# Work in Higher dimensions

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## Work

Work in higher dimensions uses the dot product (scalar product or inner product).

$$dw = \frac{F\cos\theta}{F_{parallel}}dr = |F||dr|\cos\theta = F \cdot dr$$

## Conservative Forces

For conservative forces, work is independent of the path taken. This means that to go from any one point to another, the total work done when following any and every path is the same. Since the work done to move a object from a position to the same position is zero, any path that starts and ends in the same place will have zero work done to it by conservative forces. Conservative forces have associated potential energies.

$$\oint \vec{F} \cdot \ d\vec{s} = 0$$

Since conservative forces have associated potential energy functions, the force can be recovered from the potential energy functions. The force is the negative position derivative of the potential energy.

### Force of gravity

$$\begin{aligned}
\Delta U_g &= mg\Delta y \\
\Delta U_g &= -w_g \\
\Delta U_g &= -\int F \ d\vec{r} \\
F_x &= -\frac{\partial U}{\partial x} = 0 \\
F_y &= -\frac{\partial U}{\partial y} = mg \\
F_z &= -\frac{\partial U}{\partial z} = 0
\end{aligned}$$

### Force of spring

$$\Delta U_s = -\frac{1}{2}kx^2$$
$$F_s = -\frac{dU}{dx} = -kx$$

## Non-conservative Forces

A non-conservative force is where the work depends on the path taken.

- Conservative: gravity, spring
- Non-conservative: Friction, Drag