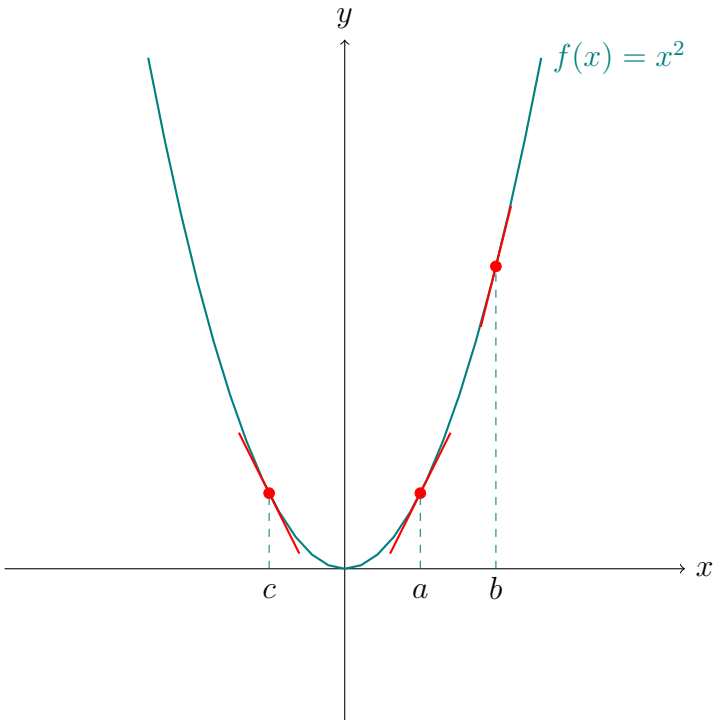
Tangent Lines: The Derivative Function

Video companion

1 Introduction

Derivative formula:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



- Slope is positive at a: $f'(a) > 0$
- Slope is positive at b and greater than at a: $f'(b) > f'(a)$
- Slope is negative at c: $f'(c) < 0$

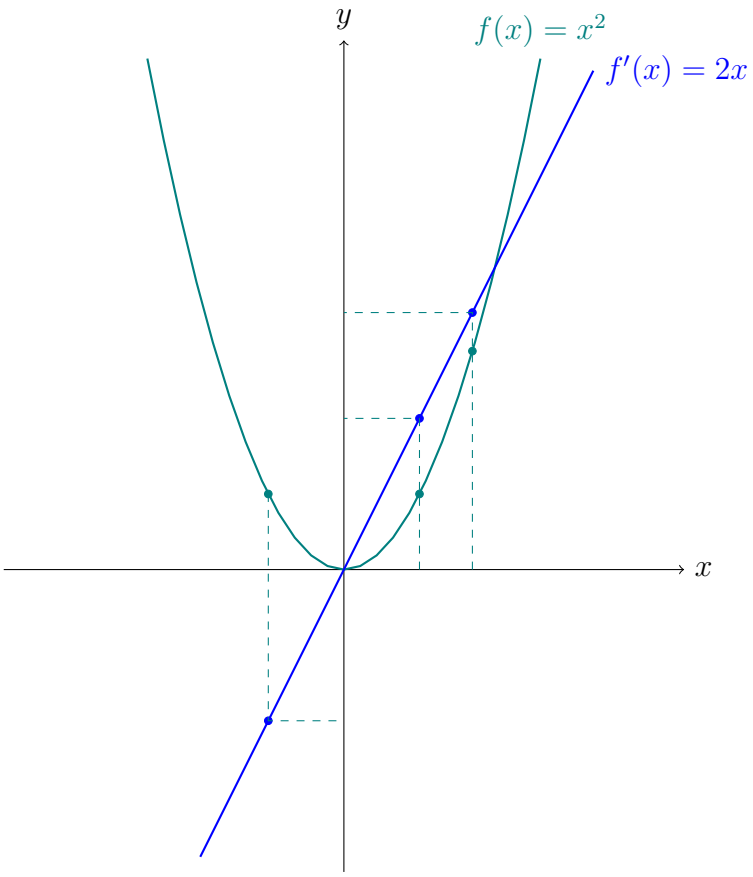
2 Calculate derivative

$$\begin{aligned} f'(a) &= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \\ &= \lim_{h \to 0} \frac{(a+h)^2 - a^2}{h} \\ &= \lim_{h \to 0} \frac{a^2 + 2ah + h^2 - a^2}{h} \\ &= \lim_{h \to 0} \frac{h(2a+h)}{h} \\ &= \lim_{h \to 0} (2a+h) \\ &= 2a \end{aligned}$$

$$\begin{aligned} f'(a) &= 2a \\ f'(b) &= 2b \\ f'(c) &= 2c \end{aligned}$$

Can verify $2a > 0$, $2b > 2a$, and $2c < 0$

3 Graph of derivative function



Next video: Finding where derivative is zero (where the tangent line to the function is horizontal) is important for optimization problems.