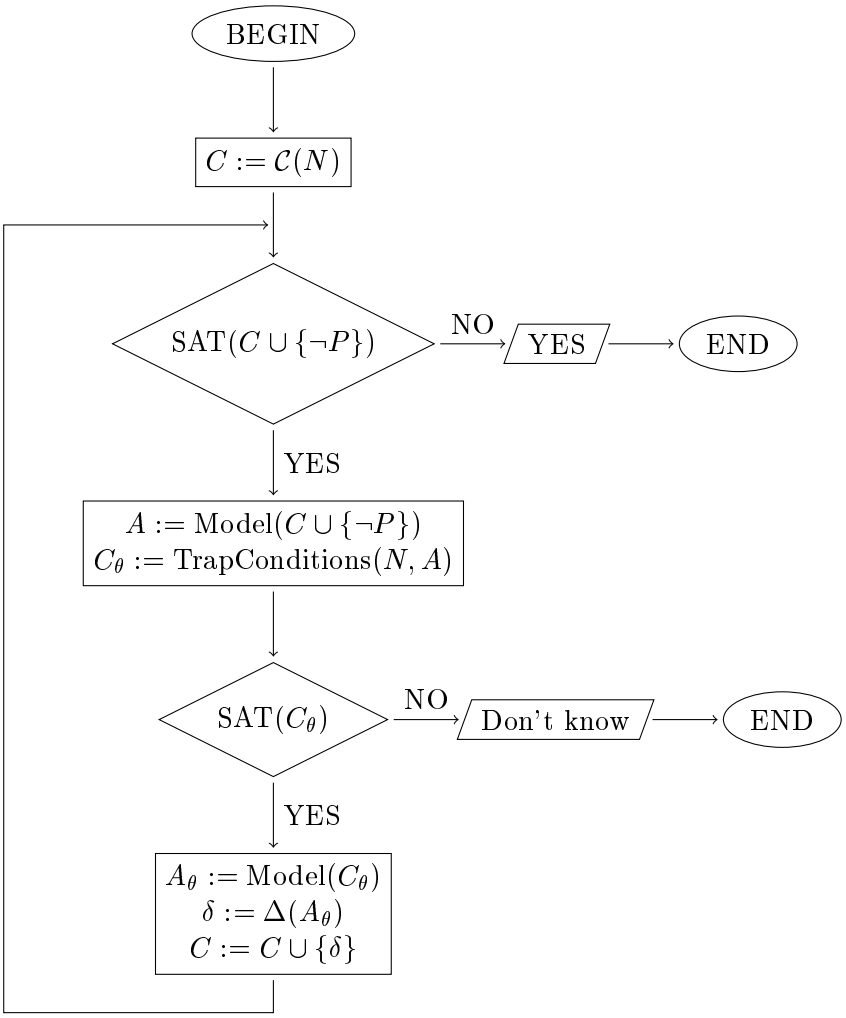
1 Peterson's Algorithm

1.1 Method



1.2 Constraints C_0

$$
\begin{array}{rl}
p_1 = 1 - u_1 & + u_6 \\
p_2 = 0 + u_1 - u_2 - u_3 & \\
p_3 = 0 \quad + u_2 + u_3 - u_4 - u_5 & \\
p_4 = 0 \quad + u_4 + u_5 - u_6 & \\
q_1 = 1 & - v_1 \quad + v_6 \\
q_2 = 0 & + v_1 - v_2 - v_3 \\
q_3 = 0 & + v_2 + v_3 - v_4 - v_5 \\
q_4 = 0 & + v_4 + v_5 - v_6 \\
(m_1 = f) = 1 - u_1 & + u_6 \\
(m_1 = t) = 0 + u_1 & - u_6 \\
(m_2 = f) = 1 & - v_1 \quad + v_6 \\
(m_2 = t) = 0 & + v_1 \quad - v_6 \\
(hold = 1) = 1 \quad + u_2 & - v_3 \\
(hold = 2) = 0 \quad - u_2 & + v_3 \\
p_4 \geq 1 & \\
q_4 \geq 1 & \\
\forall p \in S \cup T : \quad p \geq 0 &
\end{array}
$$

$$
\begin{aligned}
\delta_1 &= p_3 \vee q_2 \vee (m_2 = f) \vee (hold = 2) \\
\delta_2 &= p_2 \vee q_3 \vee (m_1 = f) \vee (hold = 1)
\end{aligned}
$$

1.3 A_1

$$
\begin{aligned}
p_1 &= 0 \\
p_2 &= 0 \\
p_3 &= 0 \\
p_4 &= 1 \\
q_1 &= 0 \\
q_2 &= 0 \\
q_3 &= 0 \\
q_4 &= 1 \\
(m_1 = f) &= 0 \\
(m_1 = t) &= 1 \\
(m_2 = f) &= 0 \\
(m_2 = t) &= 1 \\
(hold = 1) &= 1 \\
(hold = 2) &= 0 \\
u_1 &= 1 \\
u_2 &= 0 \\
u_3 &= 1 \\
u_4 &= 0 \\
u_5 &= 1 \\
u_6 &= 0 \\
v_1 &= 1 \\
v_2 &= 1 \\
v_3 &= 0 \\
v_4 &= 1 \\
v_5 &= 0 \\
v_6 &= 0
\end{aligned}
$$

1.4 A_2

$$
\begin{aligned}
p_1 &= 0 \\
p_2 &= 0 \\
p_3 &= 0 \\
p_4 &= 1 \\
q_1 &= 0 \\
q_2 &= 0 \\
q_3 &= 0 \\
q_4 &= 1 \\
(m_1 = f) &= 0 \\
(m_1 = t) &= 1 \\
(m_2 = f) &= 0 \\
(m_2 = t) &= 1 \\
(hold = 1) &= 0 \\
(hold = 2) &= 1 \\
u_1 &= 1 \\
u_2 &= 1 \\
u_3 &= 0 \\
u_4 &= 0 \\
u_5 &= 1 \\
u_6 &= 0 \\
v_1 &= 2 \\
v_2 &= 0 \\
v_3 &= 2 \\
v_4 &= 0 \\
v_5 &= 2 \\
v_6 &= 1
\end{aligned}
$$

1.5 $A_{\theta 1}$

$$
\begin{aligned}
p_1 &= 0 \\
p_2 &= 0 \\
p_3 &= 1 \\
p_4 &= 0 \\
q_1 &= 0 \\
q_2 &= 1 \\
q_3 &= 0 \\
q_4 &= 0 \\
(m_1 = f) &= 0 \\
(m_1 = t) &= 0 \\
(m_2 = f) &= 1 \\
(m_2 = t) &= 0 \\
(hold = 1) &= 0 \\
(hold = 2) &= 1
\end{aligned}
$$

1.6 $A_{\theta 2}$

$$
\begin{aligned}
p_1 &= 0 \\
p_2 &= 1 \\
p_3 &= 0 \\
p_4 &= 0 \\
q_1 &= 0 \\
q_2 &= 0 \\
q_3 &= 1 \\
q_4 &= 0 \\
(m_1 = f) &= 1 \\
(m_1 = t) &= 0 \\
(m_2 = f) &= 0 \\
(m_2 = t) &= 0 \\
(hold = 1) &= 1 \\
(hold = 2) &= 0
\end{aligned}
$$

1.7 C_θ

①

$$
\begin{aligned}
p_1 &\Longrightarrow o\_u_1 \\
p_2 &\Longrightarrow o\_u_2 \wedge o\_u_3 \\
p_3 &\Longrightarrow o\_u_4 \wedge o\_u_5 \\
p_4 &\Longrightarrow o\_u_6 \\
q_1 &\Longrightarrow o\_v_1 \\
q_2 &\Longrightarrow o\_v_2 \wedge o\_v_3 \\
q_3 &\Longrightarrow o\_v_4 \wedge o\_v_5 \\
q_4 &\Longrightarrow o\_v_6 \\
(m_1 = f) &\Longrightarrow o\_u_1 \wedge o\_v_4 \\
(m_1 = t) &\Longrightarrow o\_u_6 \\
(m_2 = f) &\Longrightarrow o\_v_1 \wedge o\_u_4 \\
(m_2 = t) &\Longrightarrow o\_v_6 \\
(hold = 1) &\Longrightarrow o\_v_3 \wedge o\_v_5 \wedge o\_u_3 \\
(hold = 2) &\Longrightarrow o\_u_3 \wedge o\_u_5 \wedge o\_v_3 \\
o\_u_1 &\Longrightarrow (p_2 \vee (m_1 = t)) \\
o\_u_2 &\Longrightarrow (p_3 \vee (hold = 1)) \\
o\_u_3 &\Longrightarrow (p_3 \vee (hold = 1)) \\
o\_u_4 &\Longrightarrow (p_4 \vee (m_2 = f)) \\
o\_u_5 &\Longrightarrow (p_4 \vee (hold = 2)) \\
o\_u_6 &\Longrightarrow (p_1 \vee (m_1 = f)) \\
o\_v_1 &\Longrightarrow (q_2 \vee (m_2 = t)) \\
o\_v_2 &\Longrightarrow (q_3 \vee (hold = 2)) \\
o\_v_3 &\Longrightarrow (q_3 \vee (hold = 2)) \\
o\_v_4 &\Longrightarrow (q_4 \vee (m_1 = f)) \\
o\_v_5 &\Longrightarrow (p_4 \vee (hold = 1)) \\
o\_v_6 &\Longrightarrow (q_1 \vee (m_2 = f))
\end{aligned}
$$

②

$$p_1 \vee q_1 \vee (m_1 = f) \vee (m_2 = f) \vee (hold = 1)$$

③$_1$

$$\neg p_4 \wedge \neg q_4 \wedge \neg(m_1 = t) \wedge \neg(m_2 = t) \wedge \neg(hold = 1)$$

③$_2$

$$\neg p_4 \wedge \neg q_4 \wedge \neg(m_1 = t) \wedge \neg(m_2 = t) \wedge \neg(hold = 2)$$

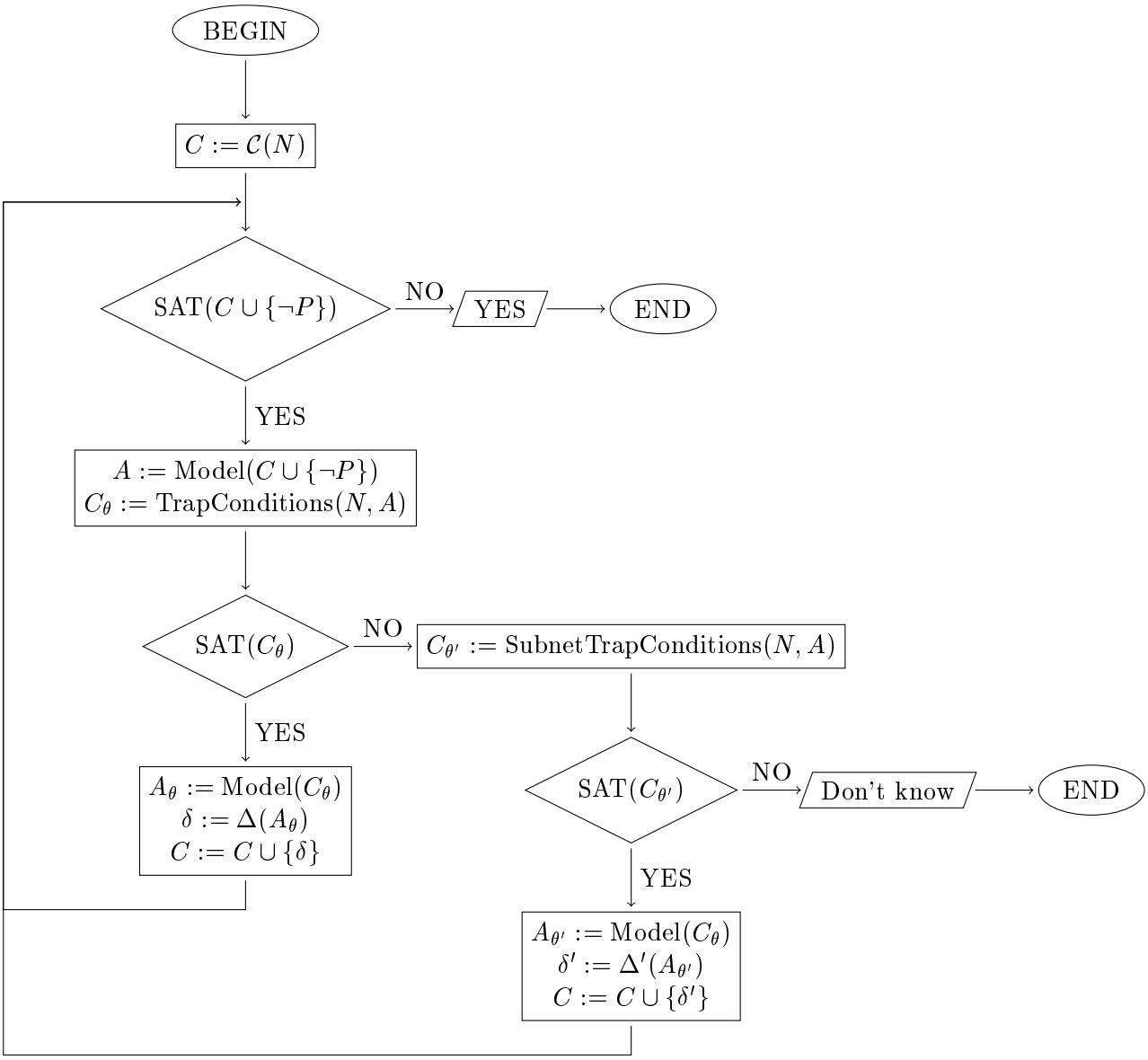
1.8 Benchmark

Give by Daniel Kroening:

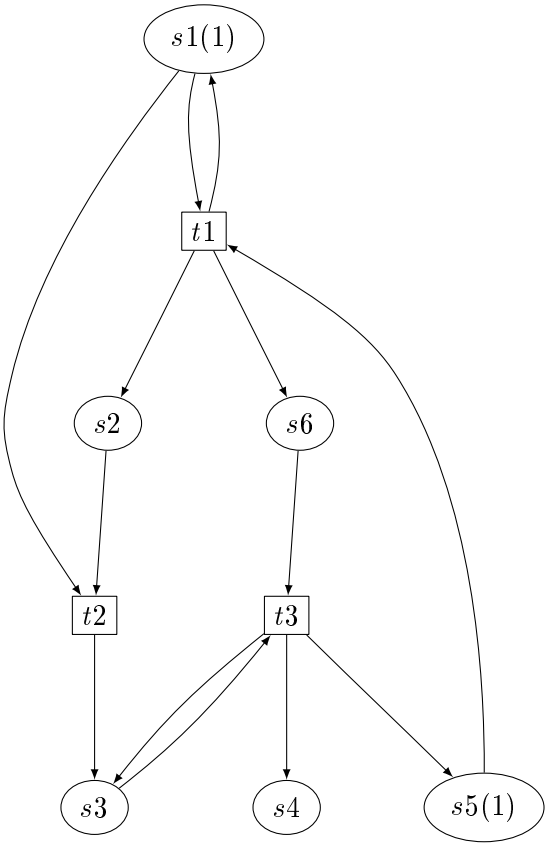
		Our tool			
		positive	don't know	timeout 10 min	
Mist	positive	8	3	0	11
	negative	0	28	0	28
	timeout 1 min	15	23	0	38
		23	54	0	77

2 Cyclic net

2.1 Method



2.2 Petri net



2.3 Constraints C_0

$$
\begin{aligned}
s_1 &= 1 - t_2 \\
s_2 &= 0 + t_1 - t_2 \\
s_3 &= 0 + t_2 \\
s_4 &= 0 + t_3 \\
s_5 &= 1 - t_1 + t_3 \\
s_6 &= 0 + t_1 - t_3 \\
s_1 &\geq 1 \\
s_2 &\geq 1 \quad s_4 \geq 1 \\
s_5 &\geq 1 \\
\forall p \in S \cup T : p &\geq 0
\end{aligned}
$$

$$\delta_1' = (t_1 > 0) \wedge (t_2 = 0) \wedge (t_3 > 0) \implies (s_3 > 0)$$

2.4 A_1

$$
\begin{aligned}
s_1 &= 1 \\
s_2 &= 1 \\
s_3 &= 0 \\
s_4 &= 1 \\
s_5 &= 1 \\
s_6 &= 0 \\
t_1 &= 1 \\
t_2 &= 0 \\
t_3 &= 1
\end{aligned}
$$

2.5 $A_{\theta' 1}$

$$
\begin{aligned}
s_1 &= 0 \\
s_2 &= 0 \\
s_3 &= 1 \\
s_4 &= 0 \\
s_5 &= 0 \\
s_6 &= 0
\end{aligned}
$$

2.6 C_θ

$$
\begin{aligned}
s_1 &\implies o\_t_1 \wedge o\_t_2 \\
s_2 &\implies o\_t_2 \\
s_3 &\implies o\_t_3 \\
s_4 &\implies true \\
s_5 &\implies o\_t_1 \\
s_6 &\implies o\_t_2 \\
o\_t_1 &\implies (s_1 \vee s_2 \vee s_6) \\
o\_t_2 &\implies s_3 \\
o\_t_3 &\implies (s_3 \vee s_4 \vee s_5)
\end{aligned}
$$

$$s_1 \vee s_5$$

$$\neg s_1 \wedge \neg s_2 \wedge \neg s_4 \wedge \neg s_5$$

2.7 $C_{\theta'}$

$$
\begin{aligned}
s_1 &\implies o\_t_1 \wedge o\_t_2 \\
s_2 &\implies o\_t_2 \\
s_3 &\implies o\_t_3 \\
s_4 &\implies true \\
s_5 &\implies o\_t_1 \\
s_6 &\implies o\_t_2 \\
o\_t_1 = (t_1 > 0) &\implies (s_1 \vee s_2 \vee s_6) \\
o\_t_2 = (t_2 > 0) &\implies s_3 \\
o\_t_3 = (t_3 > 0) &\implies (s_3 \vee s_4 \vee s_5)
\end{aligned}
$$

$$(t_1 = 1) \wedge (t_2 = 0) \wedge (t_3 = 1)$$

$$s_1 \vee s_2 \vee s_3 \vee s_4 \vee s_5 \vee s_6$$

$$\neg s_1 \wedge \neg s_2 \wedge \neg s_4 \wedge \neg s_5$$

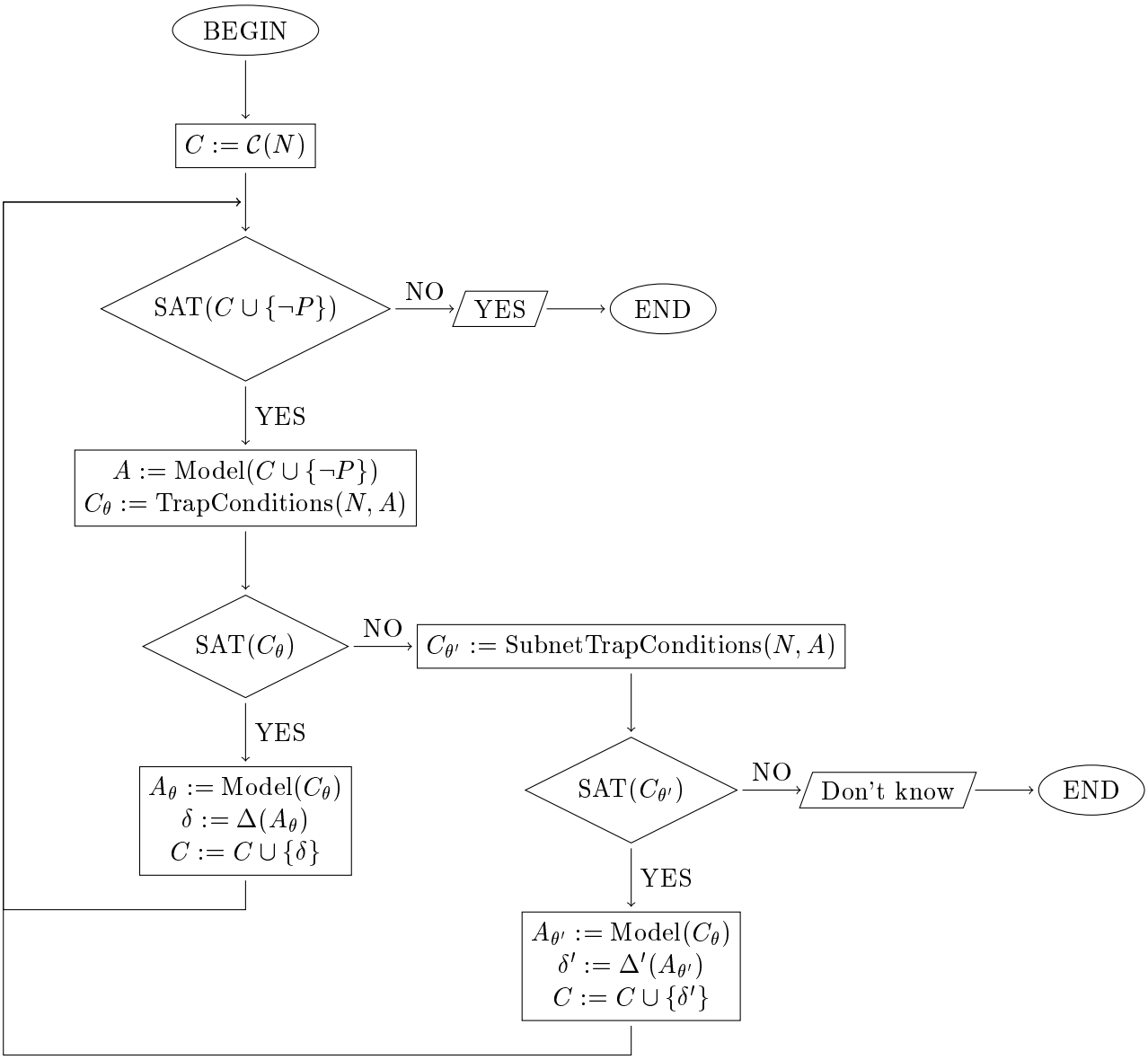
2.8 Benchmark

Give by Daniel Kroening:

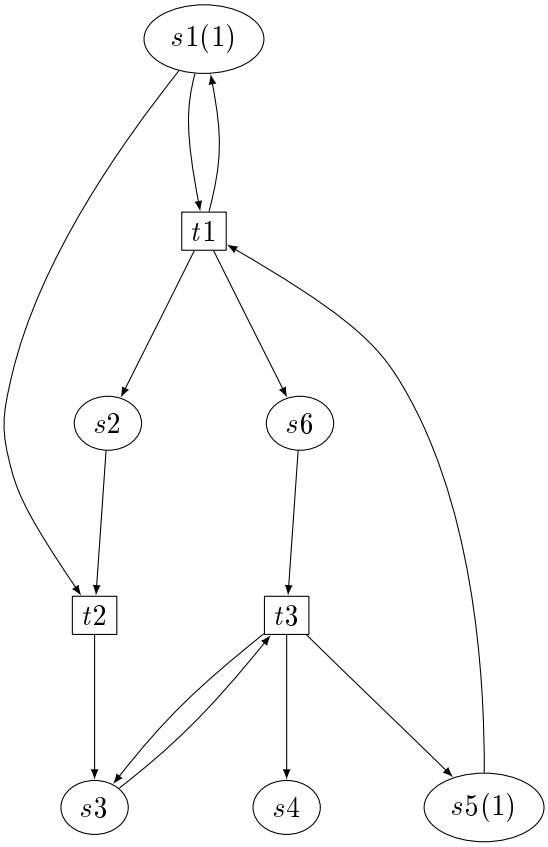
		Our tool			
		positive	don't know	timeout 10 min	
Mist	positive	8	3	0	11
	negative	0	28	0	28
	timeout 1 min	15	19	4	38
		23	50	4	77

3 Empty trap net

3.1 Method



3.2 Petri net



3.3 Constraints C_0

$$
\begin{aligned}
s_1 &= 1 - t_2 \\
s_2 &= 0 + t_1 - t_2 \\
s_3 &= 0 + t_2 \\
s_4 &= 0 + t_3 \\
s_5 &= 1 - t_1 + t_3 \\
s_6 &= 0 + t_1 - t_3 \\
s_1 &\geq 1 \\
s_2 &\geq 1 \quad s_4 \geq 1 \\
s_5 &\geq 1 \\
\forall p \in S \cup T : p &\geq 0
\end{aligned}
$$

$$\delta_1' = (t_1 > 0) \wedge (t_2 = 0) \wedge (t_3 > 0) \implies (s_3 > 0)$$

3.4 A_1

$$
\begin{aligned}
s_1 &= 1 \\
s_2 &= 1 \\
s_3 &= 0 \\
s_4 &= 1 \\
s_5 &= 1 \\
s_6 &= 0 \\
t_1 &= 1 \\
t_2 &= 0 \\
t_3 &= 1
\end{aligned}
$$

3.5 $A_{\theta' 1}$

$$
\begin{aligned}
s_1 &= 0 \\
s_2 &= 0 \\
s_3 &= 1 \\
s_4 &= 0 \\
s_5 &= 0 \\
s_6 &= 0
\end{aligned}
$$

3.6 C_θ

$$
\begin{aligned}
s_1 &\implies o\_t_1 \wedge o\_t_2 \\
s_2 &\implies o\_t_2 \\
s_3 &\implies o\_t_3 \\
s_4 &\implies true \\
s_5 &\implies o\_t_1 \\
s_6 &\implies o\_t_2 \\
o\_t_1 &\implies (s_1 \vee s_2 \vee s_6) \\
o\_t_2 &\implies s_3 \\
o\_t_3 &\implies (s_3 \vee s_4 \vee s_5)
\end{aligned}
$$

$$s_1 \vee s_5$$

$$\neg s_1 \wedge \neg s_2 \wedge \neg s_4 \wedge \neg s_5$$

3.7 $C_{\theta'}$

$$
\begin{aligned}
s_1 &\implies o\_t_1 \wedge o\_t_2 \\
s_2 &\implies o\_t_2 \\
s_3 &\implies o\_t_3 \\
s_4 &\implies true \\
s_5 &\implies o\_t_1 \\
s_6 &\implies o\_t_2 \\
o\_t_1 = (t_1 > 0) &\implies (s_1 \vee s_2 \vee s_6) \\
o\_t_2 = (t_2 > 0) &\implies s_3 \\
o\_t_3 = (t_3 > 0) &\implies (s_3 \vee s_4 \vee s_5)
\end{aligned}
$$

$$(t_1 = 1) \wedge (t_2 = 0) \wedge (t_3 = 1)$$

$$s_1 \vee s_2 \vee s_3 \vee s_4 \vee s_5 \vee s_6$$

$$\neg s_1 \wedge \neg s_2 \wedge \neg s_4 \wedge \neg s_5$$

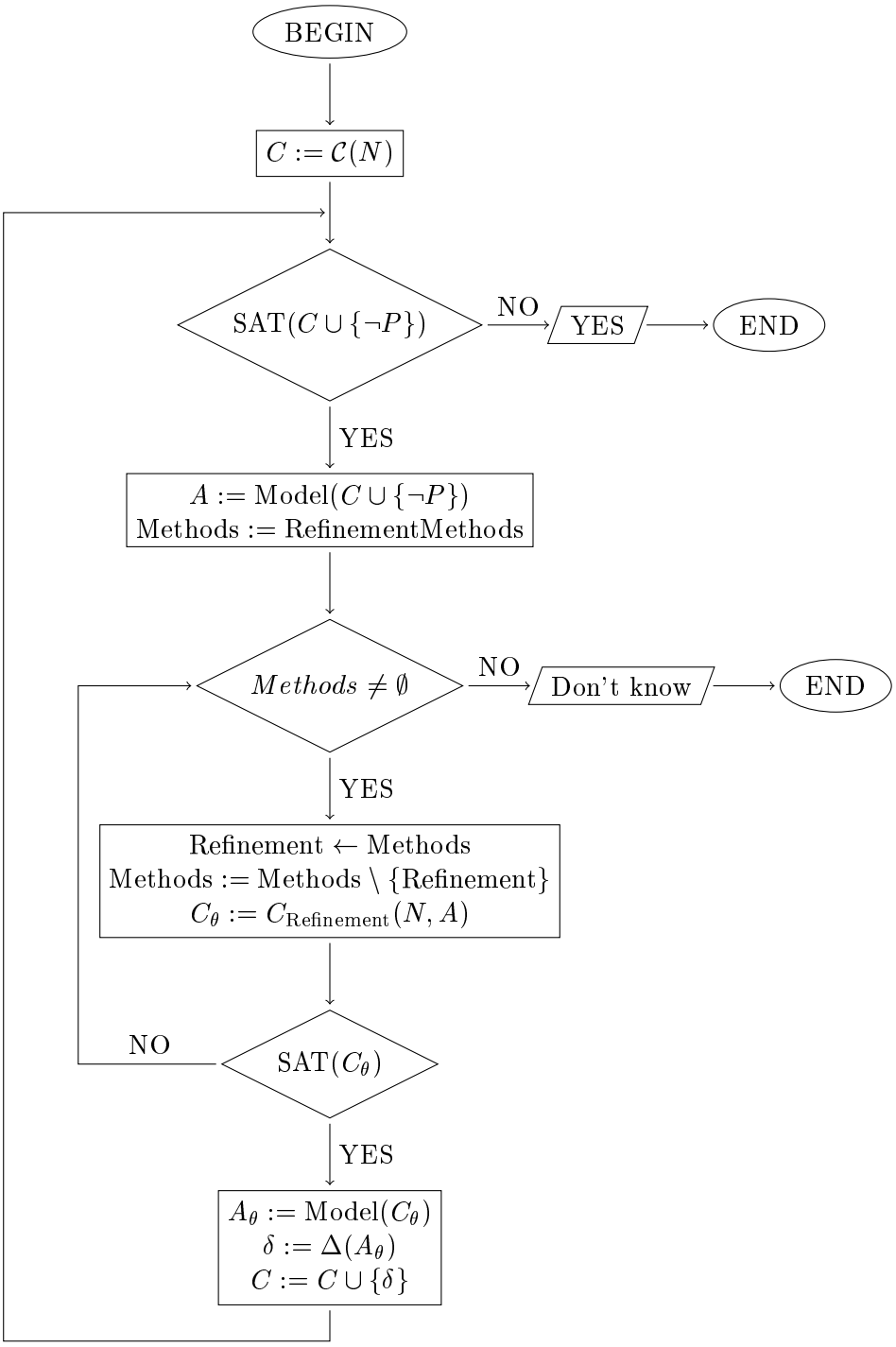
3.8 Benchmark

Give by Daniel Kroening:

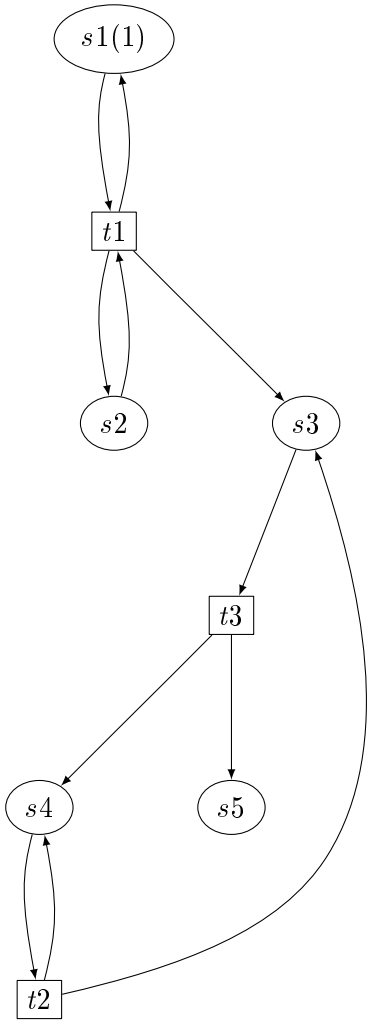
		Our tool			
		positive	don't know	timeout 10 min	
Mist	positive	8	3	0	11
	negative	0	28	0	28
	timeout 1 min	15	19	4	38
		23	50	4	77

4 Empty trap net

4.1 Method



4.2 Petri net



4.3 Constraints C_0

$$
\begin{aligned}
s_1 &= 1 \\
s_2 &= 0 \\
s_3 &= 0 + t_1 + t_2 - t_3 \\
s_4 &= 0 \qquad\quad + t_3 \\
s_5 &= 0 \qquad\quad + t_3 \\
s_5 &\geq 1 \\
\forall p \in S \cup T : p &\geq 0
\end{aligned}
$$

$$\delta_1 = (t_1 > 0) \wedge (t_3 > 0) \implies (t_1 > 0)\delta_2 \qquad = (t_1 > 0) \implies false$$

4.4 A_1

$$\begin{aligned} s_1 &= 1 \\ s_2 &= 0 \\ s_3 &= 0 \\ s_4 &= 1 \\ s_5 &= 1 \\ t_1 &= 0 \\ t_2 &= 1 \\ t_3 &= 1 \end{aligned}$$

4.5 A_2

$$\begin{aligned} s_1 &= 1 \\ s_2 &= 0 \\ s_3 &= 1 \\ s_4 &= 1 \\ s_5 &= 1 \\ t_1 &= 1 \\ t_2 &= 1 \\ t_3 &= 1 \end{aligned}$$

4.6 Empty trap $A_{\theta 1}$

$$
\begin{aligned}
s_1 &= false \\
s_2 &= false \\
s_3 &= true \\
s_4 &= true \\
s_5 &= false \\
s_6 &= false \\
o\_t_1 &= false \\
o\_t_2 &= true \\
o\_t_3 &= true \\
i\_t_1 &= true \\
i\_t_2 &= true \\
i\_t_3 &= true
\end{aligned}
$$

4.7 Empty trap $A_{\theta 2}$

$$
\begin{aligned}
s_1 &= false \\
s_2 &= true \\
s_3 &= false \\
s_4 &= false \\
s_5 &= false \\
s_6 &= false \\
o\_t_1 &= true \\
o\_t_2 &= true \\
o\_t_3 &= true \\
i\_t_1 &= true \\
i\_t_2 &= true \\
i\_t_3 &= true
\end{aligned}
$$

4.8 Benchmark

Give by Daniel Kroening:

		Our tool			
		positive	don't know	timeout 10 min	
Mist	positive	8	3	0	11
	negative	0	27	1	28
	timeout 1 min	15	16	7	38
		23	46	8	77

5 Refinement methods

5.1 TrapConditions

For a petri net N and an assignment A, find a set S that satisfies

1. S is a trap in the net N.
2. S is marked in the initial marking M_0.
3. S is unmarked in the assignment A.

For such a set S, generate a constraint $\delta = \left(\sum_{s \in S} s \geq 1\right)$, ensuring the trap is marked in any assignment.

5.2 SubnetTrapConditions

For a petri net N and an assignment A, construct a subnet N' from N that contains only the transitions that are fired in A. For the net N', find a set S that satisfies

1. S is a trap in the subnet N'.
2. S contains a place with an incoming transition in N'.
3. S is unmarked in the assignment A.

For such a set S, generate a constraint $\delta = \left(\bigwedge_{t \in T_1}(t > 0) \wedge \bigwedge_{t \in T_2}(t = 0) \implies \sum_{s \in S} s \geq 1\right)$, where T_1 are the transitions fired in A and T_2 are the transitions not fired in A. This ensures the trap is marked in the corresponding subnet.

5.3 EmptyTrapConditions

For a petri net N and an assignment A, find a set S that satisfies

1. S is a trap in the net N.
2. S is unmarked in the inital marking M_0.
3. a transition in $S^\bullet$ is fired in A
4. no transition in $S^\bullet \setminus^\bullet S$ is fired in A

For such a set S, generate a constraint $\delta = \left(\bigvee_{t \in S^\bullet}(t > 0) \implies \bigvee_{t \in {}^\bullet S \setminus S^\bullet}(t > 0)\right)$ to ensure a proper incoming transition is fired if an outgoing transition is fired where T_1 are the transitions fired in A and T_2 are the transitions not fired in A. This ensures the trap is marked in the corresponding subnet.