Kinematics and calculus

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starting with $v = \frac{dx}{dt}$

$$v = \frac{dx}{dt} \Rightarrow \int v dt = \int \frac{dx}{dt} dt = \int dx = x_f - x_i + x_0$$

The integration constant is x_0
The following relation is how position, velocity and acceleration can be found.

$$x \overset{\frac{d}{dt}}{\Longrightarrow} v \overset{\frac{d}{dt}}{\Longrightarrow} a$$

$$x \underset{\int dt}{\Longleftarrow} v \underset{\int dt}{\Longleftarrow} a$$

Motion with non-constant acceleration

to find the change in velocity during the time interval Δt, we can find the area under the $a_x(t)$ curbe

Motion with constant acceleration

When acceleration is constant, the velocity and position formulas can be simplified

$$\begin{aligned} v(t) &= v_0 + \int_0^t a(t')dt' \\ &= v_0 + a\int_0^t dt' \\ &= v_0 + at \\ x(t) &= x_0 + \int_0^t v(t')dt' \\ &= x_0 + \int_0^t v_0 + at'dt' \\ &= x_0 + \int_0^t v_0 dt' + \int_0^t at'dt' \\ &= x_0 + v_0 t + \frac{1}{2}at^2 \end{aligned}$$