Inverse Trigonometric Functions

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Domain and Range

The domain is the set of all points that results in a valid result when mapped through a function. For example, in the function $f(x) = \frac{1}{x}$, the input 0 does not produce a valid result. The range is the set of all possible outputs a function can produce.

Tips:

1. If there is a fraction, the denominator shouldn't be zero.
2. If there is a square root (or any even root), the input of the square root should be greater than or equal to zero.
3. In exponentials, the range is strictly greater than zero
4. If there is a logarithm, the input must be strictly greater than zero.

Example

$$\begin{aligned} f(x) &= \tan(x) \\ &= \frac{\sin(x)}{\cos(x)} \\ \therefore \cos(x) &\neq 0 \\ &\Rightarrow x \neq 2k\pi + \frac{\pi}{2}, 2k\pi + \frac{3\pi}{2} \\ &\text{where } k \in \mathbb{Z} \end{aligned}$$

Interval notation

A square bracket means that the interval is inclusive while a round bracket means that a interval is exclusive. Round and square brackets can be mixed to represent a interval that is closed on one side but open on another.

$$x \in [a, b] \Leftrightarrow a \leq x \leq b$$
$$x \in (a, b) \Leftrightarrow a < x < b$$

Inverse Trigonometric Functions

The sine function is not one-to-one; however, if we restrict the domain of sine to $[-\frac{\pi}{2}, \frac{\pi}{2}]$, it becomes a one-to-one function with the range $[-1, 1]$. The inverse of the sine function is $sin^{-1}(y) = x$ or $\arcsin(y) = x$

For cosine, a similar thing can be done by restricting the domain to $[0, \pi]$. $\tan(x)$ is also restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$f(x)$	Domain	Range
sin	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[-1, 1]$
cos	$[0, \pi]$	$[-1, 1]$
tan	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$(-\infty, \infty)$