The Transition- Dynamics from Government Money only to Differentiated (Crypto-) Currency Competition

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Abstract

Government money only regimes are prone to undesirable properties like hyperinflationary episodes and a resulting decline in the trading activity observed during history.

In our extension of a multi-currency New Monetarist Model (Lagos & Wright, 2005) we find that cryptocurrencies could create a competitive environment the economy can profit from. Their enlarged functionalities with value-added services and the continuous improvement due to technology upgrades help to overcome the strong network effect in the market for currency.

We state the dynamics during the transition phase from one steady state to another and find that technological improvement is the main driver for adoption as well as their different characteristics. Cryptocurrencies encourage agents to hold money and hence improve the trading activity resulting in a positive feedback loop and an increase of the currency acceptance.

The dynamics persist until the first-best steady state is reached. Nevertheless, the acceptance of a cryptocurrency does not increase that much and our analysis points towards various cryptocurrencies establishing themselves as niche monies with different characteristics and therefore serving different consumer needs in contrast to the circulating universal government currency.

Keywords: Cryptocurrency adoption and transition dynamics, Economic and monetary aspects of cryptocurrencies, Currency competition, Relation of cryptocurrencies to other payment systems, Game theoretic analysis of agents' behaviour, Welfare gains through cryptocurrencies, Network Effect, Value-added Services, Currency differentiation

1 Introduction

For more than 3.000 years issuing currencies has been a government's prerogative. The creation of a means of payment enhances trading activity but the history of money is as well a history of inflation and not every currency system works well. As Obstfeld and Rogoff (1983) point out, fiat currency regimes are prone to hyperinflationary episodes and some countries like Venezuela struggle to fulfil the task of issuing a reliable stable currency.

Therefore, economists like Hayek (1990) question the necessity of a government monopoly money and argue in favour of a competitive currency system. Similar to other markets, a profit-seeking motive would lead to better results than benevolence in issuing a currency. Competition and free-entry would discipline the behaviour of the currency issuers and provide a stronger safeguard in terms of stability and reliability.

Many economic models find undesirable properties like a decline in the purchasing power and falling trading activity, for instance, associated with a government issued currency (see Obstfeld and Rogoff (1983), Kiyotaki and Wright (1989), Lagos and Wright (2003), Fernández-Villaverde and Sanches (2016)). As Filip (2021) argues, cryptocurrency competition could help to overcome those issues. The purely peer-to-peer electronic cash system is based on a shared public ledger and avoids a central point of failure. Furthermore, since the first cryptocurrency Bitcoin (Nakamoto, 2008) was launched in 2009, more and more people got inspired and new Altcoins were developed. Cryptocurrencies improve continuously and exhibit different characteristics in order to fit consumer's needs best. Some provide a greater anonymity, faster transaction times, or lower transaction costs, for example. Besides their main original aim of being a means of payment, some tokens also offer additional services like executing smart contracts or enabling DApps by facilitating the underlying blockchain technology. The technology behind those virtual coins improves all the time and so do they.

Therefore, they are not necessarily fiduciary and their value-added services could enhance the trading activity in an economy. Furthermore, if cryptocurrencies perform the best they can, the economy profits from the socially optimum trading output as defined by Friedman (1969) and the undesirable equilibrium trajectories do not exist any more. In this first-best equilibrium competition forces the cryptocurrencies to be stable and the government needs to implement a contractionary monetary policy in order to stay in circulation. Without those additional features cryptocurrencies offer, undesirable properties always exist in any monetary arrangement. (Filip, 2021)

But cryptocurrencies struggle to gain widespread acceptance and to get into circulation as a means of payment. One reason is the strong network effect in the market for currencies (see Luther (2016), Curt (2017), McCormack (2018), Currier (2018)) preventing the economy from realizing (crypto-) currency competition gains. As proven in Filip (2020), the fully accepted legal tender government currency will always hinder cryptocurrencies from circulating in equilibrium if they do not offer some additional features. However, it is possible that different currencies circulate in one economy even though one is not fully accepted as shown by Kiyotaki and Wright (1993). This as well holds true for cryptocurrencies offering the maximum feasible additional services (Filip, 2020). So the question is if an economy can reach the Pareto optimal first-best steady state with (crypto-) currency competition and how, or if cryptocurrencies will fail to enter into circulation.

It is therefore important to analyse the possible issues and patterns of transitioning from the government money only steady state to the first-best steady state with cryptocurrency competition. So we follow Filip (2021) and introduce currency acceptance as well as the additional services of cryptocurrencies into a multi-currency version of the New Monetarist Model (Lagos & Wright, 2003). Furthermore, we account for the technology growth behind cryptocurrencies and the growing value added they provide over time and explore the possible acceptability growth due to a positive feedback loop to answer the following questions:

- Is it possible for an economy to transition from a government money only steady state to the first-best (crypto-) currency competition steady state?
- Which are the most important drivers and mechanisms for the adoption and transition and how do the respective dynamics look like?
- Which implications result for the government money and monetary policy?
- How does the acceptability of alternative cryptocurrencies evolve over time?

As a result, the enlarged functionalities can indeed help to overcome the strong network effect and enable currency competition. We state the dynamics during the transition phase from one steady state to another and find that technological improvement is the main driver. Cryptocurrencies encourage the agents to hold money and hence improve trading activity. Even though cryptocurrency tokens might not be held as a means of payment in the first place, they can be facilitated for this purpose if necessary. The output gains are observed by other agents and lead to an increase in the acceptability of cryptocurrencies. The dynamics persist until the first-best steady state is reached and the economy can clearly profit from (crypto-) currency competition. Nevertheless, our analysis points towards various cryptocurrencies establishing themselves as niche monies with different characteristics and therefore serving different consumer needs in contrast to the circulating universal government currency.

In the next section we discuss important transition determinants and the importance of the network effect in the market for cryptocurrencies according to existing literature. The behaviour and decisions of the agents and currency issuers in our model are described in section 3 as well as the resulting equilibrium conditions. Section 4 contains our analysis of the transition dynamics and section 5 emphasizes some further issues for discussion such as convergence and network risks. We conclude in section 6.

2 Cryptocurrency Adoption

Following Dowd and Greenaway (1993) the value of a particular currency to a user depends on how many others use it, thus going beyond expected return or monetary stability. They claim that network effects cannot be ignored in any monetary model. Whithin the model of Dowd and Greenaway (1993), Luther (2016) provides an explanation based on network effects and switching costs why cryptocurrencies failed to gain widespread acceptance. In the absence of either significant monetary instability or government support agents may fail to adopt this alternative currency even though it might be superior to the status quo through recent technological advantages, for instance. Also Francis Pouliot in McCormack (2018) and Filip (2020) argue that the network effect leads to a winner-takes-it-all protocol in terms of currency competition and the network effect protects the incumbent money monopoly.

But not every currency system is working well, like Venezuela's for example, and cryptocurrencies could lead to an improvement. Holding cryptocurrencies seems to be a better option than having hyperinflationary Bolivars (Salvo, 2019). Also El Salvador officially classified Bitcoin as legal tender forcing every business to accept Bitcoin for goods and services in order to enhance the economic development of the country (bbc.com, 2021). According to an analysis of Hileman (2015) the top three countries with the highest chance of cryptocurrency adoption are all economically struggling and Bitcoin may provide a solution supporting the arguments Luther (2016) raises.

Nevertheless, the overall use of cryptocurrency is far from being an accepted means of payment.

However, as discussed in Filip (2021) cryptocurrencies go beyond being a means of payment. Cryptocurrency tokens are used to enable smart contracts or run a DApp, for example[1]. So cryptocurrency can be seen as both a commodity and a currency (Pandya, Mittapalli, Gulla, & Landau, 2019). As the Government Blockchain Association (2021) points out the new technology can provide value-added services, supports public transparency and trust, improves accountability and promotes innovation to foster technical and business efficiencies. Alzahrani and Daim (2019) find in their literature review on cryptocurrency adoption decisions that the main drivers are technological curiosity, anonymity of the transactions, faster transfer of funds, lower costs, investment opportunities as well as for sure the acceptance by businesses as a payment method. Hence there exists a wider functionality and different reasons to hold and use a cryptocurrency. Expectation about performance and social factors significantly influence the acceptance of Bitcoin according to Gunawan and Novendra (2017) and Nseke (2018).

Furthermore, it is necessary that the public understands these newly available technologies in order to use it. So it is crucial in the coming decade to provide access to, communicate and educate the public about cryptocurrencies (Government Blockchain Association, 2021). Pandya et al. (2019) advise that the government should encourage companies and businesses to create knowledge and technical experts to develop further the blockchain technology and cryptocurrencies.

Over the last years the amount of Altcoins rose rapidly updating features Bitcoin offers. Many enthusiasts seek to find ways to fit consumer's needs best and thus cryptocurrencies and the blockchain technology keep improving. Litecoin, for instance, provides faster transactions, Zerocoin seeks for greater privacy and Ether offers a wider functionality in smart contracts.

Thus, Marshall Hayner in Pirus (2021) states that cryptocurrencies now hit the fourth wave of adoption. After the first three waves exuberance, speculation and utility they finally reach acceptance.

> 'Bitcoin is a game of patience. Ultimately we must focus on adoption, on technological innovation, and on education.'

Marshall Hayner in Pirus (2021)

[1] Note that cryptocurrency tokens are necessary to use these features we refer to due to the blockchain technology implemented in these services. So cryptocurrency tokens and the services considered constitute a pure bundle (see Filip (2021)).

Recent developments show that the awareness and acceptance of cryptocurrencies rises. According to a survey from The Harris Poll and Mastercard Global Foresights, Insights and Analytics (2021) with online interviews of more than 15.000 consumers in 18 countries, 67% of millennials are more open to using cryptocurrency than they were a year ago and over 75% want to learn more about this topic in order to use it. Hence it is important to analyse possible transition dynamics in an economy and to get a deeper understanding of the driving forces.

3 A New Monetarist Model with (Crypto-) Currency Competition

To analyse the emergence of a competitive currency system and the resulting dynamics for the rise of cryptocurrencies we follow Filip (2020) and use a multi-currency version of the New Monetarist Model (Lagos & Wright, 2005) with network effects. The microfoundation of this framework makes it possible to state the decisions of the various agents and issuers regarding the use of different currencies and thereby analyse the macro effects for an economy without the need of imposing cash-in-advance constraints or money in the utility function assumptions as well as any higher aggregation level as common in other models.

The economy consists of a [0,1]-continuum of agents who can work and trade to gain utility from consuming goods. In each discrete period in their infinite life they choose their optimal strategy to maximize their utility with discount factor $\beta \in (0, 1)$ for next period consumption. The use of currency is not necessary but may make trading easier and more successful. Thus, the decisions of an agent also determine how much money is optimal to hold. At the beginning their exists just one government issued currency and agents expect to use this forever.

But after a while an alternative cryptocurrency becomes available and an agent now also needs to decide *which* money he prefers to use since the currencies may differ in their value. Furthermore, as described in the previous section, the usability and liquidity service a currency provides depends crucially on the amount of buyers and sellers accepting this currency. When the alternative private currency enters the market it is not used by many agents at the beginning and hence exhibits a lower acceptability than the incumbent fully accepted legal tender government money.

Nevertheless, cryptocurrencies are not necessarily intrinsically worthless but can provide value-added services and usages beside being a means of payment. The private issuers try to fit consumers' needs best and improve their technology based currencies all the time in order to stay in business. If they manage to provide enough incentives, some agents will profit from holding this currency even though they provide a lower liquidity service due to the lower acceptability. Thus, the alternative cryptocurrency will start circulating and creates a competitive environment for the government currency. Due to the continuous technological improvement, more and more agents will profit from using this alternative currency and the acceptability rises over time. Since the government currency is defined as a fiat currency providing no additional service, it needs to implement a different monetary policy to compete with the cryptocurrency and stay in circulation in the long run.

Next, we will describe the preferences and behaviour of the agents explicitly as well as the decisions of the currency issuers in the economy. In contrast to the model of Filip (2020) we let the currency acceptability grow due to a positive feedback loop. Afterwards, we can define the overall equilibrium conditions and analyse the resulting transition dynamics in section 4.

3.1 The Life of Utility Maximizing Agents

Each agent in the economy faces a quasi-linear utility function

$$\mathcal{U}(q, x, H) = u(q) - c(q) + U(x) - AH \tag{1}$$

where $u(q)$ and $U(x)$ are the utility functions from consuming the goods q and x. $c(q)$ is the (dis-) utility from producing the good q and the good x is produced with labour input H and the linear production function $x = \omega H$. Thus, he can transform one unit of labour into ω units of the good x and thereby reduces his utility by AH. W.l.o.g. we set $\omega = 1$ and normalize $A = 1$ since it does not change the fundamental properties of the model. Furthermore, we assume that $u(q)$, $c(q)$ and $U(x)$ are C^n with $n \geqslant 2$, $u^{'} > 0$, $u^{''} < 0$, $c^{'} > 0$, $c^{''} \geqslant 0$, $u(0) = c(0) = 0$, $U^{'} > 0$, $U^{''} \leqslant 0$ and there exists $q^* \in (0, \infty)$ $s.t.$ $u^{'}(q^*) = c^{'}(q^*)$ as well as a $x^* \in (0, \infty)$ $s.t.$ $U^{'}(x^*) = 1$ and $U(x^*) > x^*$.

The good q is the specialized good produced and traded in the decentralized Day-Market subperiod (DM) whereas x denotes the general good of the centralized Night-Market subperiod (CM). These two different market types account for the fact that sometimes trading in reality is easy and relatively centralized whereas some trading activity is much more decentralized and people struggle to find a trading partner and the *right* good (Williamson & Wright, 2010b). Therefore, trading in this model during the centralized subperiod is easier. The agents are able to coordinate perfectly, take prices as given and w.l.o.g. every agent consumes and produces the same general good x.

In the decentralized subperiod each agent produces a different specialized good q and consumes only a subset of all specialized goods and never what he himself produces. The agents are randomly bilaterally matched and meet a trading partner with probability $\alpha \in [0, 1]$. These meetings are anonymous with no scope for trading future promises. With probability $\delta \in [0, 1]$ both matched agents like what the other one offers and they can barter trade their specialized goods. On the other hand the probability of a single-coincidence meeting is $\sigma \in [0, 1]$. Here one agent does not like what the other offers and they need some money in order to successfully trade. Thus, the only feasible trades in this subperiod are barter or exchange of money for the specialized good.

An agent's money holdings are denoted by $\boldsymbol{m} = (m_t^1, ..., m_t^N) \in \mathbb{R}_+^N$ where m^i is the amount of currency i one agent holds out of the N different available currencies and vice versa $\tilde{\boldsymbol{m}}$ denotes the monetary portfolio of his trading partner. In contrast to the produced goods, money $m \geqslant 0$ is storable. The amount of money the buyer pays to the seller out of his monetary portfolio to receive the quantity q is $\boldsymbol{d} = (d_t^1, ..., d_t^N) \in \mathbb{R}_+^N$.

As described above the acceptability of a currency affects the success of a trade. So we follow Filip (2020) and introduce the acceptability $\mu^i \in [0, 1]$ for each currency i capturing the probability that both agents accept the same kind of money for trading in a random bilateral single-coincidence meeting[2]. Thus, the more the currencies in an agent's portfolio are accepted, the more likely they are able to trade and leave the DM receiving a positive utility. Since the government currency g is legal tender, $\mu_t^g = 1 \geqslant \mu_t^c \;\; \forall t$ for any private (crypto-) currency $c \in C$.

At the beginning there exists only one government currency $g \in G$, i.e. $G = 1 \;\; \forall t$, $N = G = 1 \; \forall t < S$. We assume that at some point in time S an alternative cryptocurrency $c \in C$ becomes available. To focus on the dynamics this introduction causes and to enhance the presentation we as well assume $C = 1 \;\; \forall t \geqslant S$ during our baseline analysis. However, we drop this assumption later on without changing the fundamental properties and results (see Fernández-Villaverde and Sanches (2016), Filip (2020) and Filip (2021)). Hence after time S there exist two different currencies i $\in N$, $N \leqslant 2 \;\; \forall t \geqslant S$

[2]Note, that in Filip (2020) the acceptance μ is given exogenously and does not vary over time. We will change this property in our framework.

and every agent chooses how much of the government and how much of the cryptocurrency he wants to hold in his monetary portfolio $\boldsymbol{m_t} = (m_t^g, m_t^c)$. The dummy variable ψ_t^i denotes whether an agent is in possession of currency i or not. Thus,

$$\psi_t^i = \begin{cases} 1 & \text{if } m_t^i > 0 \\ 0 & \text{if } m_t^i = 0 \end{cases}$$

and $\boldsymbol{\psi_t} = (\psi_t^g, \psi_t^c)$. The currencies can differ in their endogenously defined value $\boldsymbol{\phi_t} = (\phi_t^g, \phi_t^c)$ where $\phi_t^i \geqslant 0 \;\; \forall t$ and the cryptocurrency may offer some additional service and thus an additional value $y_t^c \geqslant 0 \;\; \forall t$ per nominal unit[3]. Since the government currency is defined as a fiat currency, $y_t^g = 0 \;\; \forall t$. We capture these states in $\boldsymbol{s_t} = (\phi_t, y_t, F)$ where F is the distribution of money holdings. Because it is a government's prerogative to collect taxes, τ denotes the taxes agents need to pay.

So we can state the value functions and the agent's maximization problems during the Day-Market $V(\boldsymbol{m_t}, \boldsymbol{s_t})$ and Night-Market $W(\boldsymbol{m_t}, \boldsymbol{s_t})$ formally

$$V(\boldsymbol{m_t}, \boldsymbol{s_t}) = \alpha\sigma\boldsymbol{\mu_t}\boldsymbol{\psi_t} \int \{u[q(\boldsymbol{m_t}, \boldsymbol{\tilde{m}_t}, \boldsymbol{s_t})] + W[\boldsymbol{m_t} - \boldsymbol{d_t}(\boldsymbol{m_t}, \boldsymbol{\tilde{m}_t}, \boldsymbol{s_t})]\} \mathrm{d}F(\boldsymbol{\tilde{m}_t}) \tag{2a}$$

$$+\alpha\sigma\boldsymbol{\mu_t}\boldsymbol{\psi_t} \int \{-c[q(\boldsymbol{\tilde{m}_t}, \boldsymbol{m_t}, \boldsymbol{s_t})] + W[\boldsymbol{\tilde{m}_t} + \boldsymbol{d}(\boldsymbol{\tilde{m}_t}, \boldsymbol{m_t}, \boldsymbol{s_t})]\} \mathrm{d}F(\boldsymbol{\tilde{m}_t}) \tag{2b}$$

$$+\alpha\delta \int B(\boldsymbol{m_t}, \boldsymbol{\tilde{m}_t}, \boldsymbol{s_t}) \mathrm{d}F(\boldsymbol{\tilde{m}_t}) \tag{2c}$$

$$+(1 - 2\alpha\sigma\boldsymbol{\mu_t}\boldsymbol{\psi_t} - \alpha\delta) W(\boldsymbol{m_t}, \boldsymbol{s_t}) \tag{2d}$$

$$\begin{aligned} W(\boldsymbol{m_t}, \boldsymbol{s_t}) = & \max_{x, H, \boldsymbol{m_{t+1}}} \{U(x) - AH + \beta V(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}})\} \\ & s.t. \;\; x = \omega H + \boldsymbol{\phi_t}\boldsymbol{m_t} + \boldsymbol{y_t}\boldsymbol{m_t} - \tau - \boldsymbol{\phi_t}\boldsymbol{m_{t+1}} \end{aligned} \tag{3}$$

(2a) and (2b) describe the case of a single-coincidence meeting where the first refers to the agent being a buyer and the second to being a seller. Remember that in this scenario the use of money is necessary because one agent does not like what the other one produces and they need to exchange money $\boldsymbol{d_t}$ to trade successfully. We determine the terms of trade $\{q(\boldsymbol{m_t}, \boldsymbol{\tilde{m}_t}, \boldsymbol{s_t}), \boldsymbol{d_t}(\boldsymbol{m_t}, \boldsymbol{\tilde{m}_t}, \boldsymbol{s_t})\}$ by solving the centralized Nash bargaining

[3] As argued in Filip (2021) y_t^c could as well represent some extra cost on cryptocurrency usage if $y_t^c < 0 \;\; \forall t$. But we restrict ourselves to the value-added features and leave the analysis of $y_t^c < 0 \;\; \forall t$ in the transition phase for further research.

problem where the buyer makes take-it-or-leave-it offers to the seller. Lemma 1 in the Appendix proves that the solution is

$$q(\boldsymbol{m_t}, \boldsymbol{s_t}) = \begin{cases} \hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t}) & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ q^* & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} \geqslant c(q^*) \end{cases} \tag{4}$$

$$(\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{d_t}(\boldsymbol{m_t}, \boldsymbol{s_t}) = \begin{cases} (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ c(q^*) & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} \geqslant c(q^*) \end{cases} \tag{5}$$

and we derive several results. The traded quantity q_t and paid amount $\boldsymbol{d_t}$ do not depend on the money holdings of the seller $\boldsymbol{\tilde{m}_t}$ and they do depend on the monetary portfolio of the buyer $\boldsymbol{m_t}$ if and only if his budget constraint is binding, i.e. $(\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*)$. In this case the buyer does not hold enough money to compensate the seller for his production costs of producing the socially efficient quantity q^* where $c'(q^*) = u'(q^*)$ as defined by Friedman (1969). Thus, the buyer spends all his money $\boldsymbol{d_t} = \boldsymbol{m_t} < \frac{c(q^*)}{(\boldsymbol{\phi_t}+\boldsymbol{y_t})} = \boldsymbol{m^*}$ and receives a lower quantity $\hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t}) < q^*$.

In the other case, if the buyer holds enough money, they decide to trade the socially efficient quantity q^* and the buyer compensates the seller exactly for his production costs $\boldsymbol{d_t} = m^*$. Thus, we can define the demand for real balances $z(q_t) = (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{d_t} = (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} = c(q_t)$.

In the event of a double-coincidence meeting (2c) the randomly bilateral matched agents decide to barter trade the socially efficient quantity q^* as proven in lemma 2 in the Appendix. The symmetric Nash problem yields a unique solution and thus

$$B(\boldsymbol{m_t}, \boldsymbol{\tilde{m}_t}, \boldsymbol{s_t}) = u(q^*) - c(q^*) + W(\boldsymbol{m_t}, \boldsymbol{s_t}) \tag{6}$$

With probability $(1 - 2\alpha\sigma\boldsymbol{\mu_t}\boldsymbol{\psi_t} - \alpha\delta)$ no trade occurs (2d) and the agent leaves the Day-Market without trading.

After the decentralized Day-Market the agents enter the centralized Night-Market and face the value function $W(\boldsymbol{m_t}, \boldsymbol{s_t})$ (3). Again the agents need to decide how much they want to consume of the general good x, how much they want to work H and how much money they want to save for the next period Day-Market trading $\boldsymbol{m_{t+1}}$ in order to maximize their expected utility. Note that the additional value y_t^i a currency provides directly affects the feasible amount of the general good an agent receives in the CM. Thus, if an agent uses a currency offering some additional services $y_t^i > 0$, he profits from using this currency and gets a greater utility through greater consumption in the Night-Market beside the currencies value ϕ_t^i, taxes τ in contrast lower his consumption. Normalizing $A = 1$, substitution for H and rearranging (3) yields

$$\begin{aligned} W(\boldsymbol{m_t}, \boldsymbol{s_t}) =& \max_{\boldsymbol{m_{t+1}}}\{-\boldsymbol{\phi_t m_{t+1}} + \beta V(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}})\} \\ &+ \max_x\{U(x) - x\} \\ &+ (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} - \tau \end{aligned} \tag{7}$$

First of all we derive the solution that the quasi-linear utility function (1) rules out wealth effects and the choice of $\boldsymbol{m_{t+1}}$ is independent of an agent's money holdings $\boldsymbol{m_t}$ and the taxes he needs to pay τ. Next, $x = x^*$ where $U'(x^*) = 1$ and the agents can coordinate perfectly and trade the optimum quantity x^* in the centralized market. Furthermore, $W(\boldsymbol{m_t}, \boldsymbol{s_t})$ is linear in $\boldsymbol{m_t}$ with slope $(\boldsymbol{\phi_t} + \boldsymbol{y_t})$

$$W(\boldsymbol{m_t}, \boldsymbol{s_t}) = (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} + W(\boldsymbol{0}, \boldsymbol{s}) \tag{8}$$

Inserting all our so far derived solutions (8), (7), (6), (5) and (4) into (2) and rearranging yields the Bellman's equation

$$\begin{aligned} V(\boldsymbol{m_t}, \boldsymbol{s_t}) =& \max_{\boldsymbol{m_{+1}}}\{-\boldsymbol{\phi_t m_{t+1}} + \beta V(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}})\} \\ &+ U(x^*) - x^* + \nu(\boldsymbol{m_t}, \boldsymbol{s_t}) \\ &+ (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} - \tau \end{aligned}$$

where

$$\nu(\boldsymbol{m_t}, \boldsymbol{s_t}) = \begin{cases} \quad \alpha\sigma\boldsymbol{\mu_t\psi_t}\{u[\hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t})] - (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t}\} & \\ + \quad \alpha\sigma\boldsymbol{\mu_t\psi_t}\int\{-c[\hat{q}(\boldsymbol{\tilde{m}_t}, \boldsymbol{s_t})] + (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{\tilde{m}_t}\}\mathrm{d}F(\boldsymbol{\tilde{m}}) & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ + \quad \alpha\delta\{u(q^*) - c(q^*)\} & \\ & \\ \quad \alpha(\sigma\boldsymbol{\mu_t\psi_t} + \delta)\{u(q^*) - c(q^*)\} & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} \geqslant c(q^*) \end{cases}$$

can be interpreted as the expected return from one day of DM trade and the remaining term is the return from going to the CM.

In a next step we need to find the optimal amount an agent wants to *save* in the Night-Market in order to enhance his trading activity in the next period Day-Market, i.e. $\underset{\boldsymbol{m_{t+1}}}{max}\{-\boldsymbol{\phi_t m_{t+1}} + \beta V(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}})\}$. On the one hand, he probably profits from an additional unit of money in the next day DM single-coincidence meeting. On the other hand, he may not need money and can barter trade. Then it would be better to use the coin now, buy some additional amount of the general good and gain this utility for sure now. Furthermore, he needs to decide which money he wants to hold in his monetary portfolio in the next period after the alternative (crypto-) currency is available. The government currency will be accepted by any trading partner he meets in the random matching. Contrary, the alternative cryptocurrency may not provide the full liquidity service due to a lower acceptability, but offers some additional service an agent can profit from like a greater anonymity. Hence the F.O.C. for his optimal portfolio choice for the next period is

$$\frac{\partial V(\boldsymbol{m_t}, \boldsymbol{s_t})}{\partial m^i_{t+1}} = -\phi^i_t + \beta V^{i'}(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}}) \leqslant 0 \tag{9}$$

and it holds with equality if $m^i_{t+1} > 0$.

To compare the liquidity services each currency provides we can define the liquidity premium $l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})]$ of currency i as proven in lemma 3 in the Appendix

$$l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})] \equiv \alpha\sigma\mu^i_t(\frac{u^{'}[q_t(\boldsymbol{m_t}, \boldsymbol{s_t})]}{c^{'}[q_t(\boldsymbol{m_t}, \boldsymbol{s_t})]} - 1) \tag{10}$$

It states the marginal value of spending a coin in the DM of currency i as opposed to carrying it forward to the CM, times the probability of spending it in the DM. The acceptance μ^i_t of each currency directly affects the liquidity service it is going to provide. The more a currency is accepted, the higher is the resulting liquidity premium in equilibrium. Furthermore, the liquidity premium of each currency equals zero if the agents trade the socially efficient quantity q^* in the Day-Market.

Therefore,

$$\frac{\partial V(\boldsymbol{m_t}, \boldsymbol{s_t})}{\partial m^i_t} = (\phi^i_t + y^i_t)\{1 + l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})]\} \tag{11}$$

and we can summarize all results derived so far in one equilibrium condition for the agents determining their optimal currency choice proven in lemma 4 in the Appendix with the boundary condition $\beta(\phi^i_{t+1} + y^i_{t+1}) \leqslant \phi^i_t$ proven in lemma 5 in the Appendix.

$$\beta(\phi^i_{t+1} + y^i_{t+1})\{1 + l^i[q_{t+1}(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}})]\} = \phi^i_t \tag{12}$$

$$l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})] \begin{cases} \neq 0 & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ = 0 & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} > c(q^*) \end{cases}$$

Thus, an agent holds one coin of currency i if and only if the value of this coin $\phi^i_t > 0$ at the end of a period t (RHS) equals the discounted marginal benefit of this coin during the next period $t+1$ (LHS). Since $V(\boldsymbol{m_t}, \boldsymbol{s_t})$ is strictly concave in m^i for all $(\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*)$ given any ϕ and F there exists a unique maximizer $\boldsymbol{m_{t+1}}$ and F is degenerate for all agents in any equilibrium. Therefore, all agents in the economy hold the same *amount* of money but the decomposition of their money holdings can differ. Some agents might stick to the government currency, whereas others switch and (partly) hold the cryptocurrency as long as the equilibrium condition (12) is satisfied for each currency, i.e. the agents are indifferent in equilibrium between the circulating currencies. Otherwise, all agents would decide to hold only the currency with the greater expected benefit and there would be just this one currency in circulation in equilibrium.

Additionally, the traded amount of the specialized good strictly increases with the amount of money holdings ($q^{i'} > 0$) and thus $\hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t}) < q^*$ if $(\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*)$ because we take a look at monetary equilibria where at least one currency is valid ($\phi^i_t > 0$). As a result, in any equilibrium the behaviour and decisions of the agents imply $q(\boldsymbol{m_t}, \boldsymbol{s_t}) = \hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t}) \leqslant q^*$ and $\boldsymbol{d}(\boldsymbol{m_t}, \boldsymbol{s_t}) = \boldsymbol{m_t} \leqslant \boldsymbol{m^*_t}$ with $\boldsymbol{m^*_t} \equiv \frac{c(q^*)}{(\boldsymbol{\phi_t} + \boldsymbol{y_t})}$ at all time.

3.2 Different Currency Issuers in one Economy

The N different currencies $i \in \{1, ..., N\}$ between the agents can choose are issued by different institutions[4]. First of all, there exists a fiat currency g issued by the government. This currency behaves as standard in New Monetarist Models and many other monetary models (see Lagos and Wright (2003), Lagos and Wright (2005), Williamson and Wright (2010a),Williamson and Wright (2010b)). The government can change the nominal supply

[4]Note that from the N different available currencies in the economy not all are necessarily valued and circulating in equilibrium.

$M^g_t \in \mathbb{R}_+$ of its own currency by either taxing the agents or by changing government consumption during the CM in order to follow different monetary policy rules[5]. So the real value of the total supply of the government currency is

$$b^g_t \equiv \phi^g_t M^g_t$$

and its budget constraint yields

$$\phi^g_t M^g_t + \tau_t = \phi^g_t M^g_{t-1}$$

The value of the currency $\phi^g_t \in \mathbb{R}_+$ is determined endogenously and is equivalent to the inverse of the price level during the Night-Market. Thus, the real return on government money is

$$\rho^g_t \equiv \frac{\phi^g_t}{\phi^g_{t-1}}$$

Agents can observe the implemented monetary policy rules and form their beliefs about the behaviour of the government and its currency value. Since this currency is legal tender, the agents are forced to accept this currency and thus $\mu^g_t = 1 \;\; \forall t$.

We assume that at some point S in time a cryptocurrency c becomes available. This alternative currency can be either issued by private entrepreneurs or automata and the agents are now free to choose which currency they want to use and hold. Following the ideas of Hayek (1990) these issuers enter the market to maximize their profits, i.e. an entrepreneur's discounted lifetime utility as proposed by Fernández-Villaverde and Sanches (2016)

$$\sum_{t=S}^{\infty} \beta^t x^c_t \tag{13}$$

[5]For a detailed discussion of various monetary policies and competitive currencies, such as a money growth rule $M^g_t = (1+\omega)M^g_{t-1}$ or pegging the real value of government money supply $b^g_t \equiv \phi^g_t M^g_t = \bar{b}^g$ we refer to Fernández-Villaverde and Sanches (2016) or Filip (2021), for instance.

subject to their budget constraint

$$x_t^c = \phi_t^c \Delta_{M_t^c} + y_t^c \Delta_{M_t^c} - \sum_{i \neq c} \phi_t^i \Delta_{M_t^i} - \sum_{i \neq c} y_t^i \Delta_{M_t^i} \tag{14}$$

Therefore, they need to fit consumers' needs best and perform the best they can in order to stay in business. The real value of the total supply of the cryptocurrency at time $t \geqslant S$ is

$$b_t^c \equiv (\phi_t^c + y_t^c) M_t^c$$

The issuing entrepreneurs can choose their nominal supply $M_t^c \in \mathbb{R}_+$ and adjust it $\Delta_{M_t^c} \equiv M_t^c - M_{t-1}^c$ by buying and selling against other currencies or change their consumption in the Centralized Market according to their budget constraint (14). Issuing automata adjust their nominal supply following the pre-defined rule in their source code as common by many today existing cryptocurrencies.

Furthermore, cryptocurrency issuers may provide different additional services $y_t^c \in \mathbb{R}_+$ to their users besides the coin value $\phi_t^c \in \mathbb{R}_+$ as a means of payment as argued in the previous section[6]. Hence agents gain some additional value and thus a greater utility from using these currencies. In other words, the agents have additional and different incentives to hold a cryptocurrency coin compared to a government currency coin just held as a standard means of payment. As a consequence, cryptocurrencies are not necessarily perfect substitutes or fiat but differ in some aspects.

Moreover, cryptocurrencies are technology based and as many technologies they improve over time to fit consumers' needs best. Therefore, as the implemented technology improves, the additional service a cryptocurrency provides grows at some function

$$y_{t+1}^c = G(y_t^c)$$

with the growth rate

[6] A detailed discussion of y_t^c can be found in Filip (2021).

$$n(y_t^c) = \frac{y_{t+1}^c - y_t^c}{y_t^c} = \frac{G(y_t^c)}{y_t^c} - 1 \geqslant 0 \;\; \forall t \geqslant S \tag{15}$$

But, in contrast to the incumbent government currency, alternative currencies are not fully accepted at the beginning and they need to work on enlarging their network, so $\mu_S^c < \mu^g$. However, the network of the cryptocurrency can grow over periods. More and more agents may observe the additional profits they could gain or get inspired by the technology and possibilities cryptocurrencies offer. Thus, more agents start to use and accept this currency and the acceptability rises. We therefore define the growth rate of the acceptability

$$a(\mu_t^c) = \frac{\mu_{t+1}^c - \mu_t^c}{\mu_t^c} \;\; \forall t \geqslant S + 1 \tag{16}$$

Again, the value of all cryptocurrencies ϕ_t^c will be determined endogenously respectively. The shared public ledger and the blockchain technology permit every agent to observe the trading history and the issuer's behaviour costlessly and the agents can form their beliefs.

As Hayek (1990) imagined, the different currencies compete in a market with perfect competition and free-entry and we can analyse whether the self-interest of the profit maximizing entrepreneurs is a better motive in producing good results than benevolence in the government issued currency scenario.

The overall total supply of currency in the economy after time T is the sum of the government currency supply and the various different cryptocurrencies $c \in \{1, ..., C\}$ where $C \in N$. Since we assume $C = 1$

$$\sum_{i=1}^{N} b_t^i = b_t^g + \sum_{c=1}^{C} b_t^c = \phi_t^g M_t^g + (\phi_t^c + y_t^c) M_t^c \;\; \forall t$$

and

$$\sum_{i=1}^{N} b_t^i = z[\hat{q}_t(\boldsymbol{m_t}, \boldsymbol{s_t})] \tag{17}$$

since in any equilibrium the market-clearing condition implies that the total supply of currency needs to equal the total demand for real balances.

3.3 Analysing different Equilibria

Now we can take all so far derived results together and define the equilibrium in the economy.

Definition 1. *Any equilibrium satisfies (12) and (17) with the boundary condition* $\beta(\phi^i_{t+1} + y^i_{t+1}) \leqslant \phi^i_t$ *and* $b^i_t \geqslant 0\ \forall i \in \{1, ..., N\}$ *where* $q(\boldsymbol{m_t}, \boldsymbol{s_t}) = \hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t}) \leqslant q^*$ *and* $\boldsymbol{d}(\boldsymbol{m_t}, \boldsymbol{s_t}) = \boldsymbol{m_t} \leqslant \boldsymbol{m^*_t}$ *with* $\boldsymbol{m^*_t} \equiv \frac{c(q^*)}{(\boldsymbol{\phi_t}+\boldsymbol{y_t})}$ *at all time subject to the monetary policy the government implements and the optimal behaviour of the cryptocurrency issuers.*

At the beginning we assume that there exists only one government issued currency in the economy which is legal tender and fully accepted. Thus, $N = G = 1$, $b^i = b^g$, $\mu^g = 1\ \forall t < S$ and the equilibrium condition (12) reduces to

$$1 + l^g[\hat{q}_{t+1}(m_{t+1}, \boldsymbol{s_{t+1}})] = \frac{\phi^g_t}{\beta\phi^g_{t+1}} \tag{18}$$

with the boundary condition $\phi^g_t \geqslant \beta\phi^g_{t+1}$. Therefore, the total supply of currency in the economy equals the total supply of the government currency and the market-clearing condition (17) implies

$$z[\hat{q}_t(m_t, \boldsymbol{s_t})] = b^g_t = \phi^g_t M^g_t \tag{19}$$

Combining (18) and (19) gives

$$\beta z[\hat{q}_{t+1}(m_{t+1}, \boldsymbol{s_{t+1}})]\{1 + l^g[\hat{q}_{t+1}(m_{t+1}, \boldsymbol{s_{t+1}})]\} = z[\hat{q}_t(m_t, \boldsymbol{s_t})] \tag{20}$$

which defines the equilibrium path of the traded quantity $\hat{q}_t(m_t, \boldsymbol{s_t})$.

Definition 2. *Any equilibrium with only one government currency in circulation, i.e.* $N = G = 1$, $b^i = b^g$ *can be defined as a path* $\{\hat{q}_t(m_t, \boldsymbol{s_t})\} \in (0, q^*)$ *satisfying (20) and the boundary condition* $\phi^g_t \geqslant \beta\phi^g_{t+1}$ *where* $z[\hat{q}_t(m_t, \boldsymbol{s_t})] = b^g_t = \phi^g_t M^g_t\ \forall t < S$.

We derive several results which are exactly the same as in Lagos and Wright (2003), Filip (2020) and Filip (2021)[7].

[7] A deeper discussion of the steady state and static properties of this monetary arrangement can be found in Lagos and Wright (2005),Williamson and Wright (2010a) and Williamson and Wright (2010b), for instance.

Proposition 1. *An equilibrium with only one government money in circulation yields a monetary steady state equilibrium* $0 < \hat{q}^s < q^*$ *delivering price stability* $(\rho^g, \rho^g_{+1}) = (1, 1)$ *as well as a nonmonetary steady state* $(\rho^g, \rho^g_{+1}) = (0, 0)$ *with* $\hat{q} = 0$. *Furthermore there exists an equilibrium trajectory where* $\hat{q}_t \to 0 \;\; \forall \hat{q}_0 \in (0, q^*)$.

Proof. In a steady state equilibrium where $q_t = q_{t+1}$ the equilibrium condition (20) reduces to $\beta\{1 + l[\hat{q}(m, \boldsymbol{s})]\} = 1$ which clearly yields just one solution $\hat{q}^s$ where $(\rho^g, \rho^g_{+1}) = (1, 1) \;\; \forall t$. From the bargaining solution (4) and $l(\hat{q}^s) \neq 0$ we know that $\hat{q}^s(m, \boldsymbol{s}) < q^*$. It is easy to check that $(\rho^g, \rho^g_{+1}) = (0, 0)$ with $\hat{q} = 0$ also satisfies (20). Furthermore, (20) defines a mapping $q_{t+1} = g(q_t)$ which is single valued with $g^{'}(0) < 1$ and there exists a continuum of dynamic equilibria starting from any point $\hat{q}_0 \in (0, q^*)$ converging to $q = 0$. □

So in the monetary steady state the government currency provides a stable value $\phi^g_t = \phi^g_{t+1}$ and the traded quantity is $0 < \hat{q}^s < q^*$. Hence the agents decide to hold too little money m^g in their monetary portfolio when entering the decentralized Day-Market and thus they are not able to afford the socially optimum quantity q^* as defined by Friedman (1969) in the single-coincidence meeting. Additionally, there exist a continuum of equilibrium trajectories where the traded quantity persistently declines over time converging to zero trading activity in a nonmonetary steady state. Along these trajectories the government currency declines in its value $\rho^g_t < 1$ till zero. This scenario in a way refers to the hyperinflationary episodes fiat currencies are prone to as Obstfeld and Rogoff (1983) pointed out. Hence although the government currency provides stability in the monetary steady state, the possible equilibrium dynamics bear some risks for an economy and the traded output is strictly below the socially optimum. q^* could be reached if the government implements the optimal contractionary monetary policy but the undesirable dynamics still persist (Filip, 2021).

However, following the approach of Filip (2021) cryptocurrencies could help to overcome those issues. The wider functionality beyond the government currency encourages the agents to hold more money in their monetary portfolio because they profit from the additional service with different characteristics cryptocurrencies provide. Even though agents may not hold a cryptocurrency for being a means of payment they could facilitate those coins in the decentralized single-coincidence meeting if necessary. Thus the traded equilibrium quantity increases with the increased money holdings. If cryptocurrencies perform the best they can and provide the maximum feasible additional value y^*, there exists a unique monetary steady state with the socially optimum trading activity q^* and a stable value of the cryptocurrency $\phi_t^c = \phi_{t+1}^c$.

Therefore, we assume that at time S an alternative cryptocurrency becomes available in the economy and the agents are now free to choose which currency and how much of this currency they want to use. So $C = 1$ and $N = G + C = 2$ with $i \in \{g, c\}$ $\forall t \geqslant S$. Perfect competition and free-entry in the market for currencies forces the lifetime utility of each issuer to be zero in equilibrium in the absence of operational costs. Otherwise, new entrants will start operating, expanding the total currency supply which results in a devaluation of all circulating currencies according to the market-clearing condition and hence reducing their profits. New currencies will start circulating until the expected profit is zero as standard in many competitive market models for various segments.

Lemma 6 in the Appendix proves that the optimal strategy for any issuing entrepreneur is a constant nominal money supply $M_t^c = M_{t+1}^c = M^c$. Note that many cryptocurrencies issued nowadays by automata also provide a constant money supply in the long run. The amount of Bitcoin, for instance, increases but it approaches and will never exceed 21 million. Thus, we follow Fernández-Villaverde and Sanches (2016) and refer to issuing entrepreneurs and automata, respectively. Furthermore, the constant nominal cryptocurrency supply in equilibrium outweighs the concerns of DeLong (2013) that cryptocurrencies will expand the amount of currency in circulation without limits. As Hayek (1990) imagined, competition disciplines the behaviour of the different currency issuers.

The optimal portfolio choice of an agent (12) must hold for any circulating currency in equilibrium and implies that an agent only holds two different currencies across periods if they attain the same expected value in the next period compared to the value in the evening now, i.e. he is indifferent between those currencies he holds.

Referring to Filip (2021) it is possible to construct the first-best equilibrium with cryptocurrency competition. From the bargaining solution in the decentralized Day-Market (4) we know that $q(\boldsymbol{m_t}, \boldsymbol{s_t}) = q^*$ if and only if $(\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} \geqslant c(q^*)$. Hence the equilibrium condition for an agent implies $l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})] = 0$ and reduces to

$$\beta = \frac{\phi_t^i}{(\phi_{t+1}^i + y_{t+1}^i)} \quad \forall t \;\; \forall i$$

Note that the boundary condition defines the maximum feasible additional service a cryptocurrency can provide in order to be consistent with the equilibrium solution of this model. If the cryptocurrency provides a stable value $\phi_t^c = \phi_{t+1}^c = \phi^c \ \ \forall t$, $y^c \leqslant \frac{(1-\beta)}{\beta}\phi^c$ and the maximum feasible set is $y^* = \frac{(1-\beta)}{\beta}\phi^c$. Therefore, it is possible to construct a steady state where the cryptocurrency provides stability and performs the best it can, given any initial value of the currency

$$\beta = \frac{\phi^c}{(\phi^c + y^*)}$$

Furthermore, this steady state equilibrium is the unique equilibrium and there do not exist any trajectories with undesirable properties consistent with the initial values. So there is no decline in the traded quantity or convergence to a nonmonetary steady state.

To be consistent with this equilibrium and to circulate, the fiat government currency needs to exhibit a constant deflation rate $\rho^g > 1$ with the optimum contractionary monetary policy (Filip, 2021)

$$\beta = \frac{\phi^c}{(\phi^c + y^*)} = \frac{\phi_t^g}{\phi_{t+1}^g} = \frac{1}{\rho^g}$$

and we can summarize our findings.

Proposition 2. *There exist a unique steady state equilibrium with circulating competitive currencies and the socially optimum trading activity q^* if the cryptocurrencies perform the best they can and implement $y^* = \frac{(1-\beta)}{\beta}\phi^c$ with a stable value $\phi_t^c = \phi_{t+1}^c = \phi^c$ and the government currency provides a stable deflation rate $\rho^g > 1 \ \forall t$ satisfying the equilibrium condition (12), the market-clearing condition (17) and the boundary condition $\beta(\phi_{t+1}^i + y_{t+1}^i) \leqslant \phi_t^i$. Thus, $\boldsymbol{d}(\boldsymbol{m_t}, \boldsymbol{s_t}) = \boldsymbol{m_t} = \boldsymbol{m_t^*}$ with $\boldsymbol{m_t^*} \equiv \frac{c(q^*)}{(\boldsymbol{\phi_t} + \boldsymbol{y_t})}$.*

Proof. This follows straight from the above described computations. □

As argued by Filip (2021), this equilibrium Pareto dominates all other equilibria discussed in the analysis with competition between different cryptocurrencies and delivers the first-best solution[8]. Competition disciplines the behaviour of the different currency issuers and provides stability with a strong safeguard as Hayek (1990) imagined. The additional services cryptocurrencies provide encourage the money holdings of the agents such that

[8] Similar results hold true for a single commodity money economy as discussed in Lagos and Wright (2003) and Williamson and Wright (2010b).

the socially efficient trading output is possible during the random bilateral matching. Additionally, the socially optimal trading activity outweighs any network effect because each currency yields the same liquidity premium, irrespective of the different acceptabilities. Hence an economy would clearly profit from differentiated (crypto-) currency competition.

Nevertheless, as Filip (2020) pointed out, different acceptabilities prevent alternative currencies from getting in circulation if the traded quantity is below the socially efficient one or if the cryptocurrencies do not provide (enough) additional services. Also Luther (2016) finds in his modification of the simple model of Dowd and Greenaway (1993) that cryptocurrencies struggle to gain acceptance. The network effect protects the incumbent money and prevents competition. As a result, the strong network effect in the market for currencies may hinder the economy from establishing the first-best steady state and the economy cannot profit from the competitive scenario. It is therefore necessary to analyse the issues and patterns of transition from a government money only steady state (proposition 1) to the Pareto optimal one (proposition 2).

4 Transition Dynamics in the Economy

We start our analysis of the transition dynamics by considering a baseline scenario where one cryptocurrency gets available in the economy at time S and competes with the government currency. Afterwards we discuss the effects of various different (crypto-) currencies entering the competitive currency market and the resulting implications for monetary policy. In a last step we account for a growing acceptability of the cryptocurrencies and the effects of a possible positive feedback loop between the enhanced trading activity and the cryptocurrency usage.

4.1 Competition between Government Money and one Cryptocurrency

To keep better track of the various dynamic paths over time we first of all insert the liquidity premium (10) into the equilibrium condition for the agent's portfolio choice (12)

$$\{1+\alpha\sigma\mu^i_{t+1}(\frac{u'[q_{t+1}(\boldsymbol{m_{t+1}},\boldsymbol{s_{t+1}})]}{c'[q_{t+1}(\boldsymbol{m_{t+1}},\boldsymbol{s_{t+1}})]}-1)\}=\frac{\phi^i_t}{\beta(\phi^i_{t+1}+y^i_{t+1})} \tag{21}$$

$\forall i \in N \quad \phi^i > 0$. To enhance presentation we follow Fernández-Villaverde

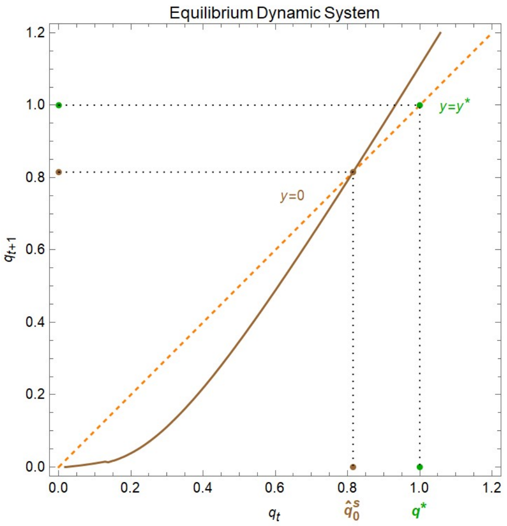

Figure 1: Equilibrium dynamic system (22) for $\hat{q}_t$ with only one government currency in circulation (proposition 1) and the socially optimum quantity q^* unique steady state achieved with the maximum feasible additional value y^* and cryptocurrency competition (proposition 2).

and Sanches (2016) and select the functional forms $u(q) = (1-\eta)^{-1}q^{1-\eta}$ and $c(q) = (1+\gamma)^{-1}q^{1+\gamma}$ where $0 < \eta < 1$ and $\gamma \geqslant 0$. (21) thus yields

$$\{1 + \alpha\sigma\mu_{t+1}^i(q_{t+1}^{-(\gamma+\eta)} - 1)\} = \frac{\phi_t^i}{\beta(\phi_{t+1}^i + y_{t+1}^i)} \tag{22}$$

In a next step we set the parameter values following Lagos and Wright (2005). Therefore, we calibrate the arrival rate near maximum $\alpha = 0.99$ so that it is very likely that an agent meets a trading partner during the decentralized trading subperiod and make barter trading rather rare with $\delta = 0.01$. The probability of a single-coincidence meeting is as big as possible $\sigma = 0.495$. Finally, set $\beta = 0.9$, $\eta = 0.5$ and $\gamma = 0.5$.

Figure 1 plots the resulting equilibrium dynamics for a system with just one fiat government currency in circulation with the steady state quantity $\hat{q}_0^s$ as well as the associated equilibrium trajectory converging to zero trading activity for any initial value as stated in proposition 1. Furthermore, it shows the socially efficient quantity q^* where $q^{-(\gamma+\eta)} = 1$ is achieved with the maximum feasible additional value y^*. Clearly $q^* > \hat{q}_0^s$ and there does not exist any undesirable path as stated in proposition 2. We therefore analyse next the transition dynamics from the fiat government money only steady

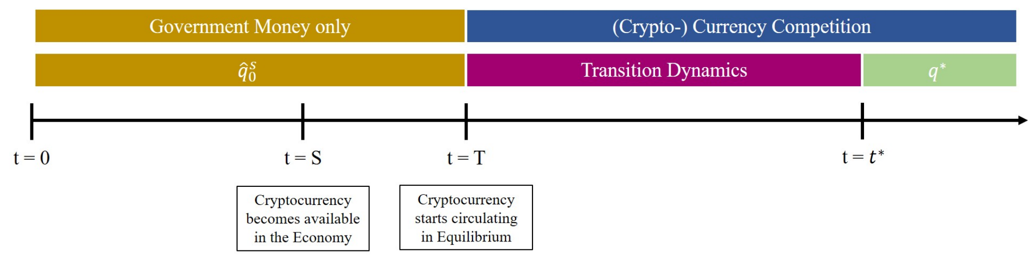

Figure 2: Timeline of the economy evolution.

state $\hat{q}{}_0^s$ to the first-best unique steady steady state q^* with a competitive cryptocurrency and the resulting implications for monetary policy.

Figure 2 sketches the time evolution of the currency arrangements and equilibria in the economy. At the beginning we assume that there is just one government currency g in circulation which is fully accepted $\mu^g = 1$ and the agents expect to use this currency forever. Hence the agents maximize their utility and make their optimal decisions as described in section 3 such that the economy is in the government money only steady state with the traded quantity $\hat{q}{}_0^s$ lower than the social optimum (proposition 1). Thus, $i = g$ with $N = G = 1$ and the government currency provides stability[9] $\phi_t^g = \phi_{t+1}^g$, i.e. $\rho_t^g = 1$ and $b_t^g = b_{t+1}^g$ with $M_t^g = M_{t+1}^g$ $\forall t$ in this steady state. Without any interruption the economy would stay there forever.

Suddenly at time S an alternative cryptocurrency c becomes available in the economy so $N = G + C = 2$ and $i \in \{g, c\}$. This new money is barely known by anyone and thus used and accepted by very few agents $\mu^c << \mu^g$. Furthermore, at the beginning as technology is in development the provided additional service is low. As a result, according to the optimal portfolio choice of an agent (22), the expected usability and utility in terms of the next period traded quantity with this alternative currency is strictly lower than the incumbent money and no agent decides to hold this currency in his portfolio across periods. Hence the cryptocurrency cannot be valued in equilibrium and the strong network effect prevents it from entering into circulation (see Filip (2020)).

[9]For sure the government could as well implement various different monetary policies as discussed in Lagos and Wright (2003), Lagos and Wright (2005), Williamson and Wright (2010a) and Williamson and Wright (2010b), for instance. But accroding to Filip (2021) undesirable dynamics always exist and therefore the optimal monetary policy is never uniquely associated with the first-best output. Thus, we assume no specific monetary policy implementation at the beginning without changing the fundamental properties of the transition dynamics to keep things simple.

However, cryptocurrencies and especially the technology behind them improve over time resulting in new possibilities and additional services they can provide to fit consumers' needs best. So the additional value grows $y^c_{t+1} = G(y^c_t)$ with the bounded decreasing positive growth rate $n(y^c_t)$ tending to zero

$$\begin{aligned} n(y^c_t) = \frac{y^c_{t+1} - y^c_t}{y^c_t} = \frac{G(y^c_t)}{y^c_t} - 1 \geqslant 0 \ \ \forall t \geqslant S \\ G(y^c_t) \geqslant y^c_t > 0 \ \ \forall y^c_t \leqslant y^* \ \ (positive) \\ n(y^c_t) \geqslant n(y^c_{t+1}) \geqslant 0 \ \ (decreasing) \\ \underset{t\to+\infty}{lim} n(y^c_t) = 0 \ \ (tends\ to\ zero) \\ \exists y^*_\infty \ \ s.t. \ \ y_t \leqslant y^*_\infty \ \ \forall t \ \ (bounded) \end{aligned} \tag{15}$$

Cryptocurrencies improve very fast at the beginning in order to get valued in equilibrium but the improvement slows as they approach the maximum feasible additional service[10] y^*. Therefore, we select the functional form

$$y_{t+1} = y_t e^{r(1-\frac{y_t}{y^*})} \tag{23}$$

with $r = 0.5$ and figure 3 plots the growth of the additional service over time.

At some point T in time the provided additional service of the cryptocurrency is large enough to compensate the lower acceptability in an agent's portfolio choice. Thus, there exists some $\hat{y}_T$ such that the expected traded quantity is $\hat{q}{}^s_0$ for a given $\mu^c << \mu^g$ and provided stability $\phi^c_t = \phi^c_{t+1}$. Now the agents are indifferent between the incumbent government currency and the alternative cryptocurrency according to their optimal portfolio choice (22) in equilibrium. Therefore, some agents start to hold the cryptocurrency in their monetary portfolio and the cryptocurrency starts circulating in the economy creating a competitive currency environment such that the market-clearing condition (17) is fulfilled and the free-entry disciplines their behaviour. So

$$\{1 + \alpha\sigma\mu^c(\hat{q}{}^{s}_0{}^{-(\gamma+\eta)} - 1)\} = \frac{\phi^c}{\beta(\phi^c + \hat{y}_T)} \equiv \{1 + \alpha\sigma\mu^g(\hat{q}{}^{s}_0{}^{-(\gamma+\eta)} - 1)\} = \frac{\phi^g}{\beta\phi^g}$$

[10]Remember that there does not exist an equilibrium if $y > y^*$ in this framework. Hence we interpret that at y^* cryptocurrencies *perform the best they can* according to the boundary condition.

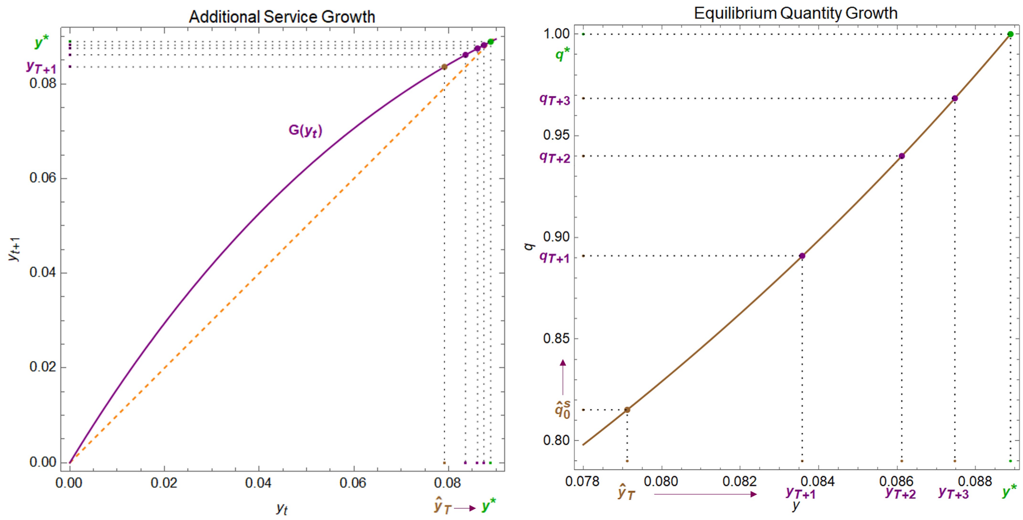

Figure 3: Growth of the additional service over time (23) and the necessary level of $\hat{y}_T$ to achieve the equilibrium traded quantity $\hat{q}_0^s$ for a given $\mu^c << \mu^g$ out of (22) as well as the resulting convergence to the first-best trading output q^* due to technology growth.

Time T. *At point T in time we derive an equilibrium for the economy where the government currency and the cryptocurrency are circulating in a competitive environment with the traded quantity $\hat{q}_0^s$. The government currency is fully accepted $\mu^g = 1$ and provides stability $\rho^g = 1$. The cryptocurrency compensates its very low acceptability $\mu^c << \mu^g$ with the additional service $\hat{y}_T$ and provides stability as well $\phi_t^c = \phi_{t+1}^c$.*

Contrary to the scenario with government money only, this no longer is a steady state. The technology behind cryptocurrencies improves even further approaching the maximum feasible additional service y^* they can provide as illustrated in figure 3. Therefore, $y_{T+1} > \hat{y}_T$ and the higher additional value of the cryptocurrency encourages the money holdings of the agents $\boldsymbol{m_{T+1}} > \boldsymbol{m_T}$ resulting in a higher traded equilibrium quantity in the next period $q_{T+1} > \hat{q}_0^s$ from (22). As described above some agents might hold the cryptocurrency only to profit from the additional service but if necessary they can facilitate the coins as a means of payment in the random bilateral single-coincidence meeting during the Day-Market and the overall traded quantity rises. In order to achieve this new equilibrium quantity as well and stay in circulation, the government currency needs to deflate $\phi_{T+1}^g > \phi_T^g$ and

$\rho^g_{T+1} > 1$ whereas the cryptocurrency is still stable $\phi^c_{T+1} = \phi^c_T$.

Time T+1. *At point $T+1$ in time we derive an equilibrium for the economy where the government currency and the cryptocurrency are circulating in a competitive environment with the traded quantity $q_{T+1} > \hat{q}{}^s_0$ due to an increase in the agents money holdings $\boldsymbol{m_{T+1}} > \boldsymbol{m_T}$ according to the increased additional service $y_{T+1} > \hat{y}_T$. The government currency is fully accepted $\mu^g = 1$ but deflates $\rho^g_{T+1} > 1$ and the cryptocurrency still provides stability $\phi^c_{T+1} = \phi^c_T$ and circulates with acceptability $\mu^c << \mu^g$.*

The technology improves even further and the additional service the cryptocurrency provides increases $y_{T+2} > y_{T+1}$, again encouraging the agent's money holdings and as a result the traded quantity in period $T+2$ increases again $q_{T+2} > q_{T+1}$ as seen in figure 3. This implies a higher necessary deflation rate of the government currency $\rho^g_{T+2} > \rho^g_{T+1} > 1$.

According to these the economy converges to the first-best unique steady state equilibrium where $q_t = q^*$ as stated in proposition 2. As the provided additional service reaches its boundary y^* after some periods, the cryptocurrency performs the best it can and encourages the agents to increase their money holding such that $\boldsymbol{m_t} = \boldsymbol{m^*}$ and they can afford the socially efficient quantity q^*. Furthermore, the socially optimum equilibrium is a steady state and all growth rates are zero, i.e. $n(y^c_t) = g(q_t) = 0$.

Time $\boldsymbol{t^*}$. *At time t^* the economy reaches the first-best equilibrium with $q^* > \hat{q}{}^s_0$, $y^* > \hat{y}_T$, $\boldsymbol{m^*} > \boldsymbol{m_T}$ and $\mu^c << \mu^g = 1\ \forall t \geqslant t^*$. The cryptocurrency is stable $\phi^c_{t^*} = \phi^c_{t^*+1}$ and the government currency provides a stable deflation rate $\rho^* > \rho^g = 1$. $n(y^c_{t^*}) = g(q_{t^*}) = 0$ and the economy is expected to stay in this steady state forever.*

Since we know from the bargaining output (4) and the optimal portfolio choice of an agent (12) that $q(\boldsymbol{m_t}, \boldsymbol{s_t}) = q^*$ implies $l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})] = 0\ \ \forall i$ we know that each currency yields the same liquidity premium irrespective of the different acceptabilities μ^i. Hence the cryptocurrency will be in circulation even though it is not fully accepted and the economy nevertheless profits from its additional service and the rising currency competition.

Some agents would benefit a lot from the additional service the cryptocurrency provides. As argued in Filip (2021) the additional service y^c can account for various characteristics like a higher level of anonymity, faster transaction times, lower costs or different token usages in smart contracts or DApps, for instance.

4.2 Various Different Competing (Crypto-) Currencies

We can drop the assumption $C = 1$ and establish equilibria with multiple circulating cryptocurrencies $C > 1$ as long as the market clearing condition (17) is fulfilled without changing the fundamental dynamic properties. According to the free-entry condition new cryptocurrencies can enter the market and circulate in equilibrium if they provide enough additional value to compensate their lower acceptability. The more they are accepted, the lower is the necessary additional service. When the additional service they offer is not sufficient, they do not get valued in the economy. Hence some cryptocurrencies might fail and lose their business in the perfect competitive currency environment.

Furthermore, notice that the *same amount* of additional value cryptocurrencies may provide $y_T = y^c \ \ \forall c \in C$ does not necessarily imply they provide the *same services*. They can still differ and are therefore not perfect substitutes but the gained benefits for an agent overall need to be the same. If $y^c = y^* \ \ \forall c \in C \ \ \forall t$, the cryptocurrencies perform the best they can but with different characteristics. So different currencies can establish themselves in an economy each being some kind of niche money with specific characteristics and additional values. Hence some cryptocurrencies might get used in some submarkets where particular characteristics are more important than others. This supports the ideas of Marshall Hayner in Pirus (2021). He expects the major utility of cryptocurrencies to exist in various niches like stablecoins, NFTs, decentralized lending/trading models and payment systems.

4.3 Implications for Monetary Policy

Furthermore, as Hayek (1990) imagined the economy would not only profit from currency competition in the sense of the socially optimum trading activity as defined by Friedman (1969) but also from the safeguard currency competition provides. The threat-of-entry and perfect competition discipline the behaviour of the circulating currencies. As a result the circulating cryptocurrencies will provide stability and the government money will have to exhibit a stable deflation rate. This outweighs the concerns of Obstfeld and Rogoff (1983) about hyperinflationary episodes fiat government monies are prone to. According to the Government Blockchain Association (2021) many investors nowadays see Bitcoin as a sustainable alternative investment protecting against the devaluation of fiat money and hedging against inflation.

The resulting constant government money deflation rate $\rho^g > 1$ in the first-best steady state is exactly the same as the deflation rate received with the optimal contractionary monetary policy seen as the Friedman rule on currency as stated in Williamson and Wright (2010b), Lagos and Wright (2005), or Filip (2021). But as Fernández-Villaverde and Sanches (2016) prove this rule is not uniquely defined with the socially optimum trading activity and there still exist equilibrium trajectories with less desirable properties in contrast to the first-best arrangement with (crypto-) currency competition.

Since in an economy with fully flexible prices and wages the costs of inflation arise from the opportunity costs of holding money balances, deflation might be optimal as argued by Friedman (1969). Contrary, if some nominal rigidities exist, theory points towards an optimal non-negative inflation rate for an economy. Furthermore, deflation complicates the conduct of monetary policy and hinders the central bank's ability to pursue countercyclical monetary policies, for instance (Bordo & Filardo, 2005). Thus, deflation is generally not seen as desirable outcome for monetary policy and necessitates further research in the context of cryptocurrency competition.

4.4 Accounting for a Growing Acceptability

In a next step we take a look at how an increasing acceptability of cryptocurrencies would evolve and how it affects the economy transition. Therefore, we assume that the agents observe the enhanced trading activity due to the cryptocurrency in circulation, learn about the new possibilities and more agents decide to accept this currency. As a result, the acceptability of the cryptocurrency increases a bit[11].

This idea is in line with the literature on positive direct and indirect network effects[12] and the positive feedback loop digital markets and technology exhibit (see Calvano and Polo (2020)). If more agents decide to accept the cryptocurrency, all users profit from the greater network and therefore further enhance the trading activity. The increased output attracts more agents, again resulting in a higher acceptability which additionally leads to a higher quantity traded.

Let $g(q_t)$ denote the growth rate of the traded quantity satisfying (22)

$$g(q_t) = \frac{q_{t+1} - q_t}{q_t} = \frac{q_{t+1}}{q_t} - 1 \geqslant 0 \ \ \forall t \geqslant T \tag{24}$$

[11] Note that the acceptability growth therefore is not endogenous but deterministic. Nevertheless, we can explore the resulting transition dynamics, changes in the equilibrium output and economy evolution. The implementation of an endogenous acceptability growth is left for further research.

[12] Since each agent can be a buyer or a seller in the random bilateral matching, a greater acceptability rises the expected quantity for both users of the two-sided platform representing direct and indirect network effects (see Filip (2020)).

We assume that the acceptability grows somehow related to the output growth with a lag of one period

$$a(\mu_t^c) = \frac{\mu_{t+1}^c - \mu_t^c}{\mu_t^c} \equiv A[g(q_{t-1})] \geqslant 0 \ \ \forall t \geqslant T+1 \tag{16}$$

Thus, the agents observe the increase in the traded quantity and react with a greater acceptability one period later.

So as in the baseline scenario described above in section 4.1, the cryptocurrency c provides enough additional value and starts circulating at time T in the economy equilibrium. Due to the technology improvement the traded quantity increases in the next period $T+1$.

But now the agents observe the benefits and in period $T+2$ the cryptocurrency will be a bit more accepted while the decentralized trading $\mu_{T+2}^c > \mu_{T+1}^c = \mu_T^c$ according to (16) as assumed. So the probability of a successful single-coincidence meeting increases as well resulting in a higher overall traded quantity in the economy due to the equilibrium condition (22). Since the technology keeps growing, $y_{T+2} > y_{T+1}$ resulting in an even higher trading output $q_{T+2} > q_{T+1}$. The overall increase in the equilibrium quantity, i.e. its growth rate, is higher compared to the baseline scenario before because q now grows according to technology growth and acceptability growth out of (22).

Time T+2. *In period $T+2$ the acceptability of the cryptocurrency increases $\mu_{T+2}^c > \mu_{T+1}^c = \mu_T^c$ due to the observed increase in the traded quantity the period before $a(\mu_{T+1}^c) \equiv A[g(q_T)]$. Again the technology improves resulting in an higher additional service provided by the cryptocurrency $y_{T+2} > y_{T+1}$. Hence the overall trading output enlarges even further $q_{T+2} > q_{T+1}$. The cryptocurrency is still stable $\phi_{T+2}^c = \phi_{T+1}^c$ while the deflation rate of the government money rises $\rho_{T+2}^g > \rho_{T+1}^g > 1$.*

As a result, again more agents decide to use the cryptocurrency and the acceptability in the next period $T+3$ increases constituting a positive feedback loop between cryptocurrency usage and trading output[13].

Figure 4 shows the necessary level of the additional service to compensate a lower acceptability and illustrates the positive relationship of the acceptability μ and the traded quantity q for fixed levels of the additional service

[13] Recall that the growth in cryptocurrency usage due to an increased trading output is deterministic but the growth of the equilibrium quantity as a result of a higher acceptability is endogenous.

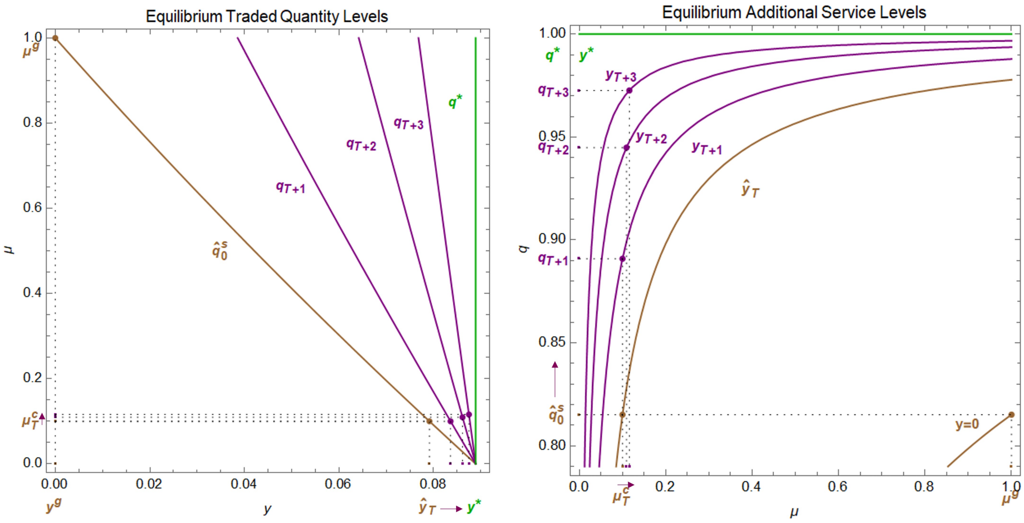

Figure 4: Necessary levels of the additional service y to compensate a lower acceptability μ_T^c and the increase in equilibrium traded quantity q_t due to increases of the acceptability μ^c and the provided additional service y according to the optimal decisions of an agent (22).

y from (22).

The main dynamics are summarized in figure 5. So the improvement of the implemented technology in cryptocurrencies increases the additional value they provide and thus encourage an agent's money holdings. Therefore, the equilibrium traded quantity in the economy increases. The observed benefits raise the acceptability of the cryptocurrency enhancing the trading activity even further. To stay valued in equilibrium, the government money needs to increase its rate of return and deflate.

Accordingly the economy again converges to the first-best output q^*. Since in the steady state the traded quantity does not increase anymore, the cryptocurrency acceptability does neither $a(\mu_{t^*}^c) = 0$.

Time $\boldsymbol{t^*}$. *At time t^* the economy reaches the first-best equilibrium with $q^* > \hat{q}_0^s$, $y^* > \hat{y}_T$, $\boldsymbol{m^*} > \boldsymbol{m_T}$, $\mu^g = 1$ and $1 \geqslant \mu_{t^*}^c > \mu_T^c$. The cryptocurrency is stable $\phi_{t^*}^c = \phi_{t^*+1}^c$ and the government currency provides a stable deflation rate $\rho^* > \rho^g = 1$. $n(y_{t^*}^c) = g(q_{t^*}) = a(\mu_{t^*}^c) = 0$ and the economy is expected to stay in this steady state forever.*

Remember that in the first-best equilibrium each currency yields the same liquidity premium, irrespective of its acceptability. This implies that it is

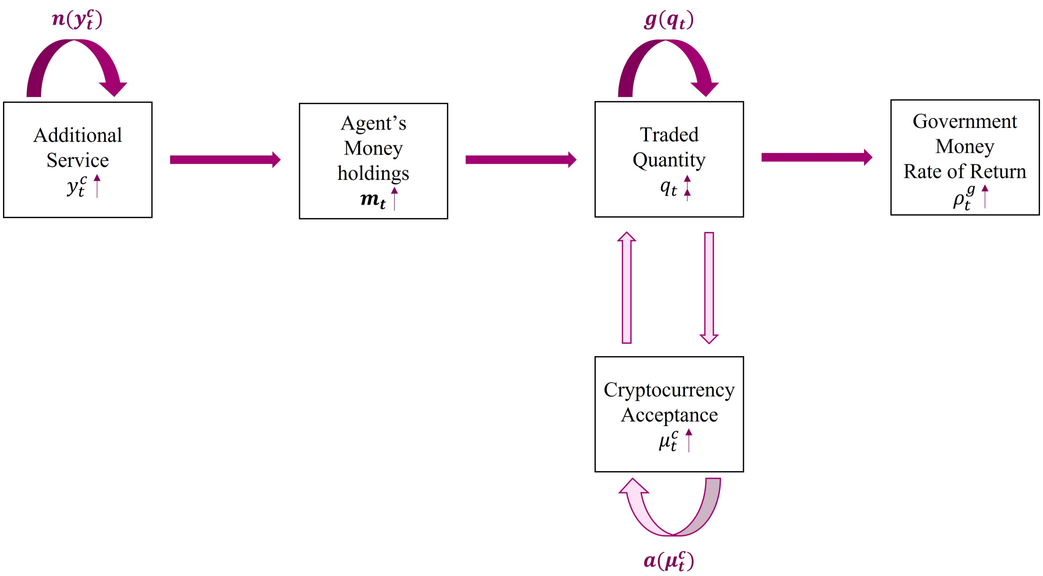

Figure 5: Summary of the main derived transition dynamics of the economy.

not necessary for the cryptocurrency to gain full acceptance in order to circulate in an economy and be valid. The growth in acceptability depends per assumption crucially on the growth of equilibrium output and the initial levels. So the worse off an economy is at the beginning, the larger the output improvement will be and the more a cryptocurrency gains acceptance. Vice versa, if an economy is quite close to the social optimum, the output gains through the cryptocurrency will not be that big and hence the overall increase in acceptability will be lower. However, these results point towards cryptocurrencies establishing themselves as a niche money rather than an universal money supporting the arguments Luther (2016) raised.

Note that the acceptability in our model defines the *overall* acceptability of the whole market in the economy. But if the market is segmented and may consists of several sub-markets, the acceptability in some sub-markets may be higher whereas it is lower in some others. As argued in section 4.2, different cryptocurrencies may establish themselves in different markets according to different characteristics, i.e. additional services provided. So the acceptability of a cryptocurrency may be higher in the respective niche it circulates compared to the other sub-markets and the overall acceptability.

This is in line with the distinction of 'global' and 'local' network effects (see Calvano and Polo (2020)). Agents care more about the adoption choices of agents they want to interact with. So agents trading in a sub-market care more about the acceptability of a cryptocurrency in this specific sub-market than about the acceptability in the whole market. Therefore, beside currency differentiation due to the additional service, local network effects may allow for multiple currencies to cohabit in the market at the same time where the local acceptabilities of different cryptocurrencies are higher than the global acceptabilities. An implementation of market segmentation and the analysis of local network effects in this context is left for further research.

5 Further Discussion

5.1 Convergence

We just briefly sketch the implicit convergence from the government money only steady state to the first-best steady state. A detailed discussion is beyond the scope of this paper but we want to highlight some important points and look forward to further research on this topic.

First of all, the speed of convergence depends crucially on the imposed functional form of technology growth. The faster the additional service grows at each discrete step, the faster the economy will converge to the social optimum. As we impose a decreasing growth rate $n(y_t^c)$, the economy converges more slowly as it approaches the first-best values.

Notice that the growth of the additional service is the main driver of convergence as it affects the quantity growth the most. The growth in acceptance of the cryptocurrency contributes little to the evolution of the economy. Thus, the additional traded quantity in equilibrium is lower due to a gain in acceptance compared to a greater additional service. However, the implied $a(\mu_t^c) = A[g(q_{t-1})]$ affects the speed of convergence as well, even though with a lower impact. The more the acceptability grows due to a growth in the traded quantity, the higher is the traded quantity next period and the faster the economy converges. But nevertheless this impact is rather small.

Additionally, the initial values at time T play an important role, i.e the initial state of the economy. Let's assume an economy is economically struggling at the beginning compared to the economy analysed in the previous section remaining in the government money only steady state $\overline{q}_0^s < \hat{q}_0^s$. If cryptocurrencies start circulating, this economy profits way more from currency competition in the first periods. The trading activity in the worse off economy is much more encouraged and results in a rather quick catch-up.

Furthermore, the overall increase in cryptocurrency acceptability is higher if the economy is worse off at the beginning as illustrated in figure 6. Therefore, as described in the previous section, a better off economy does profit less from (crypto-) currency competition and the alternative currency gains

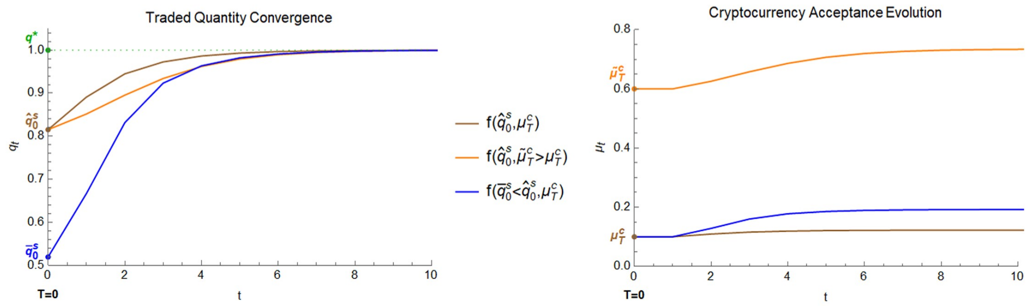

Figure 6: Equilibrium traded quantity convergence and cryptocurrency acceptance evolution over time for different economies with different initial values: $\hat{q}_0^s$ and μ_T^c like above and $\bar{q}_0^s$, $\tilde{\mu}_T^c$ where $\hat{q}_0^s > \bar{q}_0^s$ and $\mu_T^c < \tilde{\mu}_T^c$.

less acceptance over time compared to an economy which is economically struggling.

Moreover, the higher the initial acceptability of the cryptocurrency is, the lower is the necessary $\hat{y}_T$ to compensate the lower acceptability compared to the government money and achieve the same equilibrium quantity $\hat{q}_0^s$. So we assume an economy where $\tilde{\mu}_T^c > \mu_T^c$. The lower additional service results in lower quantity gains in the first periods of the transition phase but the acceptability grows more quickly and overall more as compared to an economy with a lower initial cryptocurrency acceptance (see figure 6). However, cryptocurrencies still struggle to get fully accepted.

So the gains of an economy in trading activity during convergence change over time and depend crucially on the initial values as well as the implied functional forms. For sure there is lot of space for economic analysis and we recommend it for further research.

5.2 Network Risks

Network effects and dynamics in the sense of highly interdependent systems may also lead to some serious risks for society. Due to globalization and technological revolutions like cryptocurrencies, we have a worldwide exchange of people, goods, money and information and dangerous and damaging events can spread rapidly and globally. Helbing (2013) argues that even if external shocks are absent, the decision-makers are well-skilled and do their best, systems can become unstable and create uncontrollable situations often caused by a wrong understanding due to the counter-intuitive nature of the underlying system behaviour. The systemic risk causes cascading failures in a network resulting from the connections between risks. In economic terms this could lead to a collapse of the whole financial system, for instance. The potential damage is largely determined by the size of the networked system. (Helbing, 2013)

Even Filip (2021) finds many undesirable equilibria in an economy with competitive (crypto-) currencies, like a declining purchasing power or trading activity if the additional service cryptocurrencies provide is less than the maximum feasible, highlighting the potential risks of a highly connected economy. Therefore, it would be important to take a closer look at the systemic behaviour due to the existing network risks in the complex dynamic system and try to get a better understanding of the interconnected systemic properties.

6 Conclusion

Government money only regimes are prone to undesirable properties like hyperinflationary episodes and a resulting decline in the trading activity (Lagos and Wright (2003), Fernández-Villaverde and Sanches (2016), Filip (2021)). During history some countries struggled to provide a stable and reliable national currency like Venezuela. As argued in Filip (2021) cryptocurrency competition and especially the value-added services they provide could help to overcome those issues. But according to Luther (2016), McCormack (2018) and Filip (2020) the strong network effect in the market for currencies protects the incumbent government money and therefore prevents competition and the resulting gains for an economy.

However, as cryptocurrencies are technology based inventions, they improve all the time and try to fit consumer's needs best with different characteristics and additional services. We find in our extension of a multi-currency New Monetarist Model (Lagos & Wright, 2005) that the value-added services and the continuous improvement due to technology upgrades help to overcome the strong network effect and cryptocurrencies create a competitive environment in one economy. Even though a cryptocurrency is far less accepted by trading agents than the legal tender government money, it can compensate this drawback with specific features and starts circulating in equilibrium.

As technology improves over time, more additional services are provided by the cryptocurrency encouraging the agents to increase their money holdings. Thus, cryptocurrencies enhance the trading activity in the economy which is observed by more and more agents over time. So more agents decide to accept and hold the cryptocyrrency enlarging the trading output even further. As the Government Blockchain Association (2021) argues, the public needs to be informed, educated and get access to this new technology. If they start to understand and notice the benefits, the adoption rate will grow. Hence the economy converges to the first best steady-state with the socially optimum trading activity defined by Friedman (1969) as technology improves and acceptance rises.

Nevertheless, the acceptance of the cryptocurrency does not increase that much and our model points towards cryptocurrencies establishing themselves as niche monies with different characteristics in line with the findings of Luther (2016) and Marshall Hayner in Pirus (2021). Even if a cryptocurrency is not fully accepted, the economy profits and the perfect competition in the market for currencies disciplines the behaviour of the issuers leading to a strong safeguard in terms of stability and reliability as Hayek (1990) imagined.

However, the main driver of the transitioning dynamics are the technological improvements and specific characteristics of cryptocurrencies. If a cryptocurrency does not provide enough additional value to compensate the lower acceptability and fit consumers' needs, it fails to get into circulation in the perfectly competitive environment. Furthermore, economies which are economically struggling at the government money only steady state will profit more from currency competition and cryptocurrencies will gain more acceptance supporting the findings of Hileman (2015).

In order to be valued in equilibrium the government needs to implement the optimal contractionary monetary policy at a stable deflation rate seen as the Friedman rule on currency (see Williamson and Wright (2010b), Fernández-Villaverde and Sanches (2016) and Filip (2021)).

Nonetheless, cryptocurrencies are a complex and large new topic and further research is needed to fully understand the effects and implications such a system would exhibit. As Dowd and Greenaway (1993) point out switching costs could as well prevent cryptocurrency in gaining acceptance and according to Helbing (2013) systemic properties might differ from the component properties, resulting in new systemic properties, which are important to be understood and implemented in economic models to analyse if their adoption might bear some serious risks for an economy.

7 Appendix

Lemma 1. *The bargaining solution of the single-coincidence meeting is given by (4) and (5).*

Proof. We can formally solve the agent's trading decisions by a centralized Nash solution where the buyer has bargaining power θ

$$\underset{q,\boldsymbol{d}}{max}[u(q) + W(\boldsymbol{m} - \boldsymbol{d}, \boldsymbol{s}) - W(\boldsymbol{m},\boldsymbol{s})]^{\theta}[-c(q) + W(\tilde{\boldsymbol{m}} + \boldsymbol{d}, \boldsymbol{s}) - W(\tilde{\boldsymbol{m}}, \boldsymbol{s})]^{1-\theta}$$
$$s.t.\ \ \boldsymbol{d} \leqslant \boldsymbol{m}$$

If we assume that the buyer has full bargaining power and makes take-it-or-leave-it offers to the seller, $\theta = 1$. Furthermore, using the linearity of the Night-Market value function $W(\boldsymbol{m_t}, \boldsymbol{s_t})$ (8) this simplifies nicely to

$$\underset{q,\boldsymbol{d}}{max}[u(q) - (\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{d}]$$
$$s.t.\ \ \boldsymbol{d} \leqslant \boldsymbol{m}$$
$$s.t.\ \ -c(q) + (\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{d} \geqslant 0$$

The first constraint describes the budget constraint of the buyer and the second the participation constraint of the seller. We immediately derive the results that the terms of trade $\{q(\boldsymbol{m_t}, \tilde{\boldsymbol{m}}_t, \boldsymbol{s_t}), \boldsymbol{d_t}(\boldsymbol{m_t}, \tilde{\boldsymbol{m}}_t, \boldsymbol{s_t})\}$ do not depend on the amount of money the seller holds $\tilde{\boldsymbol{m}}$ and depend on the monetary portfolio of the buyer $\boldsymbol{m}$ if and only if his budget constraint is binding.
So

$$\mathcal{L} = u(q) - (\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{d} + \lambda_1(\boldsymbol{m} - \boldsymbol{d}) + \lambda_2(-c(q) + (\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{d})$$

and the necessary and sufficient first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial q} = u^{'}(q) - \lambda_2 c^{'}(q) = 0$$
$$\frac{\partial \mathcal{L}}{\partial d^i} = -(\phi^i + y^i) - \lambda_1 + \lambda_2(\phi^i + y^i) = 0$$

λ_1 is the Lagrange multiplier on the liquidity constraint of the buyer and λ_2 the Lagrange multiplier on the participation constraint of the seller. The participation constraint of the seller is always binding, so $\lambda_2 \neq 0$ in any case.

The budget constraint of the buyer can be either binding or not to derive a solution. If $\lambda_1 = 0$, so the liquidity constraint does not bind, the solution yields $q = q^*$ and $(\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{d} = c(q^*)$.

Let $\hat{q}(\boldsymbol{m}, \boldsymbol{s})$ denote the quantity solving the F.O.C.s. If the liquidity constraint is binding $\lambda_1 \neq 0$, $q = \hat{q}(\boldsymbol{m}, \boldsymbol{s}) < q^*$ and $(\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{d} = (\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{m}$ solves the system. □

Lemma 2. *The double-coincidence meeting yields a unique solution where each agent produces q^* and money does not change hands, i.e. they barter trade.*

Proof. We can formally solve the agent's trading decisions by a symmetric Nash problem, where q_1 and q_2 are the traded goods and $\boldsymbol{\Delta}$ is the amount of money agent 1 pays agent 2, subject to their budget constraint

$$\underset{q_1, q_2, \boldsymbol{\Delta}}{max} [u(q_1) - c(q_2) - (\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{\Delta}][u(q_2) - c(q_1) + (\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{\Delta}]$$
$$s.t. \quad -\boldsymbol{m_2} \leqslant \boldsymbol{\Delta} \leqslant \boldsymbol{m_1}$$

So

$$\mathcal{L} = [u(q_1) - c(q_2) - (\phi + y)\Delta][u(q_2) - c(q_1) + (\phi + y)\Delta] + \lambda_1(\Delta - m_1) + \lambda_2(-\Delta - m_2)$$

and the first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial q_1} = u^{'}(q_1)[u(q_2) - c(q_1) + (\phi + y)\Delta] - c^{'}(q_1)[u(q_1) - c(q_2) - (\phi + y)\Delta] = 0$$
$$\frac{\partial \mathcal{L}}{\partial q_2} = u^{'}(q_2)[u(q_1) - c(q_2) - (\phi + y)\Delta] - c^{'}(q_2)[u(q_2) - c(q_1) + (\phi + y)\Delta] = 0$$
$$\frac{\partial \mathcal{L}}{\partial \Delta^i} = (\phi^i + y^i)[u(q_1) - c(q_2) - (\phi^i + y^i)\Delta^i] - (\phi^i + y^i)[u(q_2) - c(q_1) + (\phi^i + y^i)\Delta^i] + \lambda_1 - \lambda_2 = 0$$

They characterize a unique solution $q_1 = q_2 = q^*$ where $u'(q^*) = c'(q^*)$ and $\boldsymbol{\Delta} = \lambda_1 = \lambda_2 = 0$, where λ_1 & λ_2 are the multipliers on agent 1 & 2's cash constraints.

□

Lemma 3. *The liquidity premium $l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})]$ each currency i yields is (10) and defines the marginal value of spending a coin as opposed to carrying it forward, times the probability of spending it.*

Proof. The F.O.C. for the optimal portfolio choice of an agent is

$$\frac{\partial V(\boldsymbol{m_t}, \boldsymbol{s_t})}{\partial m^i_{t+1}} = -\phi^i_t + \beta V^{i'}(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}}) \leqslant 0 \tag{9}$$

and it holds with equality if $m^i_{t+1} > 0$, i.e. if an agent decides to hold currency i in his monetary portfolio.

$$\frac{\partial V(\boldsymbol{m_t}, \boldsymbol{s_t})}{\partial m^i_t} = \nu^{i'}(\boldsymbol{m_t}, \boldsymbol{s_t}) + (\phi^i_t + y^i_t) = 0 \tag{25}$$

where

$$\frac{\partial \nu(\boldsymbol{m_t}, \boldsymbol{s_t})}{\partial m^i_t} = \begin{cases} \alpha\sigma\mu^i_t\{u^{i'}[\hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t})]\hat{q}^{i'}(\boldsymbol{m_t}, \boldsymbol{s_t}) - (\phi^i_t + y^i_t)\} & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ 0 & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} > c(q^*) \end{cases}$$

From the demand for real balances $z[q(\boldsymbol{m_t}, \boldsymbol{s_t})] = (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} = c[q(\boldsymbol{m_t}, \boldsymbol{s_t})]$ we know

$$\frac{\partial z[q(\boldsymbol{m_t}, \boldsymbol{s_t})]}{\partial m^i_t} = z^{i'}[q(\boldsymbol{m_t}, \boldsymbol{s_t})]q^{i'}(\boldsymbol{m_t}, \boldsymbol{s_t}) = \phi^i_t + y^i_t$$

and thus

$$q^{i'}(\boldsymbol{m_t}, \boldsymbol{s_t}) = \frac{\phi^i_t + y^i_t}{z^{i'}[q(\boldsymbol{m_t}, \boldsymbol{s_t})]} = \frac{\phi^i_t + y^i_t}{c^{i'}[q(\boldsymbol{m_t}, \boldsymbol{s_t})]}$$

So inserting $q^{i'}(\boldsymbol{m_t}, \boldsymbol{s_t})$ into (25) rearranging gives

$$\frac{\partial V(\boldsymbol{m_t}, \boldsymbol{s_t})}{\partial m^i_t} = \begin{cases} \alpha\sigma\mu^i_t(\phi^i_t + y^i_t)\frac{u^{i'}[\hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t})]}{c^{i'}[\hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t})]} + (1 - \alpha\sigma\mu^i_t)(\phi^i_t + y^i_t) & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ (\phi^i_t + y^i_t) & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} > c(q^*) \end{cases}$$

Hence if an agent does not hold enough money to afford the socially efficient quantity q^* ($(\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*)$), the marginal benefit of a coin of currency i equals the value of spending it on the traded quantity $\hat{q}$ in the DM with probability $(\alpha\sigma\mu^i_t)$ and the value of carrying it into the CM with probability $(1 - \alpha\sigma\mu^i_t)$.

Otherwise, if $((\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} > c(q^*))$ and the socially efficient quantity q^* is traded in the Decentralized Market the value of the coin equals $(\phi_t^i + y_t^i)$ in both subperiods.

Therefore, we can define the liquidity premium of each currency i as the marginal value of spending a coin as opposed to carrying it forward, times the probability of spending it

$$l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})] \equiv \alpha\sigma\mu_t^i(\frac{u^{'}[\hat{q}_t(\boldsymbol{m_t}, \boldsymbol{s_t})]}{c^{'}[\hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t})]} - 1) \tag{10}$$

and rearranging yields

$$\frac{\partial V(\boldsymbol{m_t}, \boldsymbol{s_t})}{\partial m_t^i} = (\phi_t^i + y_t^i)\{1 + l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})]\} \tag{11}$$

with

$$l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})] \begin{cases} \neq 0 & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ = 0 & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} > c(q^*) \end{cases}$$

□

Lemma 4. *We can summarize all derived results in one equilibrium condition for the agents determining their optimal currency choice (12).*

Proof. Combining (9) and (11) gives

$$\frac{\partial V(\boldsymbol{m_t}, \boldsymbol{s_t})}{\partial m_{t+1}^i} = -\phi_t^i + \beta(\phi_{t+1}^i + y_{t+1}^i)\{1 + l^i[q_{t+1}(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}})]\} \leqslant 0$$

Since we want to analyse monetary equilibria where at least one currency is valued and held in equilibrium, $m^i > 0$ and the F.O.C. holds with equality for all valid currencies. Therefore, (12) summarizes all above derived results into one equilibrium condition for an agent determining his optimal monetary portfolio choice

$$\beta(\phi_{t+1}^i + y_{t+1}^i)\{1 + l^i[q_{t+1}(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}})]\} = \phi_t^i \tag{12}$$

$$l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})] \begin{cases} \neq 0 & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ = 0 & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} > c(q^*) \end{cases}$$

□

Lemma 5. *In any equilibrium the boundary condition is* $\beta(\phi^i_{t+1} + y^i_{t+1}) \leqslant \phi^i_t$ *at all time.*

Proof. We provide a proof by contradiction that $\beta(\phi^i_{t+1} + y^i_{t+1}) \leqslant \phi^i_t \ \ \forall t$ must hold in any equilibrium.

First, combining (9) and (25) yields

$$-\phi^i_t + \beta\nu^{i'}(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}}) + \beta(\phi^i_{t+1} + y^i_{t+1}) \leqslant 0$$

where

$$\frac{\partial\nu(\boldsymbol{m_t}, \boldsymbol{s_t})}{\partial m^i_t} = \begin{cases} \alpha\sigma\mu^i_t\{u^{i'}[\hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t})]\hat{q}^{i'}(\boldsymbol{m_t}, \boldsymbol{s_t}) - (\phi^i_t + y^i_t)\} & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ 0 & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} > c(q^*) \end{cases}$$

If we assume $\beta(\phi^i_{t+1} + y^i_{t+1}) > \phi^i_t$ at some point in time the LHS of the F.O.C. is strictly positive if $(\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} > c(q^*)$

$$\underbrace{-\phi^i + 0 + \beta(\phi^i_{+1} + y^i_{+1})}_{>0} \leqslant 0$$

This yields a contradiction and the problem has no solution.

Hence $\beta(\phi^i_{t+1} + y^i_{t+1}) > \phi^i_t$ cannot be an equilibrium and thus the boundary condition $\beta(\phi^i_{t+1} + y^i_{t+1}) \leqslant \phi^i_t$ must hold in any equilibrium for all t. □

Lemma 6. *Perfect competition and free-entry force any issuing entrepreneur to provide a constant nominal currency supply* $M^c_t = M^c_{t+1} = M^c$.

Proof. Following Fernández-Villaverde and Sanches (2016) seigniorage is the only source of income for currency issuing entrepreneurs and they maximize their lifetime utility subject to their budget constraint

$$\sum_{t=S}^{\infty} \beta^t x^c_t \tag{13}$$

$$x_t^c = \phi_t^c \Delta_{M_t^c} + y_t^c \Delta_{M_t^c} - \sum_{i \neq c} \phi_t^i \Delta_{M_t^i} - \sum_{i \neq c} y_t^i \Delta_{M_t^i} \tag{14}$$

According to the boundary condition $\beta(\phi_{t+1}^i + y_{t+1}^i) \leqslant \phi_t^i \ \ \forall t$ an issuer does not hold other currencies across periods and thus $M_t^i = 0 \quad \forall i \neq c \quad \forall t$. Furthermore, the free-entry into the market for currencies forces the lifetime utility of each entrepreneur to be zero. The lifetime utility starting from any point in time is

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} (\phi_\tau^c + y_\tau^c) \Delta_{M_\tau^c} = 0$$

Because $\beta \in (0,1) \ \ \forall t$ either $(\phi_t^c + y_t^c) = 0$, $\Delta_{M_t^c} = 0$ or both in equilibrium. $(\phi_t^c + y_t^c) = 0$ if and only if $\phi_t^c = -y_t^c$ with $y_t^c < 0$ since we assume $\phi_t^i > 0 \ \ \forall t$ for a currency to be valid in equilibrium. Furthermore, we assumed $y_t^c > 0$ and thus $(\phi_t^c + y_t^c) = 0$ yields a contradiction. Hence $\Delta_{M_t^c} = 0$ and it implies that the nominal supply of this currency stays constant over time in equilibrium $M_t^c = M_{t-1}^c = M^c$. $\square$

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