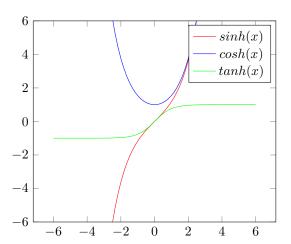
Hyperbolic Trigonometric Functions

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Hyperbolic trigonometric functions describe points on a hyperbola. The graph of a hyperbola satisfies $x^2 - y^2 = 1$ and points on the hyperbola can be described as $(\cosh x, \sinh x)$ in the same way points on a circle can be described as $(\cos t, \sin t)$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\tanh x = \frac{\sinh x}{\cosh x}$$



$$\cosh^{2}x - \sinh^{2}x = 1$$

$$\left(\frac{e^{x} + e^{-x}}{2}\right)^{2} - \left(\frac{e^{x} - e^{-x}}{2}\right)^{2}$$

$$\frac{(e^{x} + e^{-x})^{2} - (e^{x} - e^{-x})^{2}}{4}$$

$$\frac{(e^{2x} + 2e^{x}e^{-x} + e^{-2x}) - (e^{2x} - 2e^{x}e^{-x} + e^{-2x})}{4}$$

$$\frac{2 - (-2)}{4}$$

Identities

$$\sinh(-x) = -\sinh x$$

$$\cosh(-x) = \cosh x$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\frac{\cosh^2 x}{\cosh^2 x} - \frac{\sinh^2 x}{\cosh^2 x} = \operatorname{sech}^2 x$$

$$\frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = \operatorname{sech}^2 x$$

$$\frac{1}{\cosh^2 x} = \operatorname{sech}^2 x$$

$$\operatorname{sech}^2 x = \operatorname{sech}^2 x$$

Derivatives of Hyperbolic Trigonometric Functions

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

$$y = \sinh x$$

$$\frac{dy}{dx} = \frac{d}{dx} (\frac{1}{2})(e^x - e^{-x})$$

$$= (\frac{1}{2})(\frac{d}{dx}e^x - \frac{d}{dx}e^{-x})$$

$$= (\frac{1}{2})(e^x - (-e^{-x}))$$

$$= (\frac{1}{2})(e^x + e^{-x})$$

$$= \cosh x$$

One to one and derivatives

If a function's derivative is always positive or always negative for all points, then it is one-to-one. Since $\frac{d}{dx}\sinh x = \cosh x$, and $\cosh x$ is always positive, sinh is always positive and one-to-one.

Inverse

Inverse of $\sinh x$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$y = \frac{e^x - e^{-x}}{2}$$

$$2y = e^x - e^{-x}$$

$$2y = e^x (1 - e^{-2x})$$

$$2ye^{-x} = 1 - e^{-2x}$$

$$2ye^{-x} + e^{-2x} = 1$$

$$(e^{-x})^2 + 2y(e^{-x}) - 1 = 0$$

$$e^{-x} = -y \pm \sqrt{y^2 + 1}$$

$$x = -\ln\left(-y \pm \sqrt{y^2 + 1}\right)$$

$$x = -\ln\left(-y - \sqrt{y^2 + 1}\right)$$