Jeremy Williams

Direct computation by "hand"

Kit A

Question 1:

The two sensors read Negative :

Let us denote S_1, S_2 $Sensors\ and\ E\ is\ random\ variable\ for\ Extreme\ temperature$

$P(E \mid S_1 = Negative\ , S_2 = Negative)$

$$= \frac{P(\ S_1 = Negative\ , S_2 = Negative\ |E\ ) * p(E)}{P(\ S_1 = Negative\ , S_2 = Negative\ |E\ ) * p(E) + P\big(\ S_1 = Negative\ , S_2 = Negative\ |\overline{E}\ \big) * p\big(\overline{E}\big)}$$

$$\frac{P(\ S_1 = Negative \mid E\ ) * P(S_2 = Negative\ |E\ ) * p(E)}{P(\ S_1 = Negative \mid E\ ) * P(S_2 = Negative\ |E\ ) * p(E) + P\big(\ S_1 = Negative\big|\overline{E}\ \big) * P(S_2 = Negative\ |\overline{E}\ ) * p\big(\overline{E}\big)}$$

$$= \frac{0{,}97 * 0{,}97 * 0{,}2}{0{,}97 * 0{,}97 * 0{,}2 + 0{,}03 * 0{,}03 * 0{,}8} = 0{,}996188459502382$$

And 1-1 $- 0{,}996188459502382 = 0{,}00381154049761778$

0,996188459502382	0,00381154049761778

R output

```
>
>
> predOT<-querygrain(Norman.net.1,nodes=c("L","S","E","S2","L2"), type="joint")
> predOT
E
      false        true
0.996340348 0.003659652
```

Question 2:

$P(E \mid S_1 = Positive\ , S_2 = Positive\ )$

$$= \frac{P(\ S_1 = Positive\ , S_2 = Positive\ |E\ ) * p(E)}{P(\ S_1 = Positive\ , S_2 = Positive\ |E\ ) * p(E) + P\big(\ S_1 = Positive\ , S_2 = Positive\ |\overline{E}\ \big) * p\big(\overline{E}\big)}$$

$$\frac{P(\, S_1 = Positive \mid E\,) * P(S_2 = Positive \mid E\,) * p(E)}{P(\, S_1 = Positive \mid E\,) * P(S_2 = Positive \mid E\,) * p(E) + P\big(\, S_1 = Positive \mid \overline{E}\,\big) * P(S_2 = Positive \mid \overline{E}\,) * p(\overline{E})}$$

$$= \frac{0{,}01 * 0{,}01 * 0{,}2}{0{,}01 * 0{,}01 * 0{,}2 + * \, 0{,}99 * 0{,}99 * 0{,}8} = 0{,}0000255069506440505$$

And 1-1 $- 0{,}0000255069506440505 = 0{,}999974493049356$

0,0000255069506440505	0,999974493049356

R output

```
>
>
> predOT<-querygrain(Norman.net.2,nodes=c("L","S","E","S2","L2"), type="joint")
> predOT
E
       false          true
0.0000265696 0.9999734304
```

Question 3:

$P(E \mid S_1 = Positive \,, S_2 = Negative\,)$

$$= \frac{P(\, S_1 = Positive \,, S_2 = Negative \mid E\,) * p(E)}{P(\, S_1 = Positive \,, S_2 = Negative \mid E\,) * p(E) + P\big(\, S_1 = Positive \,, S_2 = Negative \mid \overline{E}\,\big) * p\big(\overline{E}\big)}$$

$$\frac{P(\, S_1 = Positive \mid E\,) * P(S_2 = Negative \mid E\,) * p(E)}{P(\, S_1 = Positive \mid E\,) * P(S_2 = Negative \mid E\,) * p(E) + P\big(\, S_1 = Positive \mid \overline{E}\,\big) * P(S_2 = Negative \mid \overline{E}\,) * p(\overline{E})}$$

$$= \frac{0{,}97 * 0{,}01 * 0{,}2}{0{,}97 * 0{,}01 * 0{,}2 + * \, 0{,}03 * 0{,}99 * 0{,}8} = 0{,}0754863813229572$$

And 1-1 $- 0{,}0754863813229572 = 0{,}924513618677043$

0,0754863813229572	0,924513618677043

Output R

```
> predOT<-querygrain(Norman.net.3,nodes=c("L","S","E","S2","L2"), type="joint")
> predOT
E
    false      true
0.0783848 0.9216152
```

Kit B

Question1:

The two sensors read Negative :

Let us denote S_1, S_2 $Sensors\ and\ E\ is\ random\ variable\ for\ Extreme\ temperature$

$P(E \mid S_1 = Negative\ , S_2 = Negative)$

$$= \frac{P(\,S_1 = Negative\ , S_2 = Negative\ |E\ ) * p(E)}{P(\,S_1 = Negative\ , S_2 = Negative\ |E\ ) * p(E) + P\big(\,S_1 = Negative\ , S_2 = Negative\ |\overline{E}\ \big) * p(\overline{E})}$$

$$\frac{P(\,S_1 = Negative \mid E\ ) * P(S_2 = Negative\ |E\ ) * p(E)}{P(\,S_1 = Negative \mid E\ ) * P(S_2 = Negative\ |E\ ) * p(E) + P\big(\,S_1 = Negative|\overline{E}\ \big) * P(S_2 = Negative\ |\overline{E}\ ) * p(\overline{E})}$$

$$= \frac{0{,}97 * 0{,}97 * 0{,}2}{0{,}97 * 0{,}97 * 0{,}2 + 0{,}03 * 0{,}03 * 0{,}8} = 0{,}996188459502382$$

And 1-1 $- 0{,}996188459502382 = 0{,}00381154049761778$

0,996188459502382	0,00381154049761778

R output

```
>
>
> predOT<-querygrain(Norman.net.1,nodes=c("L","S","E","S2","L2"), type="joint")
> predOT
E
     false        true
0.996340348 0.003659652
```

Question 2:

$P(E \mid S_1 = Positive\ , S_2 = Positive\ )$

$$= \frac{P(\,S_1 = Positive\ , S_2 = Positive\ |E\ ) * p(E)}{P(\,S_1 = Positive\ , S_2 = Positive\ |E\ ) * p(E) + P\big(\,S_1 = Positive\ , S_2 = Positive\ |\overline{E}\ \big) * p(\overline{E})}$$

$$\frac{P(\,S_1 = Positive \mid E\ ) * P(S_2 = Positive\ |E\ ) * p(E)}{P(\,S_1 = Positive \mid E\ ) * P(S_2 = Positive\ |E\ ) * p(E) + P\big(\,S_1 = Positive\ |\overline{E}\ \big) * P(S_2 = Positive\ |\overline{E}\ ) * p(\overline{E})}$$

$$= \frac{0{,}01 * 0{,}01 * 0{,}2}{0{,}01 * 0{,}01 * 0{,}2 + * \, 0{,}99 * 0{,}99 * 0{,}8} = 0{,}000025506950644050\,5$$

And 1-1 $-$ 0,0000255069506440505 $=$ 0,999974493049356

0,0000255069506440505	0,999974493049356

R output

```
>
>
> predOT<-querygrain(Norman.net.2,nodes=c("L","S","E","S2","L2"), type="joint")
> predOT
E
       false         true
0.0000265696 0.9999734304
```

Question 3:

$P(E \mid S_1 = Positive\ , S_2 = Negative\ )$

$$= \frac{P(\,S_1 = Positive\ , S_2 = Negative\ |E\ ) * p(E)}{P(\,S_1 = Positive\ , S_2 = Negative\ |E\ ) * p(E) + P\big(\,S_1 = Positive\ , S_2 = Negative\ |\overline{E}\ \big) * p\big(\overline{E}\big)}$$

$$\frac{P(\,S_1 = Positive \mid E\ ) * P(S_2 = Negative\ |E\ ) * p(E)}{P(\,S_1 = Positive \mid E\ ) * P(S_2 = Negative\ |E\ ) * p(E) + P\big(\,S_1 = Positive\ |\overline{E}\ \big) * P(S_2 = Negative\ |\overline{E}\ \big) * p\big(\overline{E}\big)}$$

$$= \frac{0{,}97 * 0{,}01 * 0{,}2}{0{,}97 * 0{,}01 * 0{,}2 + * \, 0{,}03 * 0{,}99 * 0{,}8} = 0{,}0754863813229572$$

And 1-1 $-$ 0,0754863813229572 $=$ 0,924513618677043

0,0754863813229572	0,924513618677043

Output R

```
> predOT<-querygrain(Norman.net.3,nodes=c("L","S","E","S2","L2"), type="joint")
> predOT
E
    false      true
0.0783848 0.9216152
```

In which of the three cases the answer is the same for the two kits? Why?

We can see that all three cases are the same for the two kits according to the probabilities.

(a) The two sensors read negative.

Kit A

```
## Independence network: Compiled: TRUE Propagated: TRUE
##   Nodes: chr [1:6] "E" "B" "S" "S" "S2" "S2"
##   Evidence:
##   nodes is.hard.evidence hard.state
## 1     S              TRUE      false
## 2    S2              TRUE      false
##   pEvidence: 0.187265


## E
##       false        true
## 0.996340348 0.003659652
```

Kit B

```
## Independence network: Compiled: TRUE Propagated: TRUE
##   Nodes: chr [1:7] "E" "B" "B2" "S" "S" "S2" "S2"
##   Evidence:
##   nodes is.hard.evidence hard.state
## 1     S              TRUE      false
## 2    S2              TRUE      false
##   pEvidence: 0.185877


## E
##       false        true
## 0.996340348 0.003659652
```

(b) The two sensors read positive.

Kit A

```
## Independence network: Compiled: TRUE Propagated: TRUE
##   Nodes: chr [1:6] "E" "B" "S" "S" "S2" "S2"
##   Evidence:
##   nodes is.hard.evidence hard.state
## 1     S              TRUE       true
## 2    S2              TRUE       true
##   pEvidence: 0.005901


## E
##        false         true
## 0.0000265696 0.9999734304
```

Kit B

```
## Independence network: Compiled: TRUE Propagated: TRUE
##   Nodes: chr [1:7] "E" "B" "B2" "S" "S" "S2" "S2"
##   Evidence:
##   nodes is.hard.evidence hard.state
## 1     S             TRUE       true
## 2    S2             TRUE       true
##   pEvidence: 0.000590


## E
##        false         true
## 0.0000265696 0.9999734304
```

(c) One sensor reads positive while the other reads negative.

Kit A

```
## Independence network: Compiled: TRUE Propagated: TRUE
##   Nodes: chr [1:6] "E" "B" "S" "S" "S2" "S2"
##   Evidence:
##   nodes is.hard.evidence hard.state
## 1     S             TRUE       true
## 2    S2             TRUE      false
##   pEvidence: 0.000509


## E
##     false      true
## 0.0783848 0.9216152
```

Kit B

```
## Independence network: Compiled: TRUE Propagated: TRUE
##   Nodes: chr [1:7] "E" "B" "B2" "S" "S" "S2" "S2"
##   Evidence:
##   nodes is.hard.evidence hard.state
## 1     S             TRUE       true
## 2    S2             TRUE      false
##   pEvidence: 0.000687


## E
##     false      true
## 0.0783848 0.9216152
```

Looking at all three cases, they would be exactly the same since, we are emulating by a sensor reading (Using and Testing with the strength of the soft evidence).

As we know, for any Bayes factor, we can choose the false positive and negative rates of the sensor so its reading will have exactly the same effect on belief as that of the soft evidence. This emulation of soft evidence by hard evidence on an auxiliary variable (sensor) is also known as the method of virtual evidence.

Therefore, we can say that we are using the method of virtual evidence in this problem.