

# Invertible Matrix Theorem

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A linear transformation  $T : \mathbb{R}^n \mapsto \mathbb{R}^n$  is invertible if there is a linear transformation  $S : \mathbb{R}^n \mapsto \mathbb{R}^n$  such that  $S(T(x)) = x$  and  $T(S(x)) = x$  for all  $x \in \mathbb{R}^n$ . If the transformation  $T : \mathbb{R}^n \mapsto \mathbb{R}^n$  is invertible, then the standard matrix for  $T$  is invertible.

If  $A$  is an  $n \times n$  matrix, then the following are equivalent:

- a)  $A$  is invertible
- b)  $A$  can be row reduced to the identity matrix.
- c)  $A$  has  $n$  pivot positions.
- d)  $Ax = 0$  has only the trivial solution.
- e) The columns of  $A$  form a linearly independent set.
- f) The linear transformation  $x \mapsto Ax$  is one-to-one.
- g) The columns of  $A$  span  $\mathbb{R}^n$ .
- h) the linear transformation  $x \mapsto Ax$  is onto
- i) there is an  $n \times n$  matrix  $C$  such that  $CA = I$
- j) there is an  $n \times n$  matrix  $D$  such that  $AD = I$
- k)  $A^T$  is invertible

Important note: since  $A^T$  is invertible, anything you can say about a column of a invertible matrix, you can also say about the rows of a matrix.