section 1.2, Gaussian Elimination

Row Reduction Echelon form (RREF):

=> we say matrix is in RREF if it meets all 4 conditions...

- All rows of 0 are at very last row ← only if any
- First entry (Not 0) of all rows is 1 (leading 1's)
- Leading 1's is to the right of higher leading 1
- All entries above/below leading 1's is 0

Row Reduction Algorithm: ← optional

=> How do we know which of 3 operations to do...

Not always followed by is good to know if stuck

1. If all entries are 0, RREF is achieved
2. Find first column with possible leading 1
3. divide first row by the row & column entry to get leading 1
4. Add/subtract multiples of first row from below rows (make 0's)
5. Repeat 1-4 until RREF

Solving Systems :

=> suppose that we have RREF aug. matrix ...

$$\left[\begin{array}{cccc|c} ax_1 & ax_2 & \cdots & ax_n & b_1 \\ ax_1 & ax_2 & \cdots & ax_n & b_2 \\ & \vdots & & & \vdots \end{array}\right]$$

any column w/leading 1 => **leading variable**

Not leading => **free variable**

we can express as "any values" s, t, u parameters for leading variables

=> solving an aug. matrix means it must be in RREF

RREF aug. matrix

if in form **[0 ... 0 1 1]** => No sol'ns (inconsistant)

* leading varibles are expressed with free variables

we generally will write the sol'tn as a column ...

$$x = \begin{bmatrix} x_1 = \cdots \\ x_2 = \cdots \\ x \ldots \\ t \\ s \\ \vdots \end{bmatrix}$$

← important later on

Rank :

=> given a matrix (e.g. $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$), the Rank of A is # of leading 1's

rank(A)

=> given a system of eq'tns, the rank is the rank of aug. matrix

Given that we know the rank(r) of a $m \times n$ system ...

1. system is inconsistent => No sol'tn
2. system is consistent => $n = r$ => one unique sol'tn
3. " " consistent => $r < n$ => ∞ many sol'tns
   => sol'tns have $n-r$ parameters

$$\left[\begin{array}{ccc|c} \cdots & & \cdots & \cdots \\ 0 & \cdots & 0 & a \end{array}\right]$$ where $a \in \mathbb{R}$