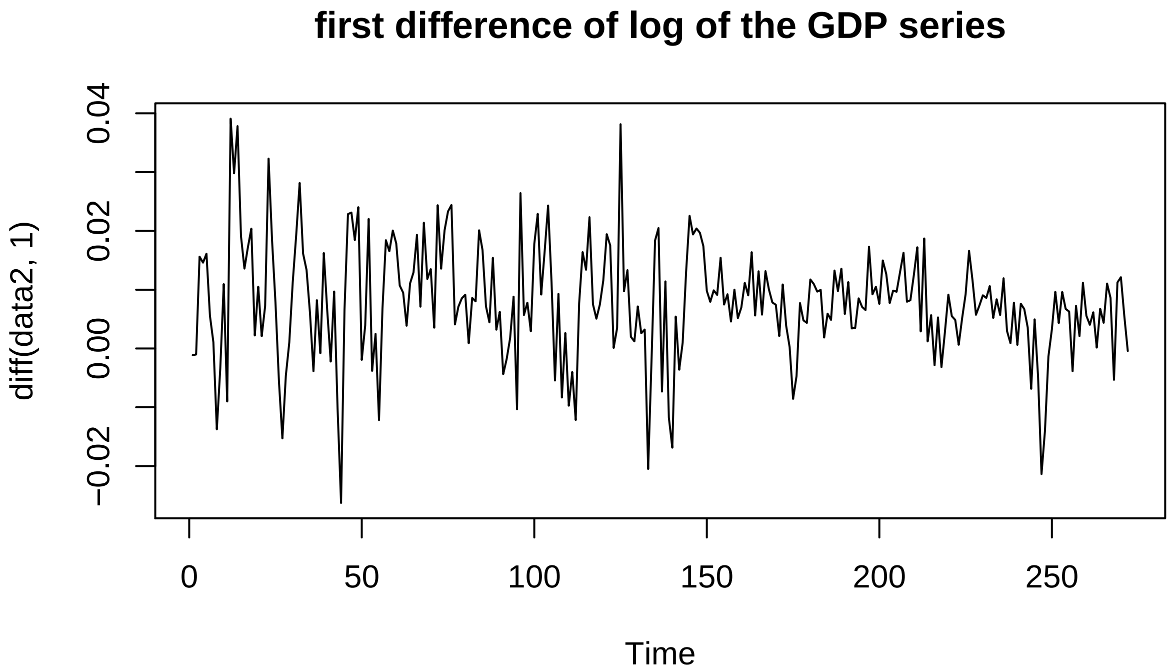


The timeseries plot shows that the series seems to be stationary.From the ACF/PACF plot for Housing Starts Series, we see that ACF cuts off after lag 2, and PACF dies down. Therefore, an ARIMA(0,0,2) would be reasonable.

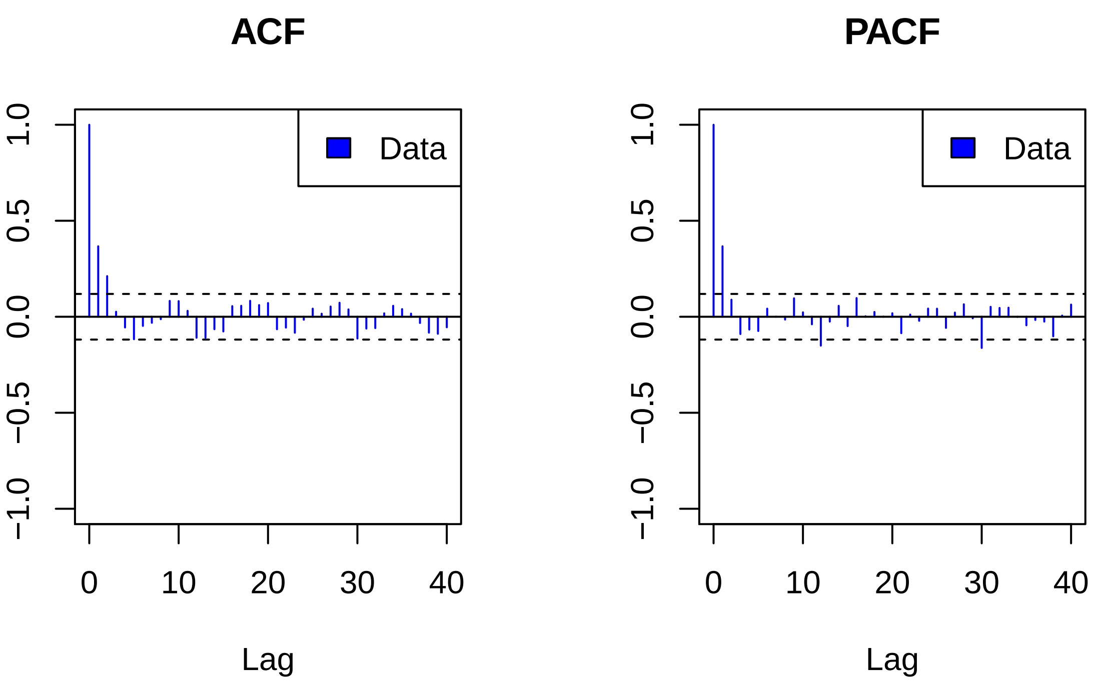
```r
ts.plot(data2,main = 'log of the GDP series')
```



```
ts.plot(diff(data2,1),main = 'first difference of log of the GDP series')
```



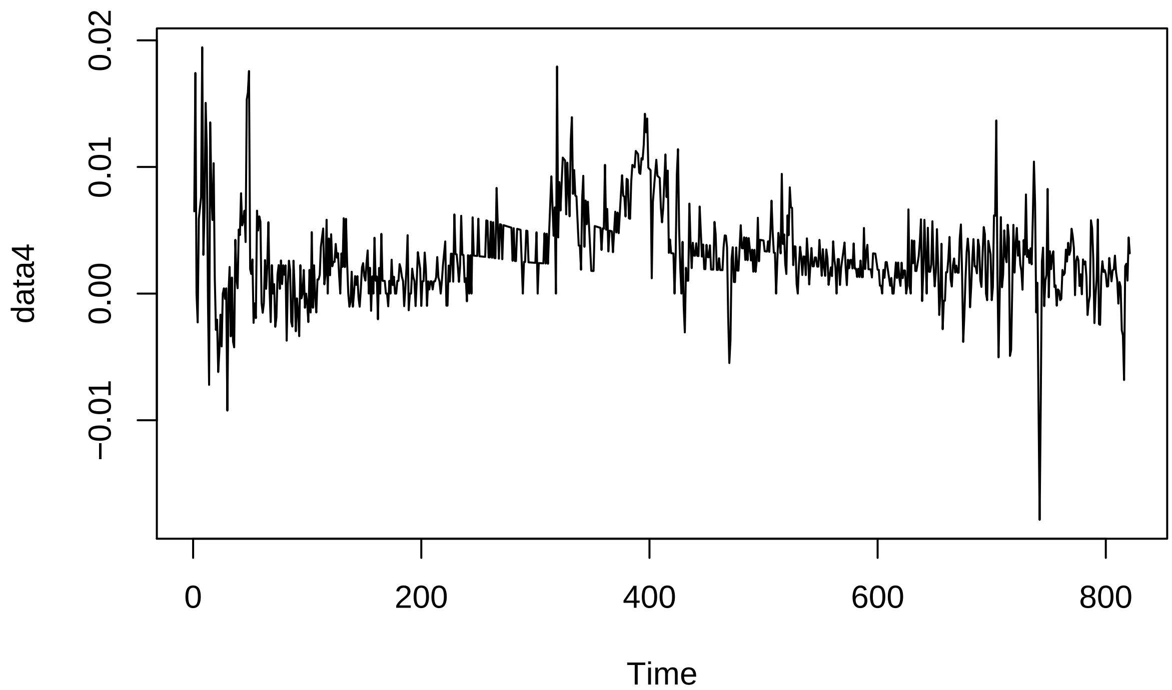

```
plota(diff(data2,1))
```



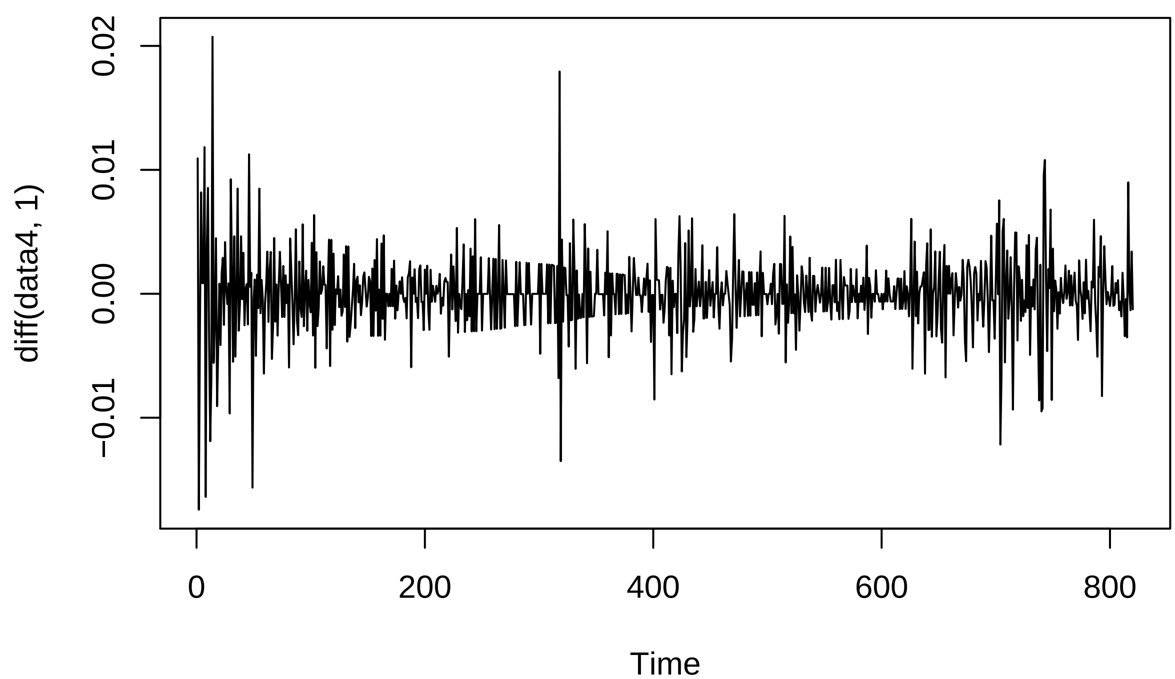
The time series plot for log of the GDP series shows that the series is nonstationary. The difference of the series seems to be stationary. From the ACF/PACF plot for first difference of log of the GDP series, we see that PACF cuts off after lag 11, and ACF dies down. Therefore, an ARIMA(11,1,0) would be reasonable.

For the first difference of log GDP, we have already see it's an ARIMA(11,0,0).

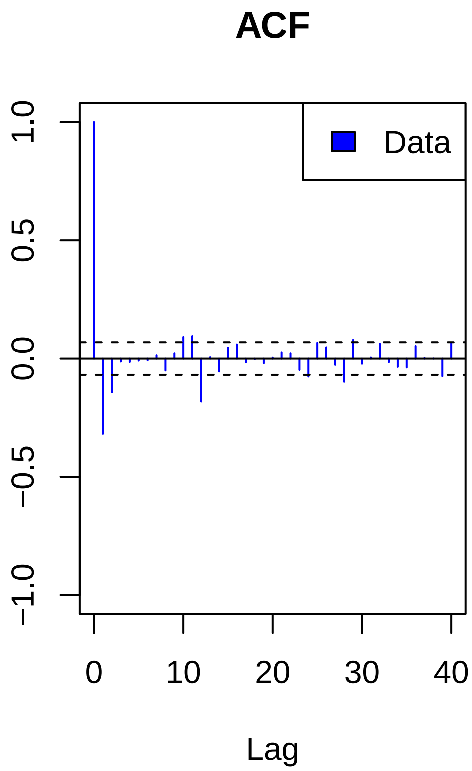
```
ts.plot(data4)
```

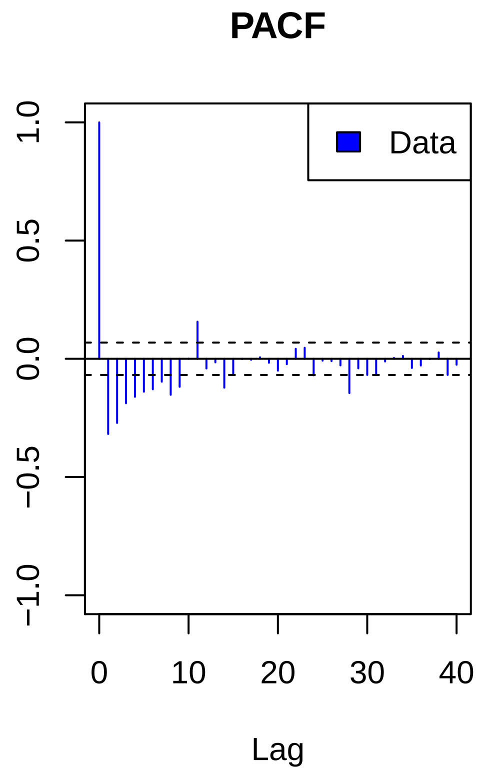



```
ts.plot(diff(data4,1))
```




```
plota(diff(data4,1))
```





The time series plot for the first difference of log CPI series shows that the series is nonstationary.The difference of the series seems to be stationary. The ACF/PACF shows that the ACF seems to die down, and PACF approximately cuts off after lag 11. The ARIMA(11,1,0) model may be appropriate.

3)

```
rho = acf(data3,plot = FALSE);r1 = rho$acf[2]
b1 = (1-sqrt(1 - 4*r1^2))/(2*r1);b2 = (1+sqrt(1 - 4*r1^2))/(2*r1)
c(b1,b2)
```

```
## [1] 0.437268 2.286927
```

For an MA(1) process,

$$b = \frac{1 \pm \sqrt{1 - 4\rho_1^2}}{2\rho_1}$$

Since we know that in order for an MA(1) to be invertible, b should be less than 1. Thus, we choose b1 = 0.437268. $X_t = \epsilon_t + 0.437268\epsilon_{t-1}$

4)

By Yule-walker equation for the AR(1) model, $r_1 = \hat{a_1} \times r_0, r_0 = 1$
Therefore, $\hat{a_1} = r_1 = 0.3670809, X_t = 0.3670809X_{t-1} + \epsilon_t$
To check stationary, we need to solve $Z = 0.3670809$. $|Z| < 1$, The model is stationary.

5)

a) For the AR(2) model, we need to solve:

$$\begin{bmatrix} r_2 \\ r_1 \end{bmatrix} = \begin{bmatrix} r_1 & r_0 \\ r_0 & r_1 \end{bmatrix} \begin{bmatrix} \hat{a_1} \\ \hat{a_2} \end{bmatrix}$$

$$\begin{bmatrix} \hat{a_1} \\ \hat{a_2} \end{bmatrix} = \begin{bmatrix} r_1 & r_0 \\ r_0 & r_1 \end{bmatrix}^{-1} \begin{bmatrix} r_2 \\ r_1 \end{bmatrix}$$

```
r0 = 1
r1 = rho$acf[2]
r2 = rho$acf[3]
a = solve(cbind(c(r1,r0),c(r0,r1)))%*%rbind(r2,r1);a
```

```
##            [,1]
## [1,] 0.33447193
## [2,] 0.08883324
```

Therefore,

$$a_1 = 0.33447193, a_2 = 0.08883324$$

$$X_t = 0.33447193X_{t-1} + 0.08883324X_{t-2} + \epsilon_t$$

b) The solution to

$$Z^2 = 0.33447193Z + 0.08883324$$

is $z_1 = -0.17447, z_2 = 0.50897$. Both

$$|z_1| < 1$$

and

$$|z_2| < 1$$

means the AR(2) model is stationary

c) The forecast of fisrt difference of log GDP is

$$D_{2015_{q1}} = a_1 D_{2014_{q4}} + a_2 D_{2014_{q3}}$$

```
n = length(data3)
predict_diff = a[1,1]*data3[n] + a[2,1]*data3[n-1];predict_diff
```

```
## [1] 0.0003432605
```

```
predict = tail(data2,1)+predict_diff;predict
```

```
## [1] 9.698508
```

The forecast of the first difference is 0.0003432605, the log gdp of 2014 q4 is 9.698165. Thefore the forecast of the 2015 q1 is 9.698165+0.0003432605 = 9.698508