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Direct computation by "hand"

Kit A

Question 1:

The two sensors read Negative:

Let us denote
S_1, S_2 *are Sensors*, B *is a Battery, and* E *is a random variable for Extreme temperature*

$P(E \mid S_1 = F, S_2 = F)$

$$= \frac{[B * [P(S_1 = F, S_2 = F \mid E)] \;\; + \bar{B}] * p(E)}{B * \left[P(S_1 = F, S_2 = F \mid E) * p(E) + P\left(S_1 = F, S_2 = F \mid \overline{E}\right) * p\left(\overline{E}\right)\right] + \bar{B}}$$

$$\frac{B * [P(S_1 = F \mid E) * P(S_2 = F \mid E)] + \bar{B} * 1}{B * \left[P(S_1 = F \mid E) * P(S_2 = F \mid E) * p(E) + P\left(S_1 = F \middle| \overline{E}\right) * P\left(S_2 = F \mid \overline{E}\right) * p\left(\overline{E}\right)\right] + \bar{B}}$$

$$= \frac{[0.9 * (0{,}03 * 0{,}03) + 0.1 * 1] * 0.2}{0{,}9 * (0{,}03 * 0{,}03 * 0{,}2 + 0{,}99 * 0{,}99 * 0{,}8) \; + \; 0.1} = 0.02502004$$

And $1 - 0.02502004 = 0.97498$

0.97498	0.02502004

R output

```
predOT<-querygrain(Norman.net.1,nodes=c("E"), type="marginal")
predOT
```

```
## $E
## E
##      false       true
## 0.97497996 0.02502004
```

```
predOT$E[["true"]]
```

```
## [1] 0.02502004
```

Question 2:

$P(E \mid S_1 = T, S_2 = T)$

$$= \frac{B * [P(S_1 = T, S_2 = T|E) * p(E)]}{P(S_1 = T, S_2 = T|E) * p(E) + P(S_1 = T, S_2 = T|\overline{E}) * p(\overline{E})}$$

$$\frac{P(S_1 = T \mid E) * P(S_2 = T|E) * p(E)}{[P(S_1 = T \mid E) * P(S_2 = T|E) * p(E) + P(S_1 = T|\overline{E}) * P(S_2 = T|\overline{E}) * p(\overline{E})]}$$

$$= \frac{0{,}9 * [0{,}97 * 0{,}97 * 0{,}2]}{0{,}9 * [0{,}97 * 0{,}97 * 0{,}2 + 0{,}01 * 0{,}01 * 0{,}8]} = 0.9995751$$

And $1 - 0.9995751 = 0.0004249442$

0.0004249442	0.9995751

R output

```
predOT<-querygrain(Norman.net.2,nodes=c("E"), type="marginal")
predOT
```

```
## $E
## E
##        false         true
## 0.0004249442 0.9995750558
```

```
predOT$E[["true"]]
```

```
## [1] 0.9995751
```

Question 3:

$P(E \mid S_1 = T, S_2 = F)$

$$= \frac{B * [P(S_1 = T, S_2 = F|E) * p(E)]}{B * [P(S_1 = T, S_2 = F|E) * p(E) + P(S_1 = T, S_2 = F|\overline{E}) * p(\overline{E})}$$

$$\frac{B * [P(S_1 = T \mid E) * P(S_2 = F|E) * p(E)]}{B * [P(S_1 = T \mid E) * P(S_2 = F|E) * p(E) + P(S_1 = T|\overline{E}) * P(S_2 = F|\overline{E}) * p(\overline{E})]}$$

$$= \frac{0{,}97 * 0{,}03 * 0{,}2}{0{,}97 * 0{,}03 * 0{,}2 + 0{,}01 * 0{,}99 * 0{,}8} = 0.4235808$$

And $1 - 0.4235808 = 0.5764192$

0.5764192	0.4235808

Output R

```
predOT<-querygrain(Norman.net.3,nodes=c("E"), type="marginal")
predOT
```

```
## $E
## E
##     false      true
## 0.5764192 0.4235808
```

```
predOT$E[["true"]]
```

```
## [1] 0.4235808
```

Kit B

Question1 and 2

Based on the above calculations… It is easy to see that:

$P(E \mid S_1 = T, S_2 = T)$

$$= \frac{B * [P(S_1 = T, S_2 = T|E) * p(E)]}{B * [P(S_1 = T, S_2 = T\ |E) * p(E)] + B * [P\left(S_1 = T, S_2 = T\ |\overline{E}\right) * p\left(\overline{E}\right)]}$$

$$\frac{P(S_1 = T \mid E) * P(S_2 = T\ |E) * p(E)}{[P(S_1 = T \mid E) * P(S_2 = T\ |E) * p(E) + P\left(S_1 = T\ |\overline{E}\right) * P(S_2 = T\ |\overline{E}) * p\left(\overline{E}\right)]}$$

$$= \frac{0{,}9 * [0{,}97 * 0{,}97 * 0{,}2]}{0{,}9 * [0{,}97 * 0{,}97 * 0{,}2] + 0{,}9 * [0{,}01 * 0{,}01 * 0{,}8]} = 0.9995751$$

And $1 - 0.9995751 = 0.0004249442$

0.0004249442	0.9995751

R output

```
predOT<-querygrain(Norman.net.2,nodes=c("E"), type="marginal")
predOT
```

```
## $E
## E
##        false         true
## 0.0004249442 0.9995750558
```

```
predOT$E[["true"]]
```

```
## [1] 0.9995751
```

$P(E \mid S_1 = F, S_2 = F)$

$$= \frac{[B * [P(S_1 = F, S_2 = F \mid E)] \quad * p(E)}{B * [P(S_1 = F, S_2 = F \mid E) * p(E)] + B * [P(S_1 = F, S_2 = F \mid \overline{E}) * p(\overline{E})]}$$

It is easy to see that:

$P(E \mid S_1 = F, S_2 = F)$ = 0.004089033

And 1 - 0.004089033 = 0.0004249442

0.99591	0.004089033

R output

```
predOT<-querygrain(Norman.net.1,nodes=c("E"), type="marginal")
predOT
```

```
## $E
## E
##       false        true
## 0.995910967 0.004089033
```

```
predOT$E[["true"]]
```

```
## [1] 0.004089033
```

Question 3:

$P(E \mid S_1 = T, S_2 = F)$

$$\frac{B * [P(S_1 = T \mid E) * P(S_2 = F \mid E) * p(E)] + B * [P(S_1 = T \mid E) * \bar{B}] * p(E)}{\begin{array}{c} B * [P(S_1 = T \mid E) * P(S_2 = F \mid E) * p(E)] + \bar{B} * [B * P(S_1 = T \mid E) * p(\overline{E})] + \\ B * [P(S_1 = T \mid \overline{E}) * P(S_2 = F \mid \overline{E}) * p(\overline{E})] + \bar{B} * [B * P(S_1 = T \mid \overline{E}) * p(\overline{E})] \end{array}}$$

$$= \frac{(0{,}9 * 0{,}97 * 0{,}9 * 0{,}03 + 0{,}97 * 0{,}9 * 0{,}1) * 0.2}{\begin{array}{c} 0{,}9 * 0{,}97 * 0{,}9 * 0{,}03 * 0{,}2 + 0{,}9 * 0{,}97 * 0{,}2 * 0{,}1 \\ + 0{,}9 * 0{,}99 * 0{,}9 * 0{,}01 * 0{,}8 + 0{,}9 * 0{,}01 * 0{,}8 * 0{,}1 \end{array}} = 0.7565559$$

And $1 - 0.7565559 = 0.2434441$

0.2434441	0.7565559

Output R

```
predOT<-querygrain(Norman.net.3,nodes=c("E"), type="marginal")
predOT
```

```
## $E
## E
##     false      true
## 0.2434441 0.7565559
```

```
predOT$E[["true"]]
```

```
## [1] 0.7565559
```

In which of the three cases the answer is the same for the two kits? Why?

We can see that, from the above calculation by hand and R-code below, $P(E \mid S_1 = T, S_2 = T)$ are the same for the two kits according to the probabilities. This is because the batteries does not affect the calculations (i.e. would cancel out - 0.9/0.9 = 1) for both Kits in this problem.

Kit A

```
#Question 2
Norman.net.2<-setEvidence(Norman.net,nodes=c("S","S2"),states=c("true","true"))

Norman.net.2
```

```
## Independence network: Compiled: TRUE Propagated: TRUE
##   Nodes: chr [1:4] "E" "B" "S" "S2"
##   Evidence:
##   nodes is.hard.evidence hard.state
## 1     S              TRUE       true
## 2    S2              TRUE       true
##   pEvidence: 0.169434
```

```
predOT<-querygrain(Norman.net.2,nodes=c("E"), type="marginal")
predOT
```

```
## $E
## E
##        false         true
## 0.0004249442 0.9995750558
```

```
predOT$E[["true"]]
```

```
## [1] 0.9995751
```

Kit B

```
#Question 2
Norman.net.2<-setEvidence(Norman.net,nodes=c("S","S2"),states=c("true","true"))

Norman.net.2
```

```
## Independence network: Compiled: TRUE Propagated: TRUE
##   Nodes: chr [1:5] "E" "B" "B1" "S" "S2"
##   Evidence:
##   nodes is.hard.evidence hard.state
## 1     S              TRUE       true
## 2    S2              TRUE       true
##   pEvidence: 0.152491
```

```
predOT<-querygrain(Norman.net.2,nodes=c("E"), type="marginal")
predOT
```

```
## $E
## E
##        false         true
## 0.0004249442 0.9995750558
```

```
predOT$E[["true"]]
```

```
## [1] 0.9995751
```