Transformations

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Transformation

A transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a function which assigns to every vector in $\mathbb{R}^n$ some vector in $\mathbb{R}^m$. If T is a function, we write $T : \mathbb{R}^n \mapsto \mathbb{R}^m$. The transformation T has a domain of $\mathbb{R}^n$ and a codomain of $\mathbb{R}^m$. The range of T is the set of all things from $T(x)$ can output. Note that the range is a subset of the codomain.

$$T_A : \mathbb{R}^n \mapsto \mathbb{R}^m$$
$$T_A(x) = Ax$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$
$$T_A : \mathbb{R}^3 \mapsto \mathbb{R}^3 \qquad T_B : \mathbb{R}^3 \mapsto \mathbb{R}^2$$

Even though T_A and T_B have the same "range", it is not the same. Although $\mathbb{R}^3$ contains many planes, it formally does not contain $\mathbb{R}^2$. $\mathbb{R}^2 \nsubseteq \mathbb{R}^3$
Consider:

$$A = \begin{bmatrix} 4 & 0 \\ 1 & 1 \\ 0 & 2 \end{bmatrix}$$
$$T_A : \mathbb{R}^2 \mapsto \mathbb{R}^3$$

The range of A is a two dimensional plane embedded in three dimensional space.