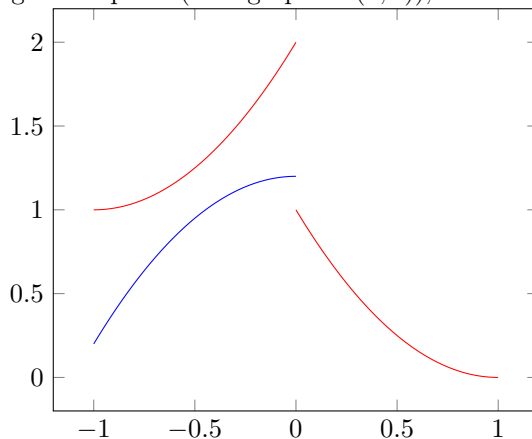


Maximum and Minimum

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A absolute maximum is the maximum value that a function can produce. The absolute minimum is the minimum value that a function can produce. A local maximum is a point where any direction in the input space will produce a lesser value and likewise for local minimum. A point can be both a absolute maximum and local maximum. If function is undefined to the left or right of a point (blue graph at $(0, 1)$), then it cannot be an local max or min.



If a function has a "max", but it is on a open interval and thus not part of the graph, then it is not considered a max.

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

Fermat's Theorem

If f has a local maximum or minimum at c , and $f'(c)$ exists, then $f'(c) = 0$. Note that if the derivative is zero, that does not necessary mean that a local extreme exists. Local extreme values can also happen when $f'(c) \neq 0$. When the derivative $f'(c)$ is zero or does not exists, then c is considered a critical number.

Finding Maximum and Minimum

The absolute maximum and minimum of a continuous function on the closed interval $[a, b]$ can be found by following these steps:

1. Find the values of f at the critical numbers of f in (a,b)
2. Find the values of f at the endpoints of the interval
3. The largest of the values from steps 1 and 2 is the absolute maximum and the least of the values is the absolute minimum on the interval $[a,b]$

Example

$$f(x) = 2 \cos x + \sin 2x$$

$$f'(x) = -2 \sin x + 2 \cos 2x$$

$$0 = -\sin x + \cos 2x$$

$$0 = -\sin x + \cos^2 x - \sin^2 x$$

$$0 = -\sin x + 1 - \sin^2 x - \sin^2 x$$

$$0 = -2 \sin^2 x - \sin x + 1$$

$$\sin x = \frac{-1 \pm 3}{4}$$

$$\sin x = -1, \frac{1}{2}$$

$$x = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$