Curve sketching

Patrick Chen

Oct 23, 2024

Example

$$xe^{\frac{1}{x}}$$

1. Check the domain of the function.

$$y_1 = x \qquad\qquad D = \mathbb{R}$$
$$y_2 = \frac{1}{x} \qquad\qquad D = \{x | x \in \mathbb{R}, x \neq 0\}$$
$$y_3 = e^x \qquad\qquad D = \mathbb{R}$$
$$y_4 = e^{\frac{1}{x}} \qquad\qquad D = \{x | x \in \mathbb{R}, x \neq 0\}$$
$$y = xe^{\frac{1}{x}} \qquad\qquad D = \{x | x \in \mathbb{R}, x \neq 0\}$$

2. Find the x and y intercepts.

$$f(0)$$

Since $x \neq 0$, there is no y-intercept.

$$xe^{\frac{1}{x}} = 0$$

Since $x \neq 0$ and e^x is never zero, there is no x-intercept

3. Symmetry

- Even functions are function that are symmetric about the y-axis. $f(-x) = f(x)$
- Odd functions are functions that are an symmetric about the origin. It can also be thought of as being the same graph when rotated half a revolution. $f(-x) = -f(x)$

$$f(-x) = -xe^{-\frac{1}{x}}$$
$$f(-x) \neq f(x)$$
$$f(-x) \neq -f(x)$$

$f(x)$ is neither even nor odd.

4. Periodic

  - A function is periodic if $f(x + p) = f(x)$ for all x in the domain, where p is a positive constant. The smallest value p satisfying this is called the period.

$$f(x + p) = (x + p)e^{\frac{1}{x+p}}$$

This function is not periodic because no such p exists.

5. Find the asymptotes

  - Horizontal asymptotes occur when the limit of a function as the input approaches infinite remains finite. If the limit approaches infinity or negative infinity, then there is no horizontal asymptotes.

$$\lim_{x \to \infty} f(x) = L$$

$$\text{or}$$

$$\lim_{x \to -\infty} f(x) = L$$

  - Vertical asymptotes are where the limit as the function approaches some value is infinity or negative infinity

$$\lim_{x \to a^+} f(x) = \pm\infty$$

$$\text{or}$$

$$\lim_{x \to a^-} f(x) = \pm\infty$$

Horizontal asymptotes of $xe^{\frac{1}{x}}$

$$\lim_{x \to \infty} xe^{\frac{1}{x}}$$
$$= \infty \cdot e^{\frac{1}{\infty}}$$
$$= \infty \cdot e^0$$
$$= \infty \cdot 1$$
$$= \infty$$

$$\lim_{x \to -\infty} xe^{\frac{1}{x}}$$
$$= -\infty \cdot e^{\frac{1}{-\infty}}$$
$$= -\infty \cdot e^0$$
$$= -\infty \cdot 1$$
$$= -\infty$$

For vertical asymptotes of $xe^{\frac{1}{x}}$, we can check the restrictions in the domain of the function.

$$\lim_{x\to 0^+} xe^{\frac{1}{x}} = \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}}{\frac{1}{x}}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{-\frac{1}{x^2}e^{\frac{1}{x}}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0^+} e^{\frac{1}{x}} = e^{\frac{1}{0}} = e^{\infty} = \infty$$

$$\lim_{x\to 0^-} xe^{\frac{1}{x}} = \lim_{x\to 0^-} \frac{e^{\frac{1}{x}}}{\frac{1}{x}}$$

$$\overset{H}{=} \lim_{x\to 0^-} \frac{-\frac{1}{x^2}e^{\frac{1}{x}}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0^-} e^{\frac{1}{x}} = e^{\frac{1}{-0}} = e^{-\infty} = 0$$

6. Intervals of increase or decrease and local maximums and minimums

$$f'(x) = e^{\frac{1}{x}} + x(-\frac{1}{x^2}e^{\frac{1}{x}})$$

$$= e^{-\frac{1}{x}}(1 - \frac{1}{x})$$

$$0 = e^{-\frac{1}{x}}(1 - \frac{1}{x})$$

$$x = 1$$

	$x < 0$	$x = 0$	$0 < x < 1$	$x = 1$	$1 < x$
		$m = \infty$		local min	
f'	+	DNE	-	0	+

7. Concavity and inflection points

$$f''(x) = \frac{1}{x^3}e^{\frac{1}{x}}$$

	$x < 0$	$x = 0$	$0 < x < 1$	$x = 1$	$1 < x$
		$m = \infty$		local min	
f"	-	DNE	+		+

3