BAYESIAN NETWORKS – 1st Exercise Block 1

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6) Dr Ann Nicholson spends 60% of her work time in her office. The rest of her work time is spent elsewhere. When Ann is in her office, half the time her light is off (when she is trying to hide from students and get research done). When she is not in her office, she leaves her light on only 5% of the time. 80% of the time she is in her office, Ann is logged into the computer. Because she sometimes logs into the computer from home, 10% of the time she is not in her office, she is still logged into the computer.

(a) Construct a Bayesian Network to represent the scenario just described.

Answer:

Given that, V = {A, B, C},

We define:

A= Dr Ann Nicholson in office, A' = Dr Ann Nicholson not in office
B = Light is on, B'= Light is off
C = Logged into the office computer, C'= Not Logged into the office computer

P(A) = 0.6, P(A') = 1 – 0.6 = 0.4

P(B/A) = 0.5, P(B'/A) = 1 – 0.5 = 0.5

P(B/A') = 0.05, P(B'/A') = 1 – 0.05 = 0.95

P(C/A) = 0.8, P(C'/A) = 1 – 0.8 = 0.2

P(C/A') = 0.1, P(C'/A) = 1 – 0.1 = 0.9

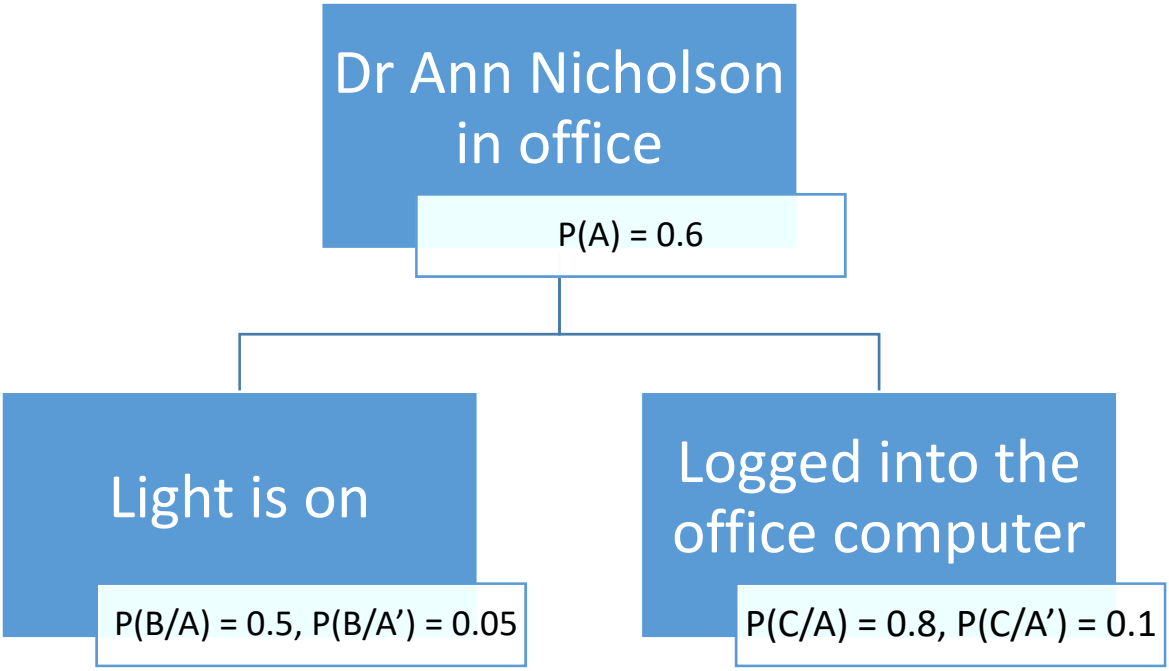
We rewrite the order of the variables, based on the P values above:

V = {B, C, A},
E = {(B, A), (C, A)},
Γ = (V, E) is a DAG

Then, (Γ, P) verifies the Markov condition if B and C are conditionally independent given A:

P(B, C/A) = P(B/A) P(C/A)

We can also construct it graphically:



(b) Suppose a student checks Dr Nicholson's login status and sees that she is logged on. What effect does this have on the student's belief that Dr Nicholson's light is on?

Step 1 –> P(C) = P(C/A)* P(A)] + P(C/A')*(A')

P(C) = (0.8 * 0.6) + (0.1 * 0.4) = 0.48 + 0.04 = **0.52**

Step 2 –> P(A/C) = [P(C/A)* P(A)]/P(C)

= (0.8 * 0.6)/0.52
= **0.923**

Step 3 -> We want to know P(B/C),

So, P(B/C) = P(B,C) / P(C)

= [P(B,C/A)*P(A) + P(B,C/A')*P(A')] / P(C)

= [P(B/A)*P(C/A)*P(A) + P(B/A')*P(C/A')*P(A')] / P(C)

= [(0.5*0.8*0.6) + (0.05*0.01*0.4)] / 0.52

= [0.24 +0.0002] / 0.52

= **0.461923**

(c) Is the evidence that Dr Nicholson's is logged on in favour of or against her light is on?

We need to find, $P(B) = P(B/A)* P(A)] + P(B/A')*(A')$
$= (0.5*0.6) + (0.05*0.4)$
$= 0.3 + 0.02$
$= \mathbf{0.32}$

$P(B/C) = \mathbf{0.461923}$

Since, $P(B) < P(B/C)$

where, $\mathbf{0.32 < 0.461923}$

Therefore, the evidence that Dr Nicholson´s is logged on is in favour of her light is on.