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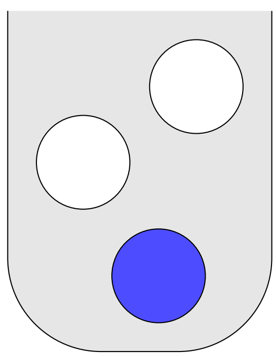
Problem Solving Methods: Using Factorial and "M Choose N"

Video companion

1 Introduction

Urn—a container you cannot see into

Example Drawing a marble from an urn containing two white and one blue marble. Can draw with or without replacement. Drawing with replacement means events are independent.



With replacement:

Draw	Probability
1 white	2/3
1 blue	1/3
2 white (in a row)	$(2/3)(2/3) = 4/9$

Without replacement:

Draw	Probability
1 white	2/3
1 blue	1/3
2 white (in a row)	$(2/3)(1/2) = 1/3$

2 Factorial

A factorial is the operation where we take a number and multiply it by each integer that is 1 less until we get down to 1.

Notation: 5! is read "five factorial." The operation means:

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120.$$

Factorial quotients:

$$\frac{7!}{5!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 7 \cdot 6 = 42$$

Convention:

$$0! = 1$$

3 "m choose n"

Draw n items from a group of m items without replacement.

Example: How many unique committees of five people from a group of ten people? In this example, "10 choose 5," $m = 10$ and $n = 5$. The notation is given by:

$$\begin{aligned}\binom{10}{5} &= \frac{10!}{5! \cdot 5!} \\ &= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2 \cdot 3 \cdot 2 \cdot 7 \cdot 3 = 252\end{aligned}$$

General formula

$$\left.\binom{m}{n} = \frac{m!}{(m-n)! \cdot n!}\right\} \text{Combination probability}$$