Volume of a Revolution

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The volume of a function rotated around the x-axis is given by:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\pi f(x_i)^2\Delta x$$
$$= \int_a^b \pi f(x)^2\ dx$$

The volume of a function rotated around the y-axis is equivalent to rotating around the x-axis, using the inverse function. The volume is given by:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\pi f^{-1}(x_i)^2\Delta x$$
$$= \int_a^b \pi f^{-1}(x)^2\ dx$$

When finding the volume of a region bounded by two by f and g:

$$\int_a^b \pi f(x)^2\ dx - \int_a^b \pi g(x)^2\ dx$$

For rotating around any arbitrary horizontal line located at y_0:

$$\int_a^b \pi(f(x)-y_0)^2 - \pi(g(x)-y_0)^2\ dx$$

Average of a function

$$f_{avg} = \lim_{n\to\infty}\frac{1}{b-a}\sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \frac{\int_a^b f(x)\ dx}{b-a}$$

Mean value theorem for Integrals

f in continuous on $[a, b]$, then there exists a number c in [a,b] such that

$$f(x) = f_{avg} = \frac{1}{b-a}\int_a^b f(x)\ dx$$
$$\int_a^b f(x)\ dx = f(c)(b-a)$$