Momentum

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Inertia

Inertia or mass is a measure of how much an object resists acceleration. It is represented by the symbol m and the SI unit is inertia is the kilogram (kg).

Systems

A system is a object of group of objects that we mentally separate from the rest of the environment. The objects we care about will guide our choice of systems. Extensive quantities are quantities that are proportional to the size of the system. Intensive quantities do not depend on the extent of the system. Isolated systems are system without inputs or outputs. Conserved quantities can only be changed by input or output.
In a system of trees, cutting down trees or planting new trees are external because they require an external effect. Trees naturally dying and reproducing are internal because they do not require external input.

$$change = input - output + creation - destruction$$

Collisions

When objects collide, they could bounce off each other, stick together, stop, deform, transfer energy, etc.

$$\Delta v \propto \frac{1}{m}$$
$$\frac{\Delta v_a}{\Delta v_b} = -\frac{m_b}{m_a}$$
$$m_a \Delta v_a = -m_b \Delta v_b$$

Linear momentum p of a particle is its mass times is velocity. Momentum is a vector, since velocity is a vector. It has units of $kg\frac{m}{s}$.

$$\vec{p} \equiv m\vec{v}$$

Momentum is transferred though interactions between objects, but cannot be created or destroyed. For an isolated system, the total momentum change is zero.

$$p_{1i} = p_{2i} = p_{1f} + p_{2f}$$
$$p_{1i} - p_{1f} = p_{2f} - p_{2i}$$
$$-\Delta p_1 = \Delta p_2$$

$$\vec{p}_{f,sys} = \vec{p}_{i,sys}$$
$$\vec{p}_{total} = \Sigma \vec{p}_i = constant$$
$$\Delta \vec{p}_{sys} = 0$$

Collisions with Friction

We need to separate the effect of friction from the effect of the collision and extrapolate the velocity of the cart.

Impulse

impulse is the change in momentum of an object.

$$\Delta \vec{p} = \vec{J}$$

Type of collisions

1. Perfectly inelastic collisions are collisions where the two objects come out stuck to each other and their final velocity is the same.
2. Inelastic collisions are collisions where kinetic energy is lost.
3. Perfectly elastic collisions are collisions where the total kinetic energy is conserved.

In all collisions, momentum is conserved.

Elastic Collisions

momentum and kinetic energy is conserved

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad \text{(p)}$$
$$\frac{1}{2} m_{1i} v_{1i}^2 + \frac{1}{2} m_{2i} v_{2i}^2 = \frac{1}{2} m_{1f} v_{1f}^2 + \frac{1}{2} m_{2f} v_{2f}^2 \qquad \text{(k)}$$

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i}) \qquad \text{rearrange variables in (p)} \qquad (1)$$
$$m_{1i} v_{1i}^2 + m_{2i} v_{2i}^2 = m_{1f} v_{1f}^2 + m_{2f} v_{2f}^2 \qquad \text{multiply (k) by two} \qquad (2)$$
$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2) \qquad \text{rearrange (2)} \qquad (3)$$
$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) \qquad \text{factor (3)} \qquad (4)$$
$$\frac{m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f})}{m_1(v_{1i} - v_{1f})} = \frac{m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i})}{m_2(v_{2f} - v_{2i})} \qquad \text{divide (4) by (1)} \qquad (5)$$
$$v_{1i} + v_{1f} = v_{2f} + v_{2i} \qquad (6)$$

$$
\begin{aligned}
v_{1f} &= v_{2f} + v_{2i} - v_{1i} \\
m_1(v_{1i} - v_{1f}) &= m_2(v_{2f} - v_{2i}) \\
m_1(v_{1i} - (v_{2f} + v_{2i} - v_{1i})) &= m_2(v_{2f} - v_{2i}) \\
m_1(v_{1i} - v_{2f} - v_{2i} + v_{1i}) &= m_2(v_{2f} - v_{2i}) \\
2m_1v_{1i} - m_1v_{2i} - m_1v_{2f} &= m_2v_{2f} - m_2v_{2i} \\
2m_1v_{1i} - m_1v_{2i} + m_2v_{2i} &= m_2v_{2f} + m_1v_{2f} \\
2m_1v_{1i} + v_{2i}(m_2 - m_1) &= v_{2f}(m_2 + m_1) \\
\frac{2m_1}{m_1 + m_2}v_{1i} + \frac{(m_1 - m_2)}{m_2 + m_1}v_{2i} &= v_{2f}
\end{aligned}
$$

$$
\begin{aligned}
v_{2f} &= v_{1i} + v_{1f} - v_{2i} \\
m_1(v_{1i} - v_{1f}) &= m_2(v_{2f} - v_{2i}) \\
m_1(v_{1i} - v_{1f}) &= m_2(v_{1i} + v_{1f} - v_{2i} - v_{2i}) \\
m_1v_{1i} - m_1v_{1f} &= m_2v_{1i} + m_2v_{1f} - 2m_2v_{2i} \\
m_1v_{1i} - m_2v_{1i} + 2m_2v_{2i} &= m_2v_{1f} + m_1v_{1f} \\
(m_1 - m_2)v_{1i} + 2m_2v_{2i} &= v_{1f}(m_1 + m_2) \\
\frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i} &= v_{1f}
\end{aligned}
$$

$$
\begin{aligned}
v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i} \\
v_{2f} &= \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}
\end{aligned}
$$

- When two carts have the same mass and the second cart is stationary, the velocity is interchanges.
- When two carts have the same mass but different velocities, the velocities still interchanges

Inelastic Collisions

Momentum is still conserved In Perfectly inelastic collisions, the final velocity is the same:

$$
\begin{aligned}
m_1v_{1i} + m_2v_{2i} &= (m_1 + m_2)v_f \\
v_f &= \frac{m_1v_{1i} + m_2v_{2i}}{m_1 + m_2}
\end{aligned}
$$