Data Science Math Skills
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Problem Solving Methods: The Sum Rule, Conditional Probability, and the Product Rule

Video companion

1 Marginal probabilities and the sum rule

Often know the joint probabilities, but don't know individual probabilities.

Table of known joint probabilities:

(X,Y)		X		
		x_1	x_2	x_3
	y_1	$P(x_1,y_1)$ 0.01	$P(x_2,y_1)$ 0.02	$P(x_3,y_1)$ 0.03
Y	y_2	$P(x_1,y_2)$ 0.10	$P(x_2,y_2)$ 0.20	$P(x_3,y_2)$ 0.49
	y_3	$P(x_1,y_3)$ + 0.04	$P(x_2,y_3)$ 0.05	$P(x_3,y_3)$ 0.06
		= 0.15		

Can refer to $P(x_1)$ as the "marginal probability of x_1" because it is in the margins of the matrix.

Sum rule: The marginal probability is equal to the sum of the joint probabilities.

For x_1, this means:

$$\begin{aligned} P(x_1) &= P(x_1,y_1) + P(x_1,y_2) + P(x_1,y_3) \\ &= 0.01 + 0.10 + 0.04 = 0.15 \end{aligned}$$

and for y_2:

$$\begin{aligned} P(y_2) &= P(x_1,y_2) + P(x_2,y_2) + P(x_3,y_2) \\ &= 0.10 + 0.20 + 0.49 = 0.79 \end{aligned}$$

Sum rule for binary probability distribution:

$$P(A) = P(A, B) + P(A, \sim B)$$

same formula, but for: 2 cases / 2+ cases

Sum rule for series of n probabilities:

$$P(A) = P(A, B_1) + P(A, B_2) + ... + P(A, B_n)$$

2 Conditional probability

Definition:
conditional probability—the probability that a statement is true given that some other statement is true with certainty.

Symbol $P(A \mid B)$ means the "probability of A given that B is true with certainty."
if 100% true

Example: What is the probability of rolling a 3 on a 6-sided die, given that the roll is odd? Of the outcomes when the roll is odd (3), one is the relevant outcome, so the probability is 1/3.

Example: What is the probability of rolling an odd, given that the roll is a 3? Of the outcomes when the roll is a 3 (1), one is the relevant outcome, so the probability is 1.

Formula for conditional probability:

$$P(A \mid B) = \frac{\text{(relevant outcomes)}}{\text{(total outcomes remaining in universe, when } B \text{ is true)}}$$

3 Product rule

Want to relate concepts of joint probability, marginal probability, and conditional probability.

Product rule:

$$P(A \mid B) = \frac{P(A, B)}{P(B)}$$

Conditional probability of A given that B is true is equal to the joint probability that A and B are true, divided by the probability that B is true.

Old definition of independence:

$$P(A, B) = P(A)P(B)$$

Dividing by $P(B)$ gives

$$P(A) = \frac{P(A, B)}{P(B)}$$

same as: $P(A|B) = \frac{P(A,B)}{P(B)}$

certain

Using the product rule gives another definition of independence.

New definition of independence:

$$P(A \mid B) = P(A)$$

Intuitively, this means knowing that B is true tells us nothing about the probabilities of A. The outcome B has no effect on A; therefore, they are independent.

Conversely, if $P(A \mid B) \neq P(A)$, then the events are *dependent*.