A Guidance of The Full-Fledged Model

Overview

Infrastructure

To run the model, ensure "Minizinc with Or-Tools" is installed on your computing device.

Note that, we run the model on *AMD Ryzen 9 7945HX* with *32* threads, using:

```
minizinc --solver CP-SAT -p 32 Modelname.mzn > RESULTS.txt
```

Instruction manual

All variables in our model (*solver* = *CP-SAT*) are integers; when floats arise, try to transform them to a integer if it`s possible.

Components

Each part of our model is separated in different `dzn` files, therefore, we have the following components:

- **Main:** `0_Deoxys. mzn` is the center control model. By enabling or disabling the functions in this main model, it`s possible to adjust the searching depth.
- **Pattern (Dis. and Ext.):** "1_differential.dzn" constraints the differential pattern of the whole attack.
  - Returning parameters: #active cells in plaintext, ciphertext r_b, r_f, #involved subkeys m_b, m_f, and $probability$.
  - The probabilistic extension is allowed, and m_b, m_f can be smaller when the state test is working in this component.
  - Open `include "1_differential.dzn";` to use this function.
- **Guessing Strategy:** "2_guess_and_determine.dzn" describes the pre-guessing strategy of subkeys.
  - Returning parameters: pre-guessed subkeys m'_b, m'_f, filters r'_b, r'_f.
  - $Gstate$ is also contain in m'_b, m'_f when the state test is working.
  - Open `include "2_guess_and_determine.dzn";` to use this function.
- **Epsilon calculation:** "3-1_epsilon_gandf-ST.dzn" and "3-2_epsilon_table-ST.dzn" give the constraints of step assignment and complexities.
  - Returning parameters: processing step of the unguessed subkeys, corresponding filters, time and memory complexity of each step.
  - When the state test is working, ST is contained as subkeys, and EGK is eliminated, in some steps.
  - Open `include "3-1_epsilon_gandf-ST.dzn";` to use the guess-and-filter approach, and `include "3-2_epsilon_table-ST.dzn";` for the pre-computation hash table approach.

Predicates: "predicates.dzn"

- `include "3-1_epsilon_gandf-ST.dzn";` is set as default.
- Many predicates are included as methods that can be used in other components.

Adjustments in Main

State test

The switch of state test:

```
int: OpenST = 0; % 1 -> Trun on State Test;
constraint if OpenST = 0
           then forall(r in 0..Rb-1, i in 0..15)(uST[r,i] = -1) /\ forall(r in
Rb+Ru+Rm+Rl..Rb+Ru+Rm+Rl+Rf-1, i in 0..15)(lST[r,i] = -1) /\
                forall(r in 0..Rb-1, i in 0..15)(uEGK[r,i] = -1) /\
                forall(r in 0..Rb-1, i in 0..15)(uESB[r,i] = -1) /\
                forall(r in 0..Rb-1, i in 0..15)(uEMC[r,i] = -1) /\
                forall(r in Rb+Ru+Rm+Rl+1..Rb+Ru+Rm+Rl+Rf, i in 0..15)(lEGK[r,i]
= -1) /\
                forall(r in Rb+Ru+Rm+Rl..Rb+Ru+Rm+Rl+Rf-1, i in 0..15)(lESB[r,i]
= -1) /\
                forall(r in Rb+Ru+Rm+Rl..Rb+Ru+Rm+Rl+Rf-2, i in 0..15)(lEMC[r,i]
= -1)
           endif;
```

epsilon calculation

Switch between two approaches of epsilon calculation:

```
% Part 3 (epsilon G and F)
% include "3-1_epsilon_gandf-ST.dzn";

% Part 3 (epsilon table)
include "3-2_epsilon_table-ST.dzn";
```

Key bridging

The following constraints of strong key bridging are applied only when an attack achieves no less than 15 rounds.

3 cases for involvements, guessing strategy, and epsilon calculation....

```
% CASE-1: Strong Key Bridges for involved key
array[0..3] of var int: vRd;
constraint forall(c in 0..3)(vRd[c] = JTable[sum(i in 0..3)
(uVSTK[0,hTable[4*c+i,1]]), sum(i in 0..3)(lVSTK[15,4*c+i])]);
var int: mb  = sum(r in 0..Rb-1, i in 0..15)(uVSTK[r,i] == 1 /\ uEGK[r,i] == -1)
+ sum(r in 0..Rb-1, i in 0..15)(uVstate[r,i]);
var int: mf  = sum(r in Rb+Ru+Rm+Rl+1..Rb+Ru+Rm+Rl+Rf, i in 0..15)(lVSTK[r,i] ==
1 /\ lEGK[r,i] == -1) + sum(r in Rb+Ru+Rm+Rl..Rb+Ru+Rm+Rl+Rf-1, i in 0..15)
(lVstate[r,i]);
var int: mk = 8*(mb + mf) - sum(c in 0..3)(vRd[c]);

% Strong Key Bridges for pre-guessed key
array[0..3] of var int: gRd;
constraint forall(c in 0..3)(
  if forall(i in 0..3)(uGSTK[0,hTable[4*c+i,1]] = uVSTK[0,hTable[4*c+i,1]] /\
lGSTK[15,4*c+i] = lVSTK[15,4*c+i])
  then gRd[c] = JTable[sum(i in 0..3)(uGSTK[0,hTable[4*c+i,1]]), sum(i in 0..3)
(lGSTK[15,4*c+i])]
  else gRd[c] = 0
  endif);
var int: mb_p = sum(r in 0..Rb-1, i in 0..15)(uGSTK[r,i]) + sum(r in 0..Rb-1, i
in 0..15)(uGstate[r,i]);
var int: mf_p = sum(r in Rb+Ru+Rm+Rl+1..Rb+Ru+Rm+Rl+Rf, i in 0..15)(lGSTK[r,i]) +
sum(r in Rb+Ru+Rm+Rl..Rb+Ru+Rm+Rl+Rf-1, i in 0..15)(lGstate[r,i]);
var int: mk_p = 8*(mb_p + mf_p) - sum(c in 0..3)(gRd[c]);

% Strong Key Bridges for epsilon calculation
array[0..3] of var -1..Step: sBigRd;
array[0..3] of var int: sRd;
constraint forall(c in 0..3)(sBigRd[c] = max(max(i in 0..3)(uSGK[0, hTable[4*c+i,
1]]), max(i in 0..3)(lSGK[15, 4*c+i])));
constraint forall(c in 0..3)(
  if sBigRd[c] >= 1
  then sRd[c] = JTable[sum(i in 0..3)(uSGK[0,hTable[4*c+i,1]] >= 1 /\
uSGK[0,hTable[4*c+i,1]] <= sBigRd[c]),
                       sum(i in 0..3)(lSGK[15,4*c+i] >= 1 /\ lSGK[15,4*c+i] <=
sBigRd[c])]
  else sRd[c] = 0
  endif);
array[1..Step] of var int: sRdS;
constraint forall(s in 1..Step)(if exists(c in 0..3)(sBigRd[c] = s) then sRdS[s]
>= 0 else sRdS[s] = 0 endif);
constraint forall(c in 0..3)(if sBigRd[c] >= 1 then sRdS[sBigRd[c]] = sRd[c]
endif);
```

Since the impact of key bridging is implicit, we give the format output to show the influence of key bridging (redundancy in different processes): easy for verify where the key bridging working in.

```
output[show(mb+mf) ++ " bytes |" ++ show(sum(c in 0..3)(vRd[c])) ++ " bits ------
"]; % involvement
output[show(mb_p+mf_p) ++ " bytes |" ++ show(sum(c in 0..3)(gRd[c])) ++ " bits --
---- "]; % pre-guessing
output[show([sRdS[s] | s in 1..Step])]; % epsilon calculation
```

Objective Functions for Elasticizing

Note that the extra variables for disabling functions exist throughout the entire model, but are constrained out (assigning constants) of solutions when they provide influences.

Main + Pattern

$$\min(a \times PrAtt + b \times (m_b + m_f))$$

Main + Pattern+ Guessing Strategy

$$constraint\ \ Time_{\epsilon} = 0$$
$$\min(a \times Time + b \times Memory + c \times Date)$$

Whole Model

$$\min(a \times Time + b \times Memory + c \times Memory_\epsilon + d \times Data)$$

Due to the long time required to solve the whole model, we provide some constraints as *TEST* for fast verification.

Examples

We give the models and the corresponding results of 2 versions of `Deoxys`, goto:

- **11-round** `Deoxys-BC-256`
- **15-round** `Deoxys-BC-384`

We give the models and the corresponding results of 2 versions of `Deoxys-AE`, goto:

- **10-round** `Deoxys-I-128-128`
- **14-round** `Deoxys-I-256-128`

We give the models and the corresponding results of 2 versions of `Deoxys-multiTK`, goto:

- **16-round** `Deoxys-TBC-512-256`
- **18-round** `Deoxys-TBC-640-256`

Alter-abling specifications for different attacks (example = 11-round rectangle attack on `Deoxys-BC-256`)

```
int: SpecDeoxys = 2; % denotes TK2
int: block_size = 128;
int: key_size = block_size * SpecDeoxys;

int: Rb = 1; % backward extension
int: Ru = 3; % upper differential
int: Rm = 2; % middle
int: Rl = 3; % lower differential
int: Rf = 2; % forward extension
```

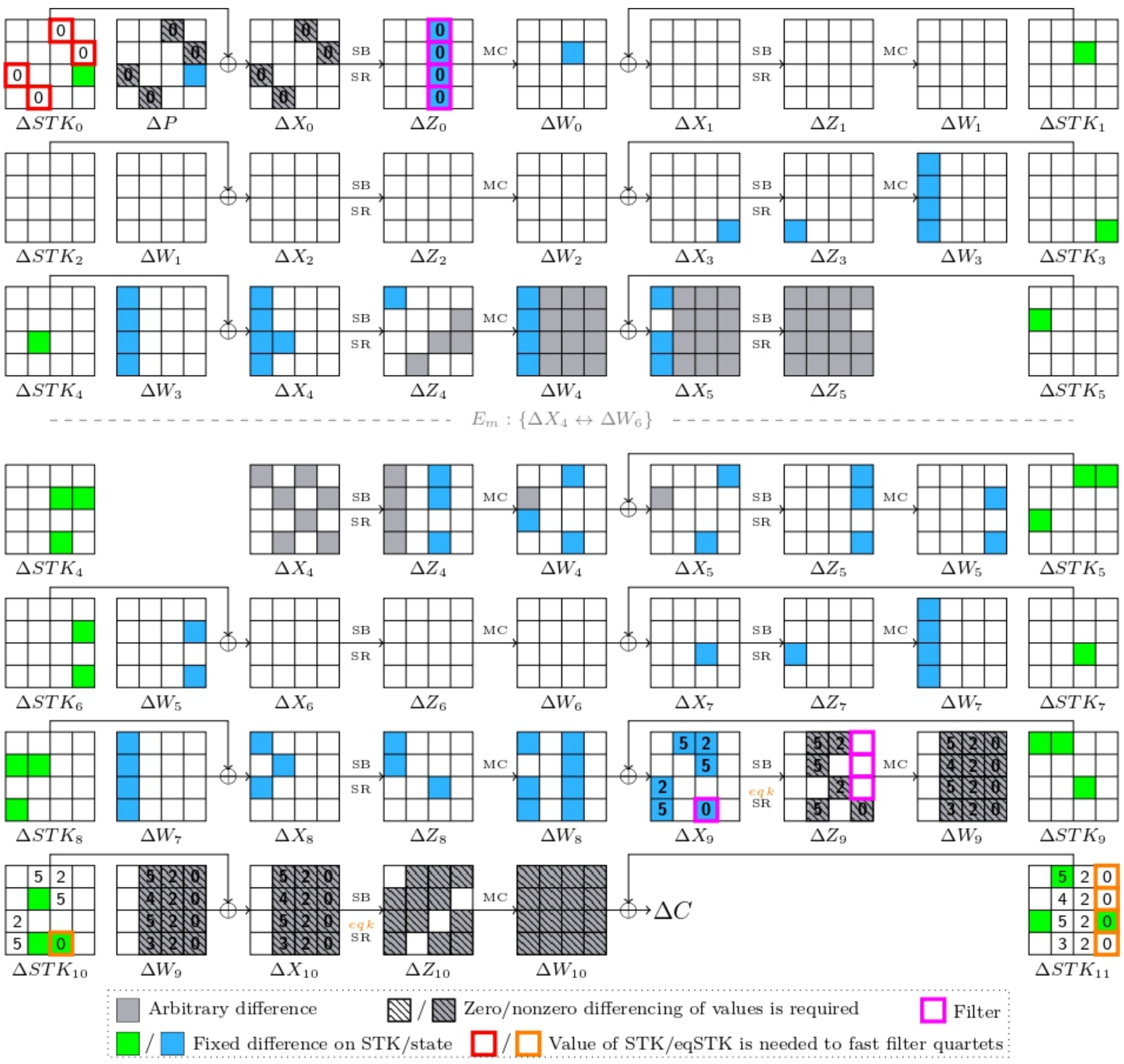
Deoxys-BC-256 (11r)

```
int: SpecDeoxys = 2;
int: block_size = 128;
int: key_size = block_size * SpecDeoxys;

int: Rb = 1;
int: Ru = 3;
int: Rm = 2;
int: Rl = 3;
int: Rf = 2;
```

Result obtained (detail):

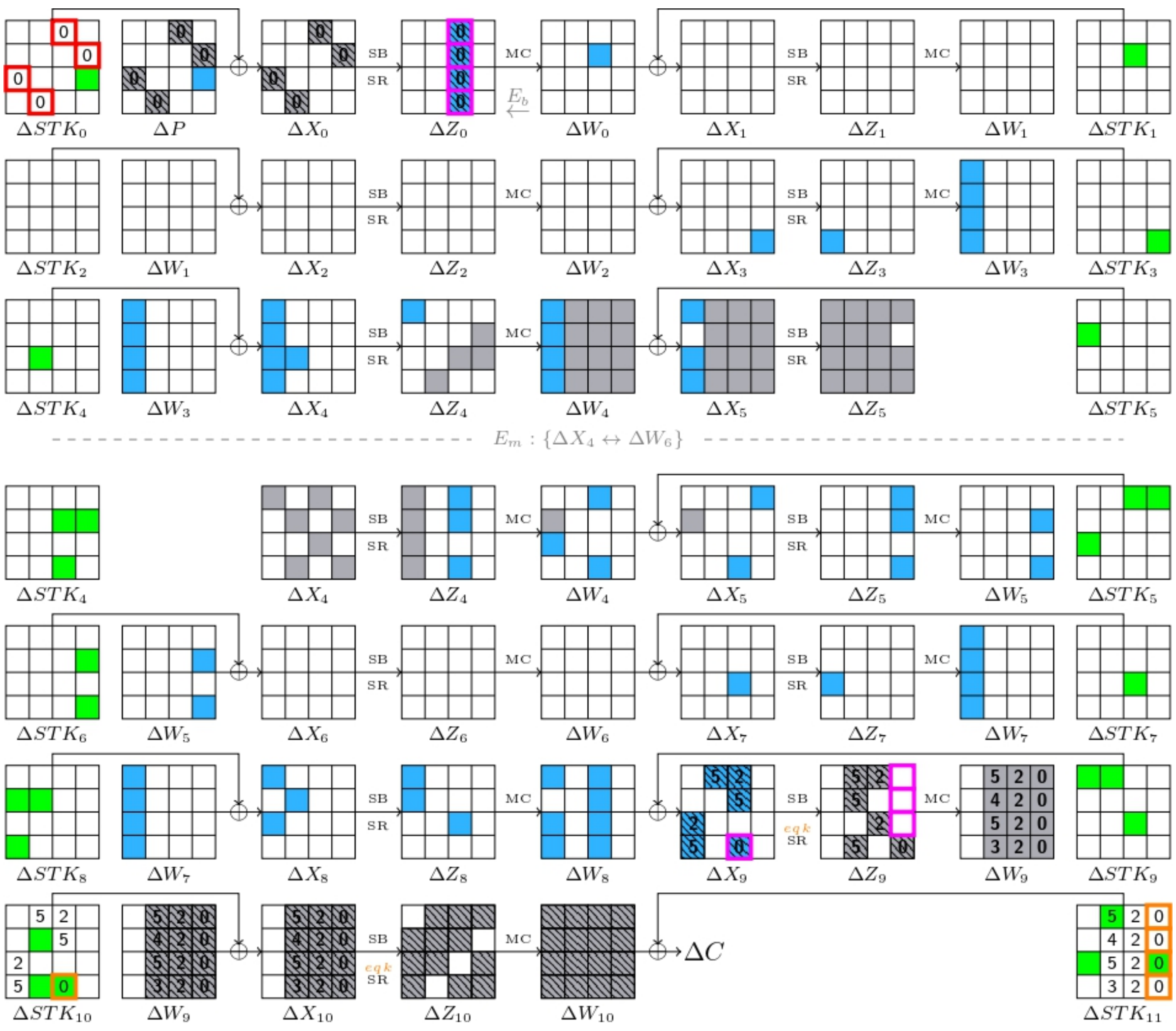
```
Complexity of Rectangle Attack on Deoxys-BC-384:

Parameters:
----------------------
rb= 4 | rb'= 4 | mb= 4 | mb'= 4
rf=12 | rf'= 4 | mf=18 | mf'= 5
----------------------
Prob.  : 112
Data   : 122
MemoryC: 123
TimeC  : 195 [T1=194 | T2u=195, T2l=253 | T3=188 (epsilon=0)]
```

Draw with TikZ latex:


ΔSTK_0
ΔP
ΔX_0
ΔZ_0
ΔW_0
ΔX_1
ΔZ_1
ΔW_1
ΔSTK_1
SB
SR
MC
$E_m : \{\Delta X_4 \leftrightarrow \Delta W_6\}$
eqk
ΔC
ΔSTK_{11}
Arbitrary difference
Zero/nonzero differencing of values is required
Filter
Fixed difference on STK/state
Value of STK/eqSTK is needed to fast filter quartets

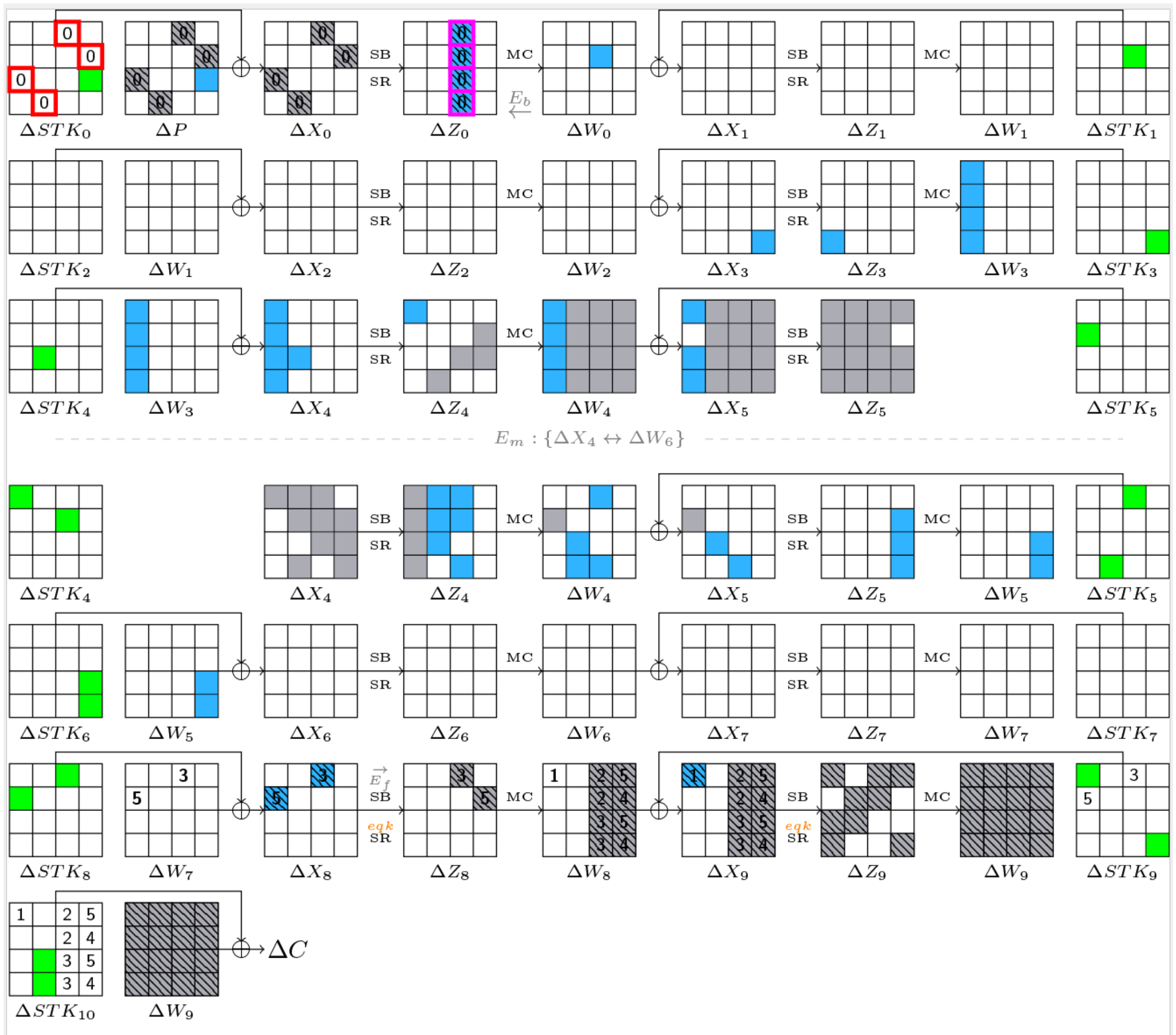
Deoxys-BC-384 (15r)

```
int: SpecDeoxys = 3;
int: block_size = 128;
int: key_size = block_size * SpecDeoxys;

int: Rb = 1;
int: Ru = 4;
int: Rm = 2;
int: Rl = 4;
int: Rf = 4;
```



Prob.: 112 (0, 14, 42, 56, 0); $r_b = 4$ $r'_b = 4$ $m_b = 4$ $m'_b = 4$; $r_f = 12$ $r'_f = 4$ $m_f = 18$ $m'_f = 5$
DataC: 122, MemoryC: 123 ($M_\epsilon = 14$), TimeC: 195 [T_1=194, T_{2u}=195 T_{2l}=253, T_3=188 (T_ϵ=0)]

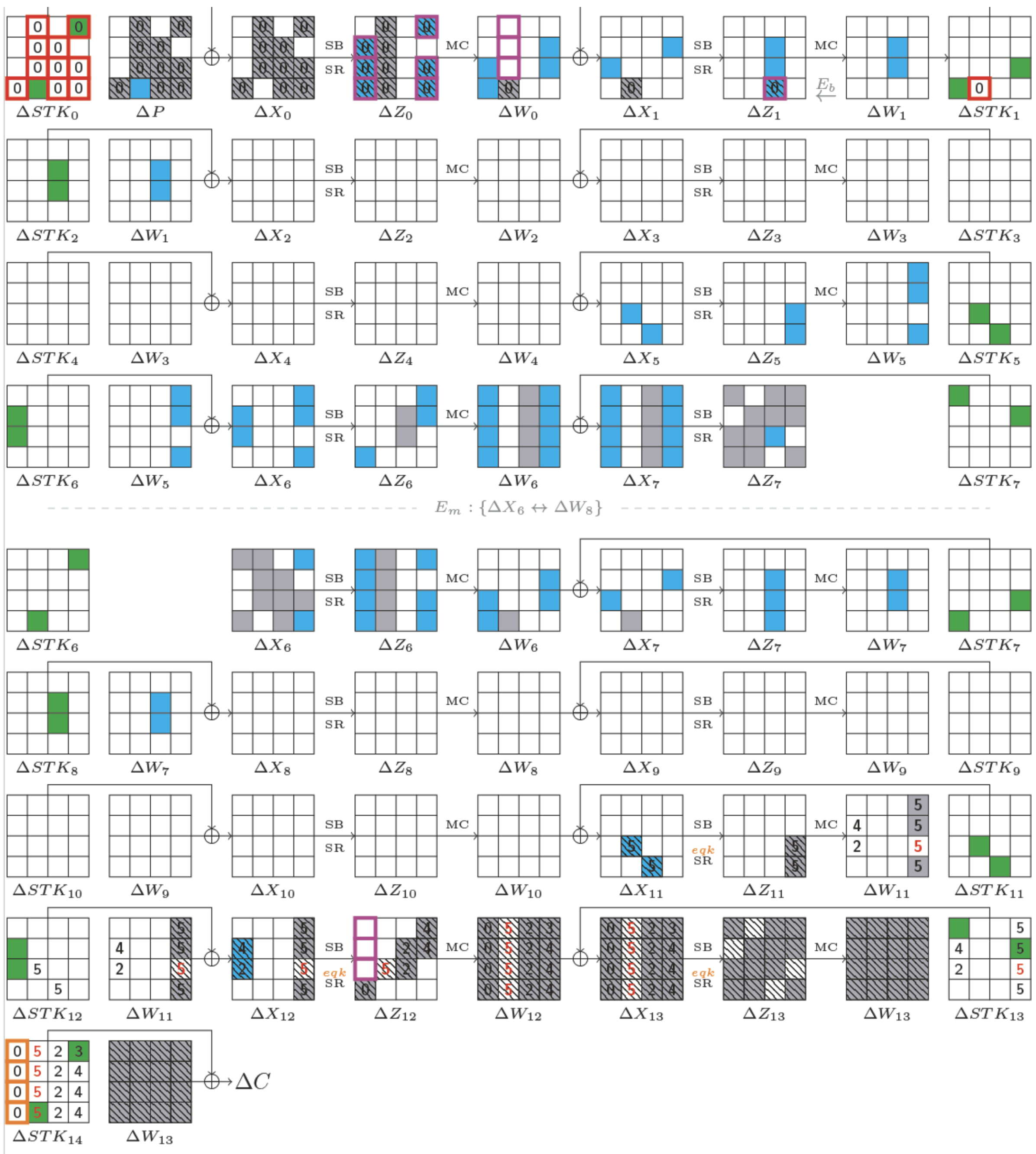
Deoxys-I-128 (10r)



Prob.: 56 (0, 14, 42, 0, 0); $r_b = 4$ $r'_b = 4$ $m_b = 4$ $m'_b = 4$; $r_f = 9$ $r'_f = 0$ $m_f = 11$ $m'_f = 0$

DataC: 94, MemoryC: 95 ($M_\epsilon = 6$), TimeC: 127 [T_1=126, T_{2u}=127 T_{2l}=165, T_3=124 (T_ϵ=2)]

Deoxys-I-256 (14r)



ΔSTK_0 ΔP ΔX_0 SB SR ΔZ_0 MC ΔW_0 ΔX_1 SB SR ΔZ_1 MC E_b ΔW_1 ΔSTK_1
ΔSTK_2 ΔW_1 ΔX_2 ΔZ_2 ΔW_2 ΔX_3 ΔZ_3 ΔW_3 ΔSTK_3
ΔSTK_4 ΔW_3 ΔX_4 ΔZ_4 ΔW_4 ΔX_5 ΔZ_5 ΔW_5 ΔSTK_5
ΔSTK_6 ΔW_5 ΔX_6 ΔZ_6 ΔW_6 ΔX_7 ΔZ_7 ΔSTK_7
$E_m : \{\Delta X_6 \leftrightarrow \Delta W_8\}$
ΔSTK_6 ΔX_6 ΔZ_6 ΔW_6 ΔX_7 ΔZ_7 ΔW_7 ΔSTK_7
ΔSTK_8 ΔW_7 ΔX_8 ΔZ_8 ΔW_8 ΔX_9 ΔZ_9 ΔW_9 ΔSTK_9
ΔSTK_{10} ΔW_9 ΔX_{10} ΔZ_{10} ΔW_{10} ΔX_{11} eqk ΔZ_{11} ΔW_{11} ΔSTK_{11}
ΔSTK_{12} ΔW_{11} ΔX_{12} eqk ΔZ_{12} ΔW_{12} ΔX_{13} eqk ΔZ_{13} ΔW_{13} ΔSTK_{13}
ΔSTK_{14} ΔW_{13} ΔC

Prob.: 102 (28, 28, 30, 0, 16); $r_b = 10$ $r'_b = 10$ $m_b = 11$ $m'_b = 11$; $r_f = 12$ $r'_f = 3$ $m_f = 19$ $m'_f = 4$

DataC: 117, MemoryC: 120 ($M_\epsilon = 15$), TimeC: 242 [T_1=237, T_{2u}=238, T_3=242 (T_ϵ=0)]

Pretty Output, Drawing Using TikZ

```
include "./format_out/1_format_out_diff.dzn"; % show differential pattern,
parameters, and probability
include "./format_out/3-2_format_out_tableST.dzn"; % show step assignment and
complexities of epsilon
include "./format_out/0_format_out_Main.dzn"; % show the necessary parameters,
complexities of the attack
include "./format_out/drawlatex.dzn"; % draw with tikz
```

Copy the LaTeX code output and paste it into a LaTeX compiler.

For SKINNY

`SKINNY` is a typical example that uses a non-MDS matrix as its linear layer; therefore, in the key recovery phase, differential-determination detection beyond value determination is necessary in the components Guessing Strategy and Epsilon Calculation.

For each differential of state, `X`, sets of variables we used in the key recovery model of `SKINNY`.

```
vX: value needed
dX: differential needed
gSTK: guessed subkey in Guessing Strategy
detX: value determined
detdiffX: differential determined
sgSTK: guessed subkey in step assignment
saX: value assigned step
sdvX: value deduced in step assignment
```

Considering new sets of variables, `detdiffX, sadiffX, sadedX, sadevX`, allows more delicate relations to be captured in our model. Specifically, in the Epsilon Calculation component, the **Sbox property** can be used to deduce subkeys that are not guessed in any process.

```
sadiffX: differential assigned step
sadedX: differential deduced using the property of Sbox property
sadevX: value deduced using the property of Sbox property
```

The stronger key bridging:

The key bridging component should be adapted to the number of attack rounds. When the attack covers more than 30 rounds, there are subkeys involved, and a full round can be deduced at no cost. To capture all involved subkeys, we use sliding windows that cover 30 rounds.

For example, for the 33-round attack, the extended rounds span in rounds 0-3, 27-33. The key bridging component relies on the five slide windows that cover the outer rounds: (0 .. 3, 27 .. 29), (1.. 3, 27 .. 30), (2 .. 3, 27 .. 31), (3 .. 3, 27 .. 32).

For classical key bridging, the subkeys in the same LANE provide at most p-cell information, which can be easily embedded into the component of stronger key bridging, where p is the number of `TK` ($p = 3$ for SKINNY-n-3n).

```
constraint forall(c in 0..15)(
    let {
        var int: vsum029 = sum(r in 0..4)(bvSTK[r,c]) + sum(r in 27..29)
(fvSTK[r,c]),
        var int: vsum130 = sum(r in 1..4)(bvSTK[r,c]) + sum(r in 27..30)
(fvSTK[r,c]),
        var int: vsum231 = sum(r in 2..4)(bvSTK[r,c]) + sum(r in 27..31)
(fvSTK[r,c]),
        var int: vsum332 = sum(r in 3..4)(bvSTK[r,c]) + sum(r in 27..32)
(fvSTK[r,c]),
        var int: vsum433 = sum(r in 4..4)(bvSTK[r,c]) + sum(r in 27..33)
(fvSTK[r,c]),
    } in (
        vL_bg[c] = max(max(vsum029, vsum130), max(max(vsum231, vsum332),
vsum433))
    )
);
vSTK_Sbg = sum(c in 0..15)(if vL_bg[c] >= 3 then 3 else vL_bg[c] endif);
```

The following context is prepared for rebuttal. (delete this sentence)

Comparison

We provide the comparison of the effect among different approaches, specifically, the state test and epsilon calculation that impact the final complexities.

State Test:

We take the 14-round attack on `Deoxys-I-256` as an example.

Epsilon Calculation:

We take the 11-round attack on `Deoxys-I-128` as an example:

Consider the multi-objective optimize

In many cases, when we optimize time complexity alone, memory complexity can exceed expectations. Therefore, the multi-objective optimization is necessary for solving, and the results we obtained that are exhibited in our paper are selected considering the time, data, and memory complexity together.

We provide a pattern that corresponds to the alternative results of the 15-round `Deoxys-BC-384`. For this pattern, the time complexity decreases slightly, whereas the memory complexity increases substantially.

Search efficiency

We provide five models for the attack instances outlined in our paper, along with the time required to solve each model. To enable fast verification, each model is equipped with pattern constraints.

Searching Strategy 1

Solving the optimal.

Before solving the entire model, we will compute lower bounds in complexity, exclude the calculation of epsilon, and collect the nearly lower bounds into a set. Then, including the calculation of epsilon to complete the model and solve the final complexity. Once the final complexity matches the lower bound, we obtain the optimal attack.

Time for lower bound searching:

11-round `Deoxys-I-128`:

10-round `Deoxys-I-128`:

14-round `Deoxys-I-256`:

Time for finding the optimal solution:

11-round `Deoxys-I-128`:

10-round `Deoxys-I-128`:

14-round `Deoxys-I-256`:

Searching Strategy 2

Elastic the model.

Since the distinguisher of SKINNY