

# Derivatives and Tangents

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## Derivatives

A line tangent to a graph takes the form:

$$y = mx + b$$

A line between two points on a graph is called a secant line. The limit as the second point approaches the first is the tangent line

$$\begin{aligned} m &= \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \\ y - a &= \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} (x - a) \\ y &= \frac{df}{dx}(a)(x - a) + f(a) \end{aligned}$$

The derivative is the slope of the tangent line. The idea is that if we zoom in far enough toward the point, the curve looks like a linear.

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$$

### Example

Find the equation of the tangent line at the point  $(3, 1)$ .

$$f(x) = \frac{3}{x}$$

$$f(x) = 3x^{-1}$$

$$m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$m = \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{3}{a+h} - \frac{3}{a} \right)$$

$$m = \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{3}{a+h} - 1 \right)$$

$$m = \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{3-a-h}{a+h} \right)$$

$$m = \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{-h}{a+h} \right)$$

$$m = \lim_{h \rightarrow 0} \left( \frac{-1}{a+h} \right)$$

$$m = \lim_{h \rightarrow 0} -\frac{1}{3+h}$$

$$m = -\frac{1}{3}$$

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$$x_1 = 3$$

$$y_1 = 1$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{3}(x - 3)$$

$$y = -\frac{1}{3}x + 2$$