## Forces and Work

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# Motions of Systems

- Internal forces is between objects in the system.
- External forces are form outside the system.

$$\begin{split} Ma_{cm} &= \sum_{j} F_{net,j} \\ &= \sum_{j} F_{net,j}^{ext} + \sum_{j} F_{net,j}^{int} \\ &= \sum_{j} F_{net,j}^{ext} + 0 \\ &= \sum_{j} F_{net,j}^{ext} \end{split}$$

Choosing a system depending on the problem can help solve the problem. When choosing a system, having friction on the boundary of the system will make analysis more difficult. Gravity will either be a change in potential energy or a force doing work on a system depending if it is in the system or not.

### Work

Work is the change in the energy of a system due to external forces. Work is a signed scalar quantity measured in joules (J) in the SI system.

$$W = \mathbf{F} \cdot \mathbf{d} = Fd\cos\theta$$
$$W = \Delta E$$

For non-constant forces, the work must be integrated

$$W = \int_{x_i}^{x_f} F \cdot dx$$

Force displacement is how far the point where the force is applied moves. In order for a force to do work, the point where the force is being applied must undergo a force displacement.

## Momentum and Energy

$$\begin{array}{ccc} \Delta p = J & \Delta E = W \\ J = 0 & \Delta E = 0 \\ J = \sum f \Delta t & W = \sum F \Delta x \end{array}$$

#### Example 1

Suppose there is a object with a horizontal for applied on a frictionless surface from point  $x_i$  to  $x_f$ .

$$W = \Delta E$$

$$= \Delta K + \Delta U + \Delta E_{th}$$

$$= \Delta K + 0 + 0$$

$$= \Delta K$$

$$F = ma_{cm}$$

$$= m \frac{dv_{cm}}{dt}$$

$$= m \frac{dv_{cm}}{dx_{cm}} \frac{dx_{cm}}{dt}$$

$$= mv_{cm} \frac{dv_{cm}}{dx_{cm}}$$

$$F dx_{cm} = mv_{cm} dv_{cm}$$

$$\int_{x_i}^{x_f} F dx_{cm} = \int_{v_i}^{v_f} mv_{cm} dv_{cm}$$

$$= \frac{1}{2} mv_{cm}^2 \Big|_{v_i}^{v_f}$$

$$= \Delta K_{cm}$$

$$F\Delta x_{cm} = \Delta K_{cm}$$

#### Example 2

Suppose there is a object with two horizontal forces in opposite directions on a frictionless surface.

$$\begin{split} W &= \Delta K \\ F_{net} &= m a_{cm} \\ F_{net} &= F_1 - F_2 \\ W &= \int_{x_i}^{x_f} \left( F_1 - F_2 \right) \, dx_{cm} \\ W_{net} &= \int_{x_i}^{x_f} F_1 \, dx_{cm} - \int_{x_i}^{x_f} F_2 \, dx_{cm} \end{split}$$

## Example 3

Suppose there is a object sliding with a friction force.  $F_k$  is a constant force. We can choose the system to be the object and the surface.

$$\Delta E = \Delta K + \Delta E_{th}$$

$$\Delta E = 0$$

$$\Delta E_{th} = -\Delta K$$

$$= -(K_f - K_i)$$

$$= K_i$$

$$\Delta K_{cm} = -\int_{x_{cm,i}}^{x_{cm,f}} f_x dx_{cm}$$

$$= -f_k \Delta x_c m$$