

# Coordinates

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## Coordinates

If  $B$  is a basis for a vector space  $V$ , then any  $v \in V$  can be written uniquely as a linear combination of the vectors in  $B$ . The notation  $[v]_B$  means the coordinates of vector  $v$  with respect to the basis  $B$ . If the basis  $B$  contains the vector  $v_1, \dots, v_n$  and  $c_1 v_1 + \dots + c_n v_n = v$ , then  $[v]_B = [c_1 \dots c_n]^T$ .

$$\begin{aligned} A &= [v_1 \quad \dots \quad v_n] \\ A[v]_B &= v \\ [v]_B &= A^{-1}v \end{aligned}$$

### Example

$$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \qquad v = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

$$\begin{aligned} c_1 v_1 + c_2 v_2 + c_3 v_3 &= v \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} &= \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \\ [v]_B &= \begin{bmatrix} -1 \\ 4 \\ -1 \end{bmatrix} \end{aligned}$$

### Example 2

$P_3$  is the set of polynomials with degree  $\leq 3$  and  $B = \{1, x, x^2, x^3\}$ .

$$\begin{aligned} p &= 1 - 3x^2 + 2x^3 \\ [p]_B &= \begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix} \end{aligned}$$