Interdependence, Contagion and Speculative Bubbles in Cryptocurrency Markets *

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Abstract

This study empirically examines the interdependence, contagion effects and asset bubbles in cryptocurrency markets during 2015-2020. After detecting several speculative bubbles, this study investigates the role of the biggest bubble – that of 2017. All the findings show an increase in both the interdependence (time-varying correlations) and the contagion (volatility spillovers) after the 2017 bubble burst. The higher the interlinkages, the easier it is to transfer risk. There is also evidence that while past contagion matters, contemporaneous contagion is more relevant. To illustrate the impact of the 2017 bubble on interdependence and contagion, this paper studies the variance and tail risks of a global minimum variance portfolio. Both risk metrics are higher in the post-bubble period than in the pre-bubble period.

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1 Introduction

In March 2017, the prices of Bitcoin, Ethereum, and Ripple – the three largest cryptocurrencies by market capitalization – started to increase at an unprecedented rate. The price of Bitcoin went from about $1,000 in early March to $19,497 on December 16, 2017 – its all-time high at the time of writing. Likewise, between mid-March 2017 and early January 2018, the value of Ethereum and Ripple increased 73 and 563 times, respectively. This unprecedented price growth was followed by a violent bursting of the cryptocurrency bubble in mid-January 2018: massive sell-offs caused prices to decline by approximately 82% in the following three months. In the aftermath of the 2017 bubble, the co-movement of cryptocurrency prices and volatility increased instead of decreasing. Has the bursting of the 2017 bubble changed the relationship among the digital currencies and caused contagion among them?

Recently, the academic literature has examined interdependence and contagion among cryptocurrencies. Some studies find interlinkages and volatility spillovers among the biggest cryptocurrencies – including Bitcoin, Ethereum, and Ripple – using Diebold and Yilmaz (2012)'s method (Corbet et al., 2018b; Yi et al., 2018; Koutmos, 2018; Ji et al., 2019), an Autoregressive Distributed Lag model (Ciaian et al., 2018), wavelet methods (Qureshi et al., 2020), and a portfolio-based analysis (Shahzad et al., 2020). Relatively less is known about interdependence (Katsiampa, 2019), or contagion (Beneki et al., 2019; Katsiampa et al., 2019; Bazán-Palomino, 2021; Caporale et al., 2021) between virtual monies based on a multivariate (BEKK) GARCH model. Further, studies focusing on asset bubbles in cryptocurrency markets are scarce. While the occurrence of past bubbles has been detected (Cheah and Fry, 2015; Cheung et al., 2015; Fry and Cheah, 2016), there is very little documented evidence of the last major bubble in 2017 (Corbet et al., 2018a; Bouri et al., 2019) or more recent bubbles.

Most previous studies focus solely on interdependence, contagion, or asset bubbles. Their analysis lacks a comparison of different methods for distinguishing between interdependence and contagion among cryptocurrencies, and for determining whether the 2017 bubble changed the interlinkages and contagion effects among the biggest cryptocurrencies. This paper aims to fill this gap by simultaneously studying interdependence, contagion, and asset bubbles. To the best of the author's knowledge, this is the first study to address the role of speculative bubbles in the relationship among Bitcoin, Ethereum, and Ripple from 2015 to 2020; to provide a comparison of different measures of interdependence and contagion; and to apply two alternative econometric tools to datestamp price bubbles in cryptocurrency markets.

A pertinent question is how to measure interdependence, contagion, or both. The existing empirical literature on financial contagion (Forbes and Rigobon, 2002; Chiang et al., 2007; Kallberg and Pasquariello, 2008; Baele and Inghelbrecht, 2010) suggests that the correlation coefficient appears to be adequate for assessing interdependence. However, a lack of consensus about how to define contagion means that there is no single technique for measuring it. For the purpose of this study, interdependence is assessed by the statistical analysis of the correlation coefficient, while contagion is assessed by the transmission of volatility (volatility spillovers) among virtual monies. Shift-contagion (Forbes and Rigobon, 2002), a significant increase in cross-market correlation after a shock to one market, lies in the middle of these two definitions.

Investigating interdependence and contagion during the different phases of the 2017 cryptocurrency bubble is relevant for risk management, portfolio allocation, derivative pricing, and the blockchain network (e.g. miners can decide which cryptocurrency to support based on the level of interdependence). Furthermore, examining whether the 2017 bubble leads to greater interdependence and contagion among the top three cryptocurrencies allows market participants and scholars to improve the prediction of market crashes and to explore the dynamics of the transmission of volatility and shocks.

During 2015-2020, cryptocurrency markets were likely to experience multiple bubble episodes. A standard definition of a bubble is large movements in asset prices unrelated to fundamentals. However, cryptocurrencies lack intrinsic values (Baek and Elbeck, 2015; Klein et al., 2018; Huang et al., 2019). For this reason, to identify boom-bust periods, this study applies the algorithms of Bai and Perron (2003) (hereafter, BP henceforth) and Phillips et al. (2015) (hereafter, PSY) because they do not need a fundamental value specification of an asset. The results of the break-dates and proximity of the price bubbles suggest that they all belong to the same speculative process, which started in March 2017 and ended in January 2018. The present paper defines this period as the 2017 bubble, the biggest bubble in cryptocurrency markets at the time of writing.

A key new finding is that the interdependence and contagion effects have been stronger in the post-2017 bubble period. The increase in the time-varying correlations suggests higher interlinkages than before, diminishing the benefits of diversification. The greater contagion in the aftermath of the 2017 bubble suggest that this is a permanent phenomenon associated with that bubble. The higher the interdependence, the easier it is to transfer idiosyncratic risk between digital currencies.

To illustrate the impact of the 2017 bubble on interdependence and contagion, this paper studies the properties of the global minimum variance portfolio (the lower risk bound). It provides evidence that the 2017 bubble magnified the variance risk and the tail risk (Gaussian and non-parametric) of the portfolio. Further, both risk metrics are bigger during the post-bubble period in comparison with the levels during the pre-bubble period. In addition, after the 2017 bubble burst, investors reallocated their money to Bitcoin and Ripple to minimize both the volatility and tail risks.

This study contributes to the existing literature in several ways. First, it sheds light on the role of the 2017 bubble in cryptocurrency markets. Previous literature has found cryptocurrency bubbles (Cheung et al., 2015; Corbet et al., 2018a; Bouri et al., 2019) but has not examined their impact on interdependence and contagion among Bitcoin, Ethereum, and Ripple.

Second, it provides a comparison of four multivariate volatility models to assess interdependence and contagion in cryptocurrency markets, a comparison that is absent in previous literature (Mighri and Alsaggaf, 2019; Beneki et al., 2019; Katsiampa et al., 2019; Bazán-Palomino, 2021; Caporale et al., 2021). The current article presents two types of models: the first can assess shift-contagion, while the second can account for a wider definition of contagion. The primary motivation for estimating the time-varying correlations and volatility series by way of different frameworks is to examine how they change under different assumptions of volatility causality (the details will be discussed in Section 3).

Third, it identifies short-lived bubbles and structural breaks after 2017. In this regard, this study complements the literature on speculative bubbles based on the Phillips-Shi-Yu method (Cheung et al., 2015; Bouri et al., 2019) because it extends the sample period and finds asset bubbles after 2017. Likewise, this study complements the scarce literature on structural breaks in Bitcoin returns (Thies and Molnár, 2018; Bouri et al., 2019), because it finds unknown break-dates not only in Bitcoin returns but also in Ethereum and Ripple returns before, during, and after the 2017 bubble.

Fourth, this paper analyzes three types of volatility transmission: past, contemporaneous, and simultaneous. In particular, contemporaneous volatility transmission between cryptocurrencies is overlooked in the literature to date, which has focused solely on past information. Fifth, in addition to the aforementioned empirical contributions, this study constitutes a methodological novelty in its examination of contemporaneous volatility transmission. It applies a 90-day rolling-window Heterogeneous Autoregressive (HAR) model of volatility to investigate the importance of bidirectional effects of volatility series and how they change during the different phases of the 2017 bubble. This methodology is particularly well-suited to dealing with nonlinearities or parameter instability in the data, as suggested by the results of structural breaks and bubble periods. To the best of the author's knowledge, this is the first study that uses a rolling-window HAR analysis to investigate volatility spillovers in cryptocurrency markets.

Last but not least, the study presents methods to distinguish between shift-contagion and contagion that could be applied in the financial contagion literature. One strand of this literature mainly applies a Dynamic Conditional Correlation GARCH (DCC-GARCH) model (e.g. Chiang et al. (2007)) to estimate the time-varying correlation and then test for shift-contagion. Another strand of this literature considers excess correlation, i.e., the increase in the correlation above what one would expect from fundamentals, to test for contagion – Kallberg and Pasquariello (2008) and Bekaert et al. (2009), just to name a few. This study applies four multivariate GARCH models, including the DCC-GARCH method, and the estimated correlation between the return residuals could be seen as a particular case of excess correlation.

The remainder of the paper proceeds as follows. Section 2 describes the data and statistics of cryptocurrency returns. Section 3 presents the econometric methods for identifying cryptocurrency bubbles, the multivariate volatility models, and the HAR model. Section 4 shows the main findings, and illustrates the impact of the 2017 bubble on interdependence and contagion. Section 5 discusses the main findings, and section 6 concludes.

2 Data and Descriptive Statistics

The data used in this study are daily closing prices of Bitcoin (BTC), Ethereum (ETH), and Ripple (XRP), the largest and most liquid cryptocurrencies. As of January 2020, the cryptocurrency market capitalization was $ 828.5 billion, and BTC (67.9%), ETH (7.3%), and XRP (4.27%) represented 79.5% of the total market value.

All prices are U.S. dollar denominated and were collected from CoinMarketCap. The sample period goes from August 8, 2015, to January 6, 2020, yielding 1,613 observations. Following the conventional approach, the cryptocurrency returns are calculated as the first difference of the natural log of each cryptocurrency price, and the returns are expressed as percentages.

Table 1 presents the basic statistics (Panel A), the optimal ARMA (p,q) specification (Panel B), and the unit root test results (Panel C) for the daily returns on BTC, ETH, and XRP. Measured by the average return and standard deviation, Bitcoin offers the same expected return as Ripple but with half the risk, while Ethereum is as risky as Ripple but with a higher expected return. Moreover, all the returns fall outside a normal distribution because the null hypothesis of the Jarque-Bera test is rejected. The negative skewness of Bitcoin indicates that an investor expects few large losses but more frequent gains, while the positive skewness of Ethereum and Ripple suggests that large returns are more likely to be negative than positive. The kurtosis of Bitcoin and Ethereum are slightly greater than 3, while Ripple exhibits a really high value resulting in extremely low or high returns.

Based on the Akaike (AIC), Schwarz (SIC) and Hannan–Quinn (HQ) information criteria, the conditional mean for Bitcoin and Ethereum is a constant and the AR(1) term in the Ripple mean equation is statistically significant [1]. In addition, the Ljung-Box test applied to the residuals from an AR(p) fit indicates that the Bitcoin innovations are not serially correlated but that the Ethereum and Ripple innovations are. The LM test for ARCH effects reveals that the null hypothesis is rejected for all the time series and at different lag values.

To evaluate the stationarity of the returns, this paper applies the Augmented Dickey-Fuller (ADF) test, the Dickey-Fuller GLS (DF-GLS) test, and the Zivot-Andrews (ZA) test. The ZA is superior to ADF and DF-GLS because it can account for one unknown structural break in the data generating process. All the tests confirm the stationarity of returns, and the ZA test finds a structural break for Bitcoin, Ethereum, and Ripple on 12/16/2017, 01/13/2018, and 01/07/2018, respectively. The break-dates are an unanticipated finding because they coincide with the dates of their maximum prices.

The present paper is interested in the joint generation mechanism of the cryptocurrencies and thus a VAR(p) model was specified for the mean equation. Based on the information criteria (AIC selects a VAR(2) model, while SIC and HQ choose a VAR(1) specification), the residuals of a VAR(1) model were obtained for two purposes: (1) to model the dynamic dependence of the variance-covariance matrix with different multivariate volatility methods, and (2) to use them as proxy variables of volatility in the causality tests (the details will be discussed later).

[1] The statistical analysis of the cryptocurrency returns reveals skewness, excess kurtosis, and other non-normal features. To account for non-normality issues, univariate GARCH(1,1) models with different distributions (normal, t-student, and skewed-t) are utilized for each return series. The three information criteria confirm that t-Student innovations better fit the three tokens: AR(0)-GARCH(1,1) for Bitcoin and Ethereum and an AR(1)-GARCH(1,1) for Ripple. The results are available upon request.

3 Methodology

3.1 Identification of Speculative Bubbles

To examine how interdependence and contagion evolve during the different phases of a bubble, this study first conducts an econometric investigation of the existence of bubbles. This study defines a bubble as an explosive behavior of an asset price. As Figure 1 shows, cryptocurrency returns (right panel) and prices (left panel) underwent rapid changes during 2015-2020.

As a starting point and motivated by the ZA test results in Table 1, the identification of more unknown structural breaks in the return series is carried out by the Bai and Perron (2003) (BP) method. One can consider the estimation of m multiple structural breaks by least squares as:

$$y_t = z_t' \delta_j + u_t \tag{1}$$

for $j = 1, ..., m+1$, where y_t is an endogenous variable (cryptocurrency returns), z_t is a matrix of explanatory variables (lagged value of cryptocurrency returns) which records the structural changes, δ_j is the vector of parameters, u_t is the disturbance term, and $t = T_{j-1,...,}T_j$ are the unknown break-dates. Bai and Perron develop a sequential procedure to estimate the number and location of break-points (T_1, ..., T_m) and the parameters (δ_1, ..., δ_{m+1}). The first break-date is identified as the one that minimizes the sum of squared errors (and the sup F statistic). Given an estimated break-point, the sample is divided into two segments, and the procedure tests for breaks in each sub-sample. If a new break is found, the process is repeated sequentially for each partition.

To complement the analysis of structural breaks, the dating algorithm of Phillips et al. (2015) (PSY) is used to overcome the asset bubble definition based on fundamental value[2]. The Generalized Supremum Augmented Dickey–Fuller (GSADF) test of PSY is based on a rolling-window ADF-style regression:

$$x_t = \alpha^0_{rw} + \alpha^1_{rw} x_{t-1} + \sum_{i=1}^{p} \phi^i_{rw} \Delta x_{t-i} + \varepsilon_t \tag{2}$$

where x_t is the cryptocurrency price, α_0, α_0, ϕ^i_{rw} are parameters estimated using OLS, p is the number of lags, ε_t is the innovation, and $rw = r_2 - r_1$ is a rolling window that starts and ends respectively with a fraction r_1 and a fraction r_2. The null is $H_0 : \alpha^1_{rw} = 1$ and the alternative is $H_a : \alpha^1_{rw} > 1$. The GSADF test statistic is $GSADF(r_0) = \sup_{r_2 \in (r_0,1)} SADF_{r_2}(r_0)$, where $SADF_{r_2}(r_0) = \sup_{r_2 \in (r_0,1)} ADF_0^{r_2}$.

The PSY procedure is designed to detect the stochastic explosive behavior of an asset price. The approach identifies the start of a speculative bubble when there is a structural break from a random walk to an explosive regime ($\alpha^1_{rw} > 1$). Similarly, the method considers the end of a bubble when it finds a structural break from an explosive regime to a random walk ($\alpha^1_{rw} = 1$). In this manner, the PSY technique allows one to avoid

[2]With regard to the fundamental value of a cryptocurrency, its price is between zero (because its intrinsic value is zero; Cheah and Fry (2015), and Cheung et al. (2015)) and a positive number, assuming that the demand is growing over time and the supply is fixed.

modeling the fundamental value of Bitcoin, Ethereum, and Ripple in order to detect when a bubble occurs and collapses.

3.2 Multivariate GARCH Models

Since correlation analysis has been used to assess interdependence between financial assets or markets, it is worthwhile to make a comparison of the estimated time-varying correlation by different multivariate volatility models. To start with, let $P_{i,t}$ be the price of cryptocurrency i at time t, $r_{i,t}$ be the logarithmic return ($\ln P_{i,t} - \ln P_{i,t-1}$), and r_t be the $k-dimensional$ ($k = 3$) multivariate return series.

Analogous to the univariate case, r_t can be decomposed as:

$$r_t = \mu_t + a_t \tag{3}$$

where $\mu_t = E(r_t \mid F_{t-1})$ is the conditional mean equation of $r_t = (r_{1,t}, r_{2,t}, r_{3,t})'$ given the available information F_{t-1}, and $a_t = (a_{1,t}, a_{2,t}, a_{3,t})'$ is the shock of the series at time t. Because this study focuses on the multivariate volatility modeling and cryptocurrencies are not directly linked to any fundamental (Baek and Elbeck (2015), Klein et al. (2018), and Huang et al. (2019)), a simple VARMA(p,q) structure is sufficient to model μ_t (Bauwens et al. (2006), and Tsay (2010)).

The volatility matrix of r_t is the conditional covariance matrix of a_t given F_{t-1}, $\Sigma_t = Cov(a_t \mid F_{t-1})$, which is a $k \times k$ positive-definite matrix. The shock can be written as $a_t = \Sigma_t^{1/2} \varepsilon_t$ where $\Sigma_t^{1/2}$ is the square-root matrix of Σ_t, and ε_t is a sequence of an independent and identically distributed (i.i.d.) random vector such that $E(\varepsilon_t) = 0$ and $Cov(\varepsilon_t) = I_k$. With regard to the probability density function (pdf) of r_t, this study assumes two common density choices of ε_t: the multivariate Gaussian distribution and the multivariate t-Student distribution.

All the multivariate volatility models provide a framework for the time evolution of the covariance matrix and, consequently, for the dynamics of the time-varying conditional correlation coefficient. The four models described below differ in the contagion assumption. This paper distinguishes between two types of models[3]. The first type includes two models that do not allow for volatility spillovers: the Dynamic Conditional Correlation GARCH framework and the t-Copula GARCH model. Despite the lack of volatility spillovers, these methods remain popular in both the industry and academia to measure interdependence and shift-contagion in financial markets. The second category covers two models that allow for volatility transmission: the Baba-Engel-Kraft-Kroner-GARCH model and the Cholesky Decomposition GARCH-type method.

3.2.1 First type: no volatility spillovers

Dynamic Conditional Correlation GARCH (DCC-GARCH). The DCC-GARCH model of Engle (2002) is built on the idea of modeling conditional variances and correlations instead of modeling Σ_t. To that

[3]For further details on all multivariate volatility specifications described in this section, readers are referred to Bauwens et al. (2006) and Tsay (2010), and to the references therein.

end, Σ_t is decomposed into conditional standard deviations (D_t) and a conditional correlation matrix (ρ_t) as:

$$\Sigma_t = D_t \rho_t D_t \ \left(\text{ or } \rho_t = D_t^{-1} \Sigma_t D_t^{-1} \right) \tag{4}$$

where $D_t = diag\left\{ \sigma_{11t}^{1/2}, ..., \sigma_{kkt}^{1/2} \right\}$ is the diagonal matrix of the k volatilities at time t, and ρ_t is the correlation matrix with $k(k-1)/2$ elements. The DCC-GARCH approach involves two steps. The first step is to individually model each element of D_t (σ_{iit}) using a univariate GARCH specification and form estimated standardized residuals. The second step is to model the pairwise conditional correlations between the standardized residuals as:

$$Q_t = (1 - \theta_1 - \theta_2) Q + \theta_1 Q_{t-1} + \theta_2 \epsilon_{t-1} \epsilon'_{t-1} \tag{5}$$

where Q_t is the covariance matrix of standardized residuals, Q is the unconditional covariance matrix of standardized residuals, and θ_1 and θ_2 are non-negative real numbers satisfying $0 < \theta_1 + \theta_2 < 1$. The correlation matrix is defined as $\rho_t = J_t Q_t J_t$, where $J_t = diag\left\{ q_{11t}^{-1/2}, ..., q_{kkt}^{-1/2} \right\}$ is a normalization matrix and $q_{ii,t}$ denotes the $(i,i)th$ element of Q_t.

It should be noted that θ_1 and θ_2 describe the dynamic dependence of the correlation matrix, and make the DCC-GARCH model very parsimonious. These two parameters are independent of the number of series to be correlated, offering a clear computational advantage.

t-Copula. Similar to DCC-GARCH models, the copula methodology is based on the specification of univariate marginal GARCH models for each asset return, and a copula function capturing the dependence between the different asset returns. The two most frequently used copulas for financial modeling are the Gaussian Copula and the t-Copula. The multivariate t-Student distribution better captures the dependence of extreme values, which is observed in cryptocurrency returns. For this reason, the present study applies a t-Copula. The idea of a t-Copula is to decompose a $k - dimensional$ joint distribution function into its $k - marginal$ distributions (the parts that only describe the marginal behavior), and a copula function that traces the dependence among the k variables (the parts that describe the dependence among variables).

Creal et al. (2011) define R_t as the $k \times k$ correlation matrix that can be decomposed as $R_t = X'_t X_t$, where X_t is an upper triangular matrix. Each element of X_t is a combination of $c_{ij,t} = cos(\theta_{ij,t})$ and $s_{ij,t} = sin(\theta_{ij,t})$ with $\theta_{ij,t}$ being a time-varying angle measured in radians and $1 \leq i < j \leq k$. Let θ_t be the $k(k-1)/2 - dimensional$ vector consisting of all the angles $\theta_{ij,t}$. The time evolution of θ_t instead of R_t can be modelled to describe the time-varying correlations. The law of motion for the angles is:

$$\theta_t = \theta_0 + \lambda_1 \theta_{t-1} + \lambda_2 \theta^*_{t-1} \tag{6}$$

where is θ_0 denotes the initial values of the angles, λ_1 and λ_2 are non-negative real numbers such that

$\lambda_1 + \lambda_2 < 1$, and θ^*_{t-1} is a local estimate of the angles using data $\{\varepsilon_{t-1}, ..., \varepsilon_{t-m}\}$ for some $m > 1$.

Overall, the method entails two steps. In the first step, $\sigma_{i,t}$ of $a_{i,t}$ is estimated via univariate GARCH models to obtain the standardized innovations. In the second step, equation (6) is fitted with a t-Copula for the standardized innovations using the pdf of a t-Copula with standardized t-Student margins.

3.2.2 Second type: volatility spillovers

Baba-Engel-Kraft-Kroner-GARCH (BEKK-GARCH). In the BEKK-GARCH(1,1) model of Engle and Kroner (1995), the covariance matrix evolves according to:

$$\Sigma_t = A_0 A_0' + A_1 a_{t-1} a_{t-1}' A_1' + B_1 \Sigma_{t-1} B_1' \tag{7}$$

where A_0 is a lower triangular matrix such that $A_0 A_0'$ is a positive-definite matrix, and A_1 and B_1 are $k \times k$ matrices. The diagonal elements of matrices A_1 and B_1 capture the impact of past innovations and past volatility, respectively. The off-diagonal elements of A_1 capture the shock transmission effects, while the off-diagonal elements of B_1 measure the volatility spillover effects. The main advantages of this approach is that it allows for dependence between the volatility series, and solves the problem of positive-definite constraint (Σ_t is positive-definite for all t). On the other hand, the main disadvantages of this model are the increasing number of parameters with the addition of more variables ($k^2 + [k(k+1)/2]$), and the fact that the parameters in A_1 and B_1 do not have a direct interpretation.

Cholesky Decomposition (Cholesky-GARCH). This framework performs linear orthogonal transformations of the shocks (a_t) via recursive least squares regressions, involving a reparametrization of Σ_t. Given that Σ_t is positive-definite, there must be a lower triangular matrix L_t with unit diagonal elements and a diagonal matrix G_t with the positive entries of the main diagonal, such that:

$$\Sigma_t = L_t G_t L_t' \tag{8}$$

where $q_{ij,t}$ is the $(i,j)th$ element of L_t and the entries of G_t contains variance components denoted by $g_{ii,t}$. The orthogonal transformation from a_t to b_t, where $b_{1t} = a_{1t}$, and b_{it} for $1 \leq i \leq k$, can be computed recursively by sequential least squares regressions as:

$$a_{it} = q_{i1,t} b_{1t} + q_{i2,t} b_{2t} + ... + q_{i(i-1),t} b_{(i-1)t} + b_{it} \tag{9}$$

for $q_{ij,t}$ satisfying $1 \leq i < k$. Note that $b_t = L_t^{-1} a_t$ and $Cov(b_t) = L_t^{-1} \Sigma_t \left(L_t^{-1}\right)' = G_t$. In matrix form, the orthogonal transformation from $a_t = (a_{1,t}, a_{2,t}, a_{3,t})'$ to $b_t = (b_{1,t}, b_{2,t}, b_{3,t})'$ is as follows:

$$
\begin{bmatrix} 1 & 0 & 0 \\ -\beta_{21} & 1 & 0 \\ -\beta_{31} & -\beta_{32} & 1 \end{bmatrix} \begin{bmatrix} a_{1,t} \\ a_{2,t} \\ a_{3,t} \end{bmatrix} = \begin{bmatrix} b_{1,t} \\ b_{2,t} \\ b_{3,t} \end{bmatrix} \tag{10}
$$

where β_{21}, β_{31}, and β_{32} are estimated by OLS. The volatility causality in the Cholesky-GARCH framework depends on the ordering in a_t (Tsay, 2010).

3.3 Heterogeneous Autoregressive (HAR) model of volatility

To delve into the idea of contemporaneous causality in volatility, this paper adopts the Heterogeneous Autoregressive (HAR) model of volatility proposed by Corsi (2009). The HAR model leads to a simple AR-type specification in the estimated volatility with the feature of considering different volatility components over different time horizons:

$$
y_t = \alpha + \alpha_d y_{t-1} + \alpha_w Y^w_{t-1} + \alpha_m Y^m_{t-1} + \beta_0 x_t + \beta_d x_{t-1} + \beta_w X^w_{t-1} + \beta_m X^m_{t-1} + \varepsilon_t \,, \tag{11}
$$

$$
Y^h_t = \frac{1}{n_h} \sum_{i=0}^{n_h-1} y_{t-i} \qquad \text{and} \qquad X^h_t = \frac{1}{n_h} \sum_{i=0}^{n_h-1} x_{t-i} \,,
$$

where $h = \{w, m\}$ denotes weekly and monthly volatility, respectively, n is the number of days within an interval of time h, Y^h_{t-1} and X^h_{t-1} are the weekly and monthly estimated volatilities, respectively, and ε_t is a random disturbance term.

Equation (11) can be seen as a multiple-factor volatility model, where the factors are directly the past estimated volatilities viewed at different frequencies. The intuition behind equation (11) is that agents with daily, weekly, and monthly trading frequencies perceive, respond to, and cause different types of volatility components.

This paper is interested in the parameter $\hat{\beta}_0$ since it captures the impact of one volatility series on the other at time t. For instance, the impact of the ETH volatility on the BTC volatility at time t. Next, a 90-day rolling window is used to estimate $\hat{\beta}_0$. The analysis based on the rolling estimation aims to help understand the transmission of volatility during the different phases of the 2017 cryptocurrency bubble. Also, this method is particularly well-suited to dealing with nonlinearities or parameter instability.

4 Empirical Results

4.1 The 2017 bubble and short-lived bubbles

As can be seen from Table 2, the BP method estimates five break-points in each return series, reported from the biggest to the smallest breaks. Chronologically, the returns underwent the first break between June and

September 2016; the second break between March and May 2017; the third break between December 2017 and January 2018; the fourth break between August and September 2018; and the fifth break between April and May 2019.

The PSY method detects 16, 19, and 8 episodes of bubbles in Bitcoin, Ethereum, and Ripple prices, respectively, and their total days of explosivity are 410, 327, and 79, respectively. The explosive periods represent a significant fraction of the price behavior of Bitcoin (25%) and Ethereum (20%), but not of Ripple (5%). The duration of the bubbles ranges from 2 to 122 days for Bitcoin, between 1 and 92 days for Ethereum, and from 1 to 31 days for Ripple. Nevertheless, Panel B of Table 2 reports bubble periods that lasted at least 7 days, because bubbles that only take 1 or 2 days could be an overfitting of the method [4].

What stands out in this table is that a significant number of bubbles occurred in 2017. Bitcoin and Ethereum prices experienced 66% and 71% of the total bubble episodes in 2017, respectively. One unanticipated finding is that all Ripple price bubbles took place consecutively between March and December 2017. Moreover, the explosive episodes of all cryptocurrencies overlapped with each other during this year. In particular, the two longest-lasting bubbles in Bitcoin, Ethereum, and Ripple occurred in 2017.

Taken together, the BP and PSY results provide important insights into the timing of the cryptocurrency speculative bubbles. Most of the breaks in the cryptocurrency returns occurred between March 2017 and January 2018. Regarding explosive behavior, all cryptocurrency prices experienced multiple bubbles between March 2017 and January 2018. Given this evidence, it is natural to assume that they all correspond to the same speculative price growth process. Each of the speculative bubble periods – the Bitcoin bubble lasted 270 days, while the Ethereum and Ripple bubbles lasted 233 days and 79 days, respectively – are added together without counting the overlapping days and considered as a single bubble. On average, the "cryptocurrency bubble" started in the first week of March 2017 and ended in the second week of January 2018. This study defines this period as the 2017 bubble.

It should be mentioned that the findings for the bubbles until 2017 are consistent with Bouri et al. (2019). However, the present study provides new evidence of explosive periods of price exuberance after 2017. The PSY procedure identifies a couple of short-lived bubbles in Bitcoin and Ethereum prices during June and July 2019, which might be related to two relevant market events in June 2019. In the first, Facebook announced its ambitious plan to create Libra, its own cryptocurrency. In the second, 'TheBoot,' a millionaire who has traded Bitcoin in BitMEX, revealed that he was able to turn US$ 3,000 into US$ 300 million by trading cryptocurrency derivatives.

[4] It might be the case that there are one- or two-days bubbles in cryptocurrency markets. For instance, on June 21, 2017, Ethereum briefly suffered a flash crash from around $319 to 10 cents within a day, but it later rebounded. Similarly, Bitcoin lost more than 30% of its value within a day, on December 22, 2017, despite a subsequent rebound.

4.2 Statistical analysis of correlation coefficients in different phases of the 2017 bubble

Once the speculative bubbles were identified, the next step is to measure interdependence and contagion and examine whether they changed over time. Figures 2, 3, and 4 show the fitted Bitcoin-Ethereum (BTC-ETH), Bitcoin-Ripple (BTC-XRP), and Ethereum-Ripple (ETH-XRP) correlation series, respectively. One of the most surprising findings is that all the multivariate volatility models produce correlation estimates above their corresponding sample means from late January 2018. After that point, there was a significant increase. As expected, the correlations of BEKK and Cholesky methods are more volatile than those of the other specifications, and the DCC and t-Copula models produce correlation series with similar patterns.

Figure 2 indicates that the correlation between Bitcoin and Ethereum showed an oscillatory pattern until the end of 2017, but was high and stable thereafter. In particular, the pattern of the BEKK correlation between Bitcoin and Ethereum until 2017 is in line with previous findings (Katsiampa et al., 2019; Beneki et al., 2019).

As for the correlation between Bitcoin and Ripple (Figure 3), it became stronger from late January 2018. However, the correlation coefficient went back to its sample mean in October 2018 and October 2019. In turn, the correlation of Ethereum with Ripple (Figure 4), on average, was low until December 2017. Since then, the correlation remained at values greater than 0.8.

To check the adequacy of the multivariate volatility models, the multivariate Ljung-Box and Lagrange Multiplier tests were applied to the standardized residuals. Both statistics fail to reject the null hypothesis (Table 3). In summary, the fitted multivariate volatility models seem adequate.

The dynamics of the correlation coefficients are at the core of contagion literature (Forbes and Rigobon, 2001; Bekaert et al., 2009). Shift-contagion is based on a significant increase in cross-market correlation. That is, the co-movement between two variables is different under extreme conditions (market turmoil) and in normal times, and it is contagion only if the correlation rises significantly. Thus, the hypothesis of shift-contagion is accepted if there is evidence of a dynamic rise in correlation coefficients. As Figures 2, 3, and 4 show, the time-varying correlations increased significantly in the aftermath of the 2017 bubble (from January 2018). The fact that all the correlations jumped above their sample means and remained at a high level after the bubble crashed is an indicator of shift-contagion.

To further examine shift-contagion, the volatility-adjusted correlation coefficients proposed by Forbes-Rigobon are calculated. Figure 5 only compares the volatility-adjusted correlation and the BEKK correlation series, but the comparison with those of the other multivariate GARCH specifications is qualitatively similar. What stands out in Figure 5 is that one cannot make a clear distinction between the two correlation coefficients; both time series are essentially the same. This finding indicates that both types of volatility models are appropriate for assessing shift-contagion, including the heteroskedasticity correction developed by Forbes and Rigobon, without dividing the sample into two or more sub-samples.

Finally, following the method used by Ratner and Chiu (2013), the BEKK estimated correlation ρ_t is extracted into a separate time series for each pair of cryptocurrencies. Then, ρ_t is regressed on dummy variables representing the different phases of the 2017 bubble:

$$\rho_t = \gamma_0 + \gamma_1 D_1(bubble) + \gamma_2 D_2(post - bubble) + \varepsilon_t, \tag{12}$$

where D_1 represents a dummy variable that captures the 2017 bubble period, and D_2 represents a dummy variable that captures the post-2017 bubble period.

Table 6 presents the coefficient estimates from the regression model specified in equation (12), and all the parameters are statistically significant at 1%. A significant positive value of γ_2 indicates that all correlation series increased considerably after the bubble crashed. This finding is consistent with the co-movement paths shown in Figure 5.

In summary, this combination of results provides support for a significant increase in both interdependence and shift-contagion in the post-2017 bubble period. It is important to note, however, that volatility could be transmitted from one market to another without requiring a significant change in cross-market relationships. The next section addresses this issue.

4.3 Volatility contagion during the different phases of the 2017 bubble

As stated before, there is no consensus on how to define contagion in financial markets (Forbes and Rigobon, 2001; Kallberg and Pasquariello, 2008; Bekaert et al., 2009), and shift-contagion is a restrictive definition (Baele and Inghelbrecht, 2010). Instead of trying to solve the contagion definitional problem, the objective of this section is to present convincing evidence of the amount of volatility contagion that took place in cryptocurrency markets during the pre-, post- and 2017 bubble period. For this purpose, this paper defines contagion as the transmission of volatility from one cryptocurrency to the other, and studies whether it changed after the bubble crashed.

To begin with, the BEKK model allows contagion to be tested for through the off-diagonal elements of the matrices A_1 and B_1, which capture the cross-asset effects of past shocks and past volatility, respectively. If the off-diagonal elements of matrices A_1 and B_1 are statistically significant, it can be argued that there is evidence of contagion. As shown in the first column of Table 4, the diagonal parameters and some of the off-diagonal parameters in matrices A_1 and B_1 are statistically significant. Because of the quadratic form of the BEKK model, there is no simple direct link between the parameters in A_1 and B_1 and the components of Σ_t. It is difficult to give an interpretation of the signs of the coefficients in matrices A_1 and B_1 because whether they are positive or negative, the quadratic form of the BEKK model generates positive contagion effects between the variables. The value of the volatility spillovers also depends on the values of shocks a_t time t. Nevertheless, these findings reinforce the hypothesis of contagion among Bitcoin, Ethereum, and Ripple based on past information (lagged variance and lagged squared shocks).

In addition, Figure 6 depicts the time evolution of the estimated coefficients of the Cholesky-GARCH model (equation 10). It is assumed that $a_{1,t} = a_{BTC,t}$, $a_{2,t} = a_{ETH,t}$, and $a_{3,t} = a_{XRP,t}$. As is evident from the figure, there is a mutual impact between cryptocurrency shocks, which started to increase from early 2018. The other orders in a_t produce similar results.

Moving on now to consider the importance of the timing of volatility transmission, the Granger causality test and the instantaneous causality test will help one to understand whether past or current information is more important at the time of contagion. To elucidate, both causality tests are applied to three proxy variables of volatility: VAR(1) squared residuals (a_t^2), VAR(1) absolute residuals ($|a_t|$), and the BEKK estimated volatility (σ_t^{BEKK}). If there is no contagion between the volatility series, one would expect both causality tests to be rejected[5].

Table 5 shows empirical evidence that while past contagion (Granger test) is relevant, contemporaneous contagion (instantaneous test) is more significant. In Granger's sense, the evidence is mixed: Bitcoin causes Ethereum and Ripple at four-order lags, while Ethereum causes Bitcoin and Ripple at eight-order lags. However, the evidence of Ripple causing Bitcoin and Ethereum in Granger's sense is strong. On the contrary, the instantaneous causality test results confirm the contemporaneous causality in volatility, which runs in both directions regardless of the number of lags and the times series (a_t^2, $|a_t|$ or σ_t^{BEKK}) used in both tests. Overall, the Granger and instantaneous test results show that cryptocurrency volatilities affect each other, especially at time t – current information helps to predict the values of volatility series. This new evidence raises a question about the timing of contagion among cryptocurrencies.

To further explore the idea of contemporaneous causality in volatility, the HAR model – equation (11) – is estimated for a_t^2, $|a_t|$ and σ_t^{BEKK} to asses the volatility transmission at time t. This paper only presents the HAR results using σ_t^{BEKK}, but the remaining results are available upon request. All parameters are statistically significant at 1%; in particular, $\hat{\beta}_0$ which indicates that there is volatility transmission among virtual currencies at time t: ETH to BTC (0.22), XRP to BTC (0.02), BTC to ETH (0.60), XRP to ETH (0.10), BTC to XRP (0.24), and ETH to XRP (0.41). As previously stated, the 90-day rolling window estimation of equation (11) allows me to understand the transmission of volatility during the different phases of the 2017 cryptocurrency bubble, and to deal with nonlinearities or parameter instability in the data suggested by the BP and PSY results.

Figure 7 plots the dynamics of the $\hat{\beta}_0$ parameters from the rolling method. It can be seen that there was a positive volatility transmission from Ethereum to Bitcoin and Ripple, and from Ripple to Bitcoin and Ethereum which increased after the bubble burst. Likewise, the volatility spillover from Bitcoin to Ethereum was relatively high in the post-bubble period. However, there is an oscillatory pattern of the volatility causality from Bitcoin to Ripple for the full sample. All in all, the dynamics of $\hat{\beta}_0$ are consistent with the hypothesis of contemporaneous and simultaneous contagion, and there is a higher volatility transmission in the post-bubble period.

[5] The Granger and instantaneous causality tests are sensitive to the number of lags included in the regression. The three information criteria select the number of lags of the explanatory and dependent variables between 3 and 8 (AIC=13, SIC=2 and HQ=8 for a_t^2; AIC=13, SIC=3, and HQ =8 for $|a_t|$; and AIC=10, SIC=1, and HQ =1 for σ_t^{BEKK}).

4.4 The impact of the 2017 bubble on the Global Minimum Variance portfolio

To provide an illustration of the impact of the 2017 bubble on interdependence and contagion in cryptocurrency markets, this paper analyses the dynamics of optimal weights, volatility and Value-at-Risk (VaR) of a Global Minimum Variance (GMV) portfolio. Without loss of generality, the GMV portfolio of cryptocurrency returns with no short sales is studied. This portfolio has the least risk among all the portfolios of risky assets that lie on the efficient frontier. Also, the optimal weights of GMV portfolio depend only on the return variances and covariances, but not on the distribution of the returns or the estimation of expected returns. The use of GMV is very common in traditional asset markets, but the adoption of this strategy in cryptocurrency markets is still in the early stages.

The GMV portfolio is the solution to the following minimization problem:

$$\begin{aligned} \min_{w_t} w_t' \Sigma_t w_t \\ s.t. \\ w_t' \mathbf{1} = 1 \\ w_{i,t} \geq 0 \end{aligned}$$

where w_t is the vector of portfolio weights at time t, Σ_t is the BEKK volatility matrix at time t, and $\mathbf{1}$ is a 3×1 column vector whose entries are ones.

Figure 8 depicts the dynamics of the optimal weights: Bitcoin (black line), Ethereum (blue line), and Ripple (red line). Note that before the collapse of the bubble, an investor would put most of her money in Bitcoin, followed by Ripple and Ethereum. However, after January 2018, Ethereum's portfolio weight decreased to almost zero and was positive only during the small bubbles of mid-2019. In addition, investors reallocated their money to Bitcoin and Ripple after the bubble burst. These two empirical findings could be explained by the higher rise of BTC-ETH and ETH-XRP relative to BTC-XRP from April 2018, and by the higher volatility transmission from Ethereum to Bitcoin and Ripple in the post-bubble period.

When it comes to risk measures, Figure 9 shows the portfolio volatility risk (black line) and its corresponding sample mean (blue line). Clearly, portfolio volatility started to increase during the 2017 bubble period and is highest during the bubble burst, while there are multiple peaks of high volatility (above the mean) after January 2018. What stands out in this figure is that the GMV standard deviation (the lower risk bound) in the post-bubble period is greater than that in the pre-bubble period. In this vein, Figure 10 illustrates the time evolution of two potential downside risks of the GMV portfolio: the Gaussian VaR (black line) and the non-parametric VaR (blue line). Both risk metrics are higher in the post-bubble period than in the pre-bubble period. Note that the non-parametric VaR indicates that losses are more extreme and frequent than those predicted by the Gaussian VaR for the full sample.

4.5 Robustness analysis

As a robustness check, it is instructive to estimate all multivariate GARCH models over two sub-samples: August 2015 – February 2017 (pre-bubble) and April 2018 – January 2020 (post-bubble). These sub-sample periods were selected to avoid overlaps in the break-dates and/or the bubble episodes of any cryptocurrency. The present study reports the BEKK-GARCH(1,1) estimates, but the results of the other models are available upon request. As indicated previously, the BEKK model allows interdependence and contagion to be tested for at the same time.

On average, in the pre-bubble period, the correlation series were low (BTC-ETH = 0.09, BTC-XRP = 0.13, and ETH-XRP = 0.09). While the post-bubble period was characterized by high correlation series, on average (BTC-ETH = 0.85, BTC-XRP = 0.74, and ETH-XRP = 0.83). It is evident that the correlation coefficients were greater from April 2018. This finding is consistent with the results of Table 6 – equation (12) estimation. In addition, the second and third columns of Table 4 show the estimated parameters in the matrices A_1 and B_1 before and after the bubble, respectively. There are more statistically significant off-diagonal parameters in matrices A_1 (A_{13}, A_{31},A_{23}, and A_{32}) and B_1 (B_{12}, B_{23}, B_{31}, and B_{32}) in the post-bubble period, revealing that past shocks and past volatility series are more relevant to explaining the dynamics of Σ_t after April 2018. As explained previously, due to the quadratic form of the BEKK model, positive contagion effects could be generated regardless of the sign of the estimated parameters. Overall, these results further support the previous finding of stronger interdependence and contagion between digital currencies after the 2017 bubble burst.

5 Discussion

The present study finds that the time-varying correlations (interdependence) and the volatility transmission (contagion) increased significantly in the post-bubble period. It could be argued that the 2017 bubble created conditions for risk transmission, and that the high interdependence in the post-bubble period facilitated the transmission of volatility among virtual monies. During 2017, the multiple bubbles were caused by successive rounds of buyers pushing prices higher and higher. In this context, sellers are less likely to sell their digital currencies, in fact and short-horizon speculators thought they could get out in time. Following this line of reasoning, either investors could not anticipate the collapse of the 2017 bubble or the high return rewarded them for the probability of a crash.

After the 2017 bubble burst, the risk materialized and investors gradually learned the negative impact of the burst bubble on their portfolios. As in traditional financial markets, investors started to follow the crowd (Chiang et al. (2007)), leading to more uniform behavior and producing a higher correlation.

The high correlation in the post-bubble period could also indicate a surge in the integration of different cryptocurrency markets. Bitcoin, Ethereum, and Ripple are traded in different markets, and the degree of connectedness between crypto-markets could have increased since the collapse of the 2017 bubble. Therefore, Bitcoin, Ethereum, and Ripple respond to common shocks, or a shock in one virtual currency can lead to a spillover to the others. The more integrated they are, the easier it is to transfer idiosyncratic risk between them, limiting the benefits of diversification in the post-2017 bubble period. The global minimum variance portfolio analysis illustrates this phenomenon. The volatility and tail risks increased during the post-bubble period in comparison with the levels during the pre-bubble period. In addition, the bubble affected the optimal portfolio weights. In order to minimize risk, investors reallocated their money to Bitcoin and Ripple after January 2018.

Another significant aspect of the correlation results is the comparison between different types of multivariate GARCH models. The estimated conditional correlations based on the multivariate volatility models that do not allow for volatility spillovers (DCC, and t-Copula) are qualitatively similar to those implied by the models that allow for volatility spillovers (BEKK, and Cholesky). However, on quantitative grounds, cross-variance and cross-covariance spillovers are risk transmission channels, and ignoring them underestimates system-wide dependencies. In this vein, the first type of models is suitable for assessing only interdependence and shift-contagion. But the second type of models measures a wider definition of contagion because they take into account the feedback across the shocks and conditional variances.

On the question of the type of volatility transmission, the instantaneous causality test results and the HAR estimates provide strong evidence of contemporaneous and simultaneous volatility spillovers. In particular, the $\hat{\beta}_0$ coefficients of the HAR model indicate changes in causal relationship. As regards past information, past contagion is less relevant, and the Granger causality results are not conclusive. Overall, this study strengthens the idea of simultaneous and contemporaneous contagion effects among Bitcoin, Ethereum, and Ripple volatility series.

Turning now to the speculative bubbles, this study estimates unknown structural breaks in the cryptocurrency returns and datestamp price bubbles during the 2015-2020 period. Failure to recognize structural breaks can lead to invalid conclusions in terms of stationarity of the return series and stability of the parameters. Similarly, failure to recognize the impact of bubbles on the relationship between digital currencies can lead to underestimation of future consequences in terms of contagion. The existence of bubbles in cryptocurrency markets is expected because, by its nature, a digital money is a speculative asset without intrinsic value. The drivers behind bubbles are a research topic that future researchers might want to take up. As mentioned earlier, this article does not attempt to go into this.

6 Conclusions

This study examines the interdependence and contagion effects among Bitcoin, Ethereum, and Ripple from 2015 to 2020, a period that is characterized by speculative bubbles. After identifying multiple bubble episodes, this study examines the impact of the 2017 bubble on the interdependence and contagion effects in cryptocurrency markets. All multivariate volatility models and the dummy-variable regression conclude that the correlations (interdependence and shift-contagion) are higher in the post-2017 bubble period, decreasing the benefits of diversification. One possible explanation is that investors' behavior converged in the post-bubble period leading to herding behavior. Another possible explanation is that cryptocurrency markets are more integrated. Both channels could be potential mechanisms for the transmission of risk.

Regarding contagion, the comparison of multivariate volatility methods shows that the first type of model can assess shift-contagion, but the second type can account for a wider definition of contagion. In addition, the causality tests and HAR results suggest that while the lagged variance and lagged squared shocks are important sources of transmission, contemporaneous feedback between volatility series is more relevant. This new evidence raises a question about the timing and direction of contagion among Bitcoin, Ethereum, and Ripple.

Given that risk transmission between cryptocurrencies has increased after 2017, policymakers should pay close attention to new bubbles and risk transmission channels from cryptocurrency markets to financial markets. Similarly, knowing the impact of bubbles is of particular importance for investors, institutions responsible for monitoring traditional financial markets, and market participants working on risk management. For instance, policymakers and investors could define appropriate policy responses and actions, respectively, to the challenges presented at different stages of cryptocurrency bubbles.

The results presented in this paper open the door for a number of promising future research. Understanding the reasons behind the bubbles is relevant for anticipating the explosive price behavior. Identifying the factors that explain the correlation coefficients will improve market-participants' approach to modeling volatility series. Finally, the methods for exploring volatility transmission may be suitable for analyzing contagion in bond and stock markets.

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Table 1: Descriptive statistics and unit roots tests.

	BTC	ETH	XRP
Panel A: Basic Statistics			
Mean	0.21	0.33	0.20
Standard Deviation	3.90	6.35	6.89
Skewness	-0.18	0.48	3.11
Kurtosis	4.72	5.29	44.15
Jarque-Bera Test	540.28***	620.85***	10954.37***
Ljung-Box(5)	2.80	26.46***	31.58***
LM-ARCH(5)	98.16***	128.02***	106.36***
Panel B: AR(p) for μ_t			
AIC	AR(0)	AR(1)	AR(1)
SIC	AR(0)	AR(0)	AR(0)
HQ	AR(0)	AR(0)	AR(0)
Panel C: Unit Root Test Statistics			
ADF	-17.48***	-15.84***	-9.47***
DF-GLS	-13.33***	-5.67***	-15.40***
ZA	-23.00***	-20.65***	-21.22***
ZA break date	12-16-2018	01-13-2018	01-07-2018

Notes: The sample period is from August 8, 2015 to January 6, 2020. BTC, ETH, and XRP denote the daily returns of Bitcoin, Ethereum, and Ripple, respectively. Further, (*), (**), and (***) represent statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 2: Detecting asset bubbles in cryptocurrencies. Structural breaks and explosive behavior results.

Panel A: Structural breaks in cryptocurrency returns – Bai and Perron (2003)

	Bitcoin returns			Ethereum returns			Ripple returns	
Position	Break-point	Break-date	Position	Break-point	Break-date	Position	Break-point	Break-date
1st	858	12-26-2017	1st	886	01-13-2018	1st	636	05-08-2017
2nd	597	03-30-2017	2nd	628	04-30-2017	2nd	395	09-09-2016
3rd	324	06-30-2016	3rd	386	08-31-2016	3rd	880	01-07-2018
4th	1340	04-12-2019	4th	1127	09-11-2018	4th	1127	09-11-2018
5th	1099	08-14-2018	5th	1360	05-10-2019	5th	1368	05-10-2019

Panel B: Explosive behavior in cryptocurrency prices – Phillips, Shi and Yu (2015)

	Bitcoin prices			Ethereum prices			Ripple prices	
# days	start	end	# days	start	end	# days	start	end
122	09-15-2017	01-15-2018	92	04-08-2017	07-09-2017	31	12-13-2017	01-13-2018
75	04-30-2017	07-14-2017	73	11-22-2017	02-03-2018	22	05-04-2017	05-26-2017
58	07-17-2017	09-13-2017	34	02-21-2016	03-26-2016	11	05-31-2017	06-11-2017
30	06-13-2019	07-13-2019	32	08-06-2017	09-07-2017	9	06-17-2017	06-26-2017
27	05-07-2019	06-03-2019	28	03-09-2017	04-06-2017			
24	05-28-2016	06-21-2016	8	02-27-2017	03-07-2017			
15	12-21-2016	01-05-2017	7	03-28-2016	04-04-2016			
13	12-03-2018	12-16-2018	7	05-14-2019	05-21-2019			
12	02-23-2017	03-07-2017						
9	11-19-2018	11-28-2018						

Notes: Bai and Perron (2003) method was applied to cryptocurrency returns. Phillips et al. (2015) method was applied to cryptocurrency prices, and bubble periods that lasted at least 7 days are reported.

Table 3: Model checking. The Multivariate Ljung-Box test and the Multivariate Lagrange Multiplier test for detecting conditional heteroscedasticity in the standardized residuals.

	BEKK	Cholesky	DCC Engel	t-Copula
Multivariate Ljung-Box				
Q(5)	41.77	30.18	31.99	10.24
Q(10)	70.39	48.85	58.62	20.67
Multivariate Lagrange Multiplier				
LM(5)	10.61*	3.18	5.83	0.68
LM(10)	12.52	4.32	8.15	2.13

Notes: $Q(m)$ and $LM(m)$ are the multivariate Ljung-Box test and the multivariate Lagrange Multiplier test at lag m, respectively. Further, (*), (**), and (***) represent statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 4: BEKK parameters

	Full Sample			Pre-Bubble			Post-Bubble		
A_1	0.374***	-0.023**	-0.003	0.428***	-0.011	0.058	0.398***	-0.115	0.081*
	-0.059	0.365***	0.030	0.065	0.397***	-0.089	0.032	0.058	0.255***
	-0.188***	0.030	0.502***	-0.266***	-0.042	0.919***	-0.500***	0.281***	0.552***
B_1	0.915***	0.003	0.003	0.894***	0.016	-0.058*	1.000***	-0.297***	0.011
	0.025	0.920***	-0.042***	-0.001	0.611***	0.024	0.391***	0.000	0.040
	0.095***	-0.021*	0.825***	0.104	0.084*	0.148***	0.276***	-0.276*	0.668***

Notes: Full Sample (August 2015 - January 2020), Pre-Bubble period (August 2015 - February 2017), and Post-Bubble period (April 2018 - January 2020). Further, (*), (**), and (***) represent statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 5: Causality test results.

	Granger				Instantaneous			
lags	1	2	4	8	1	2	4	8
Bitcoin shock as the source								
a_t^2	2.49*	1.29	2.74***	1.09	156.30***	153.32***	150.65***	153.877***
$\|a_t\|$	2.69**	1.165	2.40***	0.99	262.74***	260.35***	256.00***	261.02***
σ_t^{BEKK}	4.93***	3.04***	2.79**	1.525*	182.98***	182.185***	183.16***	185.88***
Ethereum shock as the source								
a_t^2	0.52	1.11	2.26**	3.44***	162.44***	160.15***	158.51***	164.63***
$\|a_t\|$	0.43	0.28	0.57	2.36***	264.24***	263.59***	264.57***	271.65***
σ_t^{BEKK}	2.40*	1.44	0.96	1.65**	225.39***	224.53***	223.55***	226.25***
Ripple shock as the source								
a_t^2	10.28***	5.37***	3.29***	2.15***	27.82***	28.07***	24.14***	28.55***
$\|a_t\|$	12.60***	5.36***	2.72***	2.36***	149.79***	147.24***	133.34***	139.66***
σ_t^{BEKK}	4.44**	4.25***	3.09***	2.56***	67.37**	66.52***	63.28***	64.95***

Notes: a_t^2 and $|a_t|$ are the squared residuals and absolute residuals of a VAR(1) estimation for the conditional mean (μ_t), respectively. While σ_t^{BEKK} is the estimated volatility of the BEKK-GARCH(1,1) model. Further, (*), (**), and (***) represent statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 6: Estimation results on the dummy regression of the correlation time series.

Coefficient	BTC-ETH	BTC-XRP	ETH-XRP
γ_0	0.26***	0.31***	0.28***
γ_1	0.16***	0.06***	0.11***
γ_2	0.50**	0.28***	0.46***

Note: (*), (**), and (***) represent statistical significance at the 10%, 5%, and 1% levels, respectively. Model: $\rho_t = \gamma_0 + \gamma_1 D_1(bubble) + \gamma_2 D_2(post - bubble) + \varepsilon_t$

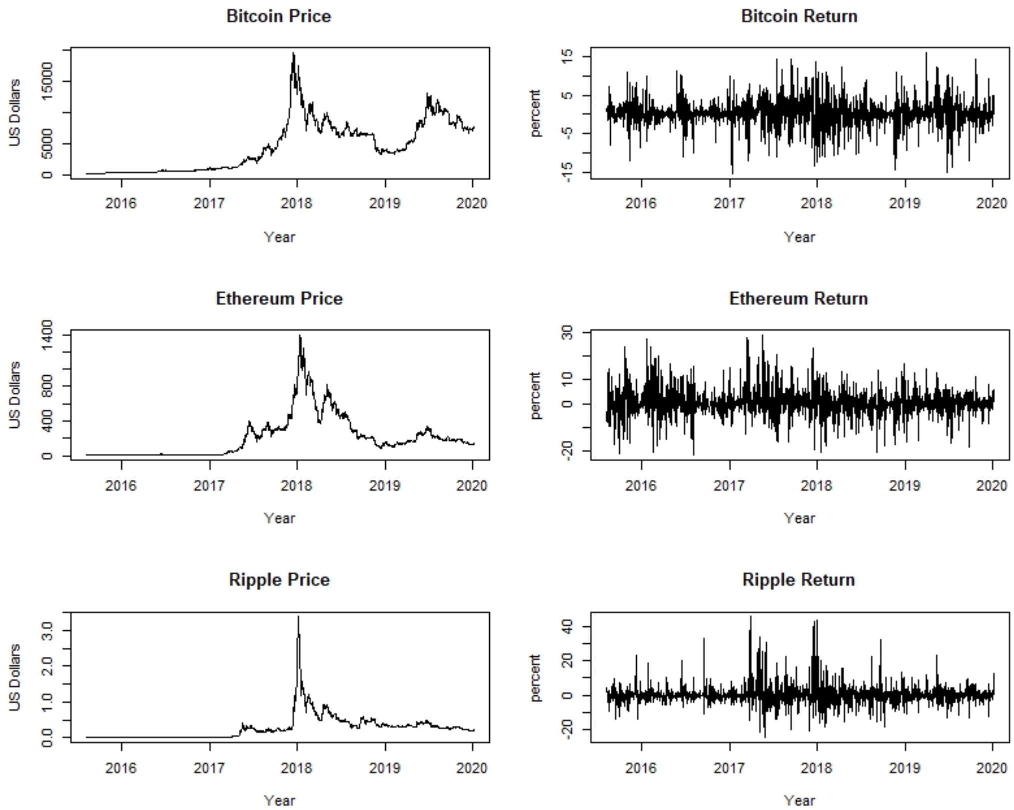

Figure 1: Daily closing prices (left panel) and daily returns (right panel) of Bitcoin, Ethereum, and Ripple.

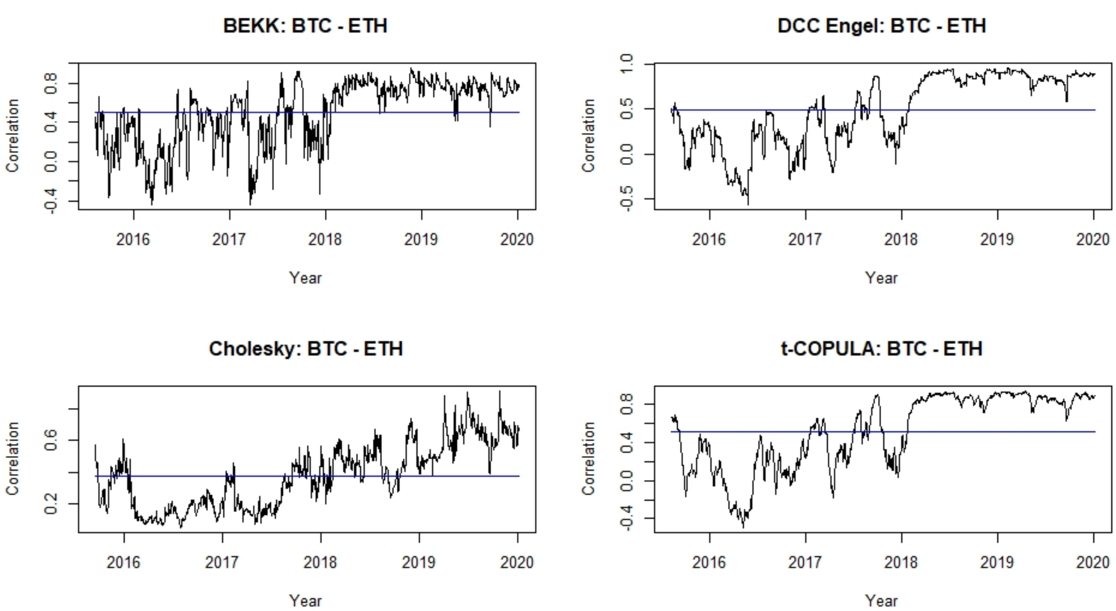

Figure 2: Estimated time-varying correlations between Bitcoin and Ethereum (BTC-ETH) for daily returns from August 2015 to January 2020. The black line denotes the fitted conditional correlation (ρ_t) and the blue line denotes its corresponding sample mean ($mean(\rho_t)$).

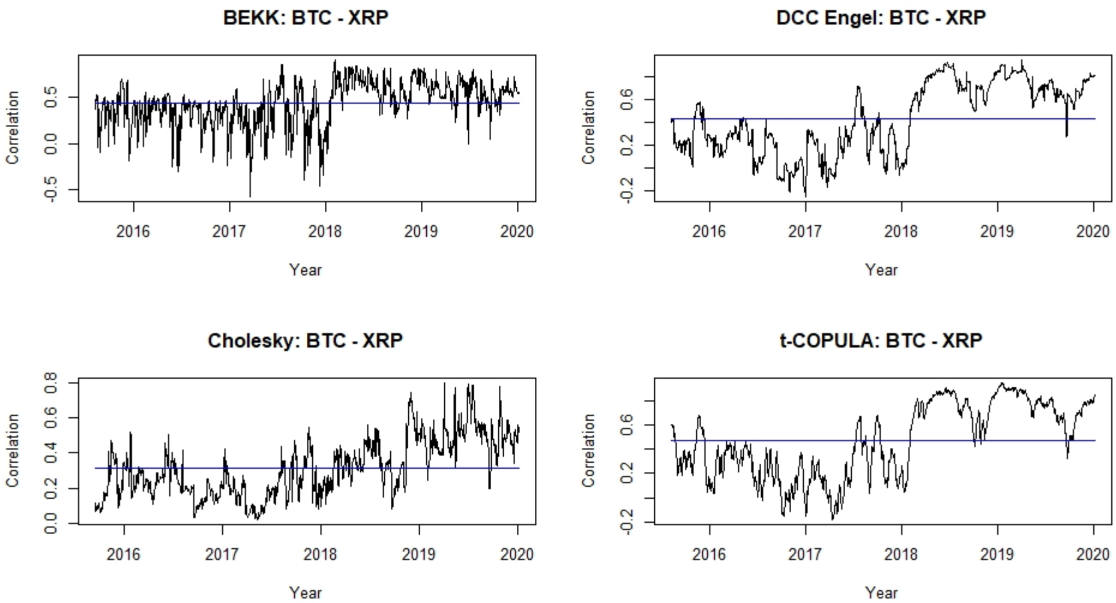

Figure 3: Estimated time-varying correlations between Bitcoin and Ripple (BTC-XRP) for daily returns from August 2015 to January 2020. The black line denotes the fitted conditional correlation (ρ_t) and the blue line denotes its corresponding sample mean ($mean(\rho_t)$).

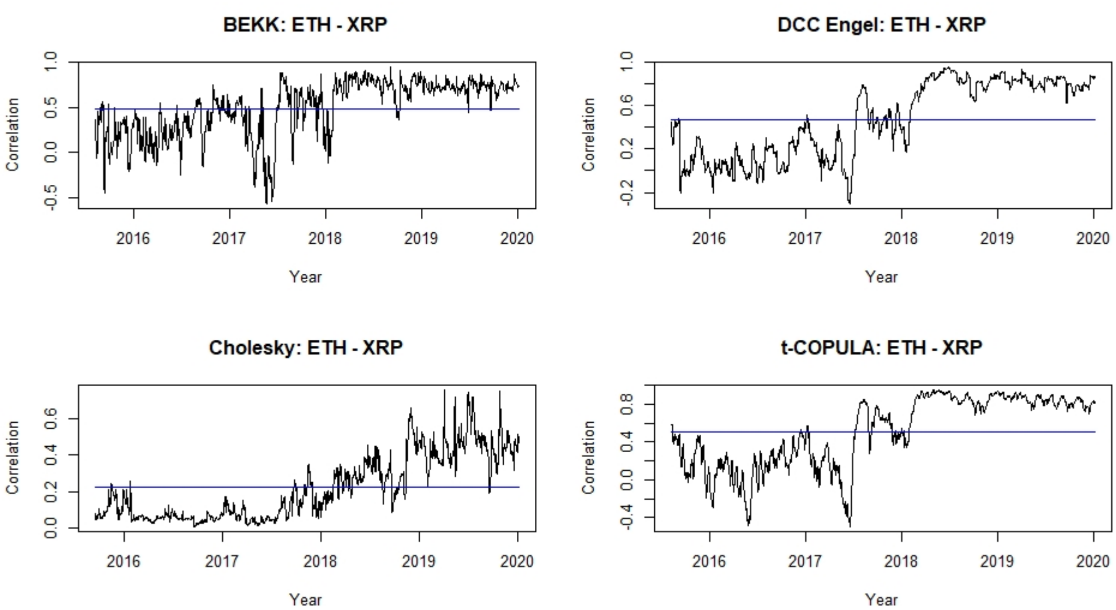

Figure 4: Estimated time-varying correlations between Ethereum and Ripple (ETH-XRP) for daily returns from August 2015 to January 2020. The black line denotes the fitted conditional correlation (ρ_t) and the blue line denotes its corresponding sample mean ($mean(\rho_t)$).

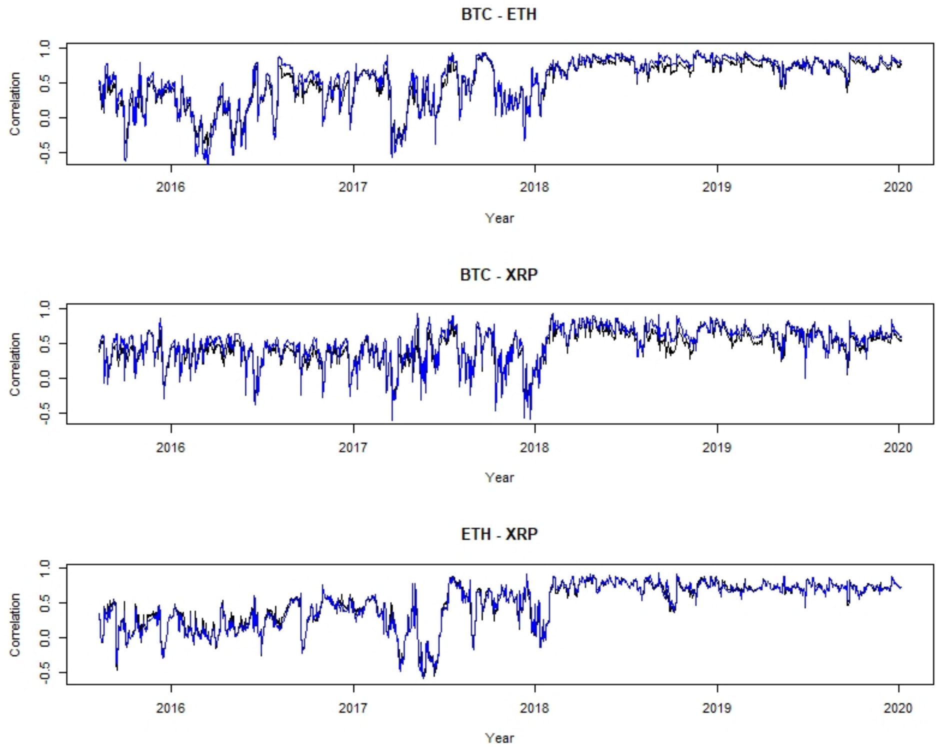

Figure 5: Estimated time-varying correlations between Bitcoin and Ethereum (BTC-ETH), Bitcoin and Ripple (BTC-XRP), and Ethereum and Ripple (ETH-XRP) for daily returns from August 2015 to January 2020. The black line denotes the fitted conditional correlation of the BEKK(1,1) model and the blue line denotes the Forbes-Rigobon adjusted correlation.

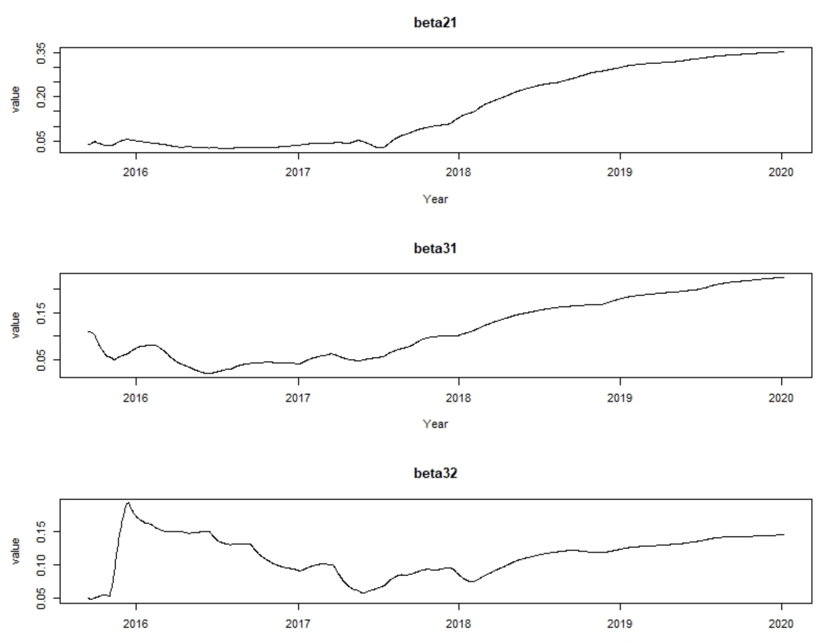

Figure 6: Estimated Cholesky-GARCH coefficients (betas): β_{21}, β_{31}, and β_{32}.

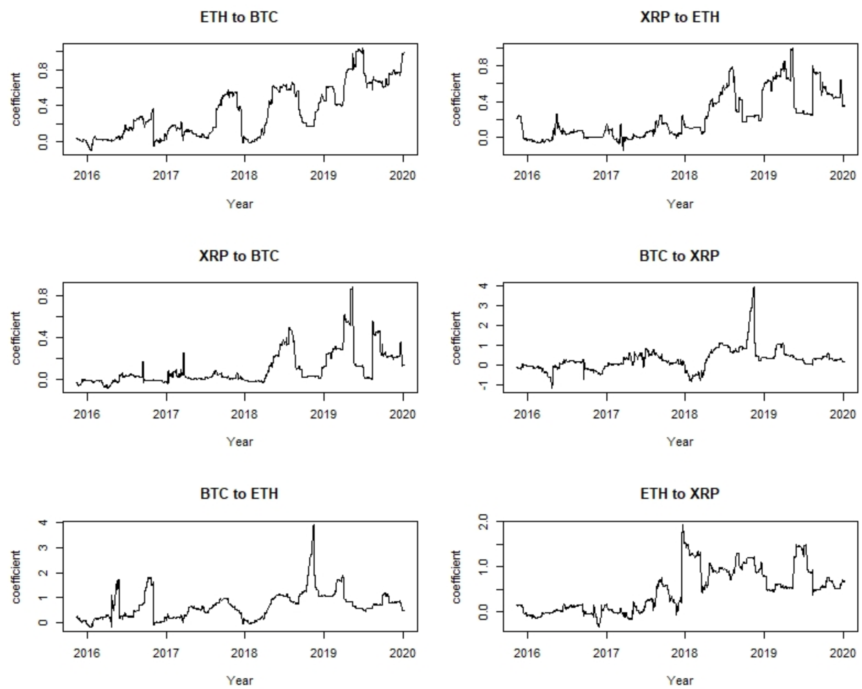

Figure 7: Estimated $\hat{\beta}_0$ coefficients of the HAR model using σ_t^{BEKK}.

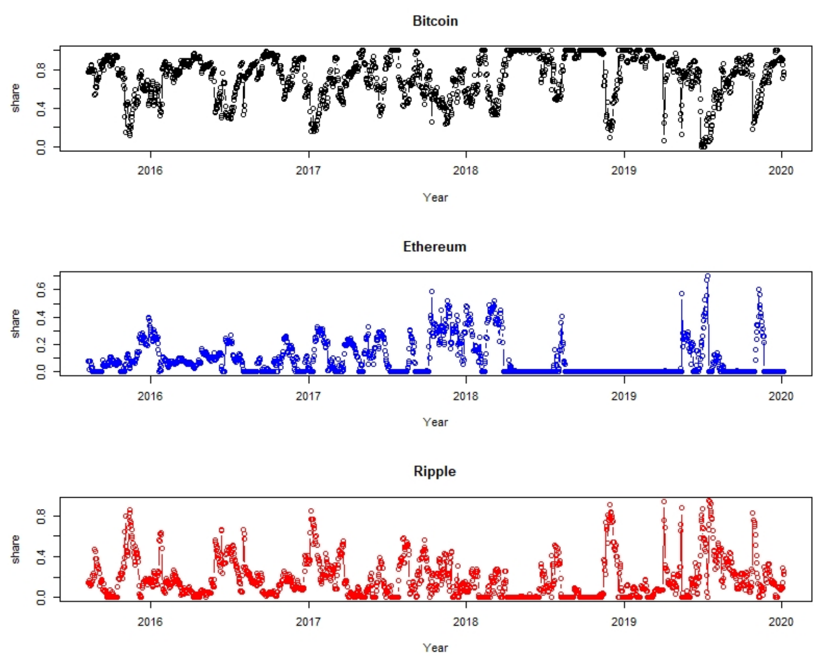

Figure 8: The estimated weights of the global minimum variance portfolio: Bitcoin (black line), Ethereum (blue line), and Ripple (red line)

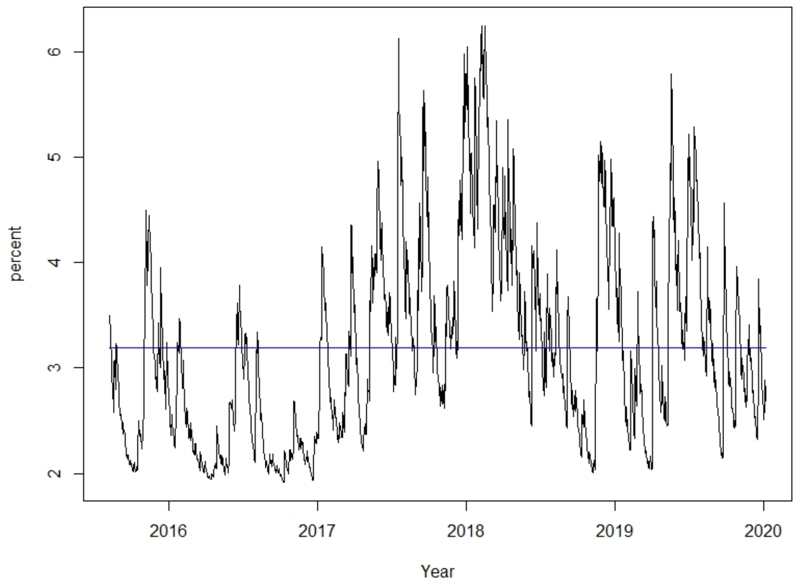

Figure 9: The estimated volatility of the global minimum variance portfolio (black line) and its corresponding sample mean (blue line)

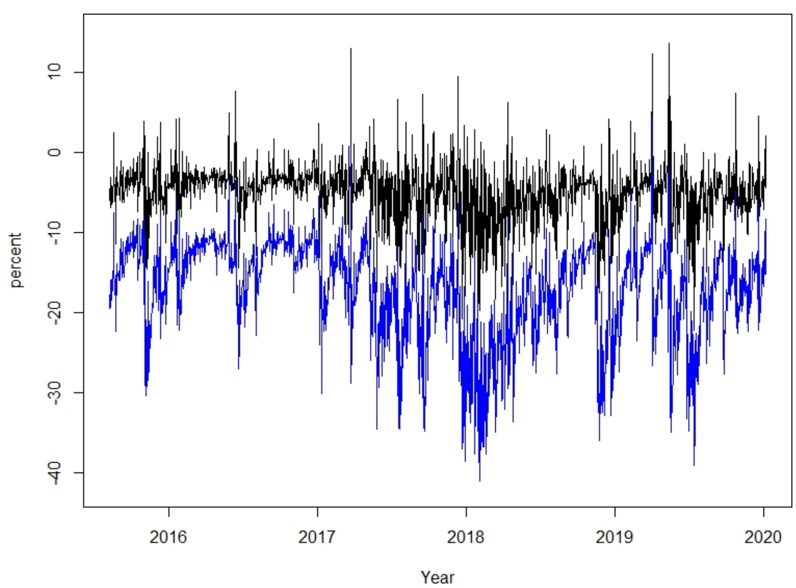

Figure 10: The estimated Gaussian (black line) and non-parametric (blue line) Value-at-Risk of the global minimum variance portfolio.