Forces and Work

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Motions of Systems

- Internal forces is between objects in the system.
- External forces are form outside the system.

$$
\begin{aligned}
Ma_{cm} &= \sum_j F_{net,j} \\
&= \sum_j F_{net,j}^{ext} + \sum_j F_{net,j}^{int} \\
&= \sum_j F_{net,j}^{ext} + 0 \\
&= \sum_j F_{net,j}^{ext}
\end{aligned}
$$

Choosing a system depending on the problem can help solve the problem. When choosing a system, having friction on the boundary of the system will make analysis more difficult. Gravity will either be a change in potential energy or a force doing work on a system depending if it is in the system or not.

Work

Work is the change in the energy of a system due to external forces. Work is a signed scalar quantity measured in joules (J) in the SI system.

$$
\begin{aligned}
W &= \mathbf{F} \cdot \mathbf{d} = Fd\cos\theta \\
W &= \Delta E
\end{aligned}
$$

For non-constant forces, the work must be integrated

$$W = \int_{x_i}^{x_f} F \cdot \ dx$$

Force displacement is how far the point where the force is applied moves. In order for a force to do work, the point where the force is being applied must undergo a force displacement.

Momentum and Energy

$$\begin{aligned} \Delta p &= J & \Delta E &= W \\ J &= 0 & \Delta E &= 0 \\ J &= \sum f \Delta t & W &= \sum F \Delta x \end{aligned}$$

Example 1

Suppose there is a object with a horizontal for applied on a frictionless surface from point x_i to x_f.

$$\begin{aligned} W &= \Delta E \\ &= \Delta K + \Delta U + \Delta E_{th} \\ &= \Delta K + 0 + 0 \\ &= \Delta K \\ F &= m a_{cm} \\ &= m \frac{dv_{cm}}{dt} \\ &= m \frac{dv_{cm}}{dx_{cm}} \frac{dx_{cm}}{dt} \\ &= m v_{cm} \frac{dv_{cm}}{dx_{cm}} \\ F \ dx_{cm} &= m v_{cm} dv_{cm} \\ \int_{x_i}^{x_f} F \ dx_{cm} &= \int_{v_i}^{v_f} m v_{cm} \ dv_{cm} \\ &= \frac{1}{2} m v_{cm}^2 \ \Big|_{v_i}^{v_f} \\ &= \Delta K_{cm} \\ F \Delta x_{cm} &= \Delta K_{cm} \end{aligned}$$

Example 2

Suppose there is a object with two horizontal forces in opposite directions on a frictionless surface.

$$\begin{aligned} W &= \Delta K \\ F_{net} &= m a_{cm} \\ F_{net} &= F_1 - F_2 \\ W &= \int_{x_i}^{x_f} (F_1 - F_2) \ dx_{cm} \\ W_{net} &= \int_{x_i}^{x_f} F_1 \ dx_{cm} - \int_{x_i}^{x_f} F_2 \ dx_{cm} \end{aligned}$$

Example 3

Suppose there is a object sliding with a friction force. F_k is a constant force. We can choose the system to be the object and the surface.

$$
\begin{aligned}
\Delta E &= \Delta K + \Delta E_{th} \\
\Delta E &= 0 \\
\Delta E_{th} &= -\Delta K \\
&= -(K_f - K_i) \\
&= K_i \\
\Delta K_{cm} &= -\int_{x_{cm,i}}^{x_{cm,f}} f_x \ dx_{cm} \\
&= -f_k \Delta x_c m
\end{aligned}
$$