

## CSE 544 Homework 3 Solutions

May 22, 2006

1. (a)  $\exists a F(d, a) \wedge (\forall a F(d, a) \Rightarrow (\forall e S(a, e) \Rightarrow L(d, e)))$   
(b)  $\exists a F(d, a) \wedge (\forall a F(d, a) \Rightarrow (\exists e S(a, e) \wedge L(d, e)))$   
(c)  $\exists a (F(d, a) \wedge (\forall e S(a, e) \Rightarrow L(d, e)))$   
(d)  $\exists a F(d, a) \wedge (\forall a F(d, a) \Rightarrow \neg(\exists e S(a, e) \wedge L(d, e)))$
2. (a) T  
(b) T  
(c) F  
(d) T  
(e) T  
(f) F
3. (a)  $\text{adom}(x) \equiv S(x) \vee \exists y (R(x, y) \vee R(y, x) \vee U(x, y) \vee U(y, x))$   
(b) i. Finite, safe. The equivalent range-restricted formula is  
 $\text{adom}(x) \wedge S(x) \wedge \forall y (\text{adom}(y) \Rightarrow \neg R(x, y))$   
ii. Finite, safe. The equivalent range-restricted formula is  
 $\text{adom}(x) \wedge S(x) \wedge \forall y (\text{adom}(y) \Rightarrow (R(x, y) \Rightarrow \exists z \text{adom}(z) \wedge (S(z) \vee U(y, z))))$   
iii. Not finite, not safe. No equivalent range-restricted formula.  
iv. Finite, not safe. No equivalent range-restricted formula.  
v. Finite, safe. The equivalent range-restricted formula is  
 $\text{adom}(x) \wedge S(x) \wedge \forall y (\text{adom}(y) \Rightarrow (U(x, y) \vee \forall z (\text{adom}(z) \Rightarrow \neg R(y, z))))$   
vi. Not finite, not safe. No equivalent range-restricted formula.
4. (a)  $\text{Parent}(x, y) :- T(x, y, z)$   
 $\text{Parent}(x, z) :- T(x, y, z)$

$\text{Same}(y, z) :- \text{Parent}(x, y), \text{Parent}(x, z)$   
 $\text{Same}(y, z) :- \text{Same}(u, v), \text{Parent}(u, y), \text{Parent}(v, z)$   
 $\text{Answer}(x) :- \text{Same}(a, x)$

- (b) The datalog program is shown below. The predicate  $\text{MovesA}(x)$  will be true for those nodes  $x$  where Alice wins if she moves first. The predicate  $\text{MovesB}(x)$  will be true for those  $x$  where Alice wins if Bob moves first.

$\text{MovesA}(x) :- \text{A}(x)$   
 $\text{MovesB}(x) :- \text{A}(x)$   
 $\text{MovesA}(x) :- \text{T}(x, y, z), \text{MovesB}(y)$   
 $\text{MovesA}(x) :- \text{T}(x, y, z), \text{MovesB}(z)$   
 $\text{MovesB}(x) :- \text{T}(x, y, z), \text{MovesA}(y), \text{MovesA}(z)$   
 $\text{Answer}(x) :- \text{MovesB}(x)$

5. (a) i. Yes. Consider the homomorphism from  $q'$  to  $q$  given by  $\{(x, x), (y, y), (z, z), (u, x), (v, y)\}$ .
- ii. No. Let  $I$  be the canonical database for query  $q$ .
- iii. Yes. We will check containment separately for two possible cases for the variables  $x, y$  in  $q$ . If  $x \neq y$ , the homomorphism given by  $\{(u, u), (x, x), (y, y)\}$  shows containment. If  $x = y$ , then the homomorphism given by  $\{(u, x), (x, v), (y, w)\}$  shows containment.
- iv. No. Consider the database instance  $I$  where  $R$  has the two tuples  $(x, y)$  and  $(y, y)$ .
- (b)  $q(x) :- R(x, y), R(y, z), R(a, b), R(b, c), R(c, d)$
- (c)  $q(x) :- R(x, y_1), R(y_1, z_1), R(a, z_1), R(x, y_2), R(y_2, z_2), R(z_2, u_2), R(y_2, b)$   
 $q'(x) :- R(x, y), R(y, z)$
6. (a) T
- (b) F
- (c) T
- (d) T
- (e) T