Inverse Trigonometric Functions

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Domain and Range

The domain is the set of all points that results in a valid result when mapped through a function. For example, in the function $f(x) = \frac{1}{x}$, the input 0 does not produce a valid result. The range is the set of all possible outputs a function can produce.

Tips:

- 1. If there is a fraction, the denominator shouldn't be zero.
- 2. If there is a square root (or any even root), the input of the square root should be greater than or equal to zero.
- 3. In exponentials, the range is strictly greater than zero
- 4. If there is a logarithm, the input must be strictly greater than zero.

Example

$$f(x) = \tan(x)$$

$$= \frac{\sin(x)}{\cos(x)}$$

$$\therefore \cos(x) \neq 0$$

$$\Rightarrow x \neq 2k\pi + \frac{\pi}{2}, 2k\pi + \frac{3\pi}{2}$$
where $k \in \mathbb{Z}$

Interval notation

A square bracket means that the interval is inclusive while a round bracket means that a interval is exclusive. Round and square brackets can be mixed to represent a interval that is closed on one side but open on another.

$$x \in [a, b] \Leftrightarrow a \le x \le b$$

 $x \in (a, b) \Leftrightarrow a < x < b$

Inverse Trigonometric Functions

The sine function is not one-to-one; however, if we restrict the domain of sine to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, it becomes a one-to-one function with the range [-1,1]. The inverse of the sine function is $\sin^{-1}(y) = x$ or $\arcsin(y) = x$

For cosine, a similar thing can be done by restricting the domain to $[0,\pi]$. $\tan(x)$ is also restricted to $[-\frac{\pi}{2},\frac{\pi}{2}]$

$$\begin{array}{lll} f(x) & \text{Domain} & \text{Range} \\ \sin & \left[-\frac{\pi}{2},\frac{\pi}{2}\right] & \left[-1,1\right] \\ \cos & \left[0,\pi\right] & \left[-1,1\right] \\ \tan & \left[-\frac{\pi}{2},\frac{\pi}{2}\right] & \left(-\infty,\infty\right) \end{array}$$