2 Harmonics:

Let's fit a constant DLM with the following specification:

$$
\begin{aligned}
Y_t &= F\theta_t + v_t & v_t &\sim N(0, \sigma_o^2) \\
\theta_t &= G\theta_{t-1} + w_t & w_t &\sim N(0, \sigma_e^2) \\
S_{jt} &= \cos(t\omega_j)S_{j,t-1} + \sin(t\omega_j)S^*_{j,t-1} \\
S^*_{jt} &= -\sin(t\omega_j)S_{j,t-1} + \cos(t\omega_j)S^*_{j,t-1}
\end{aligned}
$$

where,

$$
\begin{aligned}
\theta_t &= [S_{1t}, S^*_{1t}, S_{2t}, S^*_{2t}], \qquad F = [1, 0, 1, 0] \\
\omega_j &= \frac{2\pi j}{s}, j = 1, 2, s = 276
\end{aligned}
$$

$$
G = \begin{bmatrix} \cos(\omega_1) & \sin(\omega_1) & 0 & 0 \\ \text{-}\sin(\omega_1) & \cos(\omega_1) & 0 & 0 \\ 0 & 0 & \cos(\omega_2) & \sin(\omega_2) \\ 0 & 0 & \text{-}\sin(\omega_2) & \cos(\omega_2) \end{bmatrix}
$$

and,

$$
W = \begin{bmatrix} \sigma_e^2 & 0 & 0 & 0 \\ 0 & \sigma_e^2 & 0 & 0 \\ 0 & 0 & \sigma_e^2 & 0 \\ 0 & 0 & 0 & \sigma_e^2 \end{bmatrix}
$$

We have 5 years of data so, $t = 1 \dots T = 1650$ or $T = 1656$ depending on the pixel. To draw from the posterior distribution of $\pi(\theta_{0:t}, \sigma_o, \sigma_e | y_{1:t})$ we use a Gibbs sampler with an IG prior on the variance parameters, which is a conjugate prior when combined with the normality assumption on the observed and state space distributions. Parameterizing in terms of $1/V$ and W^{-1}, we have $1/\sigma_o \sim \mathcal{G}(a_1 = 1, b_1 = 1)$ and $1/\sigma_e \sim \mathcal{G}(a_2 = 1, b_2 = 1)$. Starting values for both $1/V$ and W^{-1} were 1. The prior on the state space equation, $N(\underline{\mathbf{0}}, 1e^{07}\underline{\mathbf{1}})$

The posteriors for the parameters are as follows:

$$
\begin{aligned}
\theta_{0:T}^{(i)} &\sim \pi(\theta_{0:T} | y_{1:T}, 1/\sigma_o^{(i-1)}, 1/\sigma_e^{(i-1)}) \\
1/\sigma_o &\sim \mathcal{G}\left(a_1 + T^*/2, b_1 + 1/2 \sum_{t=1}^{T^*} (y_t - F\theta_t)'(y_t - F\theta_t)\right) \\
1/\sigma_e &\sim \mathcal{G}\left(a_2 + 2T, b_2 + 1/2 \sum_{t=1}^{T} (\theta_t - G\theta_{t-1})'(\theta_t - G\theta_{t-1})\right)
\end{aligned}
$$

Each pixel has missing values, so T^* represents the number of non-missing values of y_t.

Simulation

Let's explore the effect of changing the evolution variance, W, on simulated values from a 2 and 3 harmonic DLM model. In both cases we assume $\theta_0 \sim N_p(m_0, C_0)$. Depending on the dimension, $m_0 = 0$ and $C_0 = \mathbb{I}$. For both the 2 and 3 harmonic DLMs, we will fix $V = 1$. For each simulation, $s = 276$ and $n = 1656$. We will vary the diagonal of the W matrix, which controls the variance of the evolution parameters.

Figure 1: Simulated 2 Harmonic

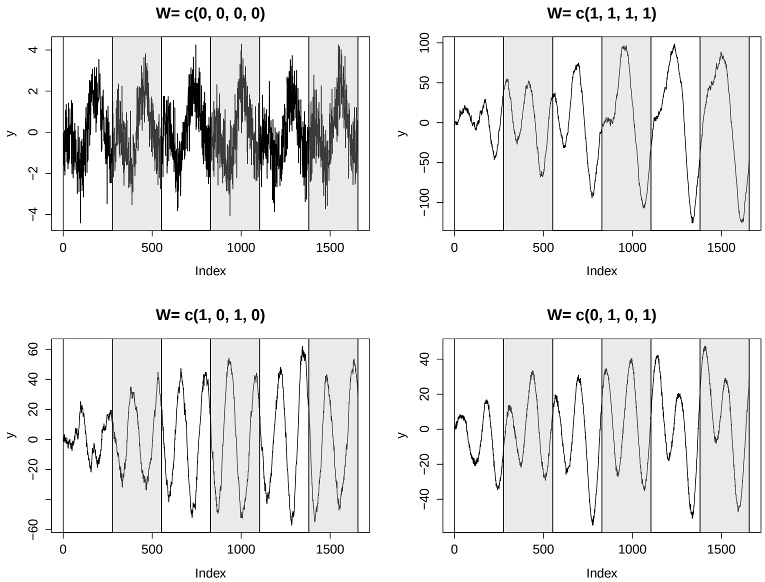


Figure 2: Simulated 2 Harmonic

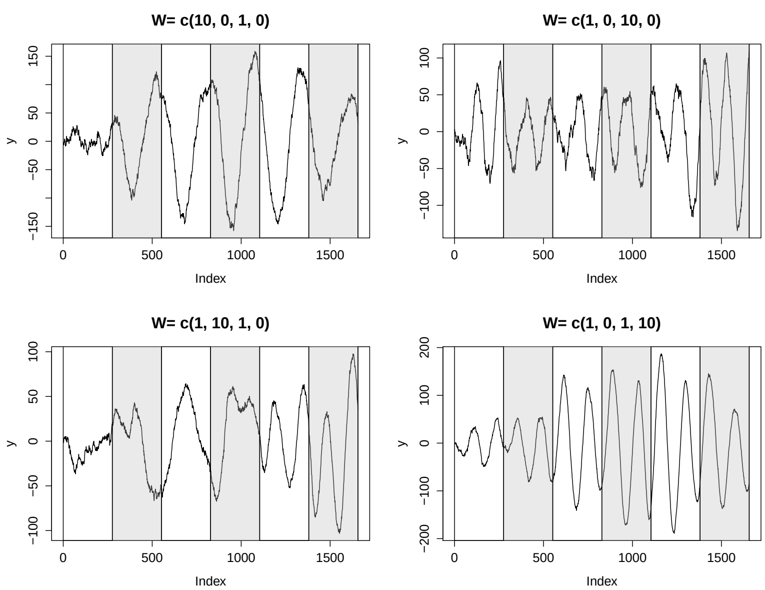


Figure 3: Simulated 3 Harmonic

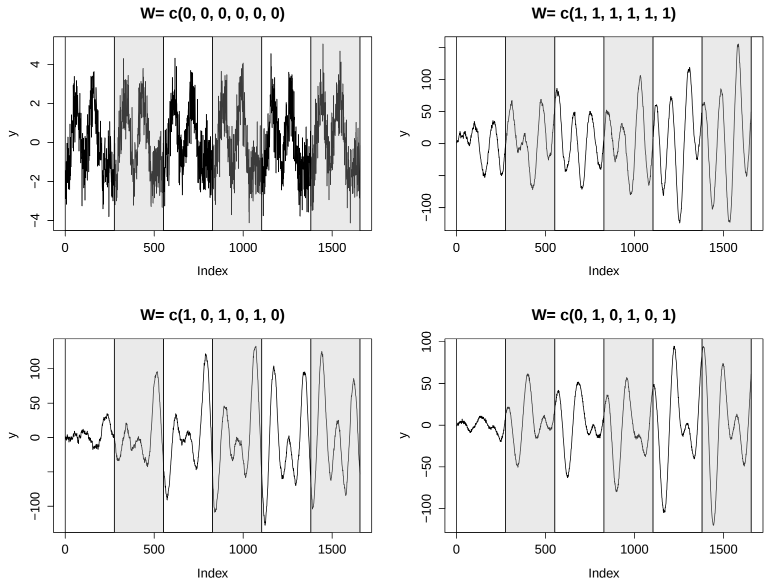
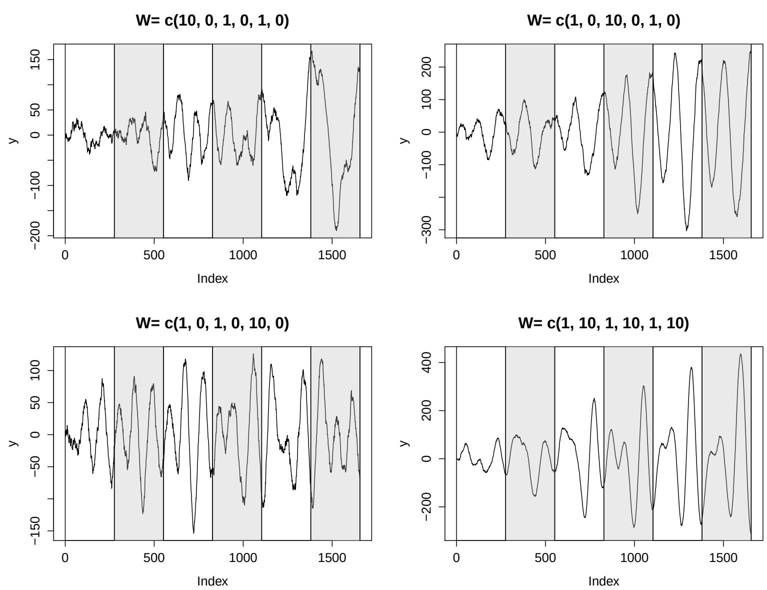


Figure 4: Simulated 3 Harmonic



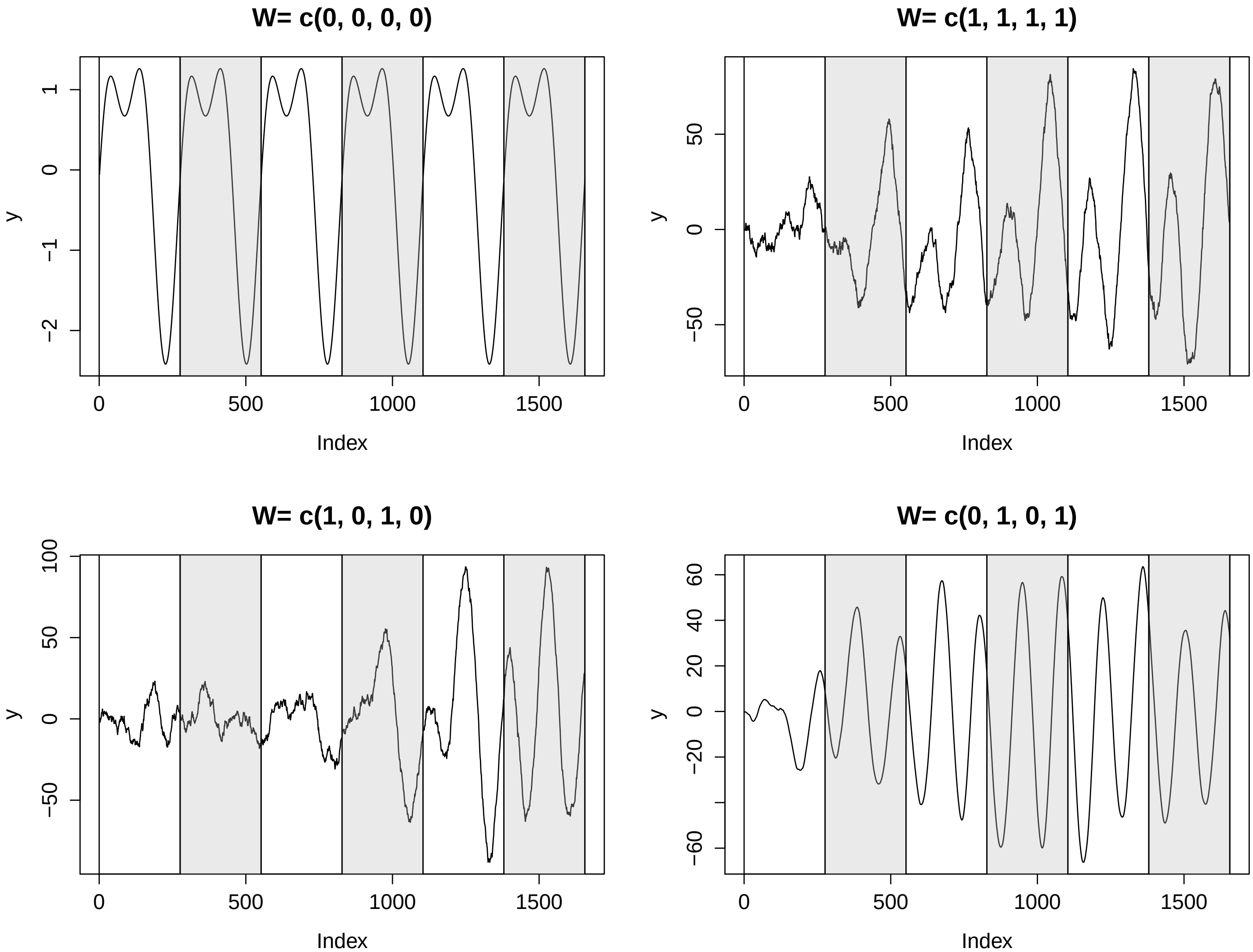
The correlation between V and W is fairly small, ρ was between .12 and .16 for all chains.

Below is the median of the states plotted above the raw data.

Figure 5: 2 Harmonic Regression for Pixel 194406

pix1_post.pdf

Figure 6: MLE Smooth States Pix 194406



For Pix 203632. Visually, this location seems to have the least noise of the 30 pixels.

Figure 7: 2 Harmonic Regression for Pixel 203632

pix30_comp.pdf

Figure 8: MLE Smooth States Pix 203632

pix30_smooth.pdf