

# Polar Coordinates

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Polar coordinates describe a point by the angle and distance.

$$\begin{aligned}r^2 &= x^2 + y^2, \quad \tan \theta = \frac{y}{x} \\x &= r \cos \theta, \quad y = r \sin \theta\end{aligned}$$

By some convention, the principal angle is  $-\pi < \theta \leq \pi$  although, a principal angle of  $0 \leq \theta < 2\pi$  is also common.

## Negative Radius

A point with a  $\theta \in (-\pi, \pi]$  and  $r > 0$  can also be expressed with a negative radius.

$$\begin{aligned}\theta &\mapsto \theta + \pi \\r &\mapsto -r\end{aligned}$$

## Example 1

$$\begin{aligned}r &= \frac{1}{\cos(\theta) + \sin(\theta)} \\r &= \frac{1}{\cos(\theta) + \sin(\theta)} \\rcos(\theta) + rsin(\theta) &= 1 \\x + y &= 1 \\y &= 1 - x\end{aligned}$$

This is a straight line with slope  $-1$  and y-intercept  $1$ .

## Example 2

$$\begin{aligned}r &= \sin \theta \\r^2 &= r \sin \theta \\x^2 + y^2 &= y \\x^2 + y^2 - 2\left(\frac{1}{2}y\right) + \left(\frac{1}{2}\right)^2 &= \left(\frac{1}{2}\right)^2 \\x^2 + \left(y - \frac{1}{2}\right)^2 &= \left(\frac{1}{2}\right)^2\end{aligned}$$

This is a circle with radius  $r = \frac{1}{2}$  and origin  $O = (0, \frac{1}{2})$ .

## Shapes in polar Coordinates

### Graphing

A graph of a polar equation  $r = g(\theta)$  is the set of all points in the plane what satisfy the equation.  $\theta$  does not need to be a principal angle and  $r$  does not need to be positive. Sometimes it is easier to convert a polar function into a Cartesian function.

### Circle

A circle is described with  $r(\theta) = c$  where  $c$  is a constant. The radius of this circle is  $|c|$ .

### Polar Tangents

The derivative of a polar function can be calculated the same way a parametric function's derivative is calculated with  $x = r \cos \theta$ , and  $y = r \sin \theta$ .

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{\frac{dr}{d\theta} \sin(\theta) + r \cos(\theta)}{\frac{dr}{d\theta} \cos(\theta) - r \sin(\theta)}$$