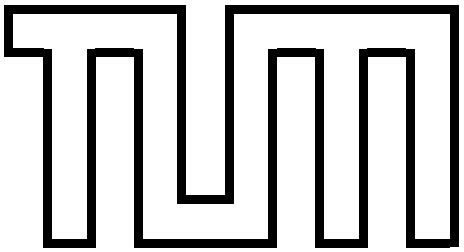

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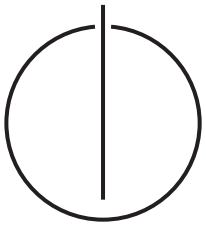

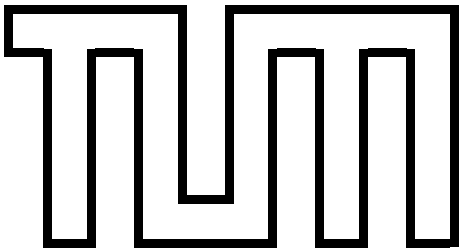

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Algorithms for refinement of modal process rewrite systems

Algorithmen zur Verfeinerung von modalen Prozessersetzungssystemen

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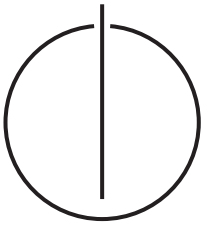

I assure the single handed composition of this bachelor's thesis only supported by declared resources.

Munich, April 2, 2013

Philipp Meyer

Contents

1 Introduction **1**

2 Theory **2**
2.1 Modal process rewrite system 2
2.2 Basic definitions 2
2.3 Modal transition system 3
2.4 Modal process rewrite system 3
2.5 Modal refinement 4
2.6 Attack sequences 4
2.7 Visibly pushdown automaton 6

3 Algorithms **10**
3.1 Description 10
3.2 Soundness and completeness 12
3.2.1 Soundness 12
3.2.2 Completeness 12
3.3 Runtime 12
3.4 Optimizations 12
3.5 Performance evaluation 12
3.6 Example 12

4 Conclusion **14**

Bibliography **15**

1 Introduction

2 Theory

2.1 Modal process rewrite system

Modal process rewrite systems [BK12] are a modal extension of process rewrite systems [May00, Esp01]. They induce a modal transition systems [BKLS09].

2.2 Basic definitions

Definition 1 (Process term)**.** The set of process terms over a set of constants $Const$ is given by

$$\frac{}{\varepsilon \in \mathcal{P}}\,(0) \qquad \frac{X \in Const}{X \in \mathcal{P}}\,(1)$$

$$\frac{p \in \mathcal{P} \qquad q \in \mathcal{P}}{p \cdot q \in \mathcal{P}}\,(S) \qquad \frac{p \in \mathcal{P} \qquad q \in \mathcal{P}}{p \| q \in \mathcal{P}}\,(P)$$

The processes expressions are considered modulo the usual structural congruence, i.e. the smallest congruence such that the operator $\cdot$ is associative, $\|$ is associative and commutative and ε is a unit for both $\cdot$ and $\|$.

Processes that can be produced just with rule 0, 1 and S, i.e. contain no $\|$, are called *sequential processes* and processes that can be produced just with rule 0, 1 and P, i.e. contain no $\cdot$, are called *parallel processes*

Definition 2 (Size of a process term)**.** The size $|p|$ of a process term p is inductively defined by

$$\begin{aligned} |\varepsilon| &= 0 \\ |X| &= 1 \\ |p \cdot q| &= |p| + |q| \\ |p \| q| &= |p| + |q| \end{aligned}$$

Process terms will be denoted by lowercase letters $p, q, r, s, t, \ldots$ while single constants are denoted by uppercase letters $P, Q, R, S, T, \ldots$.

Definition 3 (Constants of a process term)**.** The set of constants $Const(p)$ appearing in a process term p is inductively defined by

$$\begin{aligned} Const(\varepsilon) &= \emptyset \\ Const(X) &= \{X\} \\ Const(p \cdot q) &= Const(p) \cup Const(q) \\ Const(p \| q) &= Const(p) \cup Const(q) \end{aligned}$$

2.3 Modal transition system

Modal transition system definition from [BK12]:

Definition 4 (Modal transition system)**.** A *modal transition system (MTS)* over an action alphabet Act is a triple $(\mathcal{P}, \dashrightarrow, \longrightarrow)$ where $\mathcal{P}$ is a set of processes and $\longrightarrow \subseteq \dashrightarrow \subseteq \mathcal{P} \times Act \times \mathcal{P}$. An element $(p, a, q) \in \dashrightarrow$ is a *may transition*, also written as $p \overset{a}{\dashrightarrow} q$, and an element $(p, a, q) \in \longrightarrow$ is a *must transition*, also written as $p \overset{a}{\longrightarrow} q$.

2.4 Modal process rewrite system

Definition 5 (Modal process rewrite system)**.** A *process rewrite system (PRS)* over a set of constants $Const$ and action alphabet Act is a finite relation. $\Delta \subseteq \mathcal{P} \times Act \times \mathcal{P}$, elements of which are called *rewrite rules*. A *modal process rewrite system (mPRS)* is a tuple $(\Delta_{\text{may}}, \Delta_{\text{must}})$ where $\Delta_{\text{may}}, \Delta_{\text{must}}$ are process rewrite systems such that $\Delta_{\text{may}} \subseteq \Delta_{\text{must}}$.

An mPRS $(\Delta_{\text{may}}, \Delta_{\text{must}})$ induces an MTS $(\mathcal{P}, \dashrightarrow, \longrightarrow)$ as follows:

$$\frac{(p, a, p') \in \Delta_{\text{may}}}{p \overset{a}{\dashrightarrow} p'}\ (1) \qquad \frac{(p, a, p') \in \Delta_{\text{must}}}{p \overset{a}{\longrightarrow} p'}\ (2)$$

$$\frac{p \overset{a}{\dashrightarrow} p'}{p \cdot q \overset{a}{\dashrightarrow} p \cdot q}\ (3) \qquad \frac{p \overset{a}{\longrightarrow} p'}{p \cdot q \overset{a}{\longrightarrow} p' \cdot q}\ (4) \qquad \frac{p \overset{a}{\dashrightarrow} p'}{p \| q \overset{a}{\dashrightarrow} p \| q}\ (5) \qquad \frac{p \overset{a}{\longrightarrow} p'}{p \| q \overset{a}{\longrightarrow} p' \| q}\ (6)$$

2.5 Modal refinement

Definition 6 (Refinement)**.** Let $(\mathcal{P}, \dashrightarrow, \longrightarrow)$ be an MTS and $p, q \in \mathcal{P}$ be processes. We say that p *refines* q, written $p \leq_m q$, if there is a relation $\mathcal{R} \subseteq \mathcal{P} \times \mathcal{P}$ such that $(p, q) \in \mathcal{R}$ and for every $(p, q) \in \mathcal{R}$ and every $a \in Act$:

1. If $p \overset{a}{\dashrightarrow} p'$ then there is a transition $q \overset{a}{\dashrightarrow} q'$ s.t. $(p', q') \in \mathcal{R}$.
2. If $q \overset{a}{\longrightarrow} q'$ then there is a transition $p \overset{a}{\longrightarrow} p'$ s.t. $(p', q') \in \mathcal{R}$.

2.6 Attack sequences

Definition 7 (Attack transition and attack sequence)**.** Let $(\mathcal{P}, \dashrightarrow, \longrightarrow)$ be an MTS. An *attack transition* is a tupel $((p, q), S)$ with $(p, q) \in \mathcal{P}^2 = \mathcal{P} \times \mathcal{P}$ and $S \subseteq \mathcal{P}$, also written as $(p, q) \longrightarrow_a S$. For $p, q \in \mathcal{P}$, the attack transitions are given by

$$\frac{p \overset{a}{\dashrightarrow} p'}{(p, q) \longrightarrow_a \{(p', q') \mid q \overset{a}{\dashrightarrow} q'\}} (1) \qquad \frac{q \overset{a}{\longrightarrow} q'}{(p, q) \longrightarrow_a \{(p', q') \mid p \overset{a}{\longrightarrow} p'\}} (2)$$

An *attack sequence* $(p, q) \longrightarrow_a^* S$ is given by

$$\frac{(p, q) \longrightarrow_a S}{(p, q) \longrightarrow_a^* S} (3) \qquad \frac{(p, q) \longrightarrow_a^* \{(p', q')\} \uplus S \qquad (p', q') \longrightarrow_a S' \qquad S \cap S' = \emptyset}{(p, q) \longrightarrow_a^* S \uplus S'} (4)$$

The derivation of an attack sequence $(p, q) \longrightarrow_a^o S$ can be displayed in a linear form $(a_1, a_2, ..., a_n)$ consisting of attack transitions a_i with $a_1 = ((p, q), S')$ by following the linear inference chain. The length of an attack sequence is the number of inference rules applied to obtain it. An attack transition is also a sequence of length 1.

Intuitively, an attack transition $(p, q) \longrightarrow_a S$ means that from the state (p, q), there is a sequence of *attack transitions*, that is a may transition from the left side or a must transition from the right side, such that S is the set of reachable states by applying appropriate *defending transition*, that is a transition of the same type and with the same action symbol from the other side.

Lemma 1. *If* $(p, q) \longrightarrow_a^* \{(p', q')\} \uplus S$ *and* $(p', q') \longrightarrow_a^* S'$ *with* $S \cap S' = \emptyset$*, then* $(p, q) \longrightarrow_a^* S \uplus S'$*.*

Proof. Proof by induction on the length of $(p', q') \longrightarrow_a^* S'$:

1. The sequence has length 1: Then $(p', q') \longrightarrow_a S'$ and by rule 4 $(p, q) \longrightarrow_a^* S \uplus S'$.

2. The sequence has length $n > 1$ and the induction hypothesis holds for any sequence of length $< n$: Then $(p', q') \longrightarrow_a^* \{(p'', q'')\} \uplus S''$ and $(p'', q'') \longrightarrow_a S'''$ with $S' = S'' \uplus S'''$. By induction hypothesis $(p, q) \longrightarrow_a^* S \uplus (\{(p'', q'')\} \uplus S'')$ and therefore $(p, q) \longrightarrow_a^* S \uplus S'' \uplus S''' = S \uplus S'$.

□

Theorem 1. *For an MTS* $(\mathcal{P}, \dashrightarrow, \longrightarrow)$ *and processes* $p, q \in \mathcal{P}$*:*

$$(p \leq_m q) \iff \neg((p, q) \longrightarrow_a^* \emptyset)$$

Proof. $\Longrightarrow$: Assume $p \leq_m q$. Then there is a refinement relation $\mathcal{R}$. We show that for any $(p, q) \longrightarrow_a^* S$ that $(p, q) \in \mathcal{R} \Rightarrow \exists (p', q') : (p', q') \in \mathcal{S} \wedge (p', q') \in \mathcal{R}$ by induction on the length of the attack sequence $(p, q) \longrightarrow_a^* S$:

1. The sequence has length 1: Then it is an attack transition created by rule 1 or 2.

   a) If it was created with the inference rule 1 from $p \overset{a}{\dashrightarrow} p'$. If $(p, q) \in \mathcal{R}$ we get by refinement that there is a transition $q \overset{a}{\dashrightarrow} q'$ with $(p', q') \in \mathcal{R}$ and therefore $(p', q') \in S$.

   b) If it was created with the inference rule 2 from $q \overset{a}{\longrightarrow} q'$. If $(p, q) \in \mathcal{R}$ we get by refinement that there is a transition $p \overset{a}{\longrightarrow} p'$, with $(p', q') \in \mathcal{R}$ and $(p', q') \in S$.

2. The sequence has length $n > 1$ and the induction hypothesis holds for any sequence of length $< n$. Then it was created with the inference rule 4 from $(p, q) \longrightarrow_a^* \{(p', q')\} \uplus S'$ and $(p', q') \longrightarrow_a S''$ with $S = S' \uplus S''$. By the induction hypothesis we get $(p, q) \in \mathcal{R} \Rightarrow \exists (p'', q'') : (p'', q'') \in \{(p', q')\} \uplus S' \wedge (p'', q'') \in \mathcal{R}$ and $(p', q') \in \mathcal{R} \Rightarrow \exists (p''', q''') : (p''', q''') \in S'' \wedge (p''', q''') \in \mathcal{R}$. In the cases

   a) $(p', q') \neq (p'', q'')$: Then $(p, q) \in \mathcal{R} \Rightarrow (p'', q'') \in S' \subseteq S$.

   b) $(p', q') = (p'', q'')$: Then $(p, q) \in \mathcal{R} \Rightarrow (p', q') \in \mathcal{R} \Rightarrow (p''', q''') \in S'' \subseteq S$.

So for any $(p, q) \longrightarrow_a^* S \in \mathcal{A}$ with $(p, q) \in R$ we have $S \neq \emptyset$, especially for the inital one.

$\Longleftarrow$: Assume $\neg((p, q) \longrightarrow_a^* \emptyset)$. We show that $\mathcal{R} := \{(p, q) \mid \neg((p, q) \longrightarrow_a^* \emptyset\}$ is a valid refinement relation. First $(p, q) \in \mathcal{R}$, and for any $(p, q) \in \mathcal{R}$:

1. If $p \overset{a}{\dashrightarrow} p'$, then by inference rule 1 there exists $(p,q) \longrightarrow_a S$. From all $(p,q) \longrightarrow_a S$, choose the one where S is minimal with regard to the inclusion order. As $S \neq \emptyset$, there is $(p',q') \in S$ which was created from a transition $q \overset{a}{\dashrightarrow} q'$. Assuming $(p',q') \longrightarrow_a^* \emptyset$, by lemma 1 we would get $(p,q) \longrightarrow_a^* S\{p'q'\} \cup \emptyset \subsetneq S$ in contradiction to the minimality of S. So $\neg((p',q') \longrightarrow_a^* \emptyset)$ and therefore $(p',q') \in \mathcal{R}$.

2. If $q \overset{a}{\longrightarrow} q'$, by similiar argument we get a transition $p \overset{a}{\longrightarrow} p'$ for which $(p',q') \in \mathcal{R}$.

With this refinement relation we have $p \leq_m q$. □

2.7 Visibly pushdown automaton

Definition 8 (Visibly pushdown automaton)**.** A PRS is a visibly pushdown automaton (vPDA) if all processes are sequential and there is a partition $Act = Act_r \uplus Act_i \uplus Act_c$ such that each rule $(p,a,p') \in \Delta$ has the form

$$p = P \cdot S \qquad \text{and} \qquad p' = \begin{cases} Q & \text{if } a \in Act_r \quad \text{(return rule)} \\ Q \cdot T & \text{if } a \in Act_i \quad \text{(internal rule)} \\ Q \cdot T \cdot R & \text{if } a \in Act_c \quad \text{(call rule)} \end{cases}$$

The modal extension for a *modal visibly pushdown automaton (mvPDA)* is straightforward.

Definition 9 (Attack rules for mvPDA)**.** Let $(\Delta_{\text{may}}, \Delta_{\text{must}})$ be an mvPDA. We define a variant $\longrightarrow_b$ of the attack sequences obtainable from the rewrite rules. These are called *attack rules*. For every $p,q \in \mathcal{P}$, we have:

$$\frac{(p,a,p') \in \Delta_{\text{may}}}{(p,q) \longrightarrow_b \{(p',q') \mid (q,a,q') \in \Delta_{\text{may}}\}} \quad (1)$$

$$\frac{(q,a,q') \in \Delta_{\text{must}}}{(p,q) \longrightarrow_b \{(p',q') \mid (p,a,p') \in \Delta_{\text{must}}\}} \quad (2)$$

$$\frac{(p,q) \longrightarrow_b \{(p' \cdot P, q' \cdot Q)\} \uplus S \qquad (p',q') \longrightarrow_b S' \qquad \forall (p'',q'') \in S' : |p''| \leq 2}{(p,q) \longrightarrow_b S \cup \{(p'' \cdot P, q'' \cdot Q) \mid (p'',q'') \in S'\}} \quad (3)$$

$$\frac{(p,q) \longrightarrow_b \{(p',q')\} \uplus S \qquad (p',q') \longrightarrow_b S' \qquad \forall (p'',q'') \in S' : |p''| \leq 2}{(p,q) \longrightarrow_b S \cup S'} \quad (4)$$

Due to the conditions on the rewrite rules of an mvPDA and the construction of the attack rules, we can see that for any element $(p, q) \longrightarrow_b S$ it holds that $|p| = |q| = 2$ and for any $(p', q') \in S$ that $1 \leq |p'| = |q'| \leq 3$.

Lemma 2. *For an MTS generated by a mvPDA, if* $(p, q) \longrightarrow_a^* S$*, then* $(p \cdot s, q \cdot t) \longrightarrow_a^* S'$ *with* $S' = \{(p' \cdot s, q' \cdot t) \mid (p', q') \in S\}$ *for any* $s, t \in \mathcal{P}$*.*

Proof. By the MTS induction rules, we have that for every $p \overset{a}{\dashrightarrow} p'$ is generated from a $(p, a, p') \in \Delta_{\text{may}}$ and for a mvPDA therefore $|p| = 2$. Then there is only one transition from $p \cdot s$, nameley $p \cdot s \overset{a}{\dashrightarrow} p' \cdot s$ generated by the MTS induction rule 1. Also for every $q \overset{a}{\longrightarrow} q'$ there is just $q \cdot t \overset{a}{\dashrightarrow} q' \cdot t$ from $q \cdot t$.

Then the proposition is proved by induction on the length of $(p', q') \longrightarrow_a^* S'$:

1. The sequence has length 1: Then it is a single transition created from $p \overset{a}{\dashrightarrow} p'$ with $S = \{(p', q') \mid q \overset{a}{\dashrightarrow} q'\}$ and we get $p \cdot s \overset{a}{\dashrightarrow} p' \cdot s$ and $\{(p' \cdot s, q' \cdot t) \mid q \cdot t \overset{a}{\dashrightarrow} q' \cdot t\} = S'$. If the transition was created from $q \overset{a}{\longrightarrow} q'$ with $S = \{(p', q') \mid p \overset{a}{\longrightarrow} p'\}$ and we get $\{(p' \cdot s, q' \cdot t) \mid p \cdot t \overset{a}{\longrightarrow} p' \cdot t\} = S'$ Both cases yield $(p \cdot s, q \cdot t) \longrightarrow_a^* S'$.

2. The sequence has length $n > 1$ and the induction hypothesis holds for any sequence of length $< n$: Then $(p', q') \longrightarrow_a^* \{(p'', q'')\} \uplus S''$ and $(p'', q'') \longrightarrow_a S'''$ with $S = S'' \cup S'''$. By induction hypothesis $(p' \cdot s, q' \cdot t) \longrightarrow_a^* \{(p'' \cdot s, q'' \cdot t)\} \uplus \{(p''' \cdot s, q''' \cdot t \mid (p''', q'') \in S''\}$ and $(p'' \cdot s, q'' \cdot t) \longrightarrow_a \{(p''' \cdot s, q''' \cdot t) \mid (p''', q''') \in S'''\}$. Applying inference rule 4 for attack sequences yields $(p \cdot s, q \cdot t) \longrightarrow_a^* \{(p''' \cdot s, q''' \cdot t) \mid (p''', q''') \in S'' \cup S'''\} = S'$

□

Theorem 2. *For an mvPDA* $(\Delta_{may}, \Delta_{must})$ *with its induced MTS* $(\mathcal{P}, \dashrightarrow, \longrightarrow)$*, it holds that for any* $P, S, Q, T \in Const$*:*

$$(P \cdot S, Q \cdot T) \longrightarrow_a^* \emptyset \iff (P \cdot S, Q \cdot T) \longrightarrow_b \emptyset$$

Proof. $\Longrightarrow$: Assume $(P \cdot S, Q \cdot T) \longrightarrow_a^* \emptyset$ and let $(a_1, a_2, ..., a_n)$ be the linear form of a derivation of the attack sequence. Always a_1 has the form $(P \cdot S, Q \cdot T) \longrightarrow_a S$ and a_n the form $(p, q) \longrightarrow_a \emptyset$.

Our proposition is that if we can split up the sequence into subsequences which we can all compute seperately, we can also compute the whole sequence. More formally, we want to show that if there is a set of $k + 1$ indices $I = i_0, ..., i_k$ where

1. $0 = i_0 < i_1 < i_2 < ... < i_{k-1} < i_k = n$.

2. There is a sequence $(b_1, ..., b_k)$ where each b_i is an attack rule $(p, q) \longrightarrow_b S$.

3. For every $i, j \in I$ with $i < j$ the sequence $(a_{i+1}, ..., a_j)$, which generates the attack sequence $(p, q) \longrightarrow_a^* S$, the representing rule b_j is $(p, q) \longrightarrow_b S$ If $j < n$ then with $b_i = (p', q') \longrightarrow_b S'$ we require $(p', q') \in S'$. and if $j = n$ then $S = \emptyset$.

there is $(P \cdot S, Q \cdot T) \longrightarrow_b \emptyset$.

We prove this by induction on the number k:

1. $k = 1$: Then the indices are $0, n$ and the rule sequence (b_1) represents $(a_1, ..., a_n)$ generating $(P \cdot S, Q \cdot T) \longrightarrow_a^* \emptyset$. Then we have $(P \cdot S, Q \cdot T) \longrightarrow_b \emptyset$.

2. $k > 1$, and the induction hypothesis holds for any $k' < k$: Let $(b_1, ..., b_k)$ be the rule sequence. For the first rule $b_1 = (P \cdot S, Q \cdot T) \longrightarrow_b S$, there is be $(p', q') \in S$ with $|p'| = |q'| \geq 2$ because otherwise no more rules could be applied afterwards, in contradiction to $k > 1$. So this is a left-hand side rule. Also the last rule $b_k = (p', q') \longrightarrow_b \emptyset$ is a right-hand side rule.

$\Longleftarrow$: We show that for $(p, q) \longrightarrow_b S$, there is $T \subseteq S$ with $(p, q) \longrightarrow_a^* T$ by induction on the inference of $(p, q) \longrightarrow_b S$:

1. It was created by rule 1 from $(p, a, p') \in \Delta_{\text{may}}$. Then there is $p \overset{a}{\dashrightarrow} p'$. For every $(q, a, q') \in \Delta_{\text{may}}$ there is $q \overset{a}{\dashrightarrow} q'$ and for every $q \overset{a}{\dashrightarrow} q''$ it follows that $q' = q''$. Then $\{(p', q') \mid q \overset{a}{\dashrightarrow} q'\} = S$ and $(p, q) \longrightarrow_a^* S$.

2. It was created by rule 2 from $(q, a, q') \in \Delta_{\text{must}}$. Then there is $q \overset{a}{\dashrightarrow} q'$. For every $(p, a, p') \in \Delta_{\text{may}}$ there is $p \overset{a}{\dashrightarrow} p'$ and for every $p \overset{a}{\dashrightarrow} p''$ it follows that $p' = p''$. Then $\{(p', q') \mid p \overset{a}{\dashrightarrow} p'\} = S$ and $(p, q) \longrightarrow_a^* S$.

3. It was created by rule 3 from $(p, q) \longrightarrow_b \{(p' \cdot P, q' \cdot Q)\} \uplus S'$ and $(p', q') \longrightarrow_b S''$ with $S = S' \cup S'''$ for $S''' = \{(p'' \cdot P, q'' \cdot Q) \mid (p'', q'') \in S''\}$ Then by induction hypothesis there is $T' \subseteq \{(p' \cdot P, q' \cdot Q)\} \uplus S'$ with $(p, q) \longrightarrow_a^* T'$ and $T'' \subseteq S''$ with $(p', q') \longrightarrow_a^* T''$. If $(p' \cdot P, q' \cdot Q) \notin T'$, then we get $T' \subseteq S' \subseteq S$. If $(p', q') \in T'$, with lemma 2 we have regard the $(p' \cdot P, q' \cdot Q) \longrightarrow_a^* T'''$ with $T''' \subseteq S'''$. Then with lemma 1 we get that there is $T \subseteq T' \setminus \{(p', q'\} \cup T''' \subseteq S' \cup S''' = S$ with $(p, q) \longrightarrow_a^* T$.

4. It was created by rule 4 from $(p, q) \longrightarrow_b \{(p', q')\} \uplus S'$ and $(p', q') \longrightarrow_b S''$ with $S = S' \cup S''$. Then by induction hypothesis there is $T' \subseteq \{(p', q')\} \uplus S'$

with $(p, q) \longrightarrow_a^* T'$ and $T'' \subseteq S''$ with $(p', q') \longrightarrow_a^* T''$. If $(p', q') \notin T'$, then we get $T' \subseteq S' \subseteq S$. If $(p', q') \in T'$, then with lemma 1 we get that there is $T \subseteq T' \setminus \{(p', q'\} \cup T'' \subseteq S' \cup S'' = S$ with $(p, q) \longrightarrow_a^* T$.

Then if $(P \cdot S, Q \cdot T) \longrightarrow_b \emptyset$ we have $(P \cdot S, Q \cdot T) \longrightarrow_a^* \emptyset$ □

3 Algorithms

3.1 Description

Figure 3.1: Algorithm for calculating the attack rules on mvPDAs

```
1: function AttackRules(mvPDA = (Δ_may, Δ_must))
2:     rules ← ∅
3:     for P, Q, S, T ∈ Const(mvPDA), a ∈ Act(mvPDA), type ∈ {may, must} do
4:         if type = may then
5:             lhs ← (P · S, Q · T)          ▷ Attack from left-hand side for may rules
6:         else
7:             lhs ← (Q · S, P · Y)          ▷ Attack from right-hand side for must rules
8:         end if
9:         for (P · S, a, p′) ∈ Δ_type do
10:            rhs ← ∅
11:            for (Q · T, a, q′) ∈ Δ_type do
12:                if type = may then
13:                    newRhs ← (p′, q′)
14:                else
15:                    newRhs ← (q′, p′)
16:                end if
17:                rhs ← rhs ∪ {newRhs}
18:            end for
19:            rules ← rules ∪ {(lhs, rhs)}
20:        end for
21:    end for
22:    return rules
23: end function
```

Figure 3.2: Algorithm for combining attack rules

function COMBINE($lhsRule = (lhs, lhsRhsSet), rhsRule = (rhsLhs, rhsSet)$)
  $rules \leftarrow \emptyset$
  if $\forall rhs \in rhsSet : size(rhs) \leq 1$ **then**
    for $lhsRhs \in lhsRhsSet : lhsRhs = rhsLhs \cdot p$ **do**
      $newRhs \leftarrow (lhsRhsSet \setminus lhsRhs) \cup \{rhs \cdot p \mid rhs \in rhsSet\}$
      $rules \leftarrow rules \cup \{(lhs, newRhs)\}$
    end for
  end if
  return $rules$
end function

Figure 3.3: Refinement algorithm for mvPDAs

function VPDAREFINEMENT($P \cdot S, Q \cdot T, mvPDA$) $\triangleright$ $P \cdot S \leq_m Q \cdot T$ given $mvPDA$
  $initial \leftarrow [P \cdot S, Q \cdot T]$
  $rules \leftarrow$ ATTACKRULES($mvPDA$)
  while $\exists lhsRule, rhsRule \in rules :$ COMBINE$(lhsRule, rhsRule) \notin rules$ **do**
    $rules \leftarrow rules \cup$ COMBINE$(lhsRule, rhsRule)$
  end while
  return $(initial, \emptyset) \in rules$
end function

3.2 Soundness and completeness

3.2.1 Soundness

3.2.2 Completeness

3.3 Runtime

3.4 Optimizations

3.5 Performance evaluation

3.6 Example

Figure 3.4 and 3.5 define two mvPDA. The corresponding may transitions for the must transitions are implied. The problem is to decide whether $p \cdot S \leq_m q \cdot S$.

$$
\begin{aligned}
P \cdot S &\xrightarrow{\text{coin}} P \cdot M \cdot S \\
P \cdot M &\xrightarrow{\text{coin}} P \cdot M \cdot M \\
P \cdot M &\xrightarrow{\text{tea}} T \\
P \cdot M &\xrightarrow{\text{coffee}} c \\
T \cdot M &\xrightarrow{\text{tea}} T \\
T \cdot S &\xrightarrow{\text{coin}} P \cdot M \cdot S \\
c \cdot M &\xrightarrow{\text{coffee}} c \\
c \cdot S &\xrightarrow{\text{coin}} P \cdot M \cdot S
\end{aligned}
$$

Figure 3.4: mvPDA for process $P \cdot S$

$$
\begin{aligned}
Q \cdot S &\xrightarrow{\text{coin}} Q \cdot T \cdot S \\
Q \cdot S &\xrightarrow{\text{coin}} Q \cdot C \cdot S \\
Q \cdot T &\xrightarrow{\text{coin}} Q \cdot T \cdot T \\
Q \cdot C &\xrightarrow{\text{coin}} Q \cdot C \cdot C \\
Q \cdot T &\xrightarrow{\text{tea}} Q \\
Q \cdot T &\xrightarrow{\text{coffee}} Q \\
Q \cdot C &\xrightarrow{\text{tea}} Q \\
Q \cdot C &\xrightarrow{\text{coffee}} Q
\end{aligned}
$$

Figure 3.5: mvPDA for process $Q \cdot S$

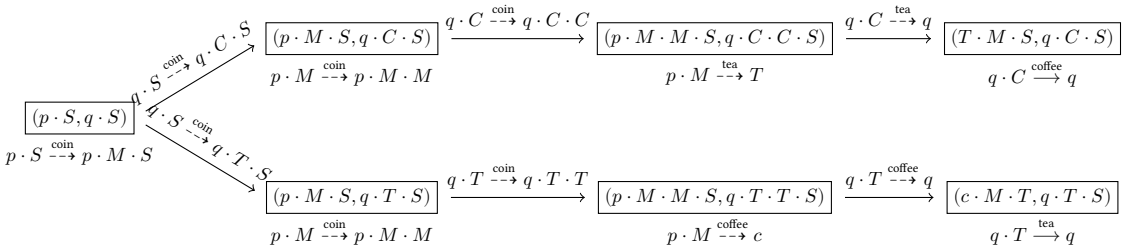

Figure 3.6: Tree for winning strategy

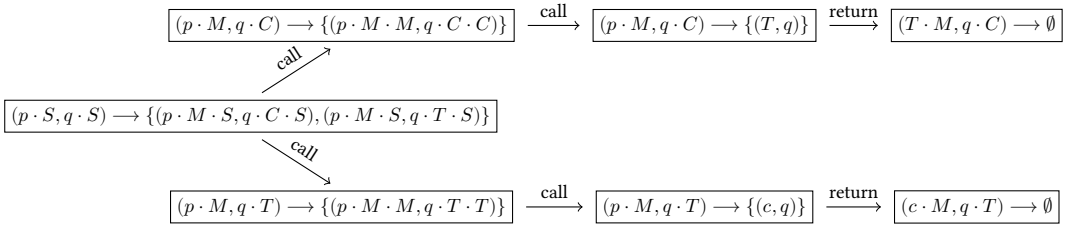

Figure 3.7: Tree for winning strategy with attack rules

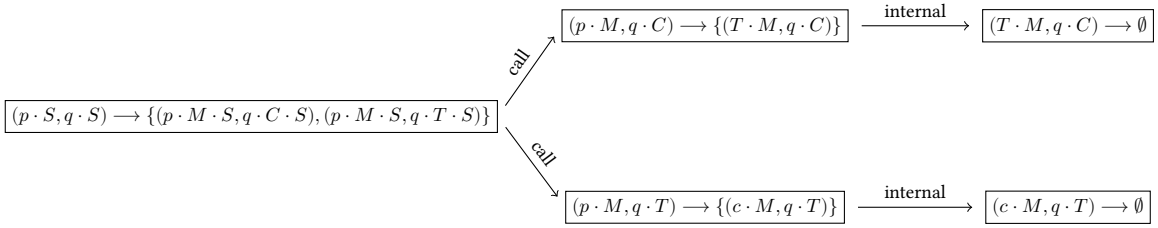

Figure 3.8: Merged tree for winning strategy with attack rules after one step

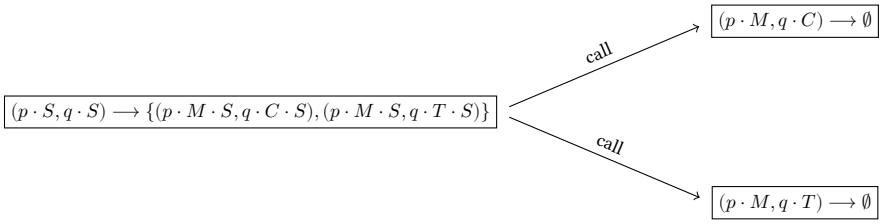

Figure 3.9: Merged tree for winning strategy with attack rules after two steps

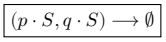

Figure 3.10: Final merged tree for winning strategy

4 Conclusion

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