Motion in Higher Dimensions

Patrick Chen

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Directional quantities are represented by vectors. Vectors hold component wise information for the direction and magnitude. 2D problems can be broken down into two 1D problems by decomposing the vectors into components. Vector components in orthogonal directions are independent. Sometimes problems are easier to solve by decomposing the vectors into a basis that is tilted with respect to the world (e.g. parallel and perpendicular components of a inclined surface).

Projectile Motion

Assumptions:

- Free fall acceleration is constant over the range of motion in the downwards direction. This is assuming a flat earth over the range of the motion and is reasonable as long as the range is small compared to the earth.
- There is no air resistance
- The projectile follows a parabolic trajectory.

$$
\begin{aligned}
v_x &= v\cos\theta \\
v_y &= v\sin\theta \\
a_x &= 0 \\
a_y &= -g \\
x(t) &= x_0 + tv_i\cos\theta \\
y(t) &= y_0 + tv_i\sin\theta - \frac{1}{2}gt^2
\end{aligned}
$$

2D motion and calculus

$$
\begin{aligned}
\vec{P} &= \int \vec{v}(t)\ dt \\
x &= \int v_x(t)\ dt \\
y &= \int v_y(t)\ dt
\end{aligned}
$$

Example

$$
\begin{aligned}
v(t) &= 4t\hat{x} + 2\hat{y}\frac{m}{s} \\
P(0) &= (1, 0) \\
P(t) &= \int 4t\hat{x} + 2\hat{y}\ dt \\
P(t) &= 2t^2\hat{x} + 2t\hat{y} + P_0 \\
P(0) &= 2(0)^2\hat{x} + 2(0)\hat{y} + P_0 \\
(1, 0) &= P_0 \\
P(t) &= 2t^2\hat{x} + 2t\hat{y} + 1\hat{x} \\
P(t) &= (2t^2 + 1)\hat{x} + 2t\hat{y}
\end{aligned}
$$