Derivatives and Tangents

Patrick Chen

Sept 16, 2024

Derivatives

A line tangent to a graph takes the form:

$$y = mx + b$$

A line between two points on a graph is called a secant line. The limit as the second point approaches the first is the tangent line

$$\begin{aligned} m &= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \\ y - a &= \lim_{x \to a} \frac{f(x) - f(a)}{x - a}(x - a) \\ y &= \frac{df}{dx}(a)(x - a) + f(a) \end{aligned}$$

The derivative is the slope of the tangent line. The idea is that if we zoom in far enough toward the point, the curve looks like a linear.

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

Example

Find the equation of the tangent line at the point (3, 1).

$$
\begin{aligned}
f(x) &= \frac{3}{x} \\
f(x) &= 3x^{-1} \\
m &= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \\
m &= \lim_{h \to 0} \frac{1}{h}\left(\frac{3}{a+h} - \frac{3}{a}\right) \\
m &= \lim_{h \to 0} \frac{1}{h}\left(\frac{3}{a+h} - 1\right) \\
m &= \lim_{h \to 0} \frac{1}{h}\left(\frac{3-a-h}{a+h}\right) \\
m &= \lim_{h \to 0} \frac{1}{h}\left(\frac{-h}{a+h}\right) \\
m &= \lim_{h \to 0} \left(\frac{-1}{a+h}\right) \\
m &= \lim_{h \to 0} -\frac{1}{3+h} \\
m &= -\frac{1}{3}
\end{aligned}
$$

$$
\begin{aligned}
m &= -\frac{1}{3} \\
x_1 &= 3 \\
y_1 &= 1 \\
y - y_1 &= m(x - x_1) \\
y - 1 &= -\frac{1}{3}(x - 3) \\
y &= -\frac{1}{3}x + 2
\end{aligned}
$$