# section 3.1-3.2, Determinant & Inverses

given 2x2 matrix A : we determined that A is invert. if and only if...

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \underbrace{ad - bc}_{\text{we will use this}} \neq 0$$

More formally, ad-bc is determinant of A, det(A)

For a $n \times n$ matrix, we'll define $(n-1)\times(n-1)$ matrix $A_{ij}$ by deleting i row and j column

e.g. $A = \begin{pmatrix} 1 & 4 & -4 \\ -3 & 8 & 2 \\ 0 & 1 & -2 \end{pmatrix} \rightarrow$ we deleted row 2 & column 3

$\Rightarrow A_{23}$ is matrix $A_{23} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

Thus ... det(A) we'll define as

more specific

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \Rightarrow \det(A) = a\det(A_{11}) - b\det(A_{12}) + c\det(A_{13})$$

since we used row 1, this is row 1 expansion

there are other rows we would expand on if they are easier than row 1

we will also alternate between + - + - for each term

## Row & Column Expansions :

row $i$ Expansion ...

$$\det(A) = (-1)^{i+1} a_{i1} \det(A_{i1}) + (-1)^{i+2} \det(A_{i2}) + \cdots$$

determine + - + - order

column $j$ Expansion ...

$$\det(A) = (-1)^{j+1} a_{1j} \det(A_{1j}) + (-1)^{j+2} \det(A_{2j}) + \cdots$$

given $n \times n$ matrix ... we say it is ...

- upper-triangular if everything below diagonal is 0
- Diagonal if everything except diagonal is 0

## Upper-triangular :

$\det(A)$ = product of diagonal entries

## Properties of Determinant :

let A & B be $n \times n$ matrices ...

* 1. if $\det(A) \neq 0 \Rightarrow A$ is invert.

2. $\det(AB) = \det(A) \cdot \det(B)$

3. $\det(A^{-1}) = \dfrac{1}{\det(A)}$ , if $A$ is invert.

4. $\det(AB) = \det(BA)$

5. $\det(A^T) = \det(A)$