Vectors

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Vector Addition

A $n \times 1$ matrix is called a column vector. The set of all vectors with n entries is called $\mathbb{R}^n$. Vectors in $\mathbb{R}^n$ can be added by adding component wise. Geometrically, two vectors added together is the resultant of putting the vectors tip-to-tail.

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{bmatrix}$$

- Vectors in $\mathbb{R}^2$ can be thought of as a point in 2D
- Vectors in $\mathbb{R}^3$ can be thought of as a point in 3D
- Vectors in higher dimensions cannot be visualized, but can still store useful information

Scalar Vector Multiplication

Multiplying a vector by a scalar will multiply every entry in the vector by the scalar.

$$\lambda \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \lambda a_1 \\ \lambda a_2 \\ \vdots \\ \lambda a_n \end{bmatrix}$$

Linear Combinations

When scalar multiples of vectors are added, it is called a linear combination of vectors:

$$\lambda_1 v_1 + \lambda_2 v_2 + \cdots + \lambda_n v_n$$

When any two vectors form a plane, any point on that plane can be expressed as a linear combination of those vectors. The span of some list of vectors is the set of all linear combination of those vectors. We can check of some vector b is in the span of some list of vectors $v_1, v_2, \ldots, v_n$ by checking if the augmented matrix $\begin{bmatrix} v_1 & v_2 & \ldots & v_n & b \end{bmatrix}$ is consistent.