

FORECASTING HOMEWORK 4

- 1) Consider the $AR(2)$ process $x_t = x_{t-1} - 5x_{t-2} + \varepsilon_t$. Determine whether the process is stationary.
- 2) Use the ACF and PACF to identify $ARIMA(p, d, q)$ models for the Housing Starts series, the log of the GDP series, the first differences of the log of the GDP series, and the first differences of the log of the CPI series (commonly known as "inflation"). Give reasons for your choices of p, d, q for each series. Do *not* try to estimate parameters. Just select p, d, q .
- 3) For the first difference of the log GDP series, use the method described in the handout for Chapter 3, Part IV, page 6 to estimate b in the invertible $MA(1)$ model $x_t = \varepsilon_t + b\varepsilon_{t-1}$.
- 4) For the first difference of the log GDP series, use the Yule-Walker equation $r_1 = \hat{a}_1 r_0$ to estimate a_1 in the $AR(1)$ model $x_t = a_1 x_{t-1} + \varepsilon_t$. Is your fitted model stationary?
- 5)
 - A) For the first difference of the log GDP series, use the two Yule-Walker equations

$$r_2 = \hat{a}_1 r_1 + \hat{a}_2 r_0$$

$$r_1 = \hat{a}_1 r_0 + \hat{a}_2 r_1$$

to estimate a_1 and a_2 in the $AR(2)$ model $x_t = a_1 x_{t-1} + a_2 x_{t-2} + \varepsilon_t$.

- B) Prove that your fitted $AR(2)$ model is stationary. (It must be stationary, since it can be proved in general that AR models estimated by solving the Yule-Walker equations are *always* stationary).
 - C) Use your fitted model to forecast the log GDP (*not* just the first difference of the log GDP, but the log GDP itself) for the first quarter of 2015. (This is a one-step-ahead forecast for log GDP, based on an $ARIMA(2, 1, 0)$ model).