Fluids

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Solids resists deformations and maintains shape and volume. A solid material is rigid. Fluids flow and are easily deformed in response to an applied force. They take the shape of the container. Liquids are nearly incompressible and gasses are compressible.

Pressure

Pressure is the amount of force per unit area. Gasses have pressure because of the collisions of the molecules with the walls of the container. In the SI system, pressure is measured in N/m^2 or Pa (Pascal). Alternatively, one atmosphere of pressure is the atmospheric pressure at mean see level at the latitude of Paris, France. 1 $atm = 1.013 \times 10^5 Pa$. The imperial unit of pressure is the psi (pounds per square inch). $1atm = 14.7psi$.

Hydrostatic Pressure

Pressure increases deeper in a liquid. Consider a cylinder of water in a swimming pool of volume $V = Ah$.

$$\begin{aligned}
&P_{top} \\
&P_{bot} = P_{top} + \Delta P \\
\sum F = 0 &= P_{bot}A - (mg + P_{top}A) \\
0 &= (P_{top} + \Delta P)A - mg - P_{top}A \\
0 &= \Delta P A - mg \\
\frac{mg}{A} &= \Delta P
\end{aligned}$$

since $m = \rho V = \rho A h$ where ρ is the density:

$$\Delta p = \rho g h$$

Pascal's Principle

A pressure change applied to an enclosed liquid is transmitted undiminished to every part of the liquid and to the wall of the container in contact with the liquid. This means that two points at the same height will experience the same pressure.

Absolute Pressure

A perfect vacuum with no molecules to collide with the walls of the container is 0 Pa. This is the "absolute zero" of pressure because it is not possible to have a lower pressure.

Gauge Pressure

Gauge pressure is the difference in pressure compared to some reference point (often the atmospheric pressure $P_1 = 1$ atm). Since it is a relative measurement, it can be either positive or negative.

$$P_{gauge} = \Delta P = P_2 - P_1$$

Example 1

When drinking through a straw, we can reduce the pressure in our lungs by -85 mmHg. Suppose the drink has a density of $0.92 \times 10^3\ kg/m^3$ What is the maximum height of straw that we could use.

$$\begin{aligned}
\Delta P &= -85\ mmHg \\
\Delta P &= -1.13 \times 10^4\ Pa \\
\rho g h &= \Delta P \\
(0.92)(9.8)h &= -1.13 \times 10^4 \\
h &= -1.25 \\
d &= 1.25 \text{ m}
\end{aligned}$$

Buoyancy

A block suspended in water will have a buoyancy force acting on it.

$$\begin{aligned}
F &= -\rho g V \\
&= -\rho g A x \\
F &\propto -x \\
k &= \rho g A
\end{aligned}$$

Since the force is proportional to the negative displacement, it is simple harmonic motion.

$$\begin{aligned}
\omega &= \sqrt{\frac{k}{m}} \\
&= \sqrt{\frac{\rho g A}{\rho V}} \\
&= \sqrt{\frac{g A}{V}}
\end{aligned}$$

Continuity Equation

For fluid flowing in a tube, mass must be conserved. This means that for incompressible fluids, the volume flowing into a tube must be the same as the volume flowing out of the tube. This means that the velocity times the cross sectional area is constant across the entire tube.

$$V_1 = V_2$$
$$\Delta x_1 A_1 = \Delta x_2 A_2$$
$$v_1 A_1 = v_2 A_2$$

The volume flow rate (Q) can be defined as:

$$Q = Av$$

Bernoulli's principle

Suppose there is pressure on both sides of a tube acting on the same volume.

$$\begin{aligned} W_{tot} &= \Delta K + \Delta U \\ P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 &= (\frac{1}{2}m_2 v_2^2 - \frac{1}{2}m_1 v_1^2) + (m_2 g y_2 - m_1 g y_1) \\ P_1 V - P_2 V &= \frac{1}{2}(\rho V)v_2^2 - \frac{1}{2}(\rho V)v_1^2 + (\rho V)g y_2 - (\rho V) g y_1 \\ P_1 - P_2 &= \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g y_2 - \rho g y_1 \\ P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 &= P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 \end{aligned}$$

This is known as Bernoulli's principle. $P + \rho g y + \frac{1}{2}\rho v^2$ is constant throughout the fluid.