

Systems of Linear Equations

Patrick Chen

Sept 3, 2024

Linear equations

a linear equation in n unknowns or variables has the form:

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b$$

Generally, 2 unknowns and 2 equations have one common solution, *but* they could have none:

$$\begin{aligned}x_1 + 3x_2 &= 2 \\x_1 + 3x_2 &= 3\end{aligned}$$

You could also have infinite solutions

$$\begin{aligned}x_1 + 3x_2 &= 2 \\2x_1 + 6x_2 &= 4\end{aligned}$$

A finite set of linear equations is called a system of linear equations.

$$\begin{aligned}a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\&\dots \\a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m\end{aligned}$$

a augmented matrix is

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{bmatrix}$$

a matrix with m rows and n columns is a $m \times n$ matrix

Examples

Linear:

$$2x_1 + 3x_2 = 4$$

Non-linear:

$$x_1^2 + x_2^2 = 1$$