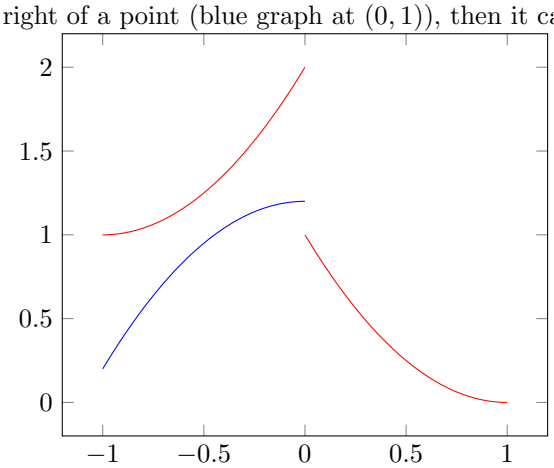
Maximum and Minimum

Patrick Chen

Oct 3, 2024

A absolute maximum is the maximum value that a function can produce. The absolute minimum is the minimum value that a function can produce. A local maximum is a point where any direction in the input space will produce a lesser value and likewise for local minimum. A point can be both a absolute maximum and local maximum. If function is undefined to the left or right of a point (blue graph at $(0, 1)$), then it cannot be an local max or min.



If a function has a "max", but it is on a open interval and thus not part of the graph, then it is not considered a max.

Extreme Value Theorem

If f is continuous on a closed interval [a,b], the then f attains an absolute maximum value f(c) and an absolute minimum value f(d) at some numbers c and d in [a,b].

Fermat's Theorem

If f has a local maximum or minimum at c, and $f'(c)$ exists, then $f'(c) = 0$. Note that if the derivative is zero, that does not necessary mean that a local extreme exists. Local extreme values can also happen when $f(c) \neq 0$. When the derivative $f'(c)$ is zero or does not exists, then c is considered a critical number.

Finding Maximum and Minimum

The absolute maximum and minimum of a continuous function on the closed interval $[a, b]$ can be found by following these steps:

1. Find the values of f at the critical numbers of f in (a,b)
2. Find the values of f at the endpoints of the interval
3. The largest of the values from steps 1 and 2 is the absolute maximum and the least of the values is the absolute minimum on the interval [a,b]

Example

$$
\begin{aligned}
f(x) &= 2\cos x + \sin 2x \\
f'(x) &= -2\sin x + 2\cos 2x \\
0 &= -\sin x + \cos 2x \\
0 &= -\sin x + \cos^2 x - \sin^2 x \\
0 &= -\sin x + 1 - \sin^2 x - \sin^2 x \\
0 &= -2\sin^2 x - \sin x + 1 \\
\sin x &= \frac{-1 \pm 3}{4} \\
\sin x &= -1, \frac{1}{2} \\
x &= \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}
\end{aligned}
$$