Partitioned Matrices

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Oct 8, 2024

A is a $(m_1 + m_2) \times (n_1 + n_2)$ matrix where A_{11} is a $m_1 \times n_1$ matrix and A_{21} is a $m_2 \times n_1$ matrix.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

B_1 should be $n_1 \times k$ and B_2 should be $n_2 \times k$. The result will be a $(m_1 + m_2) \times k$ matrix.

$$AB = \begin{bmatrix} A_{11}B_1 + A_{12}B_2 \\ A_{21}B_1 + A_{22}B_2 \end{bmatrix}$$

Important note: the order of the multiplications cannot be switched because the elements of the matrix are themselves matrices.

Example

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$

If A_{11} and A_{22} are invertible and A_{12} is any arbitrary matrix, then prove that A is invertible.