Trig Substitution

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Expression	Substitution		Identity
$\sqrt{a^2 - x^2}$	$x = a \sin x$	$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$1 - \sin^2 \theta = cos^2 \theta$
$\sqrt{a^2 + x^2}$	$x = a \tan x$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2 \theta = \sec^2 \theta$
$\sqrt{x^2 - a^2}$	$x = a \sec x$	$0 \leq \theta < \frac{\pi}{2}$	$\sec^2 \theta - 1 = \tan^2 \theta$
		$\pi \leq \theta < \frac{3\pi}{2}$	

Example 1

$$\int \frac{\sqrt{9 - x^2}}{x^2} \, dx$$

$$x = 3 \sin \theta$$
$$dx = 3 \cos \theta \; d\theta$$

$$
\begin{aligned}
&= \int \frac{\sqrt{9 - 9\sin^2\theta}}{9\sin^2\theta} 3\cos\theta \ d\theta \\
&= \int \frac{3\sqrt{1 - \sin^2\theta}}{9\sin^2\theta} 3\cos\theta \ d\theta \\
&= \int \frac{3\cos\theta}{9\sin^2\theta} 3\cos\theta \ d\theta \\
&= \int \frac{\cos^2\theta}{\sin^2\theta} \ d\theta \\
&= \int cot^2\theta \ d\theta \\
&= \int cot^2\theta + 1 - 1 \ d\theta \\
&= (-cot\theta) - \theta + c \\
&= -cot(\sin^{-1}(\frac{x}{3})) - \sin^{-1}(\frac{x}{3}) + c \\
&= -\frac{\sqrt{9 - x^2}}{x} - \sin^{-1}(\frac{x}{3}) + c
\end{aligned}
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