# Modelling the Potential Effects of Digital Currencies on Financial Inclusion in the Economies in the European Union


**Abstract**

This study models the potential short run and long run effects of digital currencies on financial inclusion in the economies in the European Union. The model is based on a novel general theory which is constructed in the present study. The symmetric versus asymmetric behaviour of these effects, their transmission mechanisms, and the cyclical (oscillatory) behaviour of financial inclusion are also rigorously analysed. Fruitful policies which flow from the analyses concerning how digital currencies can contribute towards increasing the level of financial inclusion are also proffered.



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Key words: Asymmetric Effects, Central Bank, Cryptocurrencies, Cyclical Behaviour, Cyber Security, Digital Currencies, Energy Cost, Financial Inclusion, Inequality, Symmetric Effects, and Transmission Mechanism.


## 1. Introduction

The present study rigorously addresses the following research questions: What is the state of the utilisation of information and communication technology (ICT) and digital services' utilisation among firms and consumers, as well as financial inclusion in the economies in the European Union ? What are the potential long run and short run effects of digital currencies (viz., the imminent novel central bank digital currencies and private cryptocurrencies) on financial inclusion in economies in the European Union ? Are these effects asymmetric or symmetric ? What are the forms of the structure of these asymmetric versus symmetric effects? What are the determinants and transmission mechanisms of the aforementioned long run and short run effects as well as the asymmetric versus symmetric effects ? What are the forms of the structure of potential cyclical (oscillatory) behaviour of financial inclusion, and the conditions for these forms in these economies ? What are the likely effects of conventional domestic and foreign currencies, conventional domestic currency filteration (outflows), and other factors like cyber insecurity intensity, the cost of maintaining cyber security, energy cost, the exchange rates of foreign currencies, and government administrative instruments for controlling potential evasion of inflationary tax on financial inclusion ? What fruitful policies can make digital currencies contribute significantly towards financial inclusion ?

The objectives of this study are to construct a novel general theory of the potential effects of digital currencies, conventional domestic and foreign currencies, conventional domestic currency filteration (net outflows), and the other aforementioned factors on financial inclusion, and to extract fruitful policies from the results of this study to enhance the contribution of digital currencies towards the shoring up of financial inclusion in economies in the European Union.

The theme of this study is relevant for a number of reasons. In the first place, the theme of this study is current. Digital currencies are emerging. A number of economies are currently discussing, exploring, experimenting with, or preparing to introduce, via the creation, issue and circulation of the imminent novel central bank digital currencies. Prior to this development, private cryptocurrencies had emerged, and new forms of these cryptocurrencies continue to emerge. Bitcoin, Ethereum, Ripple, Litecoin, and Diem are exemplars of private cryptocurrencies. Generally, the exchange rates in terms of United States dollars per unit of such cryptocurrencies have risen phenomenally, particularly the exchange rate of Bitcoin. Indeed, some of these cryptocurrencies have been listed on global financial asset exchange markets for example Bitcoin. It is likely that when the imminent novel central bank digital currencies are created, issued and circulated, they would be listed on global financial asset exchange markets. Private cryptocurrencies are growing in their utilisation for the satisfaction of the conventional transactionary, precautionary and speculative motives for holding conventional domestic currency (fiat money or cash).

Unfortunately, they are prone to the adverse effects of cyber insecurity and the high cost of maintaining cyber security. Besides, the operations and marketing of cryptocurrencies are very energy intensive. Their operations are thus likely to be influenced by high energy cost. It is likely that the aforementioned imminent novel central bank digital currencies and other forms of digital currencies which may be created, issued and put into circulation will circulate concurrently with the existing private cryptocurrencies and new (future) forms of cryptocurrencies, as well as conventional domestic and foreign currencies, particularly during the initial phase and transitional phase of the issue of the aforementioned central bank digital currencies. It is therefore important to analyse the potential effects of digital currencies and all these other factors on financial inclusion jointly. This has not been done in the existing relevant literature. These potential effects are not yet well understood, while the aforementioned questions which motivated this study have not been rigorously addressed jointly. The present study will therefore make a significant contribution by closing this knowledge gap in the existing relevant literature. Practical policy makers will also find this study to be useful. An enhancement of financial inclusion is likely to contribute in no small way to fostering sustainable development. The present study is very relevant.

This study is organised into five sections. The rationale for the study is presented in greater detail in Section 2; the state of the diffusion of digital services' utilisation, as well as financial inclusion, inter alia, in economies in the European Union are also delineated in this section. The novel general theory and the associated model are inextricably developed in Section 3. The results of this study are presented in Section 4, where fruitful policy implications are also proffered. Finally, concluding remarks are made in Section 5.

## 2. The Rationale of this Study

In recent times, firms in the economies in the European Union (EU) have been exhibiting profound ability to swiftly respond to some shocks which could otherwise have had undesirable dire consequences for their performance and ultimate survival. The COVID-19 global pandemic is an exemplar of such shocks. This profound ability has been made possible by the rapid growth of information and communication technology (ICT) globally during the Twenty-First Century. In economies in the European Union, it has been observed that in general, firms responded to (or implemented changes) more rapidly than they thought possible in respect of the COVID-19 pandemic. Consider the following exemplars: firms have observed, inter alia, that whereas they would have taken 585 days to respond to (or implemented changes as a result of) increasing consumer demand for online digital procurement of goods and services because of the COVID-19 pandemic, they actually took 21.9 days (approximately), representing an accelerating factor (multiple) of 27 (McKinsey 2020). This means that the actual response time was only 1/27 of what would otherwise have been the case (viz., 3.7 percent faster). Firms also observed changing customer needs and expectations, and also indicated that they had increased their expenditure on data security. Furthermore, it was observed that whereas it would have possibly responded to (or implemented changes in respect of) the increasing customer needs and expectations in 511 days, they actually took approximately 21.3 days, representing an accelerating factor (multiple) of 24 (ibid.). This means that the actual response time was only 1/24 of the time thought possible by firms (viz., about 4.2 percent faster). The corresponding accelerating factor (multiple) in respect of the aforementioned increase in expenditure on data security was observed to be 19, since the actual and expected response times were 23.5 days and 449 days respectively(ibid.). Here, therefore, the actual response time was only 1/19 of the time thought possible by firms (viz., approximately 5.3 percent factor).

Additionally, the percentages of firms which indicated that they observed changes in the aforementioned increasing customer demand for online purchasing and/or services, changing customer needs or expectations or increased spending on data security were 62, 63 and 37 percent respectively (ibid.). Moreover, it was observed that whereas 53 percent of firms perceive that the increase in customer demand for online purchasing and/or services is likely to be long lasting, 27 percent thought otherwise (ibid.). To 62 percent of firms, the aforementioned changing customer needs and expectations are likely to continue to occur whereas 18 percent perceive that this is not likely to occur (ibid.). The aforementioned increasing expenditure on data security is likely to continue (as perceived by 53 percent of the firms) whereas only 18 percent perceive otherwise (ibid.). This implies that the aforementioned swift switches to the supply of digital services and utilisation of ICT by firms are likely to continue in the economies in the European Union.

Another set of observations in respect of economies in the European Union is that the adoption of digitisation of customer interactions has accelerated by 3 years, just as it has been observed globally and in North America but less than those in the Asia-Pacific Region which is 4 years (ibid.). During the recent pre-COVID-19 Pandemic Era (specifically, June 2017, May 2018, December 2019), the average shares of customer interactions which were digital (in the EU economies) were 18 percent, 19 percent, and 32 percent respectively, whereas the corresponding average during the current COVID-19 Pandemic Era has increased to 53 percent. The corresponding global average shares were 20, 20, 36 percent respectively during the pre-pandemic period, whereas during the current pandemic period it has increased to 58 percent (ibid.). The corresponding average shares for North America are 25, 25, 41 percent prior to the pandemic and this has increased to 65 percent during the pandemic period. In the Asia-Pacific Region, the analogous percentage during the current pandemic period has been 53 percent, whereas in the pre-pandemic period, the shares were 22, 19, and 32 percent respectively (ibid.).

The corresponding average shares of products or services which are partially or completely digitised in the same period were 26, 25, 34 percent and this increased to 50 percent (ibid.). The analogous global shares (pre-COVID) were 29, 28, 35 percent versus a higher pandemic period's share of 55 percent. In North America, the shares were 33, 34, 41 percent during the pre-COVID period versus a higher share of 60 percent in the pandemic period. Whereas the pandemic share has been 54 percent in the Asia-Pacific Region, the shares were lower (viz., 31, 26, 33 percent) during the aforementioned pre-pandemic period (ibid.). The adoption acceleration from the pre-pandemic to the pandemic period was 7 years in the EU economies, as it has been globally, compared with 6 years in North America and at least 10 years in the Asia-Pacific Region.

Consumer behaviour during the current pandemic period compared with that of the pre-pandemic period reflects (or is consistent with) the behaviour of firms during these periods. The average digital adoption rate on the part of consumers in the economies in the European Union (EU) increased from 81 percent during the pre-Covid pandemic period to 94 percent during the Covid pandemic period (Fernandez, Jenkins and Vieira 2021). In addition, the digital gap from one EU economy to the other narrowed over the same period. Consumer behaviour in respect of the various industries in the EU economies show some differential. Whereas the satisfaction levels of consumers concerning the digital services procured from the entertainment, banking and social media industries are highest, those from the public sector, travel and grocery industries are lowest (ibid.). Moreover, about 70 percent of consumers plan to increase or at least maintain the level of utilisation of digital services (ibid.). There is a differential across various industries.

For regular digital services' users, the proportion (percent) of consumers who plan to maintain or increase their digital services' utilisation are 82, 74, 70, 69, 67, 67, 66, 64, 64,62 percent in the banking, entertainment, telecoms, utilities, social media, apparel, travel, grocery, insurance industry, and public sector respectively (ibid.). The corresponding proportions (percentages) in the case of new digital services' users are 82, 81, 72, 72, 72, 71, 71, 66, 65, 64 percent respectively (ibid.). The average percentages for the regular users and the new users are 70 and 72 percent respectively (ibid.).

The differential in the utilisation of digital services across various industries in the EU economies is further demonstrated in Table 1 where the proportion (percentage) of adoption rates among regular users, new users and combined users are presented. The adoption rates (combined users) are over 50 percent for the banking, entertainment, social media, telecoms and grocery industries (Table 1). Those between 30 and 50 percent are the apparel, utilities, public sector, travel and insurance industries.

**Table 1. Differential Digital Services' Adoption Rates (Percent) by Industry in EU Economies**

| Industry | Regular Users | First-Time Users | Combined |
|---|---|---|---|
| Banking | 55 | 23 | 78 |
| Entertainment | 36 | 25 | 61 |
| Social Media | 33 | 26 | 59 |
| Telecoms | 37 | 15 | 52 |
| Grocery | 29 | 22 | 51 |
| Apparel | 32 | 16 | 48 |
| Utilities | 26 | 10 | 36 |
| Public Sector | 23 | 11 | 34 |
| Travel | 28 | 6 | 34 |
| Insurance | 23 | 8 | 31 |

Source: Author's Construction Based on the Recent COVID-19 Digital Sentiment Insights Survey of McKinsey & Company, 2020 (ibid.).

The information in Table 1 further suggest that at least 30 percent of the firms in these industries have adopted digital services in the EU economies. In addition, the banking industry records the highest rate of adoption of digital services of 78 percent, implying that the supply and demand for (viz., utilisation of) digital services in economies in the European Union are substantial. This observation embodies profound implications for financial inclusion in the economies. In what follows, the state of financial inclusion in the economies in the European Union is delineated. Before this is done, however, an overview of digital currencies is presented. This is important because the observations in the previous paragraphs suggest, inter alia, that the supply and demand for digital services by firms and consumers in the economies in the European Union have increased swiftly and are likely to continue to increase over time. This implies that digital means of payments are likely to continue to be in high demand. This is where digital currencies which constitute the focus of the present study come in handy.

ADDENDUM A

The current state of the vending and procurement of digital services by firms and consumers in the economies in the European Union (EU) which has been delineated in the previous paragraphs has spawned various means of payments for digital services, in addition to conventional domestic currency (traditional fiat money or cash) and conventional other non-digital means of payments. The digital means of payments include, inter alia, digital currencies. Potentially and actually, these digital currencies are generally employed to satisfy the conventional (traditional) transactionary, precautionary, and speculative motives for holding money. These digital currencies broadly comprise private cryptocurrencies, the imminent novel central bank digital currency, Stablecoins, inter alia. Other digital payment systems include digital cards, digital bank transfer, inter alia, also exist. Additionally, non-digital payments are also employed.

Bitcoin, Ethereum, Ripple and Litecoin are exemplars of private cryptocurrencies. Recently, the digital social media services provider known as Facebook created another form of cryptocurrency referred to as Libra. In view of the limitations of Libra, it has been repackaged and renamed as Diem. The markets for private cryptocurrencies are global, such that it can generally be procured by economic operators including those in the economies in the EU, albeit their operations and transactions are cryptic and not under the regulation of the relevant Central Bank or Securities and Exchange Commission. The characteristics of cryptocurrencies and how they differ from conventional domestic currency have been discussed by researchers and practical policy practitioners (see for instance, Vigna and Casey 2015, Fung and Siu-Cheong 2017, Dabrowski and Janikolwski 2018, Ammous 2018, Carney 2018, Fung, Siu-Cheong and Hendry 2018, Bindseil 2019, Sok 2019, 2020, Alonso, Fernandez, Angel, Bas, Kaczmarck 2020, Moin, Sekniqui Sirer and Tercero-Lucas 2020, Frost, Shin and Wierts 2020, Alonso, Vazquez and Forradellas 2021, Cunha, Melo and Sebastião 2021, Sebastião, Cunha and Godinho 2021). Table 1.1 shows how the private cryptocurrency Bitcoin differs from conventional currency (viz., conventional fiat money or cash) based on nine characteristics. This table is self-explanatory.

Table 1.1. Differential Characteristics between Bitcoin and Conventional Currency (Fiat Money or Cash).

**Table 1.** Broad comparison between Bitcoin and traditional fiat money; source: author.

| Feature | Bitcoin | Fiat Money |
|---|---|---|
| Issuance | Decentralized, regulated by underlying algorithm | Centralized, regulated by central bank mandates |
| Supply | Capped at 21 million units | No hard cap Liability |
| | Not the liability of anyone | Liability of the central bank or commercial banks |
| Liquidity | Not guaranteed | Absolute |
| Stability | Volatile | Stable, except in face of hyperinflation |
| Acceptability as medium of payment | Limited | Universal in a given economy |
| Privacy of user identity | Pseudonymous, no link between addresses and natural persons | Anonymous if cash is used, Know Your Customer (KYC) enforced for accounts in commercial banks |
| Confidentiality of transactions | Ledger is public | Restricted to parties, visible to financial institutions, accessible to law enforcement |
| Geographical scope | Global | Limited |

Source: Cunha, Melo and Sebastião (2021).

The value and importance of private cryptocurrencies has increased significantly since their introduction, particularly Bitcoin. By May 2021, Bitcoin constituted 45 percent of the remarkable 2.2 trillion US dollar market capitalisation in respect of a least 3,500 private cryptocurrencies traded which were marketed on at least 370 creditable online exchanges. By the same date, the daily volume vended and procured was at least 213 billion US dollars. However, some researchers have argued that the increase and growth in the mining, vending and procurement of private cryptocurrencies are likely to atrophy, since the reward mechanism in respect of Bitcoin is likely to vary as the relevant platform (technology and its operations) for Bitcoin changes from proof-of-work to proof-of-stake consensus; and cryptocurrency mining is likely to come to a halt, while new in-chain transaction become infeasible (see for instance, Carlsten, Kalodner, Weinberg and Narayanan 2016, Easley, O'Hara and Basu 2019). On the other hand, more recent research avers conversely that, with an imminent growth in renewable energy sources which results in a dominance of renewables in the composition of energy sources, energy costs are likely to reduce significantly. Additionally, given that Bitcoin's value or price increases, coupled with the fact that Bitcoin mining is energy intensive, it flows from Bitcoin's transaction fees are likely to be adequately remunerative for Bitcoin's mining activity to continue to occur (see for instance, Phillips and Chipolina 2021, Cunha, Melo and Sebastião 2021). It is important to indicate that currently, by its inherent construction or design in respect of the built-in underlying algorithm, the supply of Bitcoin cannot exceed 21 million units ultimately. Bitcoin, by its current algorithmic design, is thus "exhaustible", until this ultimate universal supply constraint is removed and the maximum feasible supply is increased by design. Within the current design, the validation of new blocks by Bitcoin miners are rewarded with the "block reward" which is essentially a fixed number of newly minted units of Bitcoin and transaction fees (see for instance, Phillips and Chipolina 2021). The transaction fees constitute a small component of the rewards of a given miner of Bitcoin. This is approximately 6.5 percent (on average) of the total compensation of Bitcoin miners, and it is expected to increase such as to confer adequate incentives in favour of Bitcoin miners which, in turn, is likely to induce Bitcoin mining to continue (ibid.). Additionally, the block reward of Bitcoin atrophies by 50 percent of every 210,000 new blocks every four years approximately, having commenced at a block reward of 50 Bitcoin (ibid.). By the most recent halving process, the Bitcoin block reward had atrophied to 6.25 units by May 2020 (ibid.).

These first-generation private cryptocurrencies as typified by Bitcoin, have been followed by second-generation private cryptocurrencies. Both forms of private cryptocurrencies are being marketed and utilised concurrently. The strengths of the first-generation cryptocurrencies (viz., Bitcoin) notwithstanding, a number of its weaknesses have been identified. These include, inter alia, high price volatility, cryptic operations, not backed by and not under the regulation of the Central Bank or the Securities and Exchange Commission, the use of complex user identification tags which are difficult to be associated with real natural identities (viz., the property of pseudonymity), facilitator of the financing of money laundering, terrorists' financing, drug vending and procuring, not accepted as medium of exchange at all time and in all jurisdictions universally, generally not universally appreciated both as a reliable store of value and as a unit of account currently, devoid of absolute liquidity, inter alia (see for instance, Ali, Barrdear, Clews, and Southgate 2014, Bryans 2014, Hurlburt and Bojanova 2014, Yermack 2015, Ba**c**ao, Duarte, Sebastião and Redzepagic 2018, Carney 2018, Dion-Schwartz, Manheim and Johnston 2019, Foley, Karslen and Putnins 2019, Frost, Chin and Wierts 2020, Hazlett and Luther 2020, Moin, Sekniqi, Sirer and SoK 2020; Baur and Dimpfl 2021). The aforementioned second-generation private cryptocurrencies emerged to deal with some of these weaknesses and problems associated with Bitcoin and the other first-generation private cryptocurrencies, in particular, the absence of their backing from the central bank or other backing instruments (or sources), and their high price volatility with their associated facilitation of financial and other forms of economic instability.

Stablecoins typify the current second-generation private cryptocurrencies and have been designed to infuse significant stability into private cryptocurrencies via some type of backing. The problem of high price volatility which tends to vitiate the utilisation of private cryptocurrencies as a strong medium of payment has been mitigated somehow through the pegging of the price of Stablecoin to current exchange-trade assets, conventional currency (viz., fiat money or cash) or other private cryptocurrencies (see for instance, Bindseil 2019, Frost, Shin and Wierts 2020, Moin, Sekniqi, Sirer and SoK 2020, Alonso, Jorge-vazquez, and Forradellas 2021, Cunha, Melo and Sebastiao 2021). Libra (or its recently repackaged version Diem) is an exemplar of Stablecoin. This private cryptocurrency has been created in the context of the Libra Project under the auspices of the Libra Association which is a consortium under the leadership of the social media service provider Facebook the global user-base of which is as many as 2 billion (Constine 2019, Inman and Monaghan 2019, Tercero-Lucas 2019, GroB, Hertz and Schiller 2019, Schmeling 2019, Abraham and Guegan 2020, Bursztynsky 2020, Bruhl 2020). The quest for price-(value) stability by Stablecoin may not necessarily and sufficiently be satisfied, due to the specific structure of the asset utilised to back the Stablecoin and the procedure for holding these assets. Indeed, some risk could arise from the backing process (see for instance, Bindseil 2010). Like Stablecoin in general, a number of user, monetary and banking risks were delineated concerning Libra. These risks include the following: potential adverse systemic consequences, emergence of a private oligopolistic payment mechanism, potential failure of the Libra protocol and loss of confidence in its which could precipitate adverse dynamics, fraud and tax evasion with other taxation problems, cyber insecurity due to adverse cyber hacking (cyberattack, and so on), and potential adverse data property challenges and data privacy challenges due to the potential of Libra to influence control a disproportionately large number of the population in the world with its associated immense effects on the global financial system; these risks precipitated the emergence of Diem which is a repackaged second version of Libra (see for instance, Constine 2019, Inman and Monaghan 2019, GroB, Hertz and Schiller 2019), Bruhl 2020, Schmeling 2019, Abraham and Guegan 2020, Bursztynsky 2020).

In view of the phenomenal creditable performance of private cryptocurrencies and their aforementioned potential adverse effects on the effectiveness of monetary policy, coupled with the aforementioned risks of cryptocurrencies, governments in the global economy including those of economies in the European Union have been discussing, experimenting with, assessing proof-of-concept, conducting pilots and preparing to possibly issue, circulate and educate the public to utilise various forms of the imminent novel central bank digital currency. The economies in the world including economies in the European Union are at differential stages in the preparation process for the launching of this imminent novel currency. Some economies have gone very far, and some are just commencing, whereas others are sitting on the fence and waiting to learn lessons from the leaders in this regard. The initial experimentations were in respect of wholesale central bank digital currency, whereas the recent ones have been in respect of retail central bank digital currency (see for instance, Raskin and Yermack 2016, Niepelt 2018, Bordo and Levin 2017, Bank of England 2020, Panetta 2018, Meaning, Dyson Barker and Clayton 2018, Bindseil 2020, Ward 2018, Auer and Böhme 2020, Kumhof and Noone 2018, Auer, Cornelli and Frost 2020; Barontini and Holden 2019, Ward and Rochemont 2019). On the global front, Bahamas is one of the extremely few economies currently implementing its own central bank-backed (and regulated) digital currency known as the Bahamas Dollar which is indirectly pegged to the United States Dollar via its pegging to the Bahamas Dollar; this is a retail central bank digital currency which was introduced in 2020 (Gross 2020, Cunha, Soja and Themistocleous 2021).

China's exploration in respect of the introduction of its own central bank digital currency occurred in 2014, while pilot tests undertaken in April 2020. This digital currency, referred to as e-CNY/DCEP, was launched in 2021 but it is currently available in selected areas, with complete adoption of this retail central bank digital currency expected to occur during the post-2023 period, since it is still in a pilot stage in other areas. DCEP stands for Digital Currency Electronic Payment. At the time of the pilot tests in October 2020, the 200 digital Yuan given to the pilot participants to spend at merchants was equivalent to 30 United States Dollars, viz., approximately 0.15 US Dollars per 1 digital Yuan (see for instance, Kim 2020, Chorzempa 2021, Alun 2020, Arredy 2021, Popper and Li 2021). In Uruguay, the Central Bank of Uruguay initiated a pilot test in respect of the introduction (viz., issuing, circulation and testing) of retail Central Bank digital currency in November 2017. This test was completed in April 2018: this central bank-backed retail central bank digital currency is referred to as e-peso (see for instance, Berkmen, Beaton, Gershenson, Del Granado, Ishi, Kim, Kopp and Rousset 2019; McNally 2020, Wilson 2018). In East Carribean, the East Carribean Central Bank initiated a pilot test in respect of a Central Bank-backed retail central bank digital currency referred to as DCash in March 2021; this digital currency is pegged to the East Carribean Dollar which is, in turn, pegged to the United States Dollar (Eastern Carribean Central Bank 2021).

In addition to Bahamas, China, Uruguay and East Carribean, at least 55 countries had undertaken central bank-backed retail central bank digital currency experimental pilot tests by April 2021. Morever, the Bank for International Settlements' recent survey observes that at least 85 percent of the respondent Central Banks have explored the strengths and weaknesses or risks of central bank digital currency, including economies in the European Union (see for instance, Auer, Cornelli and Frost 2020, Gross 2020; Boar, Holden and Wadsworth 2020, Pocher and Veneris 2021, Boar and Wehrli 2021, Auer and Boehme 2020, HLTF-CBDC 2020, CPMI-MC 2018, Kiff, Alwazir, Davidovic, Farias, Khan Khiaonarong, Malaika, Monroe, Sugimoto, Tourpe, et al., 2020). Joint experimentations or explorations of the feasibility of introducing central bank digital currency between two or more national central banks, monetary authorities or regional monetary union/economic unions including that involving the European Union have also been undertaken. The joint Project Stella which was between the Bank of Japan and the European Central Bank, as well as the Projects Jasper/Ubin which was between the Bank of Canada and the Monetary Authority of Singapore are exemplars of these (Bank of Canada, Monetary Authority of Singapore, Bank of Japan and European Central Bank 2020).

There are a number of objectives or reasons for the issue and circulation of central bank-backed retail digital currency; these include "geographic dispersion, access to financial services, increase the banking penetration rate, financial sector become obsolete, security reasons: avoid money laundering and terrorism financing, [curtailing tax fraud], consumer protection, maintain control over monetary and macroeconomic policy, fall in use of cash (alternative), lower costs and increased efficiency of the banking system'' (Alonso, Jorge-Vazquez and Foradellas 2021, Table 1; see also Alonso, Fernandez, Angel, Bas and Kaczmarek 2020, Auer, Cornelli and Frost 2020). Furthermore, some desirable characteristics concerning central bank-backed digital currency have been delineated (see for instance, Bank of England 2021, Choi, Henry, Lehar, Reardon and Safavi-Naini 2021, Alonso, Jorge-Vazquez and Foradellas 2021, Table 1). Table 2.2 details some principal characteristics (and design objectives) of the Central Bank-backed digital currency; this table is self-explanatory (see for instance, CPMI-MC 2018, Bech and Garratt 2017, Pocher and Veneris 2021, Auer, Cornelli and Frost 2020, Auer and Böehme 2020, HLTF-CBDC 2020, Kiff, et al., op. cit., 2020, Cunha, Melo and Sebastião 2021).

Table 2.2 Some Principal Characteristics of Central Bank Digital Currency

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**Table 2.** CDBC main characteristics/design goals; Source: authors' own.

| Characteristic | Alternatives | Description |
|---|---|---|
| Application Area [43,51] | Wholesale | The currency is intended only for financial institutions which hold accounts in the central bank |
| | Retail | The currency is intended for use by the general public |
| Architecture [56] —related to Operating Model [69], Access Model [68] | Direct CBDC | CDBC is a claim on the central bank; onboarding is performed by either central bank or intermediaries (respecting KYC regulations); all payments handled by central bank |
| | Indirect/Synthetic CBDC | CBDC is a claim on an intermediary, onboarding is performed by intermediaries (respecting KYC regulations); retail payments are handled by intermediaries, wholesale payments handled by the central bank |
| | Hybrid CBDC | CDBC is a claim on the central bank; onboarding is performed by intermediaries (respecting KYC regulations); retail payments are handled by intermediaries, but the central bank periodically records all retail balances and operates a backup technical infrastructure allowing it to restart the payment system if intermediaries fail |
| | Intermediated CBDC | Like Hybrid CDBC, but the central bank maintains only a wholesale ledger, rather than all transactions |
| Access Technology [67] based on ideas from [70] | Account-based access | The value is linked to an account, with ownership tied to identity; no privacy by default |
| | Token-based access | The value is linked to demonstrated knowledge, like a digital signature, eventually stored in a hardware device; provides privacy by default |

| | | |
|---|---|---|
| Central Bank Infrastructure [67] | Conventional | The transactions are stored in a logically centralized ledger; the actual storage may be distributed, but the control over the information is centralized |
| | Distributed ledger technologies (DLT)-based | The transactions are stored in a logically distributed ledger; the control over information must be harmonized with a consensus mechanism; may make use of Blockchain technology (like R3 Corda or Quorum) |
| Interlinkages [67] | | Access is reserved to residents of a particular monetary Accessible to |
| | International | non-residents, allowing for cross-border retail payments; this is allowed by default if the access is token-based |
| Authority [69], maps partially with Infrastructure [67] and Access Technology [67] | Centralized | Only the central bank can verify and commit transactions |
| | Partially Decentralized | The central bank provides tokens to selected financial institutions to either safeguard or act as intermediaries |
| | Decentralized | The ledger is run on a DLT, allowing for decentralized transaction verification and commit |
| Availability and Limitations [69] and Restrictions on Access [68] | Unlimited usage | While theoretically possible, it may conflict with particular central bank goals due to effects on the banking sector, monetary policy, and financial stability |
| | Geographical limits | Only accessible to current residents of a monetary area |
| | Value limits | Maximum limits on the amount that can be stored in a particular account or instrument |

Source: Cunha, Melo and Sebastião 2021.

Exemplars of economies in the European Union which have been exploring and experimenting with the feasibility of issuing and circulating central bank-backed digital currency comprise Estonia, Lithuania, Finland, Norway, Sweden, Spain, Switzerland, the Netherlands, France, and the Eurozone as a whole (see for instance, Alonso, Fernandez, Angel, Bas, and Kaczmarek 2020, Auer, Cornelli and Frost 2020, Gross 2020, Alonso, Jorge-Vazquez and Foradellas 2021, Boar and Wehrli 2021, Sveriges Riksbank 2017, 2019, Bijlsma, van der Cruijsen, Jonker and Reijerink 2021).

Other payment systems exist in economies in the global economy including economies in the European Union. These comprise conventional domestic and foreign currencies (viz., fiat money or cash), card and bank transfer payments. The utilisation of cards have increased since their introduction. Indeed, the growth in demand for debit and credit cards in e-commerce transactions was facilitated by the increased demand for universally accessible, efficient, quick, secure, easier, reliable and more convenient retail payment systems. Electronic contactless payment systems, electronic wallet, retail payment systems providers like mobile network operators and fintech startups, and other non-bank participants (suppliers) in the retail payment systems like Apple Pay, PayPal and Revolut, inter alia, enhance the operations of retail digital payment systems, and constitute more recent payment systems in conjunction with bank cheques, drafts conventional domestic and foreign currencies (fiat money or cash), albeit the latter non-digital payment system has been atrophying in high-income economies including economies in the European Union like Sweden, and also in Norway (see for instance, Ley, Fottit, Honig, King, Doyle, Turan and Sonnad 2015, Arvidsson, Hedman and Segendorf 2017, Omarini 2018, Fabris 2019, Sveriges Riksbank 2019, European Central Bank 2016). The demand for e-money (electronic money), which comprises “an electronic store of monetary value on a technological device that may be widely used for making payment to entities other than the e-money issuer. The device acts as a prepaid bearer instrument which does not necessarily involve bank account in transaction” (European Central Bank 2016, p.1), is a function of the demand for credit, inter alia, and thus partially endogenous to a given open economy.

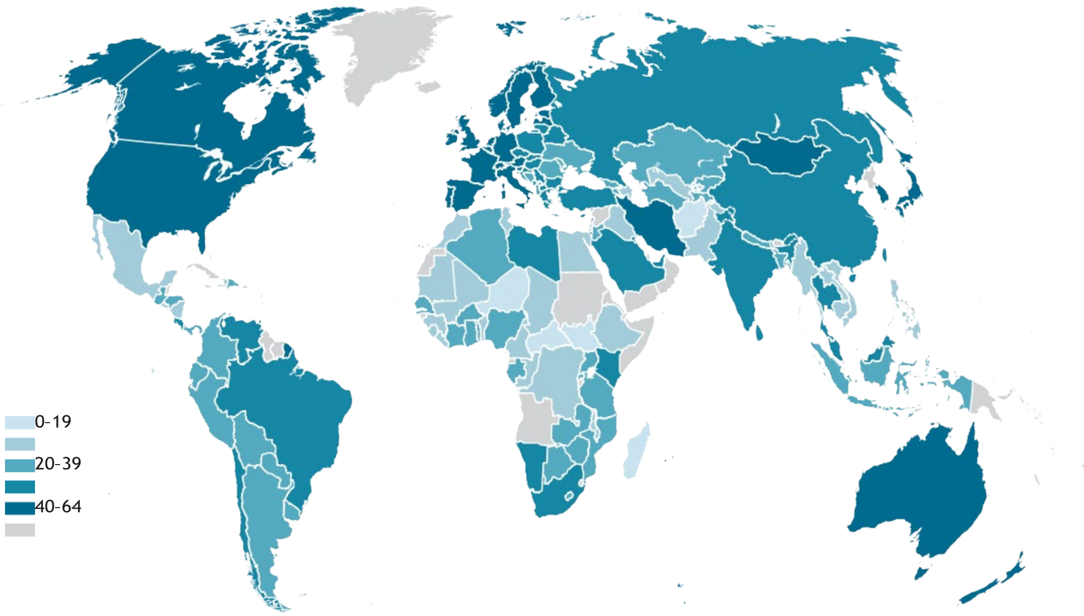
It has been indicated in the previous paragraphs that the reasons or objectives for the introduction of central bank-backed digital currency are to increase access to financial services as well as to increase the rate of banking penetration, inter alia, in an economy, and thus induce a higher level of financial inclusion. The potential and actual benefits of financial inclusion have been delineated and analysed to some extent.

Its potential in enhancing economic growth, general development, attenuation of the levels of poverty and income inequality and other forms of inequality, accelerating the rate of achievement of the United Nations' Sustainable Development Goals, inter alia (see for instance, King and Levine 1993, Beck, Levine and Loayza 2000, Clarke, Xu and Zuo 2006, Beck, Demirgu**c**-Kunt and Levine 2007, Demirgu**c**-Kunt, Levine 2009, Karlan and Morduch and 2010, Turner, Walker, Sukanya and Varghese 2012, World Bank 2014, Rojas-Suarez and Gonzalez 2010, Cull, Ehrbeck and Holle 2014, Demirgu**c**-Kunt, Klapper, Singer 2017, Demirgu**c**-Kunt, Klapper, Singer, P. van Oudheusen 2015, Park and Mercadb 2015, Sahay, Cihak, Ndiaye, Barajas, Mitra, Kyobe, Nian Mooi, and Reza Yousefi 2015, Allen, Demirguic-Kunt, Klapper, Martinez-Peria 2016, Klapper, El-Zoghbi and Hess 2016, Demirgu**c**-Kunt, Klapper, Singer 2017, Demirguc-Kunt, Klapper, Singer, Ansor and Hess 2018, Stuart 2016, Zetterli and Pellai 2016, Zimmerman and Baur 2016, Better Than Cash Alliance 2016, Claessens and Rojas-Suarez 2016, Kendall and Voorhies 2014). The present study conceptualises financial inclusion as the economic phenomenon or process of improved adult consumers', firms' and governments' enhanced secure, quick, easier, efficient, convenient, sustainable, universal availability and access to and effective use of financial services comprising adult ownership of a deposit or transactions account(s) at a bank(s), other non-bank financial institutions, mobile money service provider(s) merchant digital payments' provider(s) through which effective wholesale and retail payments are made and received, as well as savings, credit (borrowing), financial and physical investments, store of value, and formal insurance products. Additionally, its operationalisation in in the present study is a reduction in the statistical inequality of the statistical distribution of the joint attributes of financial inclusion. However, it is imperative to indicate that financial inclusion is "typically measured by ownership of an account by [adult] individuals" (Demirgu**c**-Kunt, Klapper and Singer 2018, p.3; World Bank 2018).

The state of financial inclusion in the economies in the European Union, as delineated from the recent World Bank's *Global Findex Database 2017: Measuring Financial Inclusion and the Fintech Revolution* report (World Bank 2018) is shown in Figure 2.1, where ownership of accounts by adults in economies in the European Union is contrasted with that of the rest of the world. Similarly, the number of adults who lack an account in the economies of the European Economies is contrasted with that in the rest of the world in Figure 2.2. The corresponding data on ownership of mobile phones are shown in Figures 2.3 and 2.4.

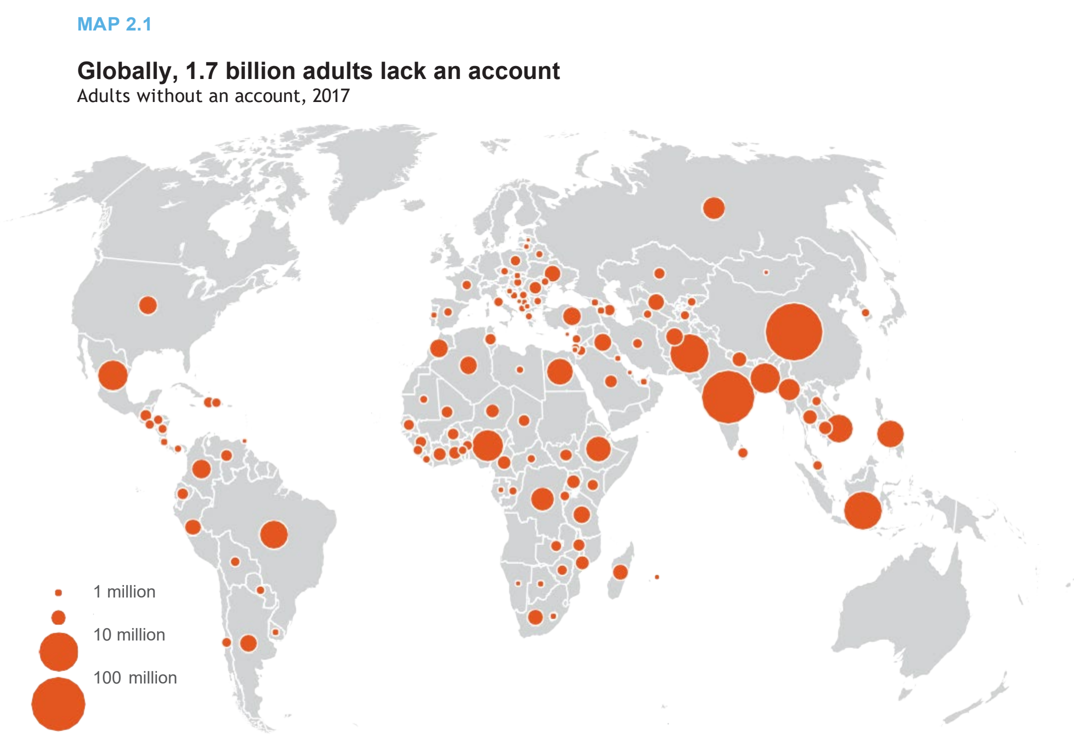
Figure 2.1

Adults with an account (%), 2017



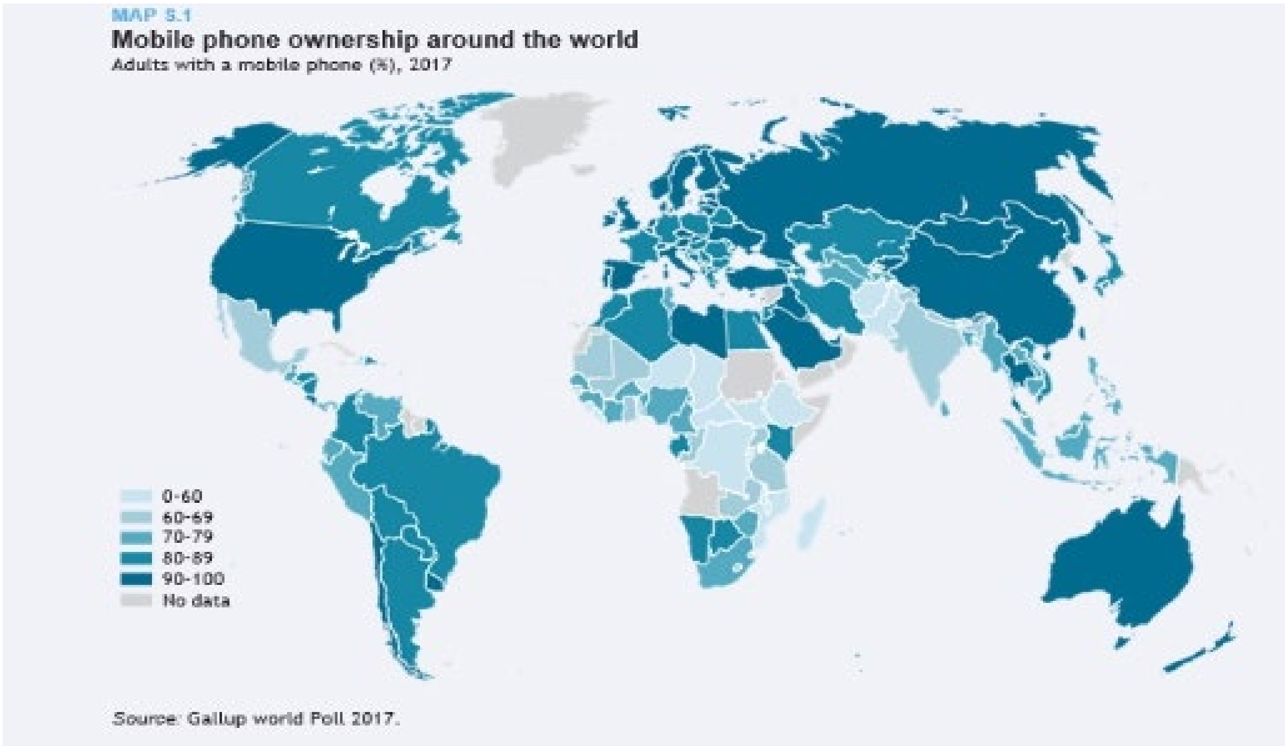

*Source:* Global Findex database (2017).

Figure 2.2



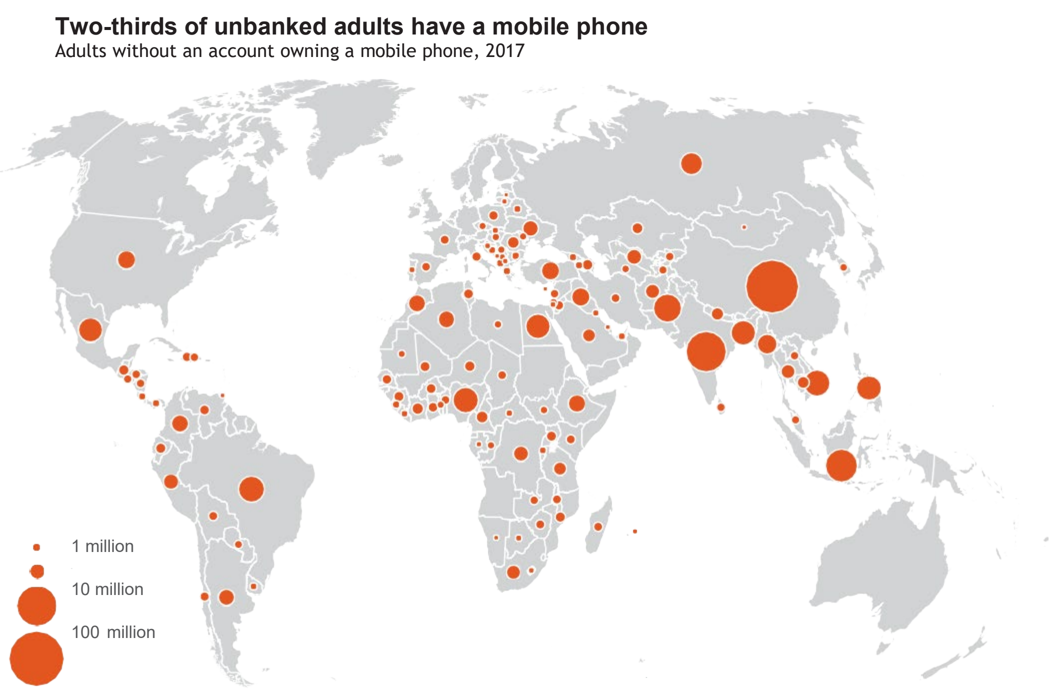

*Source:* Global Findex database (2017).

Figure 2.3




*Source:* Global Findex database (2017).

Figure 2.4




*Source:* Global Findex database (2017).

The percentages of adults who own an account, and the gaps between the richer and the poorer households in respect of account ownership in specific economies in the European Union are depicted in table 2.3. The percentage of adults who own account is 100 percent in Denmark, Finland, Netherlands, and Sweden. It is 99 percent in Belgium, Germany, and Luxembourg, whereas it is 98 percent in Austria and Estonia. It is between 95 and 97 percent (inclusive) in Ireland, 90-94 percent (inclusive) in France, Italy, Portugal, Spain, 80 and 89 percent (inclusive) in Czech Republic, Greece, Lithuania, Croatia, Poland, Cyprus, between 70 and 79 percent in Bulgaria, Hungary and Serbia, between 60 and 69 percent (inclusive) in no economy, between 50 and 59 percent in Romania, and less than 50 percent in no economy (Table 2.3). The gap between the richest 60 percent and the poorest 40 percent of households in each economy in respect of accounts ownership is shown in the last two columns of (Table 2.3), where zero points denotes an insignificant percentage gap. The economies with large gaps of more than 30 percent include Romania (33 percentage points), between 20 and 29 percent (inclusive) comprise Bulgaria (20 percentage points), between 10 and 19 (percentage points) include Czech Republic (17 percent), Hungary (12 percent), and between 1 and 9 percent (inclusive) consist of Croatia (9 percentage points), Cyprus (8 percentage points) Greece (7 percentage points) Ireland (4 percentage points), Italy (5 percentage points), Lithuania (8 percentage points) and Poland (4 percentage points), Portugal (8 percentage points). The economies in which the gap between the ownership percentage for the richest 60 percent relative to that of the poorest 40 percent of households based on household income quintile group are statistically insignificant (viz., gaps within the margin of error of the Findex Database 2017 survey: SQRT(0.25/N)*1.96/D), where N denotes population size, D denotes standard deviation, SQRT denotes square root, and '/' denotes the division sign: World Bank 2018) include Austria, Belgium, Denmark, Estonia, Finland, France, Germany, Luxembourg, The Netherlands, Spain and Sweden (Table 2.3). The actual margin of errors for the economies in the European Union are between 3.4 in Estonia and Ireland on the one hand and 3.9 in Spain on the other (World Bank 2018).

**Table 2.3. Ownership of Accounts by Adults in Economies in the European Union**

| Account Ownership (%) | Names of Economies | Gap between Richer and poorer (percentage points) | Names of Economies (with percentage points parentheses) |
|---|---|---|---|
| 100 | Denmark, Finland, Netherlands, Sweden | Over 30 | Romania (33) |
| 99 | Belgium, Germany, Luxembourg | 20 – 29<br>10 – 19 | Bulgaria (29)<br>Czech Rep (17)<br>Hungary (12)<br>Slovak Rep (10) |
| 98 | Austria, Estonia, Slovenia | 1 – 9 | Croatia (9) |
| 95 – 97 | Ireland (95)<br>Malta (97) | 1– 9 | Cyprus (8)<br>Latvia (8)<br>Lithuania (8)<br>Portugal (8) |
| 90 – 94 | France (94)<br>Italy (94)<br>Spain (94)<br>Latvia (93)<br>Portugal (92) | 1– 9 | Greece (7)<br>Italy (5)<br>Ireland (4)<br>Malta (4)<br>Poland (4) |
| 80 – 89 | Cyprus (89)<br>Poland (87)<br>Croatia (86)<br>Greece (85)<br>Slovak Rep (84)<br>Lithuania (83)<br>Czech Rep (81) | | Austria, Belgium, Denmark, Estonia, Finland, France, Germany, Luxembourg The Netherlands, Slovenia, Spain, |
| 70 – 79 | Hungary (75)<br>Bulgaria (72) | | |
| 60 – 69 | No economy | | |
| 50 – 59 | Romania (58) | | |
| Below 50 | No economy | | |

Source: Author's Construction. The data are based on The Global Findex Database 2017 published by The World Bank (World Bank 2018).

**Table 2.4. The Structure of the Statistical Distribution of Ownership of Accounts by Adults in EU Economies**

| Statistical Parameter | Empirical Estimate |
|---|---|
| Minimum | 0.00 |
| Maximum | 100.00 |
| Mean | 90.74 |
| Median | 94.00 |
| Mode | 100.00 |
| First Quintile | 83.00 |
| Second Quintile | 89.00 |
| Third Quintile | 94.00 |
| Fourth Quintile | 98.00 |
| Uppermost Quintile | 100.00 |
| Sample Size | 27 |

Source: Author's Computations. The data are based on the World Bank's Global Findex Database 2017 (World Bank 2018).

The structure of the statistical distribution of the percentage of adults who own an account is depicted in Table 2.4. This table shows the distribution is negatively skewed. This indicates high financial inclusion (Table 4 here). By way of drawing the curtain, the point is underscored that the detailed discussions (in this Section) on the diffusion and utilisation of ICT and digital services (including digital financial services) by firms and consumers in the EU economies, digital currencies (viz., first private cryptocurrencies, the imminent novel central bank digital currencies, second generation cryptocurrencies, stablecoins; inter alia), and various dimensions of financial inclusion, inter alia, aptly provide the poly-pod stand on which the rationale for the present study solidly rests. Besides, the issues which this study addresses rigorously addressed jointly in the relevant literature.

## 3. The Novel General Theory and Model

The principal axiom of the novel general theory of the potential effects of digital currencies on financial inclusion in an open economy is that the economy under study such as the economies in the European Union which constitute the focus of the present study, is characterised by the system of functions in (1) − (4):

$$Q_k - Q_k\left(\left(i_k - \dot{P}^e\right), Y, R_+, R_-\right) = 0 \quad (1)$$

$$(M - M_f) - L\left((i_k - \dot{P}^e), Y, K, Z\right) = 0 \tag{2}$$

$$Z = [V, H, C, E, X, T] \tag{2'}$$

$$Y - Y\left((i_k - \dot{P}^e), \upsilon_+, \upsilon_-\right) = 0 \tag{3}$$

and

$$\Psi_k = \frac{Q_k{}^{-1} N_{20} \int_{Q_{k,\varepsilon 0}}^{Q_{k,max}} Q_k(\cdot) f(Q_k(\cdot), \theta) dQ_k(\cdot)}{Q_k{}^{-1} N_{40} \int_0^{Q_{k,40}} Q_k(\cdot) f(Q_k(\cdot), \theta) dQ_k(\cdot)} \tag{4}$$

$$\theta = [\theta_1, \theta_2, \dots, \theta_n\ ] \tag{4'}$$

$$\Psi = \sum_k g_k \Psi_k \tag{4''}$$

where (1) refers to a novel function for the level of access to the $k\,th$ financial service like services concerning ownership of accounts, formal savings opportunities, formal borrowing (access to credit) services, formal investment services, insurance services, and the plethora of current and novel digital financial services (ATM cards, Visa cards, debit cards, credit cards, E-zwich cards, electronic bank statements, and so on), mobile phones with financial services' digital Apps, mobile money transfers, digital merchant payments, and general knowledge of and access to fintech services, Western Money Union, Ria, MoneyGram money transfers, digital and manual financial intermediation services in general, as well as digital and manual banking and non-bank financial institution services, inter alia. Access to these financial services $Q_k$, by axiom, is determined by the real interest rate $(i_k - \dot{P}^e)$, where $i$ and $\dot{P}^e$ respectively denote nominal interest rate and expected rate of inflation (viz., based on the Fisher Identity which is given by $i = r + \dot{P}^e$ where $r$ is the real interest rate). $Q_k$ is determined by the real income $Y$, and two vectors of other factors $R_+$ which exert positive effects on $Q_k$ and $R_-$ which exert negative effects on $Q_k$; $R_+, R_-$ are postulated to be exogeneous in (1).

The implicit function system in $(2) - (3)$ constitutes the equilibrium conditions in the monetary sector and the goods and services sector respectively. The condition in (2) is novel, in that it departs from the conventional classical, Keynesian and other forms of money demand functions and their respective equilibrium conditions. The present novel money demand function L $(\cdot)$ takes into account the emerging and imminent novel developments concerning digital currencies of the Twenty-First Century, inter alia. Condition (2) also incorporates the novel concepts of the conventional domestic currency filteration (net outflows) $M_f$ and effective conventional domestic currency stock or supply $(M - M_f)$, where M denotes the actual conventional domestic currency stock or supply. The argument here is that the component of $M$ which flows out (net) of the domestic economy cannot directly influence economic activities in the domestic economy (see also Fosu 2006). The a priori effects of the arguments in (1) are that $(i_k - \dot{P}^e)$ is negative or positive, that of $Y$ is positive, that of $R_+$ is positive and that of $R_-$ is negative. In (2), the effect of $(i_k - \dot{P}^e)$ is negative, that of $Y$ is positive, $K$ is negative, that of $Z_+$ is positive, and that of $Z_-$ is negative. The equilibrium condition in the goods and services sector is summarised by (3), where $U_+ and\ U_-$ are two vectors of shift factors of the aggregate demand function Y $(\cdot)$, where $U_+$ exert positive effects and $U_-$ exert negative effects on aggregate demand. In (3), the effect of $(i_k - \dot{P}^e)$ is negative.

In the present study, a novel function is employed to characterise financial inclusion. Financial inclusion is said to occur, herein, when the inequality in the distribution of levels of access of the $k\,th$ financial service exhibits a tendency of decreasing (namely, that the proportional change in $\Psi_k$ is negative, for all the $k$ financial services). The equation in (4) is the novel financial services access inequality function. It can also be referred to as the financial inclusion function. Here, $Q$ is the sum of the individual $Q_k$ $(k = 1,2, \dots, J)$. $N_{20}$ denotes the number of economic operators in the top 20 percent of the population who enjoy the most (best) access to financial services, whereas $N_{40}$ denotes the lowest 40 percent of the population which enjoy the least access to the financial services. The function $f(Q_k\,(\cdot), \theta)$ denotes the probability density function of the distribution of the levels of access to the $k\;th$ financial services $(k = 1,2, \dots, J)$. $Q_{k,40}$ denotes the highest level of $Q_k$ for the bottom 40 percent group, $Q_{k,max}$ denotes the maximum level of access to financial service k for the top 20 percent group, whereas $\theta$ denotes a vector of parameters of the probability density function $f(\cdot)$. Additionally, $g_k$ is the share of the $k\;th$ financial service in the total value of financial services. Given that (1) $-$ (4) hold, the potential marginal effects of the imminent novel central bank digital currency $K$ and the private cryptocurrencies $V$ on the inequality in the distribution of the levels of access to the $k\;th$ financial service are respectively given by (5) $-$ (6):

$$\frac{\partial \Psi_k}{\partial K} = \frac{\partial \Psi_k}{\partial Q_k} \cdot \frac{\partial Q_k}{\partial K} = \frac{\partial \Psi_k}{\partial Q_k}\left(\frac{\partial Q_k}{\partial Y} \cdot \frac{\partial Y}{\partial K} + \frac{\partial Q_k}{\partial (i_k - \dot{P}^e)} \cdot \frac{\partial (i_k - \dot{P}^e)}{\partial K}\right) \tag{5}$$

$$\frac{\partial \Psi_k}{\partial V} = \frac{\partial \Psi_k}{\partial Q_k} \cdot \frac{\partial Q_k}{\partial V} = \frac{\partial \Psi_k}{\partial Q_k}\left(\frac{\partial Q_k}{\partial Y} \cdot \frac{\partial Y}{\partial V} + \frac{\partial Q_k}{\partial (i_k - \dot{P}^e)} \cdot \frac{\partial (i_k - \dot{P}^e)}{\partial V}\right) \tag{6}$$

Similarly, the potential effects of actual conventional domestic currency $M$, conventional domestic currency filteration (net outflows) $M_f$, and the effects of the aforementioned vector $Z$ with a typical element $Z_j$ are respectively given by (7)$-$(9)

$$\frac{\partial \Psi_k}{\partial M} = \frac{\partial \Psi_k}{\partial Q_k} \cdot \frac{\partial Q_k}{\partial M} = \frac{\partial \Psi_k}{\partial Q_k}\left(\frac{\partial Q_k}{\partial Y} \cdot \frac{\partial Y}{\partial M} + \frac{\partial Q_k}{\partial (i_k - \dot{P}^e)} \cdot \frac{\partial (i_k - \dot{P}^e)}{\partial M}\right) \tag{7}$$

$$\frac{\partial \Psi_K}{\partial M_f} = \frac{\partial \Psi_k}{\partial Q_k} \cdot \frac{\partial Q_k}{\partial M_f} = \frac{\partial \Psi_k}{\partial Q_k}\left(\frac{\partial Q_K}{\partial Y} \cdot \frac{\partial Y}{\partial M_f} + \frac{\partial Q_k}{\partial (i_k - \dot{P}^e)} \cdot \frac{\partial (i_k - \dot{P}^e)}{\partial M_f}\right) \tag{8}$$

$$\frac{\partial \Psi_k}{\partial Z_j} = \frac{\partial \Psi_k}{\partial Q_k} \cdot \frac{\partial Q_k}{\partial Z_j} = \frac{\partial \Psi_k}{\partial Q_k}\left(\frac{\partial Q_K}{\partial Y} \cdot \frac{\partial Y}{\partial Z_j} + \frac{\partial Q_k}{\partial (i_k - \dot{P}^e)} \cdot \frac{\partial (i_k - \dot{P}^e)}{\partial Z_j}\right) \tag{9}$$

$$j = H, C, E, X, T$$

and the potential marginal effects of $\omega_{l_1}$ and $\omega_{l_2}$ given by (9') $-$ (9")

$$\frac{\partial \Psi_k}{\partial \omega_{l_1}} = \frac{\partial \Psi_k}{\partial Q_k}\left(\frac{\partial Q_k}{\partial Y} \cdot \frac{\partial Y}{\partial \omega_{l_1}} + \frac{\partial Q_k}{\partial (i_k - \dot{P}^e)} \cdot \frac{\partial (i_k - \dot{P}^e)}{\partial \omega_{l_1}}\right) \tag{9'}$$

$$\frac{\partial \Psi_k}{\partial \omega_{l_2}} = \frac{\partial \Psi_k}{\partial Q_k}\left(\frac{\partial Q_k}{\partial Y} \cdot \frac{\partial Y}{\partial \omega_{l_2}} + \frac{\partial Q_k}{\partial (i_k - \dot{P}^e)} \cdot \frac{\partial (i_k - \dot{P}^e)}{\partial \omega_{l_2}}\right) \tag{9''}$$

$$\omega_{l_1} = R_+, R_-;\; \omega_{l_2} = \upsilon_+, \upsilon_-$$

The marginal response of the level of inequality $\Psi_k$ in the distribution of financial services' access to $Q_k$ can be obtained from the relevant first order partial derivative from (4). The respective marginal responses of $Q_k$ to the real income and the real interest rate are obtained from (1), where the real income and the real interest rate are the equilibrium values from the equilibrium conditions in the monetary sector and goods sector in $(2) - (3)$. Invoking the Implicit Function Theorem in the context of the conditions in $(2) - (3)$, the respective potential responses of the equilibrium real income and real interest rate to the imminent novel central bank digital currencies, private cryptocurrencies, actual conventional domestic currency stock (supply), conventional domestic currency filteration (net outflows) and the other elements of vector $z$ with the typical element $z_j$ for $(j = H, C, E, X, T)$ can be obtained accordingly. Given all these pieces of information, the partial derivatives in $(5) - (9)$ can be evaluated successfully.

Taking the first order partial derivative with respect to $Q_k$ gives (10):

$$\frac{\partial \Psi_k}{\partial Q_k} = \left(Q^{-1}N_{40}\int_0^{Q_{k,40}} Q_k(\cdot)f(Q_k(\cdot),\boldsymbol{\theta})dQ_k(\cdot)\right)^{-2} \times \left[\begin{array}{c}\left(\begin{array}{c} Q^{-1}N_{40}\int_0^{Q_{k,40}} Q_k(\cdot)f(Q_k(\cdot),\boldsymbol{\theta})dQ_k(\cdot) \times \\ Q^{-1}N_{80}\int_{Q_{k,80}}^{Q_{k,max}} \frac{\partial}{\partial Q_K} Q_k(\cdot)f(Q_k(\cdot),\boldsymbol{\theta})dQ_k(\cdot)\end{array}\right) \\ -\left(\begin{array}{c} Q^{-1}N_{80}\int_{Q_{k,80}}^{Q_{k,max}} Q_k(\cdot)f(Q_k(\cdot),\boldsymbol{\theta})dQ_k(\cdot) \\ \times Q^{-1}N_{40}\int_0^{Q_{k,40}} \frac{\partial}{\partial Q_K} Q_k(\cdot)f(Q_k(\cdot),\boldsymbol{\theta})dQ_k(\cdot)\end{array}\right)\end{array}\right] \tag{10}$$

There is financial inclusion when the marginal effect of $\frac{\partial \Psi_k}{\partial Q_k}$ is negative. This can occur when the numerator in (4) increases at a lower rate than the increases' rate in the denominator, or decreases at a higher rate than the decreases in the denominator, or when the numerator decreases whereas the denominator increases, and vice versa. These occur when the algebraic difference in (10) is negative.

The aforementioned implicit function theorem invoked in respect of the implicit function system $(1) - (3)$ suggests that the endogeneous variables $(i_k - \dot{P}^e)$, $Y$ and $Q_k$ can be solved in terms of the exogeneous variables $K, V, Z, M, M_f$, where the vector $Z = [H, C, E, X, T]$ if the Jacobian matrix in (11) exhibits a determinant which does not vanish, and that the marginal effects of the aforementioned exogeneous variables on the endogeneous variables $(i_k - \dot{P}^e)$, Y and $Q_k$ are given in compact form as in (12):

$$|J(\cdot)| = \begin{vmatrix} -\frac{\partial Q_k(\cdot)}{\partial(i_k-\dot{P}^e)} & -\frac{\partial Q_k(\cdot)}{\partial Y} & 1 \\ -\frac{\partial L(\cdot)}{\partial(i_k-\dot{P}^e)} & -\frac{\partial L(\cdot)}{\partial Y} & 0 \\ -\frac{\partial Y(\cdot)}{\partial(i_k-\dot{P}^e)} & 1 & 0 \end{vmatrix} = -\left(\frac{\partial L(\cdot)}{\partial\left(i_k-\dot{P}^e\right)} + \frac{\partial L(\cdot)}{\partial Y}\cdot\frac{\partial Y}{\partial\left(i_k-\dot{P}^e\right)}\right) \neq 0 \tag{11}$$

and

$$B_1 = -(J(\cdot))^{-1}.G_1 \tag{12a}$$

$$B_2 = -(J(\cdot))^{-1}.G_2 \tag{12b}$$

$$B_3 = -(J(\cdot))^{-1}.G_3 \tag{12c}$$

$$B_4 = -(J(\cdot))^{-1}.G_4 \tag{12d}$$

$$B_{5j} = -(J(\cdot))^{-1}.G_{5j} \tag{12e}$$

$$B_{6l_1} = -(J(\cdot))^{-1}.G_{6l_1} \tag{12f}$$

$$B_{6l_2} = -(J(\cdot))^{-1}.G_{6l_2} \tag{12g}$$

where

$$B_1 = \begin{bmatrix}\frac{\partial(i_k-\dot{P}^e)}{\partial K} & \frac{\partial Y}{\partial K} & \frac{\partial Q_k}{\partial K}\end{bmatrix}' \tag{13a}$$

$$B_2 = \begin{bmatrix}\frac{\partial(i_k-\dot{P}^e)}{\partial V} & \frac{\partial Y}{\partial V} & \frac{\partial Q_k}{\partial V}\end{bmatrix}' \tag{13b}$$

$$B_3 = \begin{bmatrix}\frac{\partial(i_k-\dot{P}^e)}{\partial M} & \frac{\partial Y}{\partial M} & \frac{\partial Q_k}{\partial M}\end{bmatrix}' \tag{13c}$$

$$B_4 = \left[\frac{\partial(i_k - \dot{P}^e)}{\partial M_f} \quad \frac{\partial Y}{\partial M_f} \quad \frac{\partial Q_k}{\partial M_f}\right]' \tag{13d}$$

$$B_{5j} = \left[\frac{\partial(i_k - \dot{P}^e)}{\partial Z_j} \quad \frac{\partial Y}{\partial Z_j} \quad \frac{\partial Q_k}{\partial Z_j}\right]' \tag{13e}$$

$$j = H, C, E, X, T$$

$$B_{6l_1} = \left[\frac{\partial(i_k - \dot{P}^e)}{\partial \omega_{l_1}} \quad \frac{\partial Y}{\partial \omega_{l_1}} \quad \frac{\partial Q_k}{\partial \omega_{l_1}}\right]' \tag{13f}$$

$$B_{6l_2} = \left[\frac{\partial(i_k - \dot{P}^e)}{\partial \omega_{l_2}} \quad \frac{\partial Y}{\partial \omega_{l_2}} \quad \frac{\partial Q_k}{\partial \omega_{l_2}}\right]' \tag{13g}$$

and

$$G_1 = \left[0 \quad -\frac{\partial L(\cdot)}{\partial K} \quad 0\right]' \tag{14a}$$

$$G_2 = \left[0 \quad -\frac{\partial L(\cdot)}{\partial V} \quad 0\right]' \tag{14b}$$

$$G_3 = [0 \quad 1 \quad 0]' \tag{14c}$$

$$G_4 = [0 \quad -1 \quad 0]' \tag{14d}$$

$$G_{5j} = \left[0 \quad -\frac{\partial L(\cdot)}{\partial Z_j} \quad 0\right]' \tag{14e}$$

$$G_{6l_1} = \left[-\frac{\partial Q_k(\cdot)}{\partial \omega_{l_1}} \quad 0 \quad 0\right]' \tag{14f}$$

$$\omega_{l_1} = R_+, R_-$$

$$G_{6l_2} = \left[0 \quad 0 \quad -\frac{\partial Y(\cdot)}{\partial \omega_{l_2}}\right]' \tag{14g}$$

$$\omega_{l_2} = \upsilon_+, \upsilon_-$$

The compact matrix expressions in $(12a) - (12g)$ can be evaluated as in (15):

$$B_\alpha = -|J(\cdot)|^{-1} adj\big(J(\cdot)\big) G_\alpha \tag{15}$$

$$\alpha = 1, 2, \dots, 5j, 6l_1, 6l_2$$

where $|J(\cdot)|$ denotes the determinant of the relevant Jacobian matrix of the system in $(1) - (3)$, and adj $\big(J(\cdot)\big)$ is the adjoint of the matrix $J(\cdot)$. Hence, the partial derivatives in $(5) - (9'')$ can be evaluated using $(10), (12a) - (12g), (15)$ where B. and G. are given by $(13a) - (13g)$, and $(14a) - (14g)$ respectively. The signs of these partial derivatives can then be assessed accordingly from the a priori expectations of the respective marginal effects of the variables in $(1) - (3)$, and (10). The possible resulting signs of these potential marginal effects have been summarised in Table 1 in Section 4.

Given that the right hand side of (5)−(10) is not equal to zero, invoking the fundamental theorem of differential calculus, the total change in inequality in access to financial services $\Psi_k$ is given by (16):

$$d\Psi_k = \frac{\partial \Psi_k(\cdot)}{\partial K} dK + \frac{\partial \Psi_k(\cdot)}{\partial V} dV + \frac{\partial \Psi_k(\cdot)}{\partial M} dM + \frac{\partial \Psi_k(\cdot)}{\partial M_f} dM_f + \frac{\partial \Psi_k(\cdot)}{\partial z_j} dz_j + \frac{\partial \Psi_k(\cdot)}{\partial \omega_{l_1}} d\omega_{l_1} + \frac{\partial \Psi_k(\cdot)}{\partial \omega_{l_2}} d\omega_{l_2} \tag{16}$$

integrating (16) totally gives (17):

$$\Psi_K = \Psi_K(K, V, M, M_f, z_j, \omega_{l_1}, \omega_{l_2}) \tag{17}$$

Now, respecify $V$ as in (18a) where $\phi_V$ is given by (18b):

$$V = \phi_V V^* \tag{18a}$$

and $$\phi_V = \begin{cases} 1, & \text{if } \Delta V^* > 0 \\ 0, & \text{if } \Delta V^* = 0 \\ -1, & \text{if } \Delta V^* < 0 \end{cases} \qquad \text{(18b')(18b'')(18b''')}$$

where $\Delta V^*$ denotes the first difference of $\Delta V^* = V_t^* - V_{t-1}^*$. Similarly, respecify the other arguments $\mathrm{T} = K, M, M_f, z_j, \omega_{l_1}, \omega_{l_2}$ as (19a), where $\phi_{\mathrm{T.}}$ is given by (19b') $-$ (19b$''$) :

$$\mathrm{T} = \phi_{\mathrm{T.}}\mathrm{T.}^* \text{ and } \phi_{\mathrm{T.}} = \begin{cases} 1, & \text{if } \Delta \mathrm{T.}^* > 0 \\ 0, & \text{if } \Delta \mathrm{T.}^* = 0 \\ -1, & \text{if } \Delta \mathrm{T.}^* < 0 \end{cases} \qquad \text{(19b')(19b'')(19b''')}$$

Substituting (18a)$-$(19b$'''$) into (17) jointly as additional arguments in (17) gives (20):

$$\Psi_k - \Psi_k\left(K^*, \left(\phi_{\mathrm{T}_k} K^*\right), V^*, (\phi_V V^*), M^*, \left(\phi_{\mathrm{T}_M} M^*\right), M_f^*, \left(\phi_{\mathrm{T}_{M_f}} M_f^*\right), z_j^*, \left(\phi_{\mathrm{T}_{z_j}} z_j^*\right), \omega_{l_1}^*, \left(\phi_{\mathrm{T}_{l_1}} \omega_{l_1}^*\right), \omega_{l_2}^*, \left(\phi_{\mathrm{T}_{l_2}} \omega_{l_2}^*\right)\right) \tag{20}$$

The expressions in (18a)$-$(20) enhance the analyses of the asymmetry versus symmetry in the effects of the determinants of the inequality in access to financial services (viz., financial inclusion). When the marginal effect of $(\phi_V V^*)$ or $(\phi_{\mathrm{T.}}\mathrm{T.}^*)$ on financial inclusion $\Psi_k$ is equal to zero, then the marginal effect of the aforementioned specific determinant of financial inclusion is symmetric, in the sense that the magnitude of the marginal effect of a decrease in the level of the relevant determinant is equal to that of an increase in the level of the determinant. When the marginal effect of $(\phi_V V^*)$ or $(\phi_{\mathrm{T.}}\mathrm{T.}^*)$ is statistically not equal to zero, then the effect of the aforementioned specific determinant on financial inclusion is asymmetric, in the sense that the magnitude of the marginal effect of the aforementioned specific determinant on financial inclusion for an increase in the level of the determinant is different from the magnitude of the corresponding marginal effect of a decrease in the level of the aforementioned specific determinant of financial inclusion. As exemplars, suppose (20) forms the basis of the empirical model specified to analyse the empirical effects of digital currencies denoted herein as $K$ and $V$ on financial inclusion $\Psi_k$ and that a linear mathematical functional structure (relationship) is postulated for (20), then the marginal effects of $K$ and $V$ on $\Psi_k$ are given by (21a) and (21b) respectively:

$$\frac{\partial \Psi_k}{\partial K^*} = \alpha_K + \beta_K \phi_{\mathrm{T}_K} \tag{21a}$$

$$\frac{\partial \Psi_k}{\partial V^*} = \alpha_V + \beta_V \phi_V \tag{21b}$$

where $\alpha_K$, $\alpha_V$ are the parameters of $K^*$ and $V^*$ respectively, whereas $\beta_K$ and $\beta_V$ are the parameters of $\left(\phi_{\mathrm{T}_K} K^*\right)$ and $(\phi_V V^*)$. When $\beta_K = 0$, then the marginal effect of $K^*$ is symmetric, and when $\beta_V = 0$ then the effect of $V^*$ is also symmetric. On the contrary, asymmetry occurs when $\beta_K \neq 0$ and $\beta_V \neq 0$.

The function in (20) is based on the equilibrium relationships in (1) – (3) and (5) – (10) as well as (11) – (19). Thus, (20) is a long run equilibrium relationship. However, economic inertia in the form of trammels on the complete adjustment of the economy under study to this equilibrium state or time path prevail in some cases (for example, in the short run), and these result in the prevalence of autoregressive and distributed lag relationships between a given phenomenon and its determinants. Some economic operators may not perceive changes in the levels of the relevant determinants, or that some responses to changes in the determinants may spread over time from the current period to the next time period and to successive succeeding time periods, and so on. These precipitate the prevalence of an autoregressive distributed lag relationship. In the context of the present study, an autoregressive distributed lag relationship could prevail when some economic operators are unable to perceive changes in the prices or other attributes of digital currencies, and the aforementioned determinants of financial inclusion as specified in (20), and/or that the responses of financial inclusion to changes in its aforementioned determinants are spread over time from the current period to successive succeeding time periods, among other factors. An autoregressive distributed lag relationship of order $(q_1, q_2)$ is denoted as $ADL\ (q_1, q_2)$; the autoregressive component is denoted as $AR\ (q_1)$ and the distributed lag component is denoted as $DL(q_2)$: see also Dhrymes 1971, Hendry and Anderson 1977, Davidson, Hendry and Srba 1978, and Hendry, Pagan and Sargan 1984, for the use of autoregressive distributed lag functions in other context. Suppose (20) is specified as $ADL\ (q_1, q_2)$, then this is specified generally as (22):

$$\Psi_k = \Psi_k(\Psi_{k,t-1}, \Psi_{k,t-2}, \dots, \Psi_{k,t-q_1}, K_t^*, K_{t-1}^*, \dots, K_{t-q_{2(1)}}^*, V_t^*, V_{t-1}^*, \dots, V_{t-q_{2(2)}}^*$$

$$, \dots, \omega_{l_2,t}^*, \omega_{l_2,t-1}^*, \dots, \omega_{l_2,t-q_{2(\delta)}}^*; \left(\phi_{\mathrm{T}_k} K^*\right)_t, \left(\phi_{\mathrm{T}_k} K^*\right)_{t-1}, \dots, \left(\phi_{\mathrm{T}_k} K^*\right)_{t-q_{2(1)}}, (\phi_V V^*)_t$$

$$, (\phi_V V^*)_{t-1}, \dots, (\phi_V V^*)_{t-q_{2(2)}}, \dots, \left(\phi_{\mathrm{T}_{\omega_{l_2}}} \omega_{l_2}^*\right)_t, \left(\phi_{\mathrm{T}_{\omega_{l_2}}} \omega_{l_2}^*\right)_{t-1}, \dots, \left(\phi_{\mathrm{T}_{\omega_{l_2}}} \omega_{l_2}^*\right)_{t-q_{2(\delta)}}) \qquad (22)$$

where the index $t$ denotes time period. It is imperative to indicate that the index on all the variables in (20) is $t$. Morever, (22) can be solved to yield the long run relationship in (20).

The Granger Representation Theorem holds that, given that a long run relationship between a phenomenon and its determinants exists, then there also exists an underlying equilibrium correction function which captures the short run dynamics of the economic phenomenon under study with the short run effects of its determinants as well as the proportion of the disequilibrium in the level of the economic phenomenon which is corrected over time (Engle and Granger 1986). Invoking this Granger Representation Theorem, it follows in the context of the present study that, given that the long run equilibrium function (20) for financial inclusion exists in an economy in the European Union, then there exists an underlying equilibrium correction function from which the short run effects of digital currencies and other determinants on financial inclusion in an economy in the European Union can be computed. The long run equilibrium function (20) can also be reparameterised to yield the underlying equilibrium correction model and this is functionally given by (23):

$$\Delta\Psi_{K,t}^* = \Delta\Psi_{K,t}^*(\Delta\Psi_{K,t-1}^*, \Delta\Psi_{K,t-2}^*, \dots, \Delta\Psi_{K,t-s(1)}^*, \Delta K_t^*, \Delta K_{t-1}^*, \dots, \Delta K_{t-s(2)}^*$$

$$, \Delta\left(\phi_{\mathrm{T}_k} K^*\right)_t, \Delta\left(\phi_{\mathrm{T}_k} K^*\right)_{t-1}, \dots, \Delta\left(\phi_{\mathrm{T}_k} K^*\right)_{t-s(3)}, \Delta V_t^*, \Delta V_{t-1}^*, \dots, \Delta V_{t-s(4)}^*$$

$$, \dots, \Delta(\phi_V V^*)_t, \Delta(\phi_V V^*)_{t-1}, \dots, \Delta(\phi_V V^*)_{t-s(5)}, \dots, \Delta\omega_{l_2,t}^*, \Delta\omega_{l_2,t-1}^*, \dots, \Delta\omega_{l_2,t-s(6)}^*$$

$$, \dots, \Delta\left(\phi_{\mathrm{T}_{\omega_{l_2}}} \omega_{l_2}^*\right)_t, \Delta\left(\phi_{\mathrm{T}_{\omega_{l_2}}} \omega_{l_2}^*\right)_{t-1}, \dots, \Delta\left(\phi_{\mathrm{T}_{\omega_{l_2}}} \omega_{l_2}^*\right)_{t-s(7)}, u_t) \qquad (23)$$

where $\Delta$ denotes first difference, $\Delta\Psi^*_{k,t} = \Psi^*_{k,t} - \Psi^*_{k,t-1}$, and so on; $\Delta\Psi^*_{k,t-1}$ denotes $\Delta\Psi^*_{K,t}$ lagged one period, $\Delta\Psi^*_{k,t-s}$ denotes $\Delta\Psi^*_{k,t}$ lagged $(t-s)$ periods, and so on; $\Delta(\phi_V V^*)_t = (\phi_V V^*)_t - (\phi_V V^*)_{t-1}$ and $\Delta\left(\phi_V V^*\right)_{t-s}$ denotes $\Delta(\phi_V V^*)_t$ lagged $(t-s)$ periods, and so on; $U_t$ denotes the equilibrium correction term: $U_t = \Psi^*_{k,t} - \widehat{\Psi}^*_{k,t}$ where $\Psi^*_{k,t}$ is the actual (observed) value, and $\widehat{\Psi}^*_{k,t}$ is the estimated predicted value obtained from the long run equilibrium function.

The short run effects of digital currencies $K$ and $V$ on financial inclusion $\Psi_k$ are the respective parameters of $\Delta K^*_t, \Delta K^*_{t-1}, \Delta K^*_{t-2}, \Delta\left(\phi_{\mathrm{T}_k} K^*\right)_t, \Delta\left(\phi_{\mathrm{T}_k} K^*\right)_{t-1}, \Delta\left(\phi_{\mathrm{T}_k} K^*\right)_{t-2}, \Delta V^*_t, \Delta V^*_{t-1}, \Delta V^*_{t-2}$ $, \Delta(\phi_V V^*)_t, \Delta(\phi_V V^*)_{t-1}, \Delta(\phi_V V^*)_{t-2}$ in the equilibrium correction model in (23). The corresponding effects of the other determinants can be similarly computed.

The long run effects of digital currencies K and V on financial inclusion $\Psi_k$ which are the relevant parameters of (20) and computed as in (21a) – (21g) can also be computed from the relevant parameters of the autoregressive distributed lag model (22). In this regard, the long run marginal effects of K and V on $\Psi_k$ are respectively given by (24a) – (24b):

$$\frac{\partial\Psi_k}{\partial K^*} = \left(1 - \sum_{m=1}^{q_1} \gamma_m\right)^{-1} \left(\sum_{r=0}^{q_{2(1)}} \left(\alpha_{K,r} + \beta_{K,r}\phi_K\right)\right) \tag{24a}$$

$$\frac{\partial\Psi_k}{\partial V^*} = \left(1 - \sum_{m=1}^{q_1} \gamma_m\right)^{-1} \left(\sum_{h=0}^{q_{2(2)}} \left(\alpha_{V,h} + \beta_{V,h}\phi_V\right)\right) \tag{24b}$$

where $\gamma_m$ $(m = 1,2,\dots,q_1)$ denotes the respective parameters of $\Psi^*_{k,t-m}$; $\alpha_{K,r}$ denotes the respective parameters of $K^*_{t-r}$ $\left(r = 0,1,2,\dots,q_{2(1)}\right)$; $\beta_{k,r}(K,r)$ are the respective parameters of $\left(\phi_{\mathrm{T}_k} K^*\right)_{t-r}$ where $(r = 0,1,2,\dots,q_{2(2)})$, $\alpha_{V,h}$ denotes the respective parameters of $V^*_{t-h}$ and $\beta_{V,h}$ denotes the respective parameters of $(\phi_V V^*)_{t-h}$ where $(h = 0,1,2,\dots,q_{2(2)})$. The corresponding long run marginal effects of the other determinants of financial inclusion $\Psi_k$ can be computed in a similar manner.

Consider now, the analysis of the structure of the cyclical (oscillatory) behaviour of financial inclusion $\Psi_k$. In this regard, the relevant function which constitutes the basis of this analysis is the autoregressive component of the autoregressive distributed lag function in (22). Let this autoregressive component be a linear expression given by (25a):

$$\Psi^*_{k,t} = \gamma_0 + \gamma_1\Psi^*_{k,t-1} + \gamma_2\Psi^*_{k,t-2} + \cdots + \gamma_{q_1}\Psi^*_{k,t-q_1} \tag{25a}$$

Rearranging the terms gives (25b):

$$\Psi^*_{k,t} - \gamma_1\Psi^*_{k,t-1} - \gamma_2\Psi^*_{k,t-2} - \cdots - \gamma_{q_1}\Psi^*_{k,t-q_1} = \gamma_0 \tag{25b}$$

Leading (25b) by $q_1$ periods gives (25c):

$$\Psi^*_{k,t+q_1} - \gamma_1\Psi^*_{k,t+(q_1-1)} - \gamma_2\Psi^*_{k,t+(q_1-2)} - \cdots - \gamma_{q_1}\Psi^*_{k,t} = \gamma_0 \tag{25c}$$

Let the relevant corresponding homogeneous linear difference equation be given by (25d):

$$\Psi^*_{k,t+q_1} + b_1\Psi^*_{k,t+(q_1-1)} + b_2\Psi^*_{k,t+(q_1-2)} + \cdots + b_{q_1}\Psi^*_{k,t} = 0 \tag{25d}$$

where $b_1 = -\gamma_1, b_2 = -\gamma_2, \dots, b_{q_1} = -\gamma_{q_1}$. The corresponding characteristic equation which flows from the homogeneous linear difference equation (25d) is given by (25e):

$$n^{q_1} + b_1 n^{q_1 - 1} + b_2 n^{q_1 - 2} + \dots + b_{q_1 - 1} n + b_{q_1} = 0 \tag{25e}$$

Equation (25e) is essentially a $q_1$ $th$ degree polynomial in $n$; there are $q_1$ linearly independent solutions for this polynomial, in the sense that the roots of (25e) are $q_1$ in number. The nature of these roots determine the nature of the dynamics of financial inclusion $\Psi_k^*$ (viz., the time profile or time path of $\Psi_k^*$) including the structure of the cyclical (oscillatory) behaviour (and its stability or otherwise) of financial inclusion. In this regard, there are four principal possibilities; these are given by (26a) – (28e):

I. a real root $n_j$ with multiplicity 1 yields a time profile of financial inclusion $\Psi_{k,t}^*$ given by (26a):

$$\Psi_{k,t}^* = \Lambda n_i^t \tag{26a}$$

II. a real root $n_j$ with multiplicity $P$ gives the time profile of financial inclusion in (26b):

$$\Psi_{k,t}^* = \left( \begin{array}{c} B_1 n_j^t \\ B_2 t n_j^t \\ B_p t^{p-1} n_j^t \end{array} \right. \qquad \text{(26b')(26b'')(26b''')}$$

III. a pair of complex roots $n_l = \xi + i \; and \; \bar{n}_l = \xi - i\mu$ with multiplicity 1 gives rise to the solutions which yield the time profile of financial inclusion in (26c') – (26c''):

$$\Psi_{k,t}^* = \left( \begin{array}{l} C_1 \lambda^t \cos \theta t \\ C_2 \lambda^t \sin \theta t \end{array} \right. \qquad \text{(26c') – (26c'')}$$

where $\xi, \mu$ are real constants, $i = \sqrt{-1}$, and $\lambda$, $\cos\theta$, and $\theta$ are respectively given by (26c'''), (26c$^{IV}$) and (26c$^{V}$):

$$\lambda = \sqrt{\xi^2 + \mu^2} \tag{26c'''}$$

$$\cos\theta = \frac{\xi}{\lambda} \tag{26c$^{IV}$}$$

$$\theta = [0, \pi] \tag{26c$^{V}$}$$

$\cos\theta$ denotes the cosine of angle $\theta$, and $\sin\theta$ denotes the sine of angle $\theta$; $\pi = \frac{22}{7}$; $\theta$ is measured in $\pi$ radians.

IV. a pair of complex roots $n_e = \tau + i\mathrm{v} \; and \; \bar{n}_e = \tau - i\mathrm{v}$ with multiplicity $\mathbf{T}$ yields the solutions which reflect the time profile of financial inclusion given by (27a) – (27f):

$$
\Psi_{k,t}^{*} = \begin{cases} D_1 v, & (27a) \\ D_2 \Omega, & (27b) \\ D_3 t v, & (27c) \\ D_4 t \Omega, & (27d) \\ D_{2,\mathbf{T}} t^{\mathbf{T-1}} v, & (27e) \\ D_{2,\mathbf{T}} t^{\mathbf{T-1}} \Omega & (27f) \end{cases}
$$

where $v, \Omega, \Upsilon$ cos $\varphi$ and $\varphi$ are given by (28a) – (28e):

$$v = \Upsilon \cos \varphi t \tag{28a}$$

$$\Omega = \Upsilon \sin \varphi t \tag{28b}$$

$$\Upsilon = \sqrt{\tau^2 + \mathrm{v}^2} \tag{28c}$$

$$\cos \varphi = \frac{\tau}{\Upsilon} \tag{28d}$$

$$\varphi = [0, \pi] \tag{28e}$$

where $\pi = \frac{22}{7}$ and all the other variables are as already indicated $\Lambda, B, C, D, \Upsilon$ are arbitrary constants; $\tau, \mathrm{v}$ are constant; $\varphi$ is angle measured in $\pi$ radians.

The stability of the dynamic system in (25a), (25b), (25c), (25d), or (25e) is determined as follows. By definition, (25c) is stable (specifically, globally asymptotically stable) if the general solution of (25d), which is $C_1 u_1 + C_2 u_2 + \cdots + C_y u_y + \cdots + C_y u_y(t)$ approaches 0 as $t \rightarrow \infty$, for all values of $C_1, C_2, \ldots, C_n$. If all the characteristic roots of (25c) – (25d), have moduli less than 1, then (25c) – (25d) is stable and conversely, if the dynamics of the system in (25d) is stable, then a necessary and sufficient the relevant moduli are less than 1. Condition for all the characteristic roots of (25d), (25e) to have moduli less than 1 is given by the Schur Theorem, and this is given by (29a) – (29c) jointly, in the context of the present study:

$$\begin{vmatrix} 1 & b_{q_1} \\ b_{q_1} & 1 \end{vmatrix} > 0 \tag{29a}$$

$$\begin{vmatrix} 1 & 0 & b_{q_1} & b_{q_1 - 1} \\ b_1 & 1 & 0 & b_{q_1} \\ b_{q_1} & 0 & 1 & b_1 \\ b_{q_1 - 1} & b_{q_1} & 0 & 1 \end{vmatrix} > 0 \tag{29b}$$

and

$$\begin{vmatrix} 1 & 0 & \cdots & 0 & b_{q_1} & b_{q_1-1} & \cdots & b_1 \\ b_1 & 1 & \cdots & 0 & 0 & b_{q_1} & \cdots & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{q_1-1} & b_{q_1-2} & \cdots & 1 & 0 & 0 & \cdots & b_{q_1} \\ b_{q_1} & 0 & \cdots & 0 & 1 & b_1 & \cdots & b_{q_1-1} \\ b_{q_1-1} & b_{q_1} & \cdots & 0 & 0 & 1 & \cdots & b_{q_1-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_1 & b_2 & \cdots & b_{q_1} & 0 & 0 & \cdots & 1 \end{vmatrix} > 0 \tag{29c}$$

where (29a) – (29c) are the determinants of the respective partitioned matrices;
and $b_1, b_2, b_{q_1}, b_{q_1-1}, b_{q_1-2}$ are obtained from (25e) which are, in turn, based on (25d) as well as (25a) – (25c).

The scenarios in Case III and Case IV which involve trigonometric functions give rise to cyclical behaviour of financial inclusion $\Psi_k^*$. The structure of the cyclical behaviour (oscillatory behaviour) of financial inclusion is given by (26c') and (26c") and (27a) – (27f) jointly with (28a) – (28e). In addition, when the stability condition in (29a) – (29c) hold jointly then the cycles in the time profile of financial inclusion converge; on the contrary, when this condition does not hold jointly (viz., showing the absence of stability), then the cycles (oscillations or fluctuations) in the time profile of financial inclusion diverge over time. When case I or case II holds, then the time profile of financial inclusion is not characterised by cyclical behaviour.

## 4. Results and Policy Implications

The novel results of the rigorous analyses of the potential effects of digital currencies (viz., the imminent novel central bank digital currencies, and private cryptocurrencies), conventional domestic currency (the traditional fiat money), conventional foreign currency, conventional domestic currency filteration (net outflows), and other factors (viz., cyber insecurity levels, the cost of maintaining cyber security, energy cost, exchange rates of conventional foreign currencies, and the government administrative instruments for controlling potential inflationary tax evasion on financial inclusion herein conceptualised as a decline in the inequality in the statistical distribution of access to financial services in the economy under study, are summarised in Table 1, where $\alpha = K, V, M, M_f, Z,$ and $Z = [H, C, E, X, T]$. A negative $\frac{\partial \Psi_k}{\partial A\propto}$ is indicative of the prevalence of financial inclusion.

**Table 1. Results of the Potential Effects of Digital Currencies (viz., the Imminent the Novel Central Bank Digital Currency and Private Cryptocurrencies), and Other Factors on Financial Inclusion'**

| | When $\frac{\partial \Psi_k}{\partial Q_k}$ is negative | | When $\frac{\partial \Psi_k}{\partial Q_k}$ is positive | |
|---|---|---|---|---|
| Variable[1] $A_\alpha$ | If $\frac{\partial W_k}{\partial A_\alpha}$ is[2] | Then $\frac{\partial \Psi_k}{\partial A_\alpha}$ is | If $\frac{\partial W_k}{\partial A_\alpha}$ is[2] | Then $\frac{\partial \Psi_k}{\partial A_\alpha}$ is |
| K | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |
| V | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |
| M | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |
| $M_f$ | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |
| H | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |
| C | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |
| E | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |
| X | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |
| T | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |
| $R_+$ | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |
| $R_-$ | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |
| $Z_+$ | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |
| $Z_-$ | $+$ | $-$ | $+$ | $+$ |
| | $(-)$ | $(+)$ | $(-)$ | $(-)$ |

Source: Author's Computations.

1. See the axioms in Section 3 for the meanings of these variables and their symbols.

2. $\frac{\partial W_k}{\partial A_\propto} = \frac{\partial Q_k}{\partial A_\propto}\left(\frac{\partial Q_k}{\partial Y} \cdot \frac{\partial Y}{\partial A_\propto} + \frac{\partial Q_k}{\partial (i_k - \dot{P}^e)} \cdot \frac{\partial (i_k - \dot{P}^e)}{\partial A_\propto}\right)$

It is evident from Table 1 that the Potential marginal effects of the variables considered in this study (including digital currencies) on financial inclusion is complex, as a number of possibilities indeed occur. Consider two exemplars herein. In the first place, if the imminent novel central bank digital currency is created, issued, and circulated jointly with conventional currency and the level of the former currency is increased and this stimulates an increase in the level of access to financial services, but this increased access accrues largely to the top 20 percent of the population (in terms of having better access or having more access), then financial inclusion is likely to be compromised. On the contrary, if this increased access accrues largely to the bottom 40 percent of the population (in terms of the level of access to financial services), then financial inclusion is likely to increase. A comparison of the results in Rows 1 and 2 jointly on the one hand and Columns 3 and 4 jointly on the other epitomises this for $K$ (viz., the imminent novel central bank digital currencies). Similar results can be obtained for private cryptocurrencies as seen the corresponding or analogous Rows 3 and 4. The effects of the other variables on financial inclusion are also summarised in Table 1.

It is imperative to underscore the point that the marginal effects of the levels of access to financial services on the inequality in the statistical distribution of access to financial services and therefore financial inclusion, depends upon the mathematical structure of the statistical distribution of access to financial services, the parameter of this distribution, the level of access to financial services, and the response of the algebraic product of the probability density function of access to financial services and the level of access to financial services. There is therefore the need to characterise the structure of this density function, on the empirical front. It is also imperative to characterise the empirical structures of $(1) - (3)$ in the theory in the previous section.

Among other uses, the results in Table 1 can be employed to design and formulate countervailing policies in cases where the direction of the change in a given variable is likely to compromise the achievement of financial inclusion. An exemplar which comes in handy here is the following. Suppose a given economy is prone to a high level of cyber insecurity and this is likely to induce a compromise in the achievement of financial inclusion for example the case of Row 10 in Columns 1 and 2. Alternatively, cost of energy could be subsidised or a transition to environmentally friendly carbon neutral non-fossil-based renewable energy produced and vended at lower costs could be promoted in order to foster financial inclusion. This is the case in Row 14 of Columns 1 and 2 in table 1.

In the same vein, conventional domestic currency filteration (net outflows) which precipitates lower levels of access to financial services and a potential compromise of financial inclusion, could be countered with the aforementioned policies on countervailing reduction in the cost of maintaining cyber security and reduction in energy cost. The same arguments can be made here. This is the case in Row 8 for Column 1 and 2 for which the countervailing policies could be based on Row 12 of Columns 1 and 2, and Row 14 of Columns 1 and 2.

Additionally, a number of other policies could be implemented so that the utilisation of digital currencies could stimulate high levels of financial inclusion based on the results in Table 1. However, suffice it to say at this juncture, that as a final exemplar, consider the following. Suppose the need for non-hyperinflationary seignorage is likely to potentially precipitate lower levels of access to financial services and thus lower levels of financial inclusion, the aforementioned countervailing policies could be implemented; this is the case of Row 18 of Columns 1 and 2, and the countervailing policies based on Row 12 of Columns 1 and 2, and Row 14 of Columns 1 and 2.

By way of drawing the curtain in respect of this section, it should be noted that in an economy where the response of the inequality of access to financial services to the level of financial services is positive, similar countervailing policies could be implemented. These could be based on the relevant Rows of Columns 3 and 4 in Table 1.

Exemplars of fruitful policies which are likely to enhance the contributions of digital currencies (viz., the imminent novel central bank digital currency and private cryptocurrencies) towards higher levels of financial inclusion comprise lower levels of the cost of maintaining cyber security, harshly punishing (via the prosecution of) cyber fraudsters, lower energy costs via subsidies, promoting the production and vending of non-fossil-based environmentally-friendly carbon neutral renewable energy at lower costs, lower levels of non-hyper-inflationary seignorage (and lower levels of instruments to counter potential inflationary tax evasion), reducing cyber insecurity, and the lowering of the cost of maintaining cyber security. These policies also include reducing the levels of autonomous and marginal rates on existing and novel digital taxes, financial services taxes, cutting the levels of inordinate bank transactionary charges, capital gains taxes, increasing the levels of education of economic operators in respect of financial service availability and access, increasing access and availability of computers, mobile phones with financial Apps, internet access increase in the levels of income and reduction in income inequality, the promotion of E-commerce, e-finance, fintech, and fiscal incentives, to mention but a few).

Another set of results recorded in the present study include the following. In the first place, the economics in the European Union have been observed to exhibit profound ability to change or implement swift responses to some shocks like the recent COVID-19 health pandemic which would otherwise have had undesirable dire consequences for their performance and ultimate survival. As two exemplars of this, it was observed in Section 2 of the present study, that firms in economies in the European Union took about 2-9 days to respond to increases in consumer demand for online-digital procurement of goods and services, whereas it would have possibly taken them 585 days to respond to these increases normally. Similarly, it took firms 21.3 days to respond to customer needs and expectations concerning their utilisation of digital services, whereas it would have been possibly taken firms 511 days to respond to these changes under normal circumstances thought to be possible (see the COVID-19 Digital Sentiment Insights Survey of McKinsey and Co., 2020). Increased availability and access to digital information and communication technology (ICT) and a narrowing of the digital gap have served as catalysts for these swift responses, and these firms have also increased their expenditure on digital data security (ibid.).

Furthermore, the average share of products and services which have been digitised partially or completely has increased, while the adoption of customer interactions has accelerated by 3 years in economies in the economies in the European Union. The combined average share (percentage) of regular users and first time users of digital services has been observed to be highest in the banking industry and this share has been estimated to be as high 78 percent in the economies in the European Union. Moreover, the aforementioned observed increases in the expected high levels of consumer demand for online digital procurement and utilisation of digital services by firms are perceived by consumers and firms to be long lasting. All these results jointly suggest the following: that these is a need to sustain the availability and access to digital services, ICTs like mobile phones with the relevant Apps, digital tablets, desktop and laptop computers with the relevant software, internet connectivity with reliable access and speed as well as affordable charges, inter alia, digital payment services, digital mobile money services' provision; that there is the need to increase financial inclusion on a sustainable basis: recall that the highest average share of the level of digitisation in the banking industry is the highest in the economies in the European Union; that there is the need to ensure that the cost of maintenance of cyber security (including digital security) is affordable to consumers and firms: recall that firms in these economies have increased expenditures on digital data security; that the cost of utilisation of energy needs to be affordable: recall that the generation and utilisation of digital services is energy intensive; and that there is a need to sustain rigorous research like the potential effects of digital currencies and other digital and non-digital payments on financial inclusion. The existing foundations which provide financial support for scientific research as well as governments need to support generously, rigorous scientific research such as the present study, as well as future rigorous research which are a continuation of the present study urgently.

The results of the present study further include, inter alia, derivations of the conditions for the short run and long run potential effects of digital currencies (viz., the imminent novel central bank digital currencies and private cryptocurrencies) on financial inclusion in the EU economies to be asymmetric and for the corresponding potential effects to be symmetric. If the effects are symmetric, then the magnitudes of these potential effects are equal for increases and decreases in the levels of digital currencies. This implies that the policies for shoring up financial inclusion to a given target level involving increases in the levels of utilisation of digital currencies needs to be equal to those involving decreases in the levels of utilisation of digital currencies than for decreases (increases), and lower (higher) for decreases (increases) in the utilisation of digital currencies than for increases in digital currency utilisation. Hence, to shore up financial inclusion, policy needs to take needs to into account these magnitudes in the asymmetric effects.

The conditions for the prevalence of a cyclical (oscillatory or fluctuating) behaviour of financial inclusion are also derived in the present study. When these conditions actually prevail in the economies of the European Union, then the policies directed at sustaining increased levels of financial inclusion need to be counter-cyclical. As an exemplar, during periods of upswings in the levels of the inequality in the distribution of access to financial services (viz., reflecting lower levels of financial inclusion), the counter-cyclical policies which need to be implemented are those which attenuate or vitiate the levels of inequality in financial services' access. The results in Table 4.1 could be employed ingeniously or creatively to design and implement such counter-cyclical policies, and vice versa.

Additionally, the present study's results include a delineation of the rationale for the creation, issue and circulation of the aforementioned novel central-bank digital currency. This includes the need to increase access to financial services, to increase banking penetration, to reduce the financial challenges of firms, consumers and public organisations due to wide geographic dispersion of inhabitants per square kilometer, to increase digital readiness so as to avoid the potential of private cryptocurrencies to precipitate the obsolescence of the currently prevailing financial sector, to lower the costs and while augmenting the level of efficiency of the banking system, to maintain the central bank's control over monetary policy and macroeconomy policy, to prevent or reduce the level of money laundering and terrorism financing and for general security reasons, to achieve and sustain the digital protection of firms and consumers so as to fill in any vacuum created by a dwindling of the utilisation of conventional fiat money or cash to satisfy the conventional transactionary, precautionary and speculative motives for holding fiat money. All these reasons are inextricably linked to the promotion of financial inclusion. The utilisation of mobile money transfers and private cryptocurrencies tend to induce a vitiation of the utilisation of cash. All these have the need to analyse the potential effects of digital currencies on financial inclusion in the present study; there is therefore an urgent need to continue conducting such rigorous scientific research and those pieces of rigorous theoretical and empirical research which focus on themes related to the present study. As an exemplar, the continuation of the present study and related further scientific research themes could benefit a great deal from generous financial support for their apt, prompt and immediate execution.

## 5. Concluding Remarks

This study has modelled, in a novel manner, the potential short run and long run effects of digital currencies (viz., the imminent novel central bank digital currencies and private cryptocurrencies) on financial inclusion in the economies in the European Union. The model is based on a novel general theory which is constructed in the present study. The novel conditions for symmetric and asymmetric behaviour of these effects, their transmission mechanisms, and the novel conditions for the prevalence of cyclical (oscillatory) behaviour of financial inclusion and its structure are also rigorously analysed. The current state of adoption of ICTs and digital services by firms and consumers, and the perceptions and expectations concerning these, digital currencies, and financial inclusion in economies in the European Union, as well as the potential effects of cyber insecurity, the cost of maintaining cyber security, energy costs, government administrative instruments for controlling potential evasion of a non-hyperinflationary tax (for generating seignorage), conventional domestic currency filteration (net outflows), and conventional foreign currencies' exchange rates on financial inclusion in these economies are also rigorously analysed in the present study. Finally, fruitful policies which flow from the analyses concerning how digital currencies and other factors can contribute towards increasing the level of financial inclusion are carefully proffered. A table which delineates a novel typology of economies in respect of the potential effects of digital currencies and other factors on financial inclusion which comes in handy in the efficient design and implementation of these fruitful policies is also proferred in the present study.

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