Newton's Method

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Newton's method is a method of finding roots for a equation. It works by finding where a tangent line to a function intercepts with the x axis, then repeating for the tangent line at the x value where the previous tangent intercepts the x axis. If evaluated at an initial guess, Newton's method will refine the guess, making it closer and closer to the actual root of a equation.

Let x_1 be an initial guess and x_2 be the x coordinate where the tangent line at x_1 intercepts the y-axis.

$$\begin{aligned}
y - f(x_1) &= f'(x_1)(x - x_1) \\
0 - f(x_1) &= f'(x_1)(x_2 - x_1) \\
-f(x_1) &= f'(x_1)x_2 - f'(x_1)x_1 \\
f'(x_1)x_1 - f(x_1) &= f'(x_1)x_2 \\
\frac{f'(x_1)x_1 - f(x_1)}{f'(x_1)} &= x_2 \\
x_1 - \frac{f(x_1)}{f'(x_1)} &= x_2 \\
x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)}
\end{aligned}$$

Thus the formula for refining guesses is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$