1 Peterson's Algorithm

1.1 Method

```
                         ( BEGIN )
                             |
                             v
                      [ C := C(N) ]
                             |
          +----------------->|
          |                  v
          |             / SAT(C ∪ {¬P}) \  --NO-->  / YES /  ----->  ( END )
          |             \               /
          |                  |
          |                 YES
          |                  v
          |      [ A := Model(C ∪ {¬P})       ]
          |      [ C_θ := TrapConditions(N, A) ]
          |                  |
          |                  v
          |             / SAT(C_θ) \  --NO-->  / Don't know /  ----->  ( END )
          |             \          /
          |                  |
          |                 YES
          |                  v
          |         [ A_θ := Model(C_θ) ]
          |         [   δ := Δ(A_θ)     ]
          |         [  C := C ∪ {δ}     ]
          |                  |
          +------------------+
```

$C := \mathcal{C}(N)$

$\mathrm{SAT}(C \cup \{\neg P\})$

$A := \mathrm{Model}(C \cup \{\neg P\})$
$C_\theta := \mathrm{TrapConditions}(N, A)$

$\mathrm{SAT}(C_\theta)$

$A_\theta := \mathrm{Model}(C_\theta)$
$\delta := \Delta(A_\theta)$
$C := C \cup \{\delta\}$
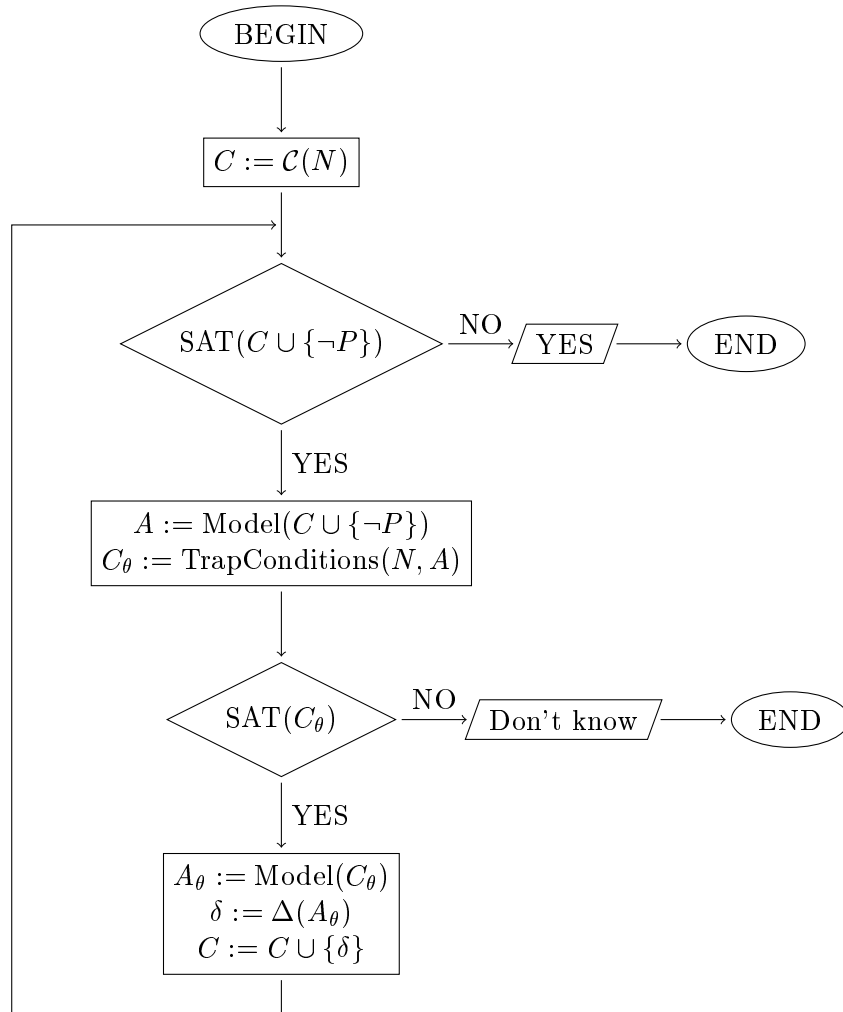
1.2 Constraints C_0

$$
\begin{aligned}
p_1 &= 1 - u_1 && + u_6 \\
p_2 &= 0 + u_1 - u_2 - u_3 \\
p_3 &= 0 \quad\quad + u_2 + u_3 - u_4 - u_5 \\
p_4 &= 0 \quad\quad\quad\quad\quad + u_4 + u_5 - u_6 \\
q_1 &= 1 && - v_1 && + v_6 \\
q_2 &= 0 && + v_1 - v_2 - v_3 \\
q_3 &= 0 && \quad + v_2 + v_3 - v_4 - v_5 \\
q_4 &= 0 && \quad\quad + v_4 + v_5 - v_6 \\
(m_1 = f) &= 1 - u_1 && + u_6 \\
(m_1 = t) &= 0 + u_1 && - u_6 \\
(m_2 = f) &= 1 && - v_1 && + v_6 \\
(m_2 = t) &= 0 && + v_1 && - v_6 \\
(hold = 1) &= 1 \quad + u_2 && - v_3 \\
(hold = 2) &= 0 \quad - u_2 && + v_3 \\
p_4 &\geq 1 \\
q_4 &\geq 1 \\
\forall p \in S \cup T : \quad p &\geq 0
\end{aligned}
$$

$$
\begin{aligned}
\delta_1 &= p_3 \vee q_2 \vee (m_2 = f) \vee (hold = 2) \\
\delta_2 &= p_2 \vee q_3 \vee (m_1 = f) \vee (hold = 1)
\end{aligned}
$$

1.3 A_1

$$p_1 = 0$$
$$p_2 = 0$$
$$p_3 = 0$$
$$p_4 = 1$$
$$q_1 = 0$$
$$q_2 = 0$$
$$q_3 = 0$$
$$q_4 = 1$$
$$(m_1 = f) = 0$$
$$(m_1 = t) = 1$$
$$(m_2 = f) = 0$$
$$(m_2 = t) = 1$$
$$(hold = 1) = 1$$
$$(hold = 2) = 0$$
$$u_1 = 1$$
$$u_2 = 0$$
$$u_3 = 1$$
$$u_4 = 0$$
$$u_5 = 1$$
$$u_6 = 0$$
$$v_1 = 1$$
$$v_2 = 1$$
$$v_3 = 0$$
$$v_4 = 1$$
$$v_5 = 0$$
$$v_6 = 0$$

1.4 A_2

$$p_1 = 0$$
$$p_2 = 0$$
$$p_3 = 0$$
$$p_4 = 1$$
$$q_1 = 0$$
$$q_2 = 0$$
$$q_3 = 0$$
$$q_4 = 1$$
$$(m_1 = f) = 0$$
$$(m_1 = t) = 1$$
$$(m_2 = f) = 0$$
$$(m_2 = t) = 1$$
$$(hold = 1) = 0$$
$$(hold = 2) = 1$$
$$u_1 = 1$$
$$u_2 = 1$$
$$u_3 = 0$$
$$u_4 = 0$$
$$u_5 = 1$$
$$u_6 = 0$$
$$v_1 = 2$$
$$v_2 = 0$$
$$v_3 = 2$$
$$v_4 = 0$$
$$v_5 = 2$$
$$v_6 = 1$$

1.5 $A_{\theta 1}$

$$p_1 = 0$$
$$p_2 = 0$$
$$p_3 = 1$$
$$p_4 = 0$$
$$q_1 = 0$$
$$q_2 = 1$$
$$q_3 = 0$$
$$q_4 = 0$$
$$(m_1 = f) = 0$$
$$(m_1 = t) = 0$$
$$(m_2 = f) = 1$$
$$(m_2 = t) = 0$$
$$(hold = 1) = 0$$
$$(hold = 2) = 1$$

1.6 $A_{\theta 2}$

$$p_1 = 0$$
$$p_2 = 1$$
$$p_3 = 0$$
$$p_4 = 0$$
$$q_1 = 0$$
$$q_2 = 0$$
$$q_3 = 1$$
$$q_4 = 0$$
$$(m_1 = f) = 1$$
$$(m_1 = t) = 0$$
$$(m_2 = f) = 0$$
$$(m_2 = t) = 0$$
$$(hold = 1) = 1$$
$$(hold = 2) = 0$$

1.7 C_θ

①

$$p_1 \implies o\_u_1$$
$$p_2 \implies o\_u_2 \wedge o\_u_3$$
$$p_3 \implies o\_u_4 \wedge o\_u_5$$
$$p_4 \implies o\_u_6$$
$$q_1 \implies o\_v_1$$
$$q_2 \implies o\_v_2 \wedge o\_v_3$$
$$q_3 \implies o\_v_4 \wedge o\_v_5$$
$$q_4 \implies o\_v_6$$
$$(m_1 = f) \implies o\_u_1 \wedge o\_v_4$$
$$(m_1 = t) \implies o\_u_6$$
$$(m_2 = f) \implies o\_v_1 \wedge o\_u_4$$
$$(m_2 = t) \implies o\_v_6$$
$$(hold = 1) \implies o\_v_3 \wedge o\_v_5 \wedge o\_u_3$$
$$(hold = 2) \implies o\_u_3 \wedge o\_u_5 \wedge o\_v_3$$
$$o\_u_1 \implies (p_2 \vee (m_1 = t))$$
$$o\_u_2 \implies (p_3 \vee (hold = 1))$$
$$o\_u_3 \implies (p_3 \vee (hold = 1))$$
$$o\_u_4 \implies (p_4 \vee (m_2 = f))$$
$$o\_u_5 \implies (p_4 \vee (hold = 2))$$
$$o\_u_6 \implies (p_1 \vee (m_1 = f))$$
$$o\_v_1 \implies (q_2 \vee (m_2 = t))$$
$$o\_v_2 \implies (q_3 \vee (hold = 2))$$
$$o\_v_3 \implies (q_3 \vee (hold = 2))$$
$$o\_v_4 \implies (q_4 \vee (m_1 = f))$$
$$o\_v_5 \implies (p_4 \vee (hold = 1))$$
$$o\_v_6 \implies (q_1 \vee (m_2 = f))$$

②

$$p_1 \vee q_1 \vee (m_1 = f) \vee (m_2 = f) \vee (hold = 1)$$

③₁

$$\neg p_4 \wedge \neg q_4 \wedge \neg (m_1 = t) \wedge \neg (m_2 = t) \wedge \neg (hold = 1)$$

③₂

$$\neg p_4 \wedge \neg q_4 \wedge \neg (m_1 = t) \wedge \neg (m_2 = t) \wedge \neg (hold = 2)$$
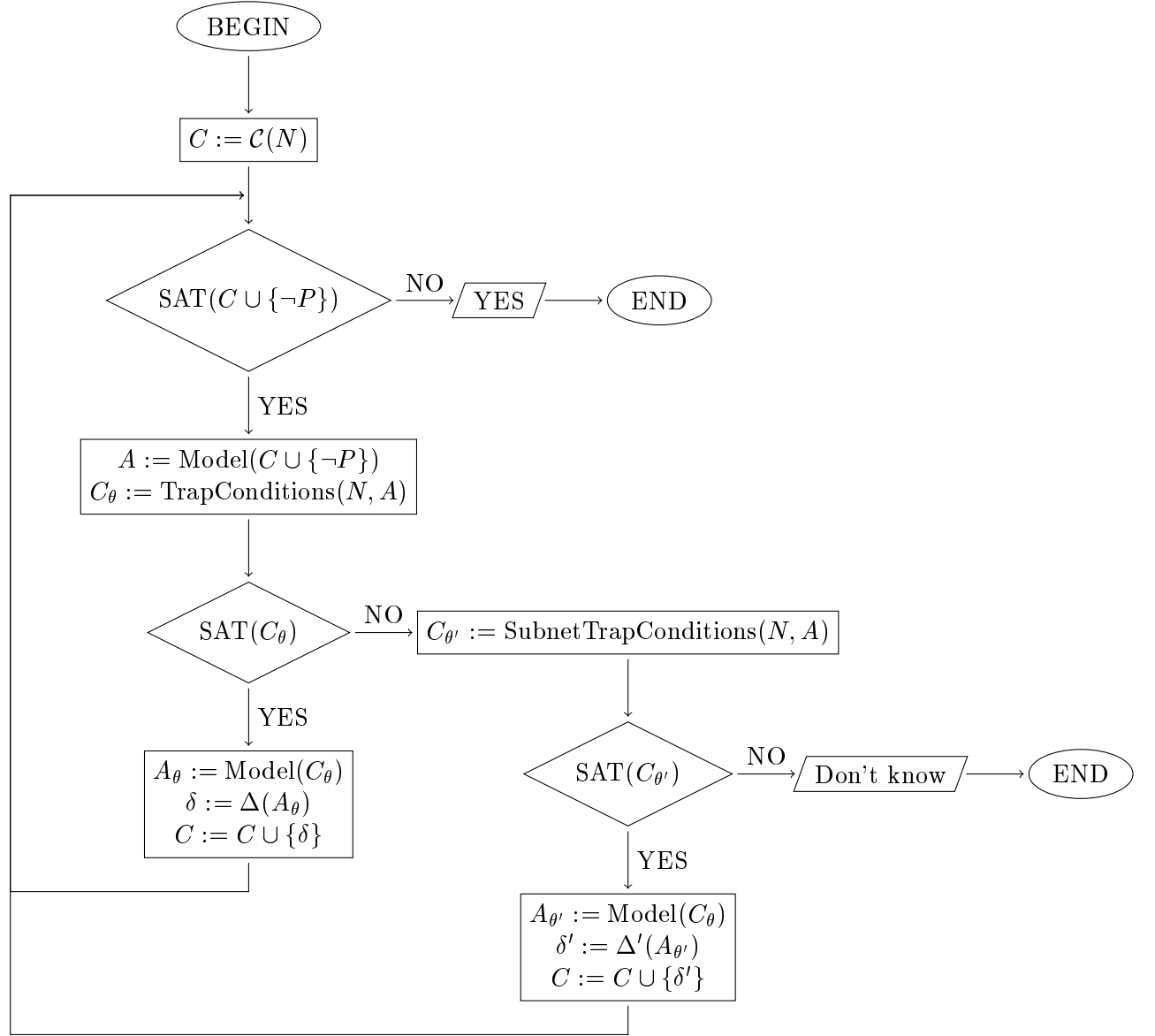
1.8 Benchmark

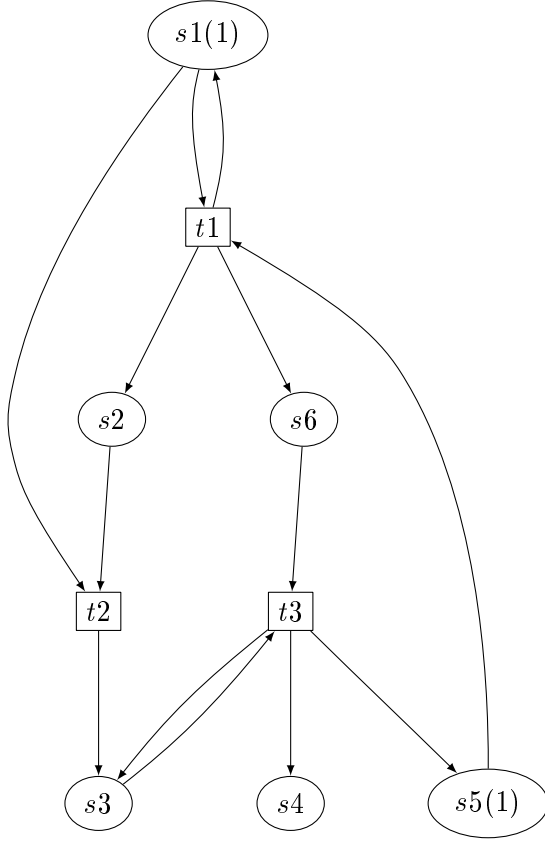Give by Daniel Kroening:

		Our tool			
		positive	don't know	timeout 10 min	
Mist	positive	8	3	0	11
	negative	0	28	0	28
	timeout 1 min	15	23	0	38
		23	54	0	77

2 Cyclic net

2.1 Method

```
                      ┌─────────┐
                      │  BEGIN  │
                      └─────────┘
                           │
                    ┌──────────────┐
                    │ C := C(N)    │
                    └──────────────┘
                           │
                           ▼
                    ╱ SAT(C ∪ {¬P}) ╲ ──NO──▶ / YES / ──▶ ( END )
                    ╲               ╱
                           │
                          YES
                           │
              ┌────────────────────────────┐
              │ A := Model(C ∪ {¬P})        │
              │ C_θ := TrapConditions(N, A)  │
              └────────────────────────────┘
```

$C := \mathcal{C}(N)$

$\mathrm{SAT}(C \cup \{\neg P\})$

$A := \mathrm{Model}(C \cup \{\neg P\})$
$C_\theta := \mathrm{TrapConditions}(N, A)$

$\mathrm{SAT}(C_\theta)$

$C_{\theta'} := \mathrm{SubnetTrapConditions}(N, A)$

$A_\theta := \mathrm{Model}(C_\theta)$
$\delta := \Delta(A_\theta)$
$C := C \cup \{\delta\}$

$\mathrm{SAT}(C_{\theta'})$

$A_{\theta'} := \mathrm{Model}(C_\theta)$
$\delta' := \Delta'(A_{\theta'})$
$C := C \cup \{\delta'\}$

Don't know

2.2 Petri net

2.3 Constraints C_0

$$
\begin{aligned}
s_1 &= 1 && - t_2 \\
s_2 &= 0 + t_1 - t_2 \\
s_3 &= 0 && + t_2 \\
s_4 &= 0 && + t_3 \\
s_5 &= 1 - t_1 && + t_3 \\
s_6 &= 0 + t_1 && - t_3 \\
s_1 &\geq 1 \\
s_2 &\geq 1 \quad s_4 \geq 1 \\
s_5 &\geq 1 \\
\forall p &\in S \cup T : p \geq 0
\end{aligned}
$$

$$
\delta_1' = (t_1 > 0) \wedge (t_2 = 0) \wedge (t_3 > 0) \implies (s_3 > 0)
$$

2.4 A_1

$$s_1 = 1$$
$$s_2 = 1$$
$$s_3 = 0$$
$$s_4 = 1$$
$$s_5 = 1$$
$$s_6 = 0$$
$$t_1 = 1$$
$$t_2 = 0$$
$$t_3 = 1$$

2.5 $A_{\theta'1}$

$$s_1 = 0$$
$$s_2 = 0$$
$$s_3 = 1$$
$$s_4 = 0$$
$$s_5 = 0$$
$$s_6 = 0$$

2.6 C_θ

$$
\begin{aligned}
s_1 &\implies o\_t_1 \wedge o\_t_2 \\
s_2 &\implies o\_t_2 \\
s_3 &\implies o\_t_3 \\
s_4 &\implies true \\
s_5 &\implies o\_t_1 \\
s_6 &\implies o\_t_2 \\
o\_t_1 &\implies (s_1 \vee s_2 \vee s_6) \\
o\_t_2 &\implies s_3 \\
o\_t_3 &\implies (s_3 \vee s_4 \vee s_5)
\end{aligned}
$$

$$s_1 \vee s_5$$

$$\neg s_1 \wedge \neg s_2 \wedge \neg s_4 \wedge \neg s_5$$

2.7 $C_{\theta'}$

$$s_1 \implies o\_t_1 \wedge o\_t_2$$
$$s_2 \implies o\_t_2$$
$$s_3 \implies o\_t_3$$
$$s_4 \implies true$$
$$s_5 \implies o\_t_1$$
$$s_6 \implies o\_t_2$$
$$o\_t_1 = (t_1 > 0) \implies (s_1 \vee s_2 \vee s_6)$$
$$o\_t_2 = (t_2 > 0) \implies s_3$$
$$o\_t_3 = (t_3 > 0) \implies (s_3 \vee s_4 \vee s_5)$$

$$(t_1 = 1) \wedge (t_2 = 0) \wedge (t_3 = 1)$$

$$s_1 \vee s_2 \vee s_3 \vee s_4 \vee s_5 \vee s_6$$

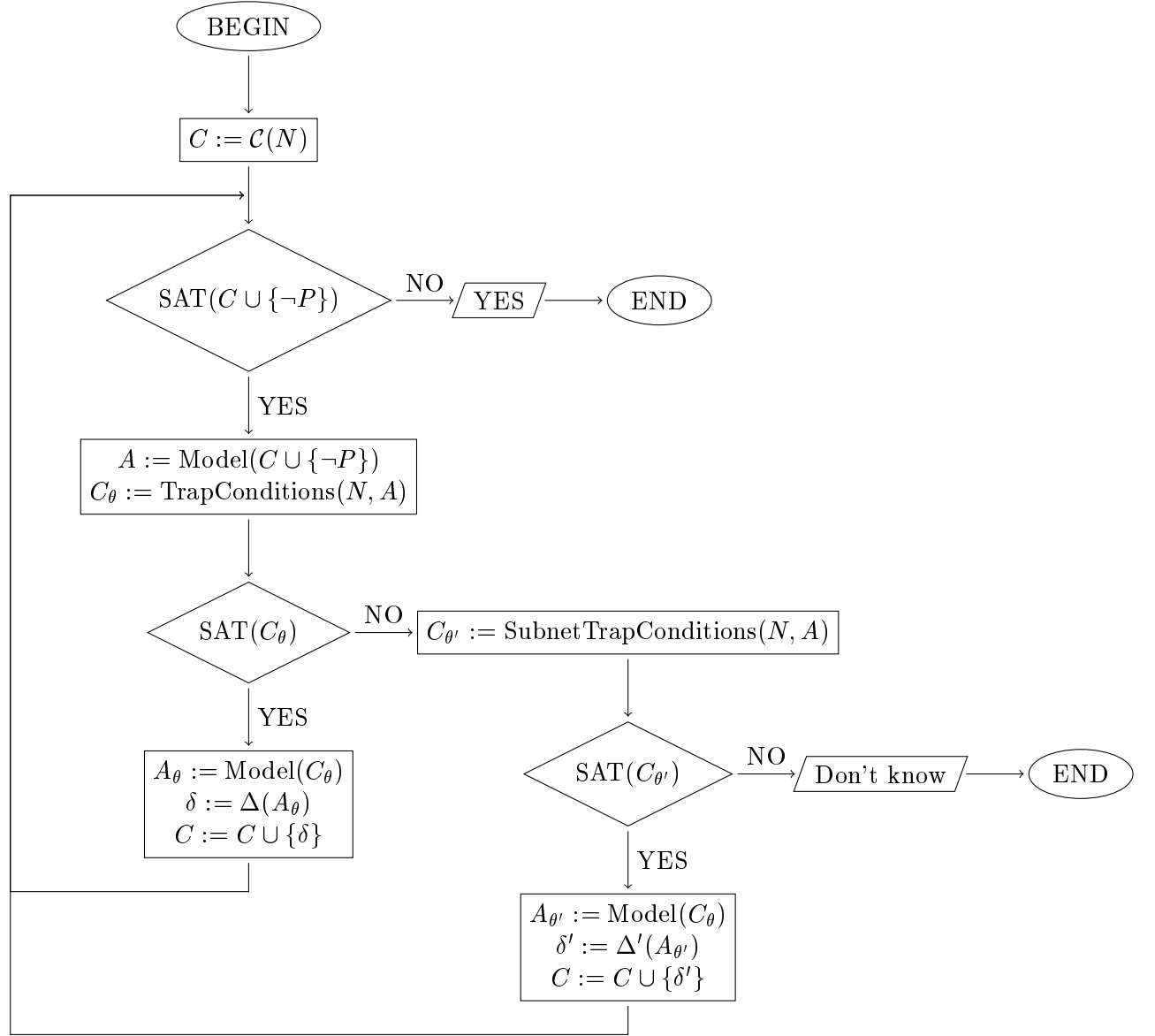$$\neg s_1 \wedge \neg s_2 \wedge \neg s_4 \wedge \neg s_5$$

2.8 Benchmark

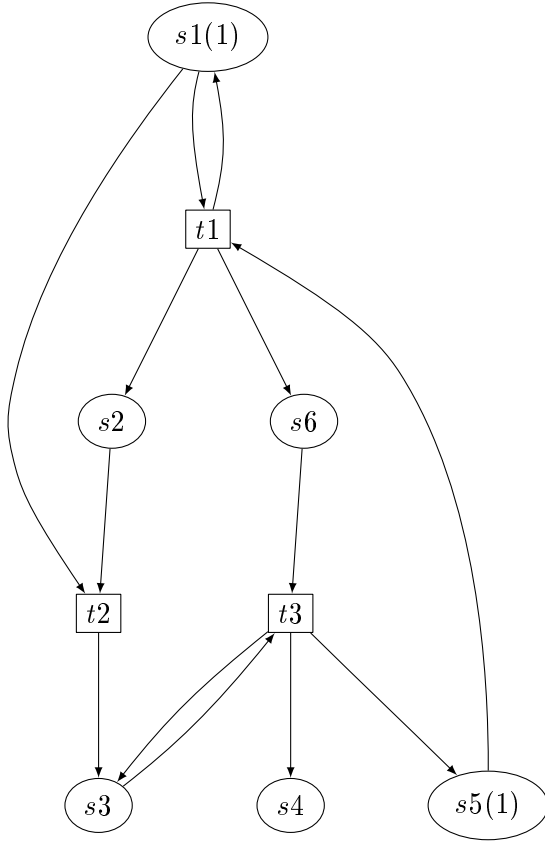Give by Daniel Kroening:

		Our tool			
		positive	don't know	timeout 10 min	
Mist	positive	8	3	0	11
	negative	0	28	0	28
	timeout 1 min	15	19	4	38
		23	50	4	77

3 Empty trap net

3.1 Method

BEGIN

$C := \mathcal{C}(N)$

$\text{SAT}(C \cup \{\neg P\})$ — NO → YES → END

YES

$A := \text{Model}(C \cup \{\neg P\})$
$C_\theta := \text{TrapConditions}(N, A)$

$\text{SAT}(C_\theta)$ — NO → $C_{\theta'} := \text{SubnetTrapConditions}(N, A)$

YES

$A_\theta := \text{Model}(C_\theta)$
$\delta := \Delta(A_\theta)$
$C := C \cup \{\delta\}$

$\text{SAT}(C_{\theta'})$ — NO → Don't know → END

YES

$A_{\theta'} := \text{Model}(C_\theta)$
$\delta' := \Delta'(A_{\theta'})$
$C := C \cup \{\delta'\}$

3.2 Petri net

3.3 Constraints C_0

$$
\begin{aligned}
s_1 &= 1 && - t_2 \\
s_2 &= 0 + t_1 - t_2 \\
s_3 &= 0 && + t_2 \\
s_4 &= 0 && && + t_3 \\
s_5 &= 1 - t_1 && && + t_3 \\
s_6 &= 0 + t_1 && && - t_3 \\
s_1 &\geq 1 \\
s_2 &\geq 1 \quad s_4 \geq 1 \\
s_5 &\geq 1 \\
\forall p &\in S \cup T : p \geq 0
\end{aligned}
$$

$$
\delta_1' = (t_1 > 0) \wedge (t_2 = 0) \wedge (t_3 > 0) \implies (s_3 > 0)
$$

3.4 A_1

$$s_1 = 1$$
$$s_2 = 1$$
$$s_3 = 0$$
$$s_4 = 1$$
$$s_5 = 1$$
$$s_6 = 0$$
$$t_1 = 1$$
$$t_2 = 0$$
$$t_3 = 1$$

3.5 $A_{\theta'1}$

$$s_1 = 0$$
$$s_2 = 0$$
$$s_3 = 1$$
$$s_4 = 0$$
$$s_5 = 0$$
$$s_6 = 0$$

3.6 C_θ

$$s_1 \implies o\_t_1 \land o\_t_2$$
$$s_2 \implies o\_t_2$$
$$s_3 \implies o\_t_3$$
$$s_4 \implies true$$
$$s_5 \implies o\_t_1$$
$$s_6 \implies o\_t_2$$
$$o\_t_1 \implies (s_1 \lor s_2 \lor s_6)$$
$$o\_t_2 \implies s_3$$
$$o\_t_3 \implies (s_3 \lor s_4 \lor s_5)$$

$$s_1 \lor s_5$$

$$\lnot s_1 \land \lnot s_2 \land \lnot s_4 \land \lnot s_5$$

3.7 $C_{\theta'}$

$$s_1 \implies o\_t_1 \wedge o\_t_2$$
$$s_2 \implies o\_t_2$$
$$s_3 \implies o\_t_3$$
$$s_4 \implies true$$
$$s_5 \implies o\_t_1$$
$$s_6 \implies o\_t_2$$
$$o\_t_1 = (t_1 > 0) \implies (s_1 \vee s_2 \vee s_6)$$
$$o\_t_2 = (t_2 > 0) \implies s_3$$
$$o\_t_3 = (t_3 > 0) \implies (s_3 \vee s_4 \vee s_5)$$

$$(t_1 = 1) \wedge (t_2 = 0) \wedge (t_3 = 1)$$

$$s_1 \vee s_2 \vee s_3 \vee s_4 \vee s_5 \vee s_6$$

$$\neg s_1 \wedge \neg s_2 \wedge \neg s_4 \wedge \neg s_5$$

3.8 Benchmark

Give by Daniel Kroening:

		Our tool			
		positive	don't know	timeout 10 min	
Mist	positive	8	3	0	11
	negative	0	28	0	28
	timeout 1 min	15	19	4	38
		23	50	4	77

4 Empty trap net

4.1 Method

```
                    ┌─────────┐
                    │  BEGIN  │
                    └─────────┘
                         │
                         ▼
                  ┌─────────────┐
                  │ C := C(N)   │
                  └─────────────┘
```

$$C := \mathcal{C}(N)$$

SAT$(C \cup \{\neg P\})$ — NO → YES → END

YES

$$A := \text{Model}(C \cup \{\neg P\})$$
$$\text{Methods} := \text{RefinementMethods}$$

$Methods \neq \emptyset$ — NO → Don't know → END

YES

Refinement $\leftarrow$ Methods
$$\text{Methods} := \text{Methods} \setminus \{\text{Refinement}\}$$
$$C_\theta := C_{\text{Refinement}}(N, A)$$

NO   SAT(C_θ)

YES

$$A_\theta := \text{Model}(C_\theta)$$
$$\delta := \Delta(A_\theta)$$
$$C := C \cup \{\delta\}$$
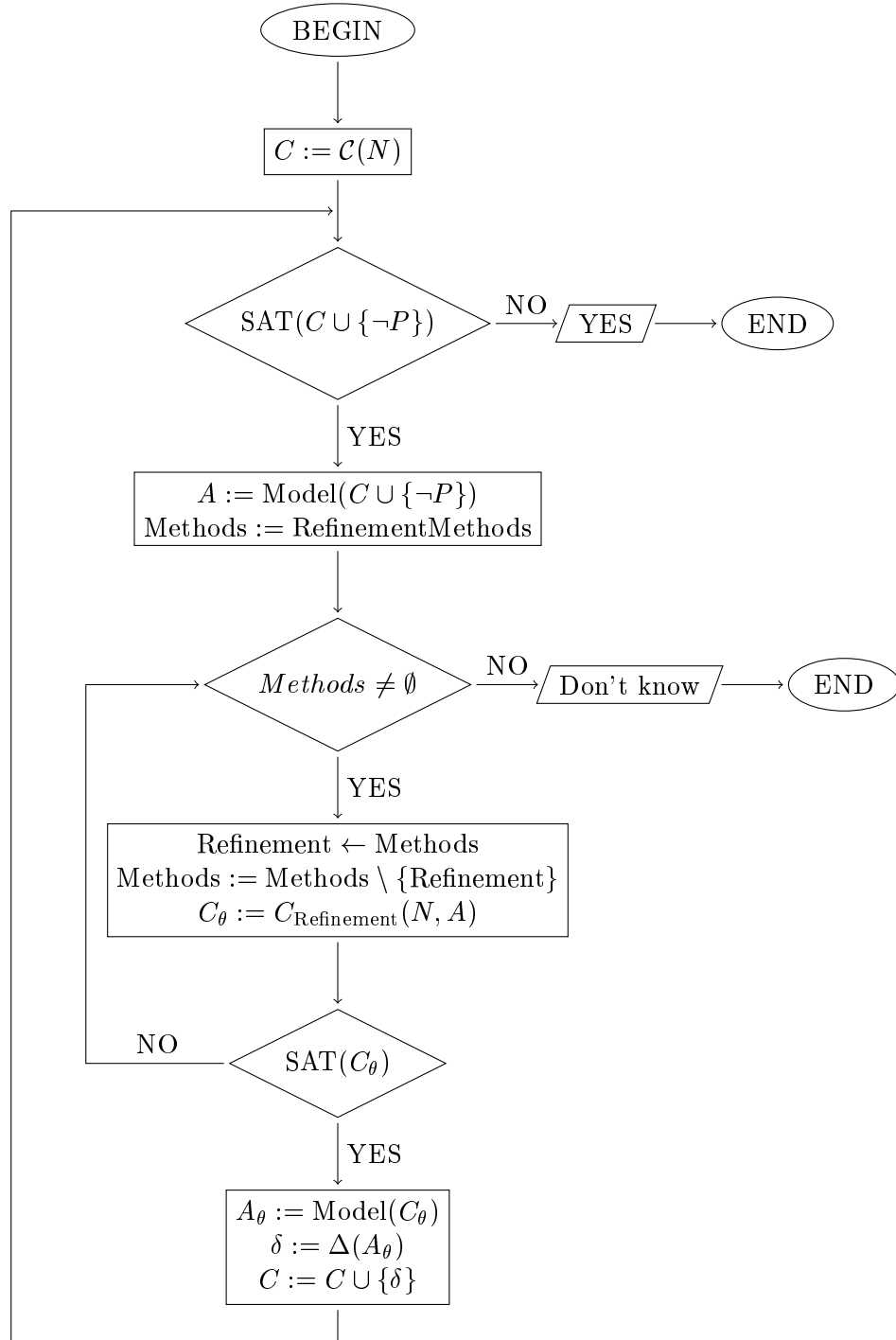
4.2 Petri net
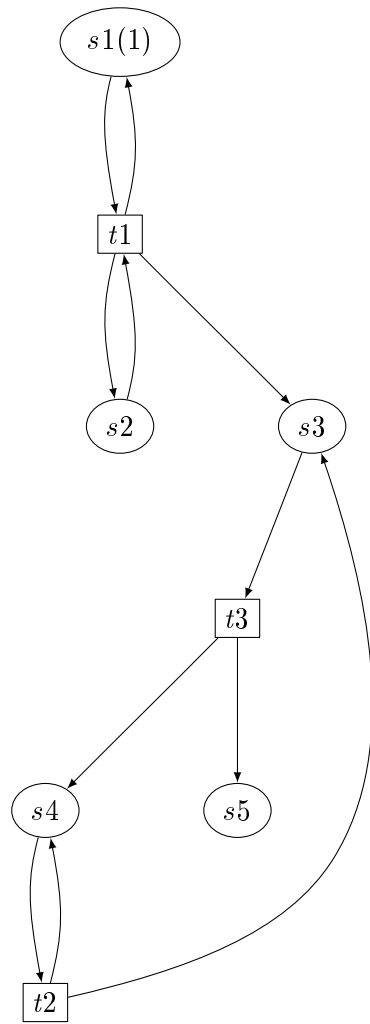
4.3 Constraints C_0

$$s_1 = 1$$
$$s_2 = 0$$
$$s_3 = 0 + t_1 + t_2 - t_3$$
$$s_4 = 0 \qquad\qquad + t_3$$
$$s_5 = 0 \qquad\qquad + t_3$$
$$s_5 \geq 1$$
$$\forall p \in S \cup T : p \geq 0$$

$$\delta_1 = (t_1 > 0) \wedge (t_3 > 0) \implies (t_1 > 0)\delta_2 \qquad = (t_1 > 0) \implies false$$

4.4 A_1

$$s_1 = 1$$
$$s_2 = 0$$
$$s_3 = 0$$
$$s_4 = 1$$
$$s_5 = 1$$
$$t_1 = 0$$
$$t_2 = 1$$
$$t_3 = 1$$

4.5 A_2

$$s_1 = 1$$
$$s_2 = 0$$
$$s_3 = 1$$
$$s_4 = 1$$
$$s_5 = 1$$
$$t_1 = 1$$
$$t_2 = 1$$
$$t_3 = 1$$

4.6 Empty trap $A_{\theta 1}$

$$s_1 = false$$
$$s_2 = false$$
$$s_3 = true$$
$$s_4 = true$$
$$s_5 = false$$
$$s_6 = false$$
$$o\_t_1 = false$$
$$o\_t_2 = true$$
$$o\_t_3 = true$$
$$i\_t_1 = true$$
$$i\_t_2 = true$$
$$i\_t_3 = true$$

4.7 Empty trap $A_{\theta 2}$

$$s_1 = false$$
$$s_2 = true$$
$$s_3 = false$$
$$s_4 = false$$
$$s_5 = false$$
$$s_6 = false$$
$$o\_t_1 = true$$
$$o\_t_2 = true$$
$$o\_t_3 = true$$
$$i\_t_1 = true$$
$$i\_t_2 = true$$
$$i\_t_3 = true$$

4.8 Benchmark

Give by Daniel Kroening:

		Our tool			
		positive	don't know	timeout 10 min	
Mist	positive	8	3	0	11
	negative	0	27	1	28
	timeout 1 min	15	16	7	38
		23	46	8	77