Relative Motion

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Center of mass

The center of mass is a point at the average of the masses. A multi-object system may be replaced with a point mass at this point to simplify the model. Object with any symmetry will lie on the axis of symmetry. For a uniform object, the center of geometry and mass is distributed evenly around the center of mass.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$
$$x_{CM} = \frac{\sum_i m_i \vec{r_i}}{\sum_i m_i}$$

The derivative of center of mass with respect to time is the total momentum of the collection. Since the total momentum is constant, the velocity of the center of mass will also remain constant.

$$Mr_{CM} = \sum m_i r_i$$
$$\Downarrow \frac{d}{dt}$$
$$Mv_{CM} = \sum m_i v_i = p_{total}$$

Relative motion

Since velocity is relative, it needs to be measured with respect to a frame of reference.

$$V_{AC} = V_{AB} + V_{BC}$$
$$V_{AB} = -V_{BA}$$

V_{AC} is the velocity of C relative to A. When changing reference frames, the graph of velocities are shifted up or down.

When a collision is observed from the center of mass reference frame, the collision looks like a head on collision. If the two masses are the same, the two masses will be the same speed.

Kinetic Energy

$$
\begin{aligned}
v_{cm} &= 0 \\
K_E &= K_Z + \frac{1}{2}Mv_{cm}^2 \\
\Delta K_E &= \Delta K_Z
\end{aligned}
$$

K_Z is the kinetic energy in the center of mass frame. $\frac{1}{2}MV_{cm}^2$ is the kinetic energy of the center of mass in the earth frame. The difference of the kinetic energy in the center of mass frame is called the convertible kinetic energy. During a collision, the kinetic energy cannot be lower than the kinetic energy of the center of mass because the kinetic energy of the center of mass is the lowest kinetic energy possible to conserve momentum. This means that the convertible kinetic energy is the only kinetic energy that can be converted into potential energy during the collisions, hence the name convertible.

$$
\begin{aligned}
K_E &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \\
K_{CM} &= \frac{1}{2}(m_1 + m_2)v_{cm}^2 \\
K_Z &= \frac{1}{2}m_1v_{z1}^2 + \frac{1}{2}m_2v_{z2}^2
\end{aligned}
$$

Galilean Relativity

$$
\begin{aligned}
r &= r' + r_0 & x_{AC} &= x_{AB} + x_{BC} \\
v &= v' + v_0 & v_{AC} &= v_{AB} + v_{BC} \\
a &= a' + a_0 & a_{AC} &= a_{AB} + a_{BC}
\end{aligned}
$$

Since it is an inertial reference frame, $a = 0$.