Matrix Equations

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Suppose $v_1 \dots v_m$ are vectors in $\mathbb{R}^n$. Let $A = [v_1 \dots v_m]$. If A is a $n \times m$ matrix and x is a vector in $\mathbb{R}^m$. We define $Ax = x_1 v_1 + x_2 v_2 + \cdots + x_m v_m$.

$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

When we create a matrix, it describes a linear combination of its columns

Determining if a vector is in the span of a set of vectors

$$Ax = b$$
$$\text{where } A = \begin{bmatrix} v_1 & v_2 & \dots & v_m \end{bmatrix}$$

The above equation is equivalent to determining if b is a linear combination of $v_1 \dots v_m$. x represents the coefficients of the linear combination.
In $\mathbb{R}^n$ for any n, the span of a single non-zero vector is a line. Any two non-zero vectors will span a line or a plane. If you want to determine if vectors $v_1 \dots v_m$ in $\mathbb{R}^n$, row reduce the matrix formed by all the vectors. If every row in the matrix has a leading 1, it will span all of $\mathbb{R}^n$.