Data Science Math Skills
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Bayes' Theorem: Updating with New Data

Video companion

1 Updating probabilities

Bayes' theorem allows for updating probabilities based on new data.

From previous solution:

Process		P(white marble)
Urn 1	A_1	20%
Urn 2	A_2	10%

$P(\text{Urn 1} \mid \text{3 white marbles in a row}) = 8/9$
$P(\text{Urn 2} \mid \text{3 white marbles in a row}) = 1/9$

New information: We draw a fourth marble that is also white.

$P(A_1)$ and $P(A_2)$ become our *new prior probabilities* or *new priors.* because we're starting on choosing #4, not initial

$$
\begin{aligned}
&P(\text{urn 1} \mid \text{3 white marbles in a row, } \textit{and} \text{ a 4th}) \\
&= \frac{P(\text{white} \mid \text{urn 1})P(\text{urn 1})}{P(\text{white} \mid \text{urn 1})P(\text{urn 1}) + P(\text{white} \mid \text{urn 2})P(\text{urn 2})} \\
&= \frac{(0.2)\boxed{(8/9)}}{(0.2)\boxed{(8/9)} + (0.1)\boxed{(1/9)}}
\end{aligned}
$$

$P(\text{urn 1}) = 94.12\%$ $\uparrow$
$P(\text{urn 2}) = 5.88\%$ $\downarrow$

2 Technical vocabulary

Technical vocabulary of Bayesian inverse probability:

$$\underbrace{P(\theta_i \mid D)}_{\text{posterior probability}} = \frac{\overbrace{P(D \mid \theta_i)}^{\text{likelihood}} \; \overbrace{P(\theta_i)}^{\text{prior probability}}}{\underbrace{P(D)}_{\text{marginal probability}}}$$

before any data: $\boxed{\%/n}$ (2 urns = 50%)
after new data: the $\boxed{\text{old posterior}}$ probability

posterior probability—probability after new data is observed
prior probability—probability before any data is observed or before new data is observed
likelihood—standard forward probability of data given parameters
marginal probability—probability of the data

$P(D) = \sum_{n=1}^{n} P(D \mid A_n) \cdot P(A_n)$