section 2.1 - 2.3, Matrix Operations

=> Matrices are equal if same size & corresponding entries are equal ($A_{ij} = B_{ij}$, for all i, j)

Special Matrices:

=> Zero matrix: all entries are 0

=> Identity matrix: $n \times n$ w/ 1s along main diagonals & 0 in other entries

Scaling & Adding Matrices:

- Adding => $(A+B)_{ij} = A_{ij} + B_{ij}$ more simple than said
- Scaling => $(cA)_{ij} = c \cdot A_{ij}$

basic properties

=> $A+B = B+A$ | $A + 0 = A$

$(A+B)+C = A+(B+C)$ | $c(A+B) = cA + cB$

$(c+d)A = cA + dA$ | $(kl)A = k(lA)$

Transposes & Symmetric Matrices:

=> Transpose ... A^T

for $m \times n$, A => A^T is $n \times m$

switching rows & columns

e.g. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

- $(A+B)^T = A^T + B^T$
- $(kA)^T = k(A^T)$
- $(A^T)^T = A$

let A be $n \times n$ matrix...

- Symmetric $\Leftarrow A^T = A$
- Skew-symmetric $\Leftarrow A^T = -A$

Matrix Multiplication :

$\Rightarrow (AB)_{ij} = \sum_{l=1}^{n} a_{il} b_{lj}$ ← basically $x \cdot y = a_1b_1 + a_2b_2 + \dots + a_nb_n$

only works if # of columns A = # of rows B

$m \times n$ $n \times l$

Properties...

- $A(BC) = (AB)C$
- $A(B+C) = AB + AC$
- $(A+B)C = AC + BC$
- $I_m A = A$
- $A I_n = A$
- $(AB)^T = B^T A^T$

Connection to Systems :

$\Rightarrow$ let A be coefficients
x be column of unknowns
b be column of constant

$\Rightarrow$ system expressed $Ax = b$

e.g. $\begin{array}{l} x+2y+3z=4 \\ 5x+6y+7z=9 \end{array}$

$\Rightarrow A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$

$Ax = b$ has sol'tn if & only if b is linear combination of A

$Ax = b$

$\Rightarrow xA + yA = b$

(linear combination)

- let x_p be a sol'tn to $Ax = b$ $\Rightarrow$ any sol'tn to system $Ax = b$ is $x = x_p + \underbrace{x_0}$

x_0 is $Ax = 0$ homogeneous system

x_0 will have parameters