

# Volume of a Revolution

Patrick Chen

Nov 13, 2024

The volume of a function rotated around the x-axis is given by:

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sum_{i=1}^n \pi f(x_i)^2 \Delta x \\ &= \int_a^b \pi f(x)^2 dx \end{aligned}$$

The volume of a function rotated around the y-axis is equivalent to rotating around the x-axis, using the inverse function. The volume is given by:

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sum_{i=1}^n \pi f^{-1}(x_i)^2 \Delta x \\ &= \int_a^b \pi f^{-1}(x)^2 dx \end{aligned}$$

When finding the volume of a region bounded by two by  $f$  and  $g$ :

$$\int_a^b \pi f(x)^2 dx - \int_a^b \pi g(x)^2 dx$$

For rotating around any arbitrary horizontal line located at  $y_0$ :

$$\int_a^b \pi (f(x) - y_0)^2 - \pi (g(x) - y_0)^2 dx$$