## Derivatives and Tangents

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## **Derivatives**

A line tangent to a graph takes the form:

$$y = mx + b$$

A line between two points on a graph is called a secant line. The limit as the second point approaches the first is the tangent line

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$
$$y - a = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} (x - a)$$
$$y = \frac{df}{dx} (a)(x - a) + f(a)$$

The derivative is the slope of the tangent line. The idea is that if we zoom in far enough toward the point, the curve looks like a linear.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

## Example

Find the equation of the tangent line at the point (3, 1).

$$\begin{split} f(x) &= \frac{3}{x} \\ f(x) &= 3x^{-1} \\ m &= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \\ m &= \lim_{h \to 0} \frac{1}{h} (\frac{3}{a+h} - \frac{3}{a}) \\ m &= \lim_{h \to 0} \frac{1}{h} (\frac{3}{a+h} - 1) \\ m &= \lim_{h \to 0} \frac{1}{h} (\frac{3}{a+h} - 1) \\ m &= \lim_{h \to 0} \frac{1}{h} (\frac{3}{a+h}) \\ m &= \lim_{h \to 0} \frac{1}{h} (\frac{-h}{a+h}) \\ m &= \lim_{h \to 0} (\frac{-1}{a+h}) \\ m &= \lim_{h \to 0} -\frac{1}{3+h} \\ m &= -\frac{1}{3} \end{split}$$

$$m = -\frac{1}{3}$$

$$x_1 = 3$$

$$y_1 = 1$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{3}(x - 3)$$

$$y = -\frac{1}{3}x + 2$$