Section 1.1 , Systems of Linear Equation

variables

An equation is linear if expressed in form $c_1x_1 + c_2x_2 + \dots + c_nx_n = b$

coefficients

constant

* invalid linear eq'tns are if the variables are not left as seen in form (invalid e.g. $\sqrt{x}$ OR xy)

System of equations :

given systems of linear eq'tns ...

m rows

$$\begin{cases} a_{1n_1}x_1 + a_{1n_2}x_2 + a_{1n_3}x_3 + \dots a_{1n}x_n = b_1 \\ a_{2n_1}x_1 + a_{2n_2}x_2 + a_{2n_3}x_3 + \dots a_{2n}x_n = b_2 \\ \vdots \end{cases}$$

n columns

=> we call it **m×n system** (m eq'tns , n unknowns)

=> a_{ij} : i is the row # , j is column #

Solutions :

=> a solution to a system of lin. eq'tns is a solution to every equation (LS = RS)

=> **Consistency** :

- # of solutions ≥ 1 , consistent
- No solution , inconsistent

Augmented Matrices :

=> again with m rows & n columns ($m \times n$ matrix), we denote the j^{th} entry in row i as a_{ij}

i, j entry of A

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

=> An $m \times n$ matrix is defined as **augmented matrix** when system of equations form a matrix...

$$\underbrace{\begin{matrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \end{matrix}}_{\text{system of eq'tns}} \Rightarrow \underbrace{\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ & & \vdots & & \vdots \end{array}\right]}_{\text{this is aug. matrix}}$$

Row Operations :

=> we can only do any of these 3 operations...

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- Exchange order of 2 eq'tns
- Multiply both sides of an eq'tn by constant (not 0)
- Add a multiple of one eq'tn to another

=> we do this to achieve RREF