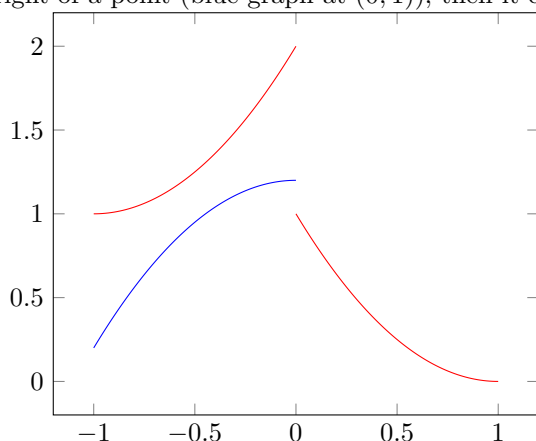


# Maximum and Minimum

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A absolute maximum is the maximum value that a function can produce. The absolute minimum is the minimum value that a function can produce. A local maximum is a point where any direction in the input space will produce a lesser value and likewise for local minimum. A point can be both a absolute maximum and local maximum. If function is undefined to the left or right of a point (blue graph at  $(0, 1)$ ), then it cannot be an local max or min.



If a function has a "max", but it is on a open interval and thus not part of the graph, then it is not considered a max.

## Extreme Value Theorem

If  $f$  is continuous on a closed interval  $[a, b]$ , then  $f$  attains an absolute maximum value  $f(c)$  and an absolute minimum value  $f(d)$  at some numbers  $c$  and  $d$  in  $[a, b]$ .

## Fermat's Theorem

If  $f$  has a local maximum or minimum at  $c$ , and  $f'(c)$  exists, then  $f'(c) = 0$ . Note that if the derivative is zero, that does not necessary mean that a local extreme exists. Local extreme values can also happen when  $f'(c) \neq 0$ . When the derivative  $f'(c)$  is zero or does not exists, then  $c$  is considered a critical number.

## Finding Maximum and Minimum

The absolute maximum and minimum of a continuous function on the closed interval  $[a, b]$  can be found by following these steps:

1. Find the values of  $f$  at the critical numbers of  $f$  in  $(a,b)$
2. Find the values of  $f$  at the endpoints of the interval
3. The largest of the values from steps 1 and 2 is the absolute maximum and the least of the values is the absolute minimum on the interval  $[a,b]$

### Example

$$f(x) = 2 \cos x + \sin 2x$$

$$f'(x) = -2 \sin x + 2 \cos 2x$$

$$0 = -\sin x + \cos 2x$$

$$0 = -\sin x + \cos^2 x - \sin^2 x$$

$$0 = -\sin x + 1 - \sin^2 x - \sin^2 x$$

$$0 = -2 \sin^2 x - \sin x + 1$$

$$\sin x = \frac{-1 \pm 3}{4}$$

$$\sin x = -1, \frac{1}{2}$$

$$x = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$