Cryptography

Hash Functions

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Hash Functions

In general, a *cryptographic hash function* consists of a polynomial-time key generator (with a security parameter 1^n as input) and a keyed hash function

$$H_k : \{0,1\}^* \to \{0,1\}^n$$

that takes a key and a binary string as input and outputs a hash value of length n.

In practice, hash functions are *unkeyed* or the key is fixed.

Collision Resistance

Since hash values are used as *message digests* or unique *identifiers*, their main requirement is *collision resistance*. A collision is given by two input values $x \neq x'$ with

$$H(x) = H(x').$$

Definition

A function $H_k = H : D \to R$, where H, k, D and R depend on a security parameter n, is called *collision resistant*, if the probability that a probabilistic polynomial-time adversary finds a collision $H(x) = H(x')$, where $x,\ x' \in D$ and $x \neq x'$, is negligible in n.

Weak Collision Resistance and Preimage Resistance

There are two related requirements:

- *Second-preimage resistance* or *weak collision resistance* means that an adversary, who is given a uniform $x \in D$, is not able to find a second preimage $x' \in D$ with $x \neq x'$ such that $H(x) = H(x')$.
- *Preimage resistance* or *one-wayness* means that an adversary, who is given a uniform $y \in R$, is not able to find a preimage $x \in D$ such that $H(x) = y$.

Obviously, collision resistance implies second-preimage resistance. Under certain conditions, collision resistance also implies one-wayness.

Cryptographic Hash Functions

Definition

A cryptographic hash function H is an efficient algorithm that takes a message m of arbitrary length as input and outputs a digest $H(m)$. A cryptographic hash function should be *collision-resistant* and *one-way*.

Random Oracle Model

An ideal hash function is called a *random oracle*. The output of a random oracle is uniformly random, unless the same input is queried twice; in this case the oracle returns the same output.

A hash function that behaves as a random oracle is required in some security theorems and proofs. However, concrete implementations of a random oracle are impossible.

Birthday Paradox

Hash values should not be too short. In fact, the *Birthday Paradox* demonstrates that collisions occur surprisingly often:

Theorem

Let k be the number of independent samples drawn from a uniform distribution on a set of size N. If $k \approx 1.2\sqrt{N}$, then the probability of a collision is around 50%.

If we consider hash values of length n and assume that they have a uniform distribution, then collisions occur after hashing around $\sqrt{2^n} = 2^{n/2}$ messages.

In order to minimize the risk of random collisions, hash values should be at least 200 bits long.

Integrity Protection

Firstly, the hash value can be used as a short *identifier* of data.

$$m \longrightarrow H(m)$$

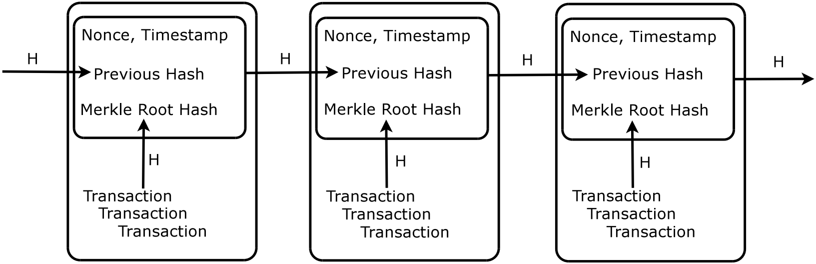
The identifier is unique as long as the hash function is collision resistant. Hashes can be used to verify the *integrity* of messages. Note that the verifier needs access to the *authentic* message digest $H(m)$.

Hashes can also be used in the construction of *message authentication codes*, which depend on a secret symmetric key.

Signature schemes first hash the message.

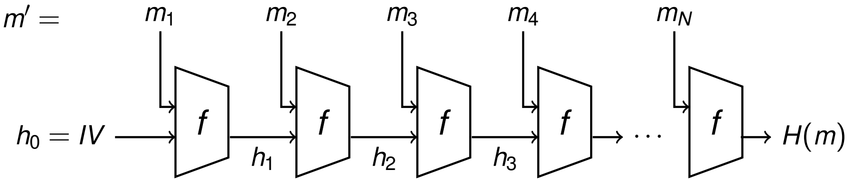
Blockchain

A *blockchain* is a sequence of linked blocks. Each block contains the hash value of the previous block. A blockchain can be used as a *distributed ledger*, which records transactions in an efficient and verifiable way. Hash values protect the integrity of the blockchain: transactions in a block cannot be modified without changing all subsequent hash values.



Merkle-Damgård Construction

The *Merkle-Damgård construction* has found widespread use, including the MD-SHA family. The Merke-Damgård transform is based on a *compression function* $f : \{0,1\}^{n+l} \to \{0,1\}^n$, which maps $n+l$ input bits to n output bits. The compression function is applied recursively. A message m is padded and its length is appended, giving m'. In each step, l bits of m' are processed and the last output defines the hash value $H(m)$.



SHA-1

SHA-1 is a Merkle-Damgård hash function based on a compression function

$$f : \{0,1\}^{160+512} \to \{0,1\}^{160}.$$

A 512-bit message block $m = W_0 \| W_1 \| \dots \| W_{15}$ is subdivided into 16 words of length 32 bits. By XOR operations and a circular left shift by one position, 64 additional words $W_{16}, \dots, W_{79}$ are generated:

$$W_j = (W_{j-16} \oplus W_{j-14} \oplus W_{j-8} \oplus W_{j-3}) \lll 1 \text{ for } 16 \leq j \leq 79$$

The 160-bit input vector $h = H_1 \| H_2 \| H_3 \| H_4 \| H_5$ is subdivided into five 32-bit words and copied to the initial status vector:

$$A \| B \| C \| D \| E \leftarrow H_1 \| H_2 \| H_3 \| H_4 \| H_5$$

SHA-1 Compression Function

Then, 80 rounds of the SHA-1 compression function are performed, which update the status words $A\|B\|C\|D\|E$. In round j, the 32-bit message word W_j is processed. A nonlinear bit-function F (defined by AND, OR, NOT and XOR operations) and a constant K are used. The function F and the constant K change every 20 rounds. The following function is used for the first 20 rounds:

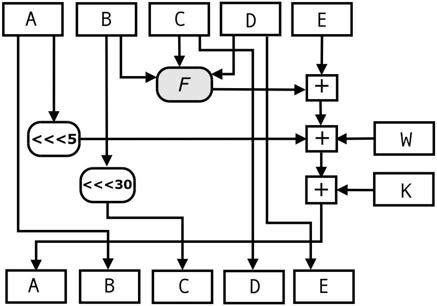
$$F(B, C, D) = (B \wedge C) \oplus (\neg B \wedge D)$$

After completing 80 rounds, the compression function outputs

$$f(h, m) = (A + H_1 \parallel B + H_2 \parallel C + H_3 \parallel D + H_4 \parallel E + H_5),$$

where $+$ denotes addition modulo 2^{32}.

SHA1 Compression Function



One round of the SHA-1 compression function f which updates the 160*-bit state. W is a* 32*-bit message chunk, F is a bit function (see above) and K a constant.* $\lll$ *denotes left-rotations and* $+$ *is addition modulo* 2^{32}.

SHA-1 Collision

In February 2017, a SHA-1 collision was found. The attack required 2^{63} SHA-1 calls, and took approximately 6500 CPU years and 100 GPU years. A prefix P was chosen and two different 1024-bit messages $M^{(1)}$ and $M^{(2)}$ were found such that

$$H(P \| M^{(1)}) = H(P \| M^{(2)}).$$

Since P is a valid preamble for PDF documents, the collision makes it possible to fabricate two different PDF files with the same SHA-1 hash value. Impressive examples have been published.

Here is the prefix P in ASCII characters:

```
%PDF-1.3.%.......1 0 obj.<</Width 2 0 R/Height 3 0 R/Type 4 0 R/
Subtype 5 0 R/Filter 6 0 R/ColorSpace 7 0 R/Length 8 0 R/BitsPer
Component 8>>.stream......$SHA-1 is dead!!!!!./..#9u.9...<L.....
```

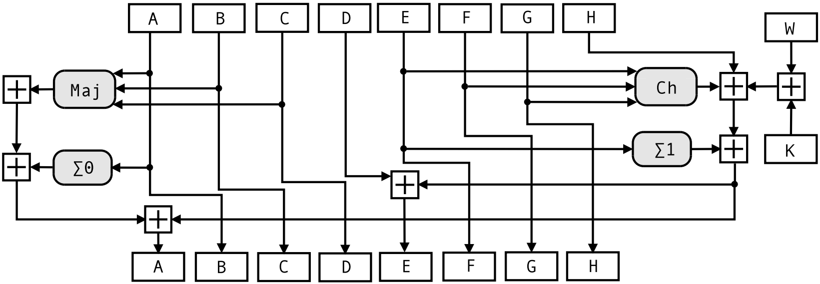
SHA-2

The SHA-2 hash functions SHA-224, SHA-256, SHA-384 and SHA-512 are constructed in a similar way to SHA-1, but use an extended internal state and larger digests. It is assumed that SHA-2 offers better protection against collision-finding attacks. SHA-2 is now widely used in security protocols and applications.

In the following, we describe the SHA-256 variant. The SHA-2 compression function f takes as input a 256-bit status vector and a 512-bit message block and outputs an updated 256-bit status:

$$f : \{0,1\}^{256+512} \rightarrow \{0,1\}^{256}$$

SHA-2 Compression Function



One round of the SHA-2 compression function f. The functions Maj, Ch, Σ_0 and Σ_1 are defined in a similar way as the function F used in SHA-1.

The 32-bit status words A, B, C, D, E, F, G, H are updated. In each of the 64 rounds, a 32-bit chunk W derived from the message block is processed. K is a 32-bit constant that depends on the round number.

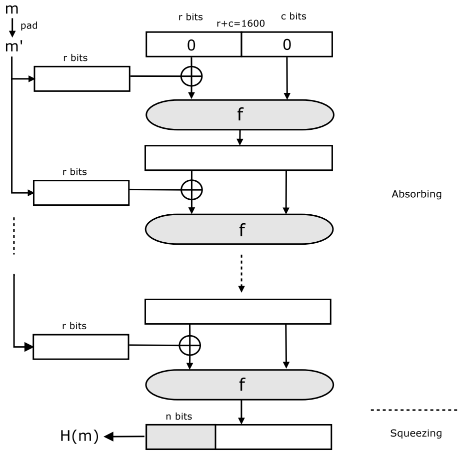
SHA-3

After a competition to construct a new secure hash function called SHA-3, *Keccak* was selected as the winner. Keccak is not of Merkle-Damgård type, but rather based on a *sponge* construction.

In 2015, the Keccak variants SHA3-224, SHA3-256, SHA3-384, SHA3-512 with output lengths between 224 and 512 bits were standardized. The SHA-3 instance of Keccak uses a three-dimensional state array of $5 \times 5 \times 64 = 1600$ bits. The unkeyed Keccak-$f[1600]$ permutation operates on the 1600-bit state array, and it is assumed that f behaves like a *random permutation*.

$$f: \{0,1\}^{1600} \to \{0,1\}^{1600}$$

Keccak Operation



Absorbing message blocks of length r and squeezing out the hash value.

SHA-3 Family

Definition

The SHA-3 family of hash functions

$$H : \{0,1\}^* \to \{0,1\}^n$$

supports output lengths $n \in \{224, 256, 384, 512\}$. Depending on n, the rate r and the capacity c are fixed, where $r + c = 1600$. The input message m is padded such that the length is a multiple of r. The padded message m' is split into blocks $m_1,\ m_2,\ \ldots,\ m_N$ of length r:

$$m' = m \| 0110\ldots01 = m_1 \| m_2 \| \ldots \| m_N$$

The state $s = s_1 \| s_2$ is initialized by the zero vector $0^r \| 0^c$.

SHA-3 Family

Definition

During the *absorbing* phase, the state is recursively updated by the message blocks m_i:

$$s_1 \| s_2 \leftarrow f(s_1 \oplus m_i \parallel s_2) \text{ for } 1 \leq i \leq N.$$

Finally, the SHA-3 hash value is obtained by a single *squeezing* operation: $H(m)$ is given by the leftmost n bits of the state.

SHA-3 Family

An advantage of the sponge construction – in comparison to the Merkle-Damgård transform – is that the hash value does *not reveal the full state*, which prevents length extension attacks.

The SHA-3 standard also defines two *extendable-output functions* (XOF) called SHAKE128 and SHAKE256, with which the output can be extended to any desired length. In this case, the Keccak-f function is applied multiple times during the squeezing phase in order to obtain the required number of output bits.

There are also keyed hash functions based on Keccak (KMAC128, KMAC256).