Transformations

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Transformation

A transformation from \mathbb{R}^n to \mathbb{R}^m is a function which assigns to every vector in \mathbb{R}^n some vector in \mathbb{R}^m . If T is a function, we write $T: \mathbb{R}^n \mapsto \mathbb{R}^n$. The transformation T has a domain of \mathbb{R}^n and a codomain of \mathbb{R}^m . The range of T is the set of all things from T(x) can output. Note that the range is a subset of the codomain.

$$T_A: \mathbb{R}^n \mapsto \mathbb{R}^m$$

 $T_A(x) = Ax$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$T_A : \mathbb{R}^3 \mapsto \mathbb{R}^3$$

$$T_B : \mathbb{R}^3 \mapsto \mathbb{R}^2$$

Even though T_A and T_B have the same "range", it is not the same. Although \mathbb{R}^3 contains many planes, it formally does not contain \mathbb{R}^2 . $\mathbb{R}^2 \nsubseteq \mathbb{R}^3$ Consider:

$$A = \begin{bmatrix} 4 & 0 \\ 1 & 1 \\ 0 & 2 \end{bmatrix}$$
$$T_A : \mathbb{R}^2 \mapsto \mathbb{R}^3$$

The range of A is a two dimensional plane embedded in three dimensional space.