Cryptography

Symmetric Encryption

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Block Ciphers and Stream Ciphers

Symmetric ciphers can be divided into *block ciphers* and *stream ciphers*. A *block cipher* encrypts a block of plaintext bits of fixed length l into a block of ciphertext bits, and vice versa:

$$E_k(m_1, m_2, \ldots, m_l) = (c_1, c_2, \ldots, c_l)$$
$$E_k^{-1}(c_1, c_2, \ldots, c_l) = (m_1, m_2, \ldots, m_l)$$

Operations modes allow the encryption of plaintexts of arbitrary length.

A *stream cipher* generates keystream $k_1, k_2, \ldots$ (a pseudorandom bit sequence) from k, and encrypts the plaintext bits $m_1, m_2, \ldots$ by XORing with the keystream. Decryption works similarly.

$$(c_1, c_2, \ldots) = (m_1, m_2, \ldots) \oplus (k_1, k_2, \ldots) = (m_1 \oplus k_1, m_2 \oplus k_2, \ldots)$$
$$(m_1, m_2, \ldots) = (c_1, c_2, \ldots) \oplus (k_1, k_2, \ldots) = (m_1 \oplus k_1 \oplus k_1, m_2 \oplus k_2 \oplus k_2, \ldots)$$

Block Ciphers

A block cipher is a *keyed family of permutations*, which is designed to behave like a pseudorandom permutation (*prp*). Block ciphers operate on a binary string of fixed length and depend on a key:

$$E: \{0,1\}^n \times \{0,1\}^l \to \{0,1\}^l$$

Today, a block length of $l = 128$ bits and key lengths between $n = 128$ and $n = 256$ bits are used.

Design of Block Ciphers

The bijective encryption functions

$$E_k : \{0,1\}^l \rightarrow \{0,1\}^l$$

should be indistinguishable from a true random permutation if k is unknown.

The construction of block ciphers is a non-trivial task. *Diffusion* and *confusion* are important design goals of block ciphers.

- Diffusion: Small input changes, e.g., only one bit, result in large output changes (avalanche effect).
- Confusion: Complex relationship between ciphertext and key, even if plaintext is known. This helps to protect the key against attacks (e.g., EAV, CPA, CCA).

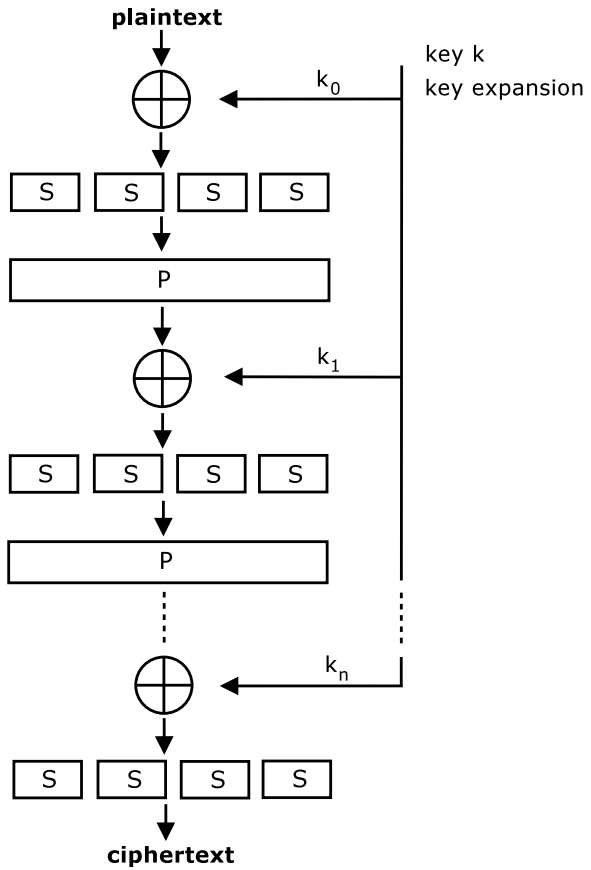
Diffusion and Confusion

Diffusion can be achieved by linear and affine maps, e.g., by XORing with pseudorandom round keys and by matrix operations (linear maps).

Confusion requires *nonlinear* and *non-affine* maps (so-called S-Boxes). Since nonlinear maps can be difficult to describe on full blocks, S-Boxes often operate only on small segments of a block and are applied in parallel, e.g., on 8-bit segments of a 128-bit block.

Substitution-Permutation Networks

In practice, *substitution-permutation networks* (SPN) and *Feistel networks* are used to construct block ciphers.



SPN: Plaintext is transformed into ciphertext in several invertible rounds.

AES Encryption

The block cipher *Rijndael* has been adopted as *Advanced Encryption Standard (AES)* and the cipher is widely used today. The standardized AES cipher has a block length of 128 bits and a 128-, 192- or 256-bit key length. The Rijndael encryption function

$$E_k : \{0,1\}^{128} \to \{0,1\}^{128}$$

is given by a SPN. The 128-bit *state* is arranged in a 4×4 matrix over $GF(2^8)$ by writing the bytes $p_0, p_1, \ldots, p_{16}$ into the columns.

$$\begin{pmatrix} p_0 & p_4 & p_8 & p_{12} \\ p_1 & p_5 & p_9 & p_{13} \\ p_2 & p_6 & p_{10} & p_{14} \\ p_3 & p_7 & p_{11} & p_{15} \end{pmatrix}$$

Each byte is interpreted as an element of the field

$$GF(2^8) = GF(2)[x]/(x^8 + x^4 + x^3 + x + 1).$$

High-level Description of AES

```
Rijndael(State, CipherKey)
{
   KeyExpansion(CipherKey, ExpandedKey)
   AddRoundKey(State,ExpandedKey[0])
   for(i = 1; i < Nr ; i++)  {     // Nr is either 10, 12 or 14
      // Round i
      SubBytes(State)
      ShiftRows(State)
      MixColumns(State)
      AddRoundKey(State,ExpandedKey[i])
   }
   // Final Round
   SubBytes(State)
   ShiftRows(State)
   AddRoundKey(State,ExpandedKey[Nr])
}
```

AES S-Box

The AES S-Box `SubBytes` is the only non-affine component of AES.

The S-Box function $S_{RD} : GF(2^8) \rightarrow GF(2^8)$ is applied to each byte of the state individually.

$$\begin{pmatrix} p_0 & p_4 & p_8 & p_{12} \\ p_1 & p_5 & p_9 & p_{13} \\ p_2 & p_6 & p_{10} & p_{14} \\ p_3 & p_7 & p_{11} & p_{15} \end{pmatrix} \xrightarrow{\text{SubBytes}} \begin{pmatrix} S_{RD}(p_0) & S_{RD}(p_4) & S_{RD}(p_8) & S_{RD}(p_{12}) \\ S_{RD}(p_1) & S_{RD}(p_5) & S_{RD}(p_9) & S_{RD}(p_{13}) \\ S_{RD}(p_2) & S_{RD}(p_6) & S_{RD}(p_{10}) & S_{RD}(p_{14}) \\ S_{RD}(p_3) & S_{RD}(p_7) & S_{RD}(p_{11}) & S_{RD}(p_{15}) \end{pmatrix}$$

S-Box Function S_{RD}

The definition

$$GF(2^8) = GF(2)[x]/(x^8 + x^4 + x^3 + x + 1)$$

gives a $GF(2)$-linear isomorphism between $GF(2^8)$ and $GF(2)^8$. The invertible S-Box function is defined by multiplicative inversion in $GF(2^8)^*$ (which is highly nonlinear) followed by an affine map:

$$S_{RD} : GF(2^8) \to GF(2^8),\ S_{RD}(a) = \begin{cases} Aa^{-1} + b & \text{for } a \neq 0 \\ b & \text{for } a = 0 \end{cases}$$

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

ShiftRows

ShiftRows is a bit-permutation and rotates the bytes in the second, third and fourth row to the left. The first row is left unchanged, the bytes in second row are rotated by one position, bytes in the third row are rotated by two positions and bytes in the fourth row are rotated by three positions. Obviously, *ShiftRows* is invertible.

$$\begin{pmatrix} p_0 & p_4 & p_8 & p_{12} \\ p_1 & p_5 & p_9 & p_{13} \\ p_2 & p_6 & p_{10} & p_{14} \\ p_3 & p_7 & p_{11} & p_{15} \end{pmatrix} \xrightarrow{\text{ShiftRows}} \begin{pmatrix} p_0 & p_4 & p_8 & p_{12} \\ p_5 & p_9 & p_{13} & p_1 \\ p_{10} & p_{14} & p_2 & p_6 \\ p_{15} & p_3 & p_7 & p_{11} \end{pmatrix}$$

MixColumns

MixColumns transforms the columns of the state by a $GF(2^8)$-linear map. The product of a constant 4×4 matrix M over $GF(2^8)$ and the state matrix defines the new state, i.e., each column of the state is multiplied with M and yields the updated column. The matrix M is invertible.

$$\begin{pmatrix} p_0 & p_4 & p_8 & p_{12} \\ p_1 & p_5 & p_9 & p_{13} \\ p_2 & p_6 & p_{10} & p_{14} \\ p_3 & p_7 & p_{11} & p_{15} \end{pmatrix} \xrightarrow{\text{MixColumns}} \underbrace{\begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix}}_{M} \cdot \begin{pmatrix} p_0 & p_4 & p_8 & p_{12} \\ p_1 & p_5 & p_9 & p_{13} \\ p_2 & p_6 & p_{10} & p_{14} \\ p_3 & p_7 & p_{11} & p_{15} \end{pmatrix}$$

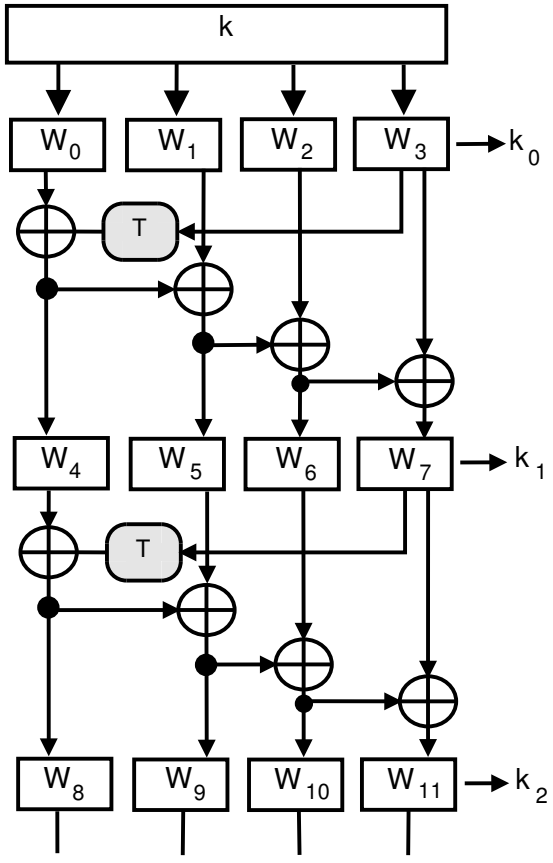
Key Scheduling

In the key expansion step, the 128-bit round keys $k_0, k_1, \ldots, k_r$ are derived from the AES key k. The key scheduling is nonlinear, which is intended to protect the cipher against *related key attacks*.

Suppose k is a 128-bit AES key. Then AES has ten rounds and eleven 128-bit round keys $k_0, k_1, \ldots, k_{10}$ are required. During key expansion, 44 words $W_0, W_1, \ldots, W_{43} \in GF(2^8)^4$ of length 32 bits are computed. The round keys are given by

$$k_i = W_{4i} \| W_{4i+1} \| W_{4i+2} \| W_{4i+3} \text{ for } i = 0, 1, \ldots, 10.$$

Key Scheduling for 128-bit AES keys



The first two rounds of 128*-bit AES key scheduling (round keys* k_0, k_1, k_2*). T is defined by* $T(W_{4i-1}) = \mathbf{S}(sh(W_{4i-1})) \oplus (RC_i, 0, 0, 0)$*, where sh rotates the bytes to the left,* $\mathbf{S} = S_{RD} \times S_{RD} \times S_{RD} \times S_{RD}$ *and* RC_i *is a round constant.*