1 Questions

To begin with

1. What is quantum Hamming weight? Why does it make sense to take the Hamming weight of a quantum state, and how is the output qubit entangled with each input state?

2. What is the rotation by Hamming value used for?

3. In Hoyer-Spalek, the constant-depth of Or and other quantum logic gates seems to depend on taking the gates from a arbitrary single-qubit rotations, rather than a fixed set. However, in the beginning sections, we do not make any such assumptions, and yet the assumption is made of constant-depth using a fixed basis. How do we get around this quantum compiling problem?

4. Useful papers to add to our reading list include the result that a rotation about the Z-axis of an irrational multiple of π, together with Hadamard and CNOT, are universal, when in fact we know that Hadamard, the T gate ($R_z(\pi/8)$), and CNOT are normally considered universal everywhere else. How do we resolve this consistency?

5. Is it worth computing exact parameters for KSV compiling? Soft failure says no, just keep it at asymptotics at this stage.

6. What is the implementation of controlled-R_z, using some standard decomposition?

2 Exact Gate in 2D CCNTC

The *exact*$[t]$ gate is defined as the $(n+1)$-qubit operation which takes $|x\rangle |y\rangle \rightarrow |x\rangle |y \oplus g(x)\rangle$ where we define $g(x) = 1$ iff $|x| = t$. It is achieved by a rotation by Hamming weight using a basic angle $\phi = 2\pi/m$, where m is determined by the *or* gate, and an added rotation of $-\phi \cdot t$.

3 Or Gate in 2D CCNTC

From Hoyer-Spalek, the OR gate

4 Rotation by Hamming Weight in 2D CCNTC

This is a basic gate in Hoyer-Spalek which makes use of unbounded-fanout. It takes as input n input qubits and an output qubit in the $|+\rangle$ state which is an equal superposition of $|0\rangle$ and $|1\rangle$. It modifies the phase on the $|1\rangle$ component of the output qubit based on the Hamming weight of the input. Since the input register $|x\rangle$ is in general a superposition of 2^n computational basis states, the output qubit is entangled to encode a rotation based on the Hamming weight of a particular input x.

Therefore, the notion of a *quantum Hamming weight* appears to be ill- defined. writes the output in a single qubit, giving the following $(n+1)$-qubit operation.

$$\sum_{x \in \mathcal{B}^n} |x\rangle \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \right) \to \sum_{x \in mathcalB^n} |x\rangle \left(\frac{|0\rangle + e^{i\phi|x|}\,|1\rangle}{\sqrt{2}} \right) \tag{1}$$

It is denoted $\mu_\phi^{|x|}$, meaning that the phase of output qubit is shifted by $|x| \cdot \phi$. Note that we do not mention what to do with the output qubit. Obviously this phase is not measurable, but under certain circumstances mentioned in the succeeding sections, a Hadamard can be applied to this gate, in which case the phase does become measurable in the Z basis. Based on the values of ϕ, and additional rotations, this can be used to implement various logic gates.

As it stands however, assuming that controlled-R_z gates can be done in constant depth, the depth of this gate is one for the Hadamard, the depth of unbounded quantum fanout (which we know to be 9), the controlled-R_z rotation, which we can assume to approximate with resolution $2^-\delta$, so that the depth according to KSV-style approximation is $O(\log^2 \delta)$, plus a depth of 6 for the unfanout. For $\delta = O(\log n')$, which is the case if we can bound the fan-in of this gate by some n' related to the larger overall problem, size, the depth then becomes $16 + O((\log \log \delta)^2)$.

The size of this gate is one for the Hadamard, plus $10n - 9$ for the fanout, plus $n \times O()$ (fact check this, we need the size for KSV), plus $3n + 2$ for the unfanout.

The width of this gate is just $2n + 1$.

5 Half-or-Nothing Gate in 2D CCNTC

The $\mu_\phi^{|x|}$ gate from the previous section is useful for implementing approximately

The basic gate from Hoyer-Spalek lets us approximate a "half-or-nothing" gate on n input qubits $x = x_1 x_2 \ldots x_n$. That is, it returns a 1 if $|x| = n/2$ and 0 if $|x| = 0$, exactly, whatever that means for quantum Hamming weight. Note that this is not a total function, since we are not always promised that x has Hamming weight of exactly half ones or all zeros. In this case, we use the μ gate previously with $\phi = m$.

What is the relationship between m and n in the case of this gate? We seem to need the idea of an *expected Hamming weight*.

Is this gate exact? Yes, for the partial function that it is.

6 Expected Hamming Weight

WRITE ABOUT THIS. This appears to be the key ingredient for doing everything else. It is contained in the proof of Theorem 4, on page 7 of Hoyer-Spalek.

7 OR Gate