Cryptography

Digital Signatures

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Signatures and their Objectives

Digital signatures are asymmetric cryptographic schemes which aim at data *integrity and authenticity*. There are some similarities to message authentication codes, but digital signatures are verified using a *public key*. Successful verification shows that the data is authentic and has not been tampered with.

Since the private key is exclusively controlled by the signer, digital signatures achieve not only data integrity and authenticity, but also *non-repudiation*.

Signatures have applications beyond integrity protection, for example in entity authentication protocols.

Signature Schemes

Definition

A *digital signature* scheme is given by:

- A message space $\mathcal{M}$,
- A space of key pairs $\mathcal{K} = \mathcal{K}_{pk} \times \mathcal{K}_{sk}$,
- A randomized key generation algorithm $Gen(1^n)$ that takes a security parameter 1^n as input and outputs a pair of keys (pk, sk),
- A signing algorithm, which may be randomized. It takes a message m and a private key sk as input and outputs a signature $s \leftarrow \text{sign}_{sk}(m)$,
- A deterministic verification algorithm that takes a public key pk, a message m and a signature s as input and outputs 1 if the signature is valid, and 0 otherwise.

Security Definition

Adversary		*Challenger/Oracle*
	$\xleftarrow{1^n,\ pk}$	$(pk, sk) \xleftarrow{\$} Gen(1^n)$
Choose m	$\xrightarrow{m}$	
	$\xleftarrow{s}$	$s = \text{sign}_{sk}(m)$
	$\vdots$	
Choose $m' \neq m$, forge a signature s'	$\xrightarrow{(m', s')}$	Verify (m', s'), output 1 or 0

Signature forgery experiment.

Secure Signature Schemes

Definition

The scheme is called *existentially unforgeable under an adaptive chosen message attack* (*EUF-CMA secure* or just *secure*), if for all probabilistic polynomial-time adversaries, the probability of successfully forging a signature is negligible in n.

Digital signatures protect the integrity and authenticity of messages and can also achieve *non-repudiation*. Since the signer alone controls the private key, they cannot deny the signature afterwards.

The verification of a digital signature requires the *authentic public key* of the signer. Although public keys can be openly shared, authenticity is not evident. A *man-in-the-middle* might replace the message, the signature and the public key with his own data.

Plain RSA Signature

Definition

The RSA signature scheme uses the same parameters as RSA encryption.

- A key generation algorithm $Gen(1^n)$ generates p, q, $N = pq$, e, d and outputs the public key $pk = (e, N)$ and the private key $sk = (d, N)$.
- The message space is $\mathcal{M} = \mathbb{Z}_N^*$.
- The deterministic signature algorithm takes sk and a message $m \in \mathcal{M}$ as input and outputs the signature

$$s = \mathrm{sign}_{sk}(m) = m^d \mod N.$$

Plain RSA Signature

Definition

- The verification algorithm takes pk, a message $m \in \mathbb{Z}_N^*$ and a signature s. It computes

$$s^e \mod N$$

and outputs 1 (valid) if $m = s^e \mod N$, and 0 otherwise.

Unfortunately, this scheme is both impractical and insecure. Firstly, the message length is limited by the size of the RSA modulus N, but in practice, one needs to sign messages of *arbitrary length*.

Secondly, the plain RSA signature scheme is insecure because new signature values can be easily forged. Furthermore, the plain RSA signature is *multiplicative*.

RSA FDH

The RSA-FDH (*Full Domain Hash*) signature is similar to the plain RSA scheme, but leverages a hash function $H : \{0,1\}^* \to \mathbb{Z}_N^*$. A message m is first hashed and then signed:

$$s = \text{sign}_{sk}(m) = H(m)^d \mod N$$

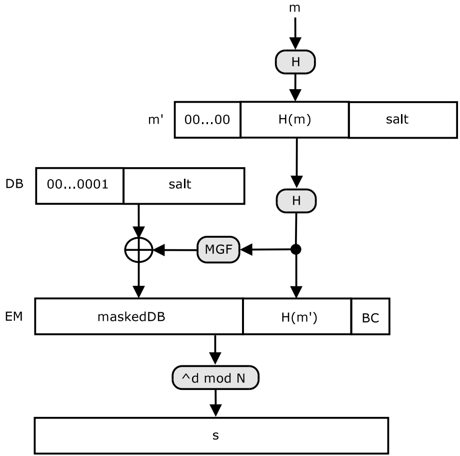
In the verification step, $H(m)$ is computed and then compared to $s^e \mod N$. The signature is valid if $H(m) = s^e \mod N$.

Obviously, the collision-resistance of H is crucial, since a collision $H(m_1) = H(m_2)$ with $m_1 \neq m_2$ can be used for a forgery.

Theorem

If H has range $\mathbb{Z}_N^*$ and is modeled as a random oracle, the RSA-FDH scheme is EUF-CMA secure under the RSA assumption.

RSA-PSS



Signing a message m using RSA-PSS (Probabilistic Signature Scheme)

Security of RSA-PSS

Since the length of cryptographic hashes is usually smaller than the size of the RSA modulus, RSA-PSS stretches the hash by randomized padding. If the salt is randomly chosen and sufficiently long, then the RSA-PSS signature is randomized and signing the same message twice using the same key will give different signature values.

The structure of RSA-PSS makes it very hard to forge a valid signature:

Theorem

The RSA-PSS signature scheme is EUF-CMA secure in the random oracle model under the RSA assumption.

Other Signature Schemes

An alternative to RSA are signature schemes which are based on the *discrete logarithm problem* in a cyclic group, similar to the Diffie-Hellman key exchange:

- ElGamal signature scheme
- DSA/DSS (Digital Signature Algorithm)
- ECDSA (Elliptic Curve Digital Signature Algorithm)

Furthermore, there are hash-based signatures schemes, for example the Lamport signature and the (extended) Merkle Signature Scheme (XMSS, RFC 8391).