Curve sketching

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Example

$$xe^{\frac{1}{x}}$$

1. Check the domain of the function.

$$\begin{aligned} y_1 &= x & D &= \mathbb{R} \\ y_2 &= \frac{1}{x} & D &= \{x|x \in \mathbb{R}, x \neq 0\} \\ y_3 &= e^x & D &= \mathbb{R} \\ y_4 &= e^{\frac{1}{x}} & D &= \{x|x \in \mathbb{R}, x \neq 0\} \\ y &= xe^{\frac{1}{x}} & D &= \{x|x \in \mathbb{R}, x \neq 0\} \end{aligned}$$

2. Find the x and y intercepts.

$$f(0)$$

Since $x \neq 0$, there is no y-intercept.

$$xe^{\frac{1}{x}} = 0$$

Since $x \neq 0$ and e^x is never zero, there is no x-intercept

3. Symmetry

- Even functions are function that are symmetric about the y-axis. $f(-x) = f(x)$
- Odd functions are functions that are an symmetric about the origin. It can also be thought of as being the same graph when rotated half a revolution. $f(-x) = -f(x)$

$$\begin{aligned} f(-x) &= -xe^{-\frac{1}{x}} \\ f(-x) &\neq f(x) \\ f(-x) &\neq -f(x) \end{aligned}$$

$f(x)$ is neither even nor odd.

4. Periodic

   - A function is periodic if $f(x+p) = f(x)$ for all x in the domain, where p is a positive constant. The smallest value p satisfying this is called the period.

$$f(x+p) = (x+p)e^{\frac{1}{x+p}}$$

   This function is not periodic because no such p exists.

5. Find the asymptotes

   - Horizontal asymptotes occur when the limit of a function as the input approaches infinite remains finite. If the limit approaches infinity or negative infinity, then there is no horizontal asymptotes.

$$\lim_{x\to\infty} f(x) = L$$
$$\text{or}$$
$$\lim_{x\to-\infty} f(x) = L$$

   - Vertical asymptotes are where the limit as the function approaches some value is infinity or negative infinity

$$\lim_{x\to a^+} f(x) = \pm\infty$$
$$\text{or}$$
$$\lim_{x\to a^-} f(x) = \pm\infty$$

   Horizontal asymptotes of $xe^{\frac{1}{x}}$

$$\begin{aligned}&\lim_{x\to\infty} xe^{\frac{1}{x}}\\ =&\infty \cdot e^{\frac{1}{\infty}}\\ =&\infty \cdot e^{0}\\ =&\infty \cdot 1\\ =&\infty\end{aligned}$$

$$\begin{aligned}&\lim_{x\to-\infty} xe^{\frac{1}{x}}\\ =&-\infty \cdot e^{\frac{1}{-\infty}}\\ =&-\infty \cdot e^{0}\\ =&-\infty \cdot 1\\ =&-\infty\end{aligned}$$

For vertical asymptotes of $xe^{\frac{1}{x}}$, we can check the restrictions in the domain of the function.

$$\begin{aligned}\lim_{x\to 0^+} xe^{\frac{1}{x}} &= \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}}{\frac{1}{x}}\\ &\overset{H}{=} \lim_{x\to 0^+} \frac{-\frac{1}{x^2}e^{\frac{1}{x}}}{-\frac{1}{x^2}}\\ &= \lim_{x\to 0^+} e^{\frac{1}{x}} = e^{\frac{1}{0}} = e^{\infty} = \infty\end{aligned}$$

$$\begin{aligned}\lim_{x\to 0^-} xe^{\frac{1}{x}} &= \lim_{x\to 0^-} \frac{e^{\frac{1}{x}}}{\frac{1}{x}}\\ &\overset{H}{=} \lim_{x\to 0^-} \frac{-\frac{1}{x^2}e^{\frac{1}{x}}}{-\frac{1}{x^2}}\\ &= \lim_{x\to 0^-} e^{\frac{1}{x}} = e^{\frac{1}{-0}} = e^{-\infty} = 0\end{aligned}$$

6. Intervals of increase or decrease and local maximums and minimums

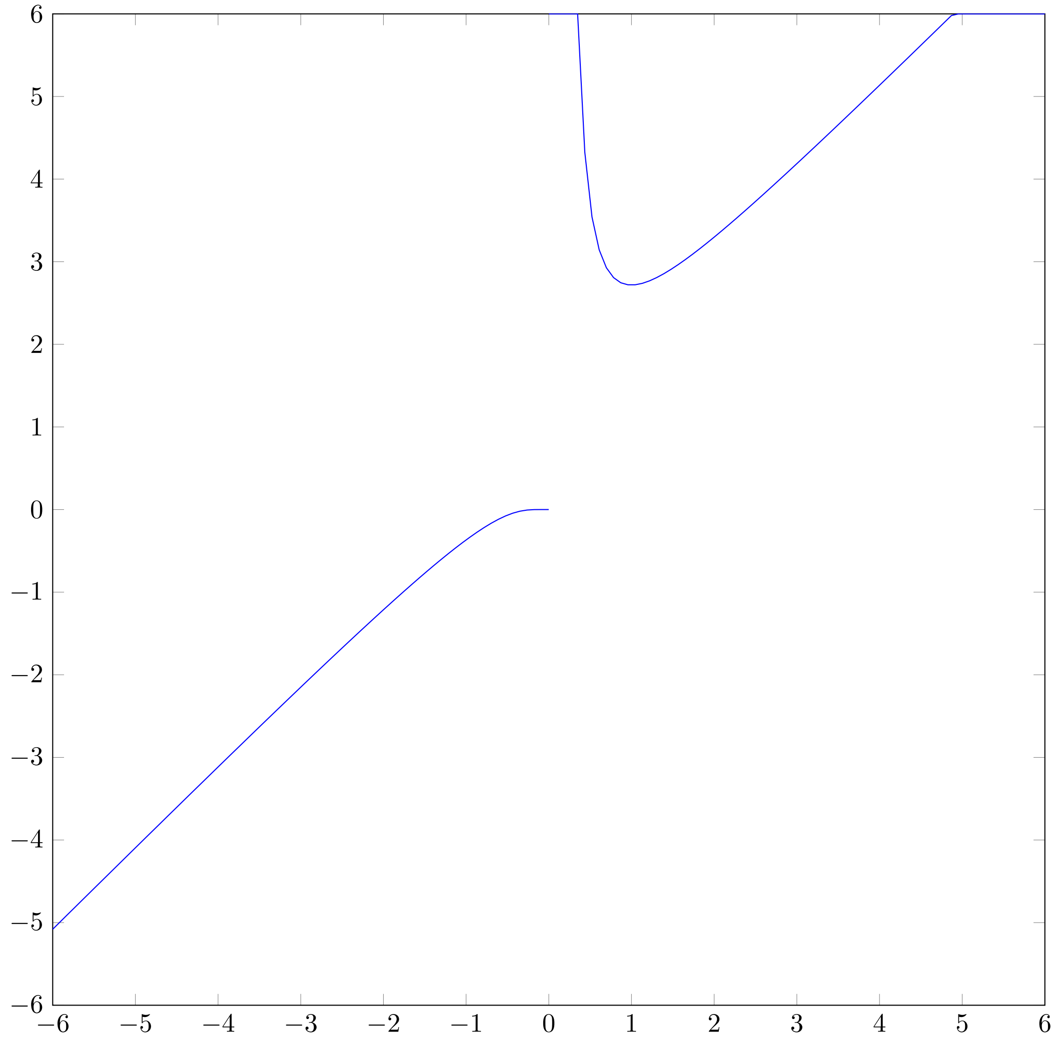
$$\begin{aligned}f'(x) &= e^{\frac{1}{x}} + x(-\frac{1}{x^2}e^{\frac{1}{x}})\\ &= e^{-\frac{1}{x}}(1-\frac{1}{x})\\ 0 &= e^{-\frac{1}{x}}(1-\frac{1}{x})\\ x &= 1\end{aligned}$$

	$x<0$	$x=0$	$0<x<1$	$x=1$	$1<x$
f'	+	$m=\infty$ DNE	-	local min 0	+

7. Concavity and inflection points

$$f''(x) = \frac{1}{x^3}e^{\frac{1}{x}}$$

	$x<0$	$x=0$	$0<x<1$	$x=1$	$1<x$
f"	-	$m=\infty$ DNE	+	local min	+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