

Trigonometry Review

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Angles

Positive angles is counter clockwise, negative angles are clockwise. In this course, we will use radians. Radians is a measure of how many times a radius fit in the arc corresponding with the angle.

$$\pi \text{ radian} = 180^\circ$$

$$\frac{\pi D}{180^\circ} = \text{rad}$$

Note that $-\frac{5\pi}{4}$ is not the same as $\frac{3\pi}{4}$ even though it ends in the same place because one is clockwise and the other is counter clockwise

Acute angles

Trig functions can be defined with right triangles

$$\begin{array}{lll} \sin\theta = \frac{\text{opp}}{\text{hyp}} & \cos\theta = \frac{\text{adj}}{\text{hyp}} & \tan\theta = \frac{\text{opp}}{\text{adj}} \\ \csc\theta = \frac{1}{\sin\theta} & \sec\theta = \frac{1}{\cos\theta} & \cot\theta = \frac{1}{\tan\theta} \end{array}$$

Special triangles: $(1, 1, \frac{\sqrt{2}}{2})$, $(1, 2, \sqrt{3})$

Obtuse and Negative angles

A point with distance r away from the origin and angle θ will have an x coordinate of $r\cos\theta$ and a y coordinate of $r\sin\theta$

$$\begin{array}{lll} \sin\theta = \frac{y}{r} & \cos\theta = \frac{x}{r} & \tan\theta = \frac{y}{x} \\ \csc\theta = \frac{r}{y} & \sec\theta = \frac{r}{x} & \cot\theta = \frac{x}{y} \end{array}$$

Trig Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Sine is the y axis coordinate so a negating the angle will negate the sine of the angle. Cosine is the x axis coordinate which doesn't change when the angle is negated

$$\begin{aligned}\sin(-\theta) &= -\sin(\theta) \\ \cos(-\theta) &= \cos(\theta)\end{aligned}$$

Sine and Cosine is a quarter rotation phase shifted

$$\begin{aligned}\cos\theta &= \sin(\theta + \frac{\pi}{2}) \\ \sin\theta &= \cos(\theta - \frac{\pi}{2})\end{aligned}$$

| | | | | | |
|--------------|---|-----------------|-------|------------------|--------|
| θ | 0 | $\frac{\pi}{2}$ | π | $\frac{3\pi}{2}$ | 2π |
| $\sin\theta$ | 0 | 1 | 0 | -1 | 0 |
| $\cos\theta$ | 1 | 0 | -1 | 0 | 1 |
| $\tan\theta$ | 0 | ud | 0 | ud | 0 |