Matrices

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A matrix with m rows and n columns is called an $m \times n$ matrix and can be considered a 2d array. If $m = n$, then the matrix is square.

$$\begin{bmatrix} a_{11} & a_{12} & \\ a_{21} & a_{22} & \dots \\ & \vdots & \end{bmatrix}$$

Two matrices are equal if they have the same dimension and all their elements are equal. The sum and difference is done entry-wise.

$$C = A + B \Leftrightarrow c_{ij} = a_{ij} + b_{ij}$$

For a $m \times k$ matrix A and a $k \times n$ matrix B then the product is a $m \times n$ matrix defined as follows. Matrix multiplication is not commutative.

$$C = AB \Leftrightarrow c_{ij} = \sum_{r=1}^{k} a_{ir} b_{rj}$$

The identity matrix (denoted by I) is a matrix with ones along the diagonal and zeros in all other elements. It has the special property that any matrix multiplied with it will result in the same matrix.

$$I \Leftrightarrow i_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$$

The power of a matrix is the repeated multiplication of that matrix with itself. A matrix to the power of zero is the identity matrix.

$$A^n = \underbrace{AA \dots A}_{n}$$

The inverse of a $n \times n$ matrix A is a matrix B such that the product of the two is the identity matrix.

$$A^{-1} A = I$$

The transpose of a $n \times m$ matrix A is a $m \times n$ matrix A^T. The transpose of a matrix if another matrix that is reflected about the main diagonal.

$$B = A^T \Leftrightarrow b_{ij} = a_{ji}$$

A symmetric matrix A is a matrix such that $A = A^T$

Zero-One Matrix

A zero-one matrix is a matrix where all the elements are either zero or one. There are some operations that work on them

- The join of A and B is
$$C = A \vee B \Leftrightarrow c_{ij} = a_{ij} \vee b_{ij}$$
- The meet of A and B is
$$C = A \wedge B \Leftrightarrow c_{ij} = a_{ij} \wedge b_{ij}$$
- The boolean product of two matrices
$$C = A \odot B \Leftrightarrow (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \dots$$