

Curve sketching

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Example

$$xe^{\frac{1}{x}}$$

1. Check the domain of the function.

$$y_1 = x \qquad D = \mathbb{R}$$

$$y_2 = \frac{1}{x} \qquad D = \{x | x \in \mathbb{R}, x \neq 0\}$$

$$y_3 = e^x \qquad D = \mathbb{R}$$

$$y_4 = e^{\frac{1}{x}} \qquad D = \{x | x \in \mathbb{R}, x \neq 0\}$$

$$y = xe^{\frac{1}{x}} \qquad D = \{x | x \in \mathbb{R}, x \neq 0\}$$

2. Find the x and y intercepts.

$$f(0)$$

Since $x \neq 0$, there is no y-intercept.

$$xe^{\frac{1}{x}} = 0$$

Since $x \neq 0$ and e^x is never zero, there is no x-intercept

3. Symmetry

- Even functions are function that are symmetric about the y-axis. $f(-x) = f(x)$
- Odd functions are functions that are an symmetric about the origin. It can also be thought of as being the same graph when rotated half a revolution. $f(-x) = -f(x)$

$$f(-x) = -xe^{-\frac{1}{x}}$$

$$f(-x) \neq f(x)$$

$$f(-x) \neq -f(x)$$

$f(x)$ is neither even nor odd.

4. Periodic

- A function is periodic if $f(x+p) = f(x)$ for all x in the domain, where p is a positive constant. The smallest value p satisfying this is called the period.

$$f(x+p) = (x+p)e^{\frac{1}{x+p}}$$

This function is not periodic because no such p exists.

5. Find the asymptotes

- Horizontal asymptotes occur when the limit of a function as the input approaches infinite remains finite. If the limit approaches infinity or negative infinity, then there is no horizontal asymptotes.

$$\lim_{x \rightarrow \infty} f(x) = L$$

or

$$\lim_{x \rightarrow -\infty} f(x) = L$$

- Vertical asymptotes are where the limit as the function approaches some value is infinity or negative infinity

$$\lim_{x \rightarrow a^+} f(x) = \pm\infty$$

or

$$\lim_{x \rightarrow a^-} f(x) = \pm\infty$$

Horizontal asymptotes of $xe^{\frac{1}{x}}$

$$\begin{aligned} & \lim_{x \rightarrow \infty} xe^{\frac{1}{x}} \\ &= \infty \cdot e^{\frac{1}{\infty}} \\ &= \infty \cdot e^0 \\ &= \infty \cdot 1 \\ &= \infty \end{aligned}$$

$$\begin{aligned} & \lim_{x \rightarrow -\infty} xe^{\frac{1}{x}} \\ &= -\infty \cdot e^{\frac{1}{-\infty}} \\ &= -\infty \cdot e^0 \\ &= -\infty \cdot 1 \\ &= -\infty \end{aligned}$$

For vertical asymptotes of $xe^{\frac{1}{x}}$, we can check the restrictions in the domain of the function.

$$\begin{aligned}\lim_{x \rightarrow 0^+} xe^{\frac{1}{x}} &= \lim_{x \rightarrow 0^+} \frac{e^{\frac{1}{x}}}{\frac{1}{x}} \\ &\stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{-\frac{1}{x^2}e^{\frac{1}{x}}}{-\frac{1}{x^2}} \\ &= \lim_{x \rightarrow 0^+} e^{\frac{1}{x}} = e^{\frac{1}{0}} = e^{\infty} = \infty\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} xe^{\frac{1}{x}} &= \lim_{x \rightarrow 0^-} \frac{e^{\frac{1}{x}}}{\frac{1}{x}} \\ &\stackrel{H}{=} \lim_{x \rightarrow 0^-} \frac{-\frac{1}{x^2}e^{\frac{1}{x}}}{-\frac{1}{x^2}} \\ &= \lim_{x \rightarrow 0^-} e^{\frac{1}{x}} = e^{\frac{1}{-0}} = e^{-\infty} = 0\end{aligned}$$

6. Intervals of increase or decrease and local maximums and minimums

$$\begin{aligned}f'(x) &= e^{\frac{1}{x}} + x\left(-\frac{1}{x^2}e^{\frac{1}{x}}\right) \\ &= e^{-\frac{1}{x}}\left(1 - \frac{1}{x}\right) \\ 0 &= e^{-\frac{1}{x}}\left(1 - \frac{1}{x}\right) \\ x &= 1\end{aligned}$$

	$x < 0$	$x = 0$	$0 < x < 1$	$x = 1$	$1 < x$
f'	+	$m = \infty$ DNE	-	local min 0	+

7. Concavity and inflection points

$$f''(x) = \frac{1}{x^3}e^{\frac{1}{x}}$$

	$x < 0$	$x = 0$	$0 < x < 1$	$x = 1$	$1 < x$
f''	-	$m = \infty$ DNE	+	local min	+

