Invertible Matrix Theorem

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A linear transformation $T : \mathbb{R}^n \mapsto \mathbb{R}^n$ is invertible if there is a linear transformation $S : \mathbb{R}^n \mapsto \mathbb{R}^n$ such that $S(T(x)) = x$ and $T(S(x)) = x$ for all $x \in \mathbb{R}^n$. If the transformation $T : \mathbb{R}^n \mapsto \mathbb{R}^n$ is invertible, then the standard matrix for T is invertible.

If A is an $n \times n$ matrix, then the following are equivalent:

- a) A is invertible
- b) A can be row reduced to the identity matrix.
- c) A has n pivot positions.
- d) $Ax = 0$ has only the trivial solution.
- e) The columns of A form a linearly independent set.
- f) The linear transformation $x \mapsto Ax$ is one-to-one.
- g) The columns of A span $\mathbb{R}^n$.
- h) the linear transformation $x \mapsto Ax$is onto
- i) there is an $n \times n$ matrix C such that $CA = I$
- j) there is an $n \times n$ matrix D such that $AD = I$
- k) A^T is invertible

Important note: since A^T is invertible, anything you can say about a column of a invertible matrix, you can also say about the rows of a matrix.