

section 1.2, Gaussian Elimination

Row Reduction Echelon form (RREF):

=> we say matrix is in RREF if it meets all 4 conditions...

- All rows of 0 are at very last row \leftarrow only if any
- First entry (not 0) of all rows is 1 (leading 1's)
- Leading 1's is to the right of higher leading 1
- All entries above/below leading 1's is 0

\swarrow optional

Row Reduction Algorithm:

=> How do we know which of 3 operations to do ...

1. If all entries are 0, RREF is achieved
2. Find first column with possible leading 1
3. divide first row by the row & column entry to get leading 1
4. Add/subtract multiples of first row from below rows (make 0's)
5. Repeat 1-4 until RREF

Not always followed by is good to know if stuck

Solving Systems :

⇒ suppose that we have RREF aug. matrix ...

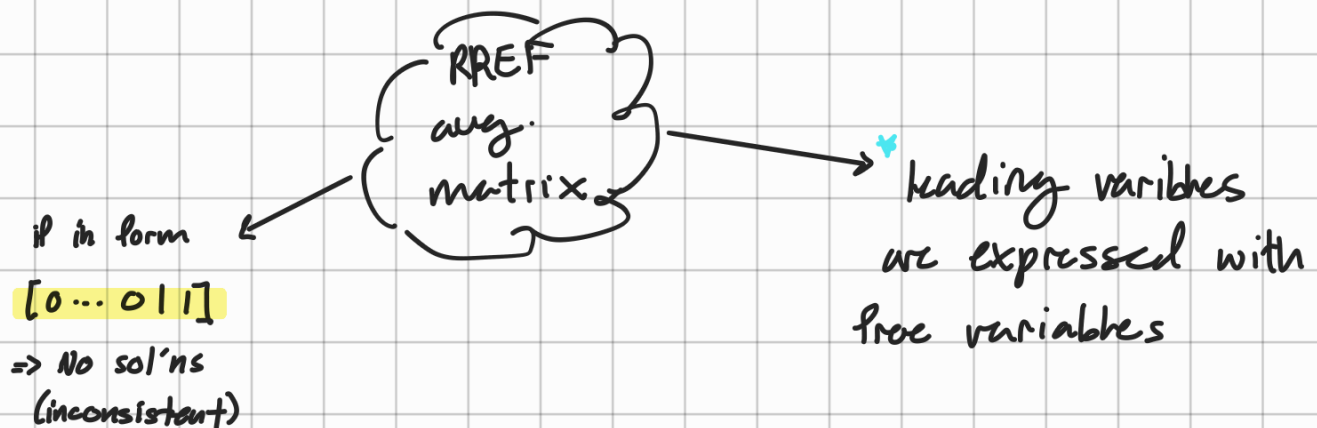
$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \end{array} \right]$$

↓
any column
w/leading 1
⇒ leading
variable

Not leading
⇒ free variable

we can express as
"any values" s, t, u
parameters for
leading variables

⇒ solving an aug. matrix means it must be in RREF



we generally will write the sol'n as a column...

$$x = \begin{bmatrix} x_1 = \dots \\ x_2 = \dots \\ x_{\dots} = \dots \\ t \\ s \\ \vdots \end{bmatrix}$$

← important later
on

Rank :

=> given a matrix (e.g. $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$), the Rank of A is # of leading 1's $\leftarrow \text{rank}(A)$


=> given a system of eq'ts, the rank is the rank of aug. matrix

Given that we know the rank(r) of a $m \times n$ system...

1. system is inconsistent => No sol'tn

2. system is consistent => $n = r$ => one unique sol'tn

3. " " consistent => $r < n$ => ∞ many sol'tns
=> sol'tns have $n - r$ parameters


$$\left[\begin{array}{ccc|c} \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & a \end{array} \right] \text{ where } a \in \mathbb{R}$$