Coordinates

Patrick Chen

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Coordinates

If B is a basis for a vector space V, then any $v \in V$ can be written uniquely as a linear combination of the vectors in B. The notation $[v]_B$ means the coordinates of vector v with respect to the basis B. If the basis B contains the vector $v_1, \dots v_n$ and $c_1v_1 + \cdots + c_nv_n = v$, then $[v]_B = [c_1 \dots c_n]^T$.

$$\begin{aligned} A &= \begin{bmatrix} v_1 & \dots & v_n \end{bmatrix} \\ A[v]_B &= v \\ [v]_B &= A^{-1}v \end{aligned}$$

Example

$$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \qquad v = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

$$\begin{aligned} c_1v_1 + c_2v_2 + c_3v_3 &= v \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} &= \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \\ [v]_B &= \begin{bmatrix} -1 \\ 4 \\ -1 \end{bmatrix} \end{aligned}$$

Example 2

P_3 is the set of polynomials with degree ≤ 3 and $B = \{1, x, x^2, x^3\}$.

$$\begin{aligned} p &= 1 - 3x^2 + 2x^3 \\ [p]_B &= \begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix} \end{aligned}$$