Fast Growth, Slow Growth:
The Rate of Growth of Continuous Processes

Video companion

1 Introduction

"Exponential rate of growth" can be a *discrete* exponential rate of growth or a *continuous* exponential rate of growth

Discrete rate of growth $\left(\boxed{1+R}\right)^t$

$\$1.00(1+r)^t$

How much money would grow in discrete intervals of time t

If $r = 100\%/\text{year}$ and $t = 1$, then would have \$2.00 after one year,
After 2 years, would have \$4.00
After 3 years, would have \$8.00...

2 Continuous exponential growth

$e^{Rt} = \left(e^R\right)^t$

Euler's constant e

100% interest per year (discrete)
50% interest for 6 months, then interest on interest for another 6 months.

Interval	Factor	Repeats		Result
1 year	$1+1$	1	$(2)^1 =$	2
6 months	1.5	2	$(1.5)^2 =$	2.25
3 months	1.25	4	$(1.25)^4 =$	2.44

As time intervals decrease, does result increase in an unlimited way?

No...

Interval	Factor	Repeats		Result
1 month	1.08	12	$(1.08)^{12} =$	2.613
1 week	1.019	52	$(1.019)^{52} =$	2.693
1 day	1.002739	365	$(1.002739)^{365} =$	2.7146
1 hour	1.000114	8760	$(1.000114)^{8760} =$	2.71813
1 minute	1.0000019	525,600	$(1.0000019)^{525{,}600} =$	2.71828
1 second	1.0000000317	31,536,000	$(1.0000000317)^{31{,}536{,}000} =$	2.71828

$e = 2.71828$, Euler's constant

Problem A baby elephant weighing 200 kg grows at a continuously compounded rate of 5%/year. How much does it weigh in 3 years?

$$(200 \text{ kg})e^{(0.05)(3)} = 232.4 \text{ kg}$$

naturally occuring,
continuous rates
of growth

3 Continuous rate of return

"Log to the base e of x" is given by the symbol $\ln(x)$, where ln stands for *natural logarithm*.

Problem Rabbit population increases in mass at a rate of 200% per year. Population starts at 10 kg. If they increase at a continuously compounded rate, how many years is it until they weigh as much as the Earth (5.972×10^{24} kg)?

$$\begin{aligned} 5.972 \times 10^{24} \text{ kg} &= (10 \text{ kg})e^{2t} \\ 5.972 \times 10^{23} &= e^{2t} \\ \ln(5.972 \times 10^{23}) &= \ln(e^{2t}) = 2t \\ \frac{\ln(5.972 \times 10^{23})}{2} &= t = 27.37 \text{ years} \end{aligned}$$