Eigenvectors and Eigenvalues

Patrick Chen

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For a $n \times n$ matrix A and a non-zero vector $u \in \mathbb{R}^n$ where $Au = \lambda u$, then λ is a eigenvalue and u is a eigenvector. The λ-eigenspace of A is the set of all eigenvectors with eigenvalues of λ. The eigenspace is a subspace of $\mathbb{R}^n$ equal to the nullspace of $A - \lambda I$. This means that if a eigenvector is scaled, it will still be an eigenvector.

$$\begin{aligned} Au &= \lambda u \\ Au - \lambda u &= 0 \\ (A - \lambda I)u &= 0 \end{aligned}$$

Since $u \neq \vec{0}$, the determinant of $A - \lambda I$ must be zero.

Example 1

Check if 6 is a eigenvalue of the following matrix.

$$\begin{aligned} A &= \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix} \\ Au &= 6u \\ Au - 6u &= 0 \\ (A - 6I)u &= 0 \\ A - 6I &= \begin{bmatrix} -5 & 3 & 2 \\ 2 & -5 & 3 \\ 3 & 2 & -5 \end{bmatrix} \\ rref(A - 6I) &= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \\ u &= \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} \\ u &= x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \end{aligned}$$

Example 2

$\lambda = 2$ is a eigenvalue

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A - 2I = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$rref(A - 2I) = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$u = x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$