section 1.3, Homogenous Systems

A **Homogeneous system** of eq'tns is in form ...

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= 0 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= 0 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= 0
\end{aligned}
$$

all systems $=0$

solutions :

trivial sol'tn

- $x_1 = 0$, $x_2 = 0$, $x_3 = 0$ ... - is always a sol'tn
- Any other system is non-trivial

Linear Combination ...

=> for columns of same size n ...

$$ax + by$$

linear combination
(e.g. $x+y$, $x-2z$, etc.)

Homogeneous system w/ rank r ...

1. if x, y are sol'tn => any linear combination of $ax+by$ is also sol'tn

2. Any sol'tn x can be expressed as a linear combination of a set of "$\underbrace{\text{basic sol'tn}}_{\text{general sol'tn}}$" $\underbrace{x_1, x_2, \dots, x_k}_{\text{parameters}}$

3. basic sol'tns found by "grouping parameter"
=> # of sol'tns = $\underbrace{n - r}_{\text{columns - rank}}$

$$x = s\begin{pmatrix}\vdots\end{pmatrix} + t\begin{pmatrix}\vdots\end{pmatrix}$$

basic sol'tn

=> Find basic sol'tns of homogeneous; steps:

1. Find general sol'tn & express in column form

2. re-write general sol'tn as column, one for each parameter

3. factor out parameter => corresponding basic sol'tn