Kinematics in 1D

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Dimensional Analysis

A dimension (unit) is an inherent property of a quantity. Most physical quantities can be expressed in terms of these fundamental dimensions: Mass, Length, Time. We can use dimensions to understand and solve problems.

Rules

1. addition and subtraction must have same dimensions.
2. log and exponential functions only apply to dimensionless quantities
3. trig functions only apply to dimensionless quantities

Example

In the equation $E = mc^2$, what is the unit of E?

$$\begin{aligned} E &= mc^2 \\ &= mass\ speed^2 \\ &= mass\ (\frac{length}{time})^2 \\ &= mass\ length^2\ time^{-2} \\ &= energy \end{aligned}$$

Consider the following equation

$$x(=?)x_0 + v_0t^2 + at$$

The first term has unit of *length* and the second term has units of *length time*. These two terms are different units and cannot be added therefore this equation is invalid.

$$\begin{aligned} x &= x_0 + v_0t^2 + at \\ &= length + length/time\ time^2 + length/time^2\ time \\ &= length + \frac{length}{time}\ time^2 + \frac{length}{time^2}\ time \\ &= length + length\ time + \frac{length}{time} \end{aligned}$$

The correct formula is

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Kinematics

Kinematics is the description of motion. These three important quantities can describe 1D motion as a function of time:

- Position: $\vec{r}(t)$ (displacement from the origin)
- Velocity: $\vec{v}(t)$ (rate of change of position)
- Acceleration $\vec{a}(t)$ (rate of change of velocity)

In 1D direction can be expressed as sign. The relations between these quantities and be found using calculus.

Vector vs Scalar

Physical quantities can be classified as scalars, vectors, etc.
Scalars are described as real numbers with units (examples: speed, temperature, mass, time, energy)
Vectors describe both a scalar (magnitude) and a direction in space (examples: velocity displacement, electric field, force)

Notation

- scalars: ordinary or italic font (i, i)
- vector: boldface, arrow notation, underline ($\mathbf{i}$, $\vec{i}$, $\underline{i}$)
- unit vector: hat ($\hat{i}$ or $\hat{x}$ for x, $\hat{j}$ or $\hat{y}$ for y, $\hat{k}$ or $\hat{z}$ for z)

Unit vectors are vectors with a magnitude of 1. It is basically a direction.

Position, distance, displacement

- Position: gives a location, need to define an origin
- Distance: a scalar that expressed the change in position, depending on the path
- Displacement: a vector that expressed the straight line distance without depending on the path

Displacement:

$$\Delta\vec{x} \equiv \vec{x}_f - \vec{x}_i$$

Vector arithmetic

- magnitude: scalar, length of vector
- scalar * vector: multiply the magnitude of vector
- vector * vector: either dot or cross product
- vector + vector: add vectors tip to tail
- vector - vector: add the negation of the subtracted vector

Velocity

Velocity is the time rate of change of position. Average velocity:

$$\vec{v}_{avg} = \frac{\Delta \vec{x}}{\Delta t}$$

The instantaneous velocity is the slope of a line tangent to curve. Thus, velocity is the time derivative of position.

$$\vec{v} = \frac{d\vec{x}}{dt}$$

Acceleration

Acceleration is the time rate of change of velocity. Average acceleration:

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

The instantaneous acceleration is the slope of a line tangent to curve.

$$\vec{a} = \frac{d\vec{v}}{dt}$$